The Role of Long-Term Contracting in Business Lending*

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April 21, 2021

Abstract

This paper studies inefficiencies arising from a lack of long-term contracting for small business lending in less developed credit markets. Drawing a unique data from a Chinese bank's department that serves small and medium-sized firms, I find most firms repeatedly take short-term loan contracts each year, and the main concern for the bank is default risk. How, and to what degree can default risk be reduced by extending contracting horizon to allow more flexible contractual arrangements? To analyze firm's default incentives and determination of contract terms, I develop a dynamic model where bank and firm repeatedly interact through short-term lending contracts under uncertainty and learning. Learning drives the dynamics of contract terms, which can explain observed features in the pane data. Estimates of the structural model imply that over seventy percent of observed defaults can be attributed to agency friction, i.e., willful borrower's default. In my counterfactual analysis, bank can design long-term contracts with state contingent terms. Optimal long-term contracts improves efficiency through two channels: First, the *intertemporal* transfer can alleviate agency frictions and reduce default incentives by front-loading prices. Second, the *intratemporal* structure specifies price discount for firms with poor performance, providing contingent insurance against risks from uncertainty. I find that long-term contracting could reduce cumulative default rates by 17.15% and expand total outputs during the sample period by 2.63%. The majority of the welfare improvement comes from the intertemporal channel.

*I am indebted to Jean-François Houde and Ken Hendricks for their guidance and continuous support throughout the research process. I am also very grateful to Dean Corbae for his helpful comments and suggestions. I would also like to thank Lorenzo Magnolfi, Alan Sorensen, Ikuo Takei, Mark Rempel, Jason Choi, Anson Zhou, Andrey Zubanov, and participants at IO lunch workshop, IO seminar, IO-Finance Reading Group, and Finance-Macro-International Workshop at UW-Madison.

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1 Introduction

Small businesses play a central role in driving economic growth. The growth and development of these firms, however, is sluggish in underdeveloped economies (Hsieh and Klenow, 2009, 2014). Since poor countries are also characterized by under-developed financial market, a vast literature is devoted to the issue involving financial frictions and firm growth[1]. It is generally recognized that financial restrictions can hinder firms' ability to use inputs efficiently and affect firm growth.[2]

Financial markets with relatively low levels of development often feature limited access to formal financial services.[3] One particular aspect largely overlooked by the literature is the lack of long-term contracting in these markets.[4] For example, while long-term financing agreements like line of credits are common in US and other developed countries, they are rarely seen in the lending market for small and medium-sized businesses in China.[5] Short-term loans with one year or less is the predominant type of loan product offered by commercial banks to small businesses. As a result, small firms in China have to repeatedly negotiate short-term loans with banks, which lead to excessive default risks and interest risks. The goal of this paper is to examine and quantify the benefits of long-term contracting on firm's financing and development, in the context of small and young firms in China.

Compared to short-term contracts which only specify lending terms for the current time period, long-term contracts involve a predetermined schedule of *future* contract terms (which can vary with time and borrower's financial status). I focus on two benefits of such schedules in long-term contracting. First, it alleviates the agency friction through its intertemporal structure. Business lending is characterized by the agency friction arising from borrower's willful *ex post* default, which poses a risk for the lender and distort the flow of capital. A long-term contract can frontload prices and improve contract terms over time, thus lessening firm's default incentives and boosting access to credit. Second, a long-term contract accommodates rate re-

[1]See Buera et al. (2015) for an excellent review of the literature.

[2]Cooley and Quadrini (2001), Huynh and Petrunia (2010), Buera et al. (2011), Arellano et al. (2012), Midrigan and Xu (2014), among others.

[3]For example, less access to savings accounts or bank loans, and lower levels of overall financial depth (King and Levine, 1993).

[4]The key difference between short-term contract and long-term contract is the presence of lender commitments. In repeated short-term financing, the lender has the unilateral option of calling in the loan or raising interest rates at the beginning of each period. In long-term financing contracts, the bank commits to the preestablished contracts (Sharpe, 1991).

[5]There are many reasons behind the observed scarcity of long-term financing arrangement for small business lending in China. One important reason is that traditional commercial banks, which are the major lenders in this market, lack financial innovation. By contrast, newly emerging FinTech companies are starting to offer small business credit lines with longer contract maturity.

duction following realization of bad states, thus (at least partially) insure firms against negative shocks.

This paper draws on a proprietary dataset from a Chinese bank, which keeps track of all loan contracts with each of its small and medium-sized corporate customers. I can identify the beginning of each firm's lending relationship with the bank, and construct a complete panel of loan contracts over a period of eight years. Several features of the data stand out. First, all contracts are short-term, ranging from six month to one year (the majority is one-year). Second, the ability of the bank to recover loans is a main concern, despite almost all loans are collateralized. This can be seen from the fact that 33.4% firms default during the sample period, and that average coverage rate on defaulted loan is around half. Third, contract terms respond to new information about firm performance: interest rates go up and loan size go down following a poor performance. Furthermore, such responses become weakened over time, which is consistent with Bayesian learning in an environment with incomplete information.

How much of the observed default is due to agency friction and how can it be reduced by longer contracting horizon and more flexible contractual arrangements? To answer these questions, I develop a dynamic structural model of firms (borrowers) and banks (lenders), in an environment featuring limited commitment and symmetric learning. In the model, firms are heterogeneous in their probability of getting negative productivity shocks, i.e., quality type. A firm's quality type is unknown to all banks and to the firm itself, but the distribution of quality type conditional on observed firm characteristics is common knowledge. Banks compete at the beginning of each period by offering single-period contracts, which consists of interest rates, loan size, and collateral coverage. Firms make default decisions at the end of the period after productivity and liquidity shock is realized. If default, the firm exits the market with a salvage value; otherwise, it enters the next period with the updated belief based on this period's productivity performance.

The over-time variations in observed contract terms and firms' performances, as well as their comovements, are important in identifying the level of uncertainty and learning process, namely type distribution and each type's probability of negative shocks. The observed collateral coverage, default rates and its comovements with shifts in funding costs, are critical in identifying parameters related with the level of agency friction, namely transaction cost associated with collateral, salvage value, and variance of liquidity shocks. A larger variance of liquidity shock means higher liquidity risk that is *innate* to the business operation, which implies that a larger fraction of observed defaults are due to intrinsic risks of the liquidity situation rather than agency friction.

The model is estimated using generalized method of moments. Estimation results show several important findings. First, while observed firm characteristics like industry account for most dispersion in initial contract terms, conditional on firm observables, firms with low unobserved type is five times as likely to perform poorly. Second, uncertainty about firm's unobserved quality types lead to mispricing of default risks and misallocation of capital. Mispricing induces three percent more defaults of high-type firms, and capital misallocation results in four percent less capital taken by high-type firms. Learning over the seven-year period reduces mispricing and misallocation by less than half. Third, collateral, as a tool to disincentivize default, is costly: the transaction costs associated with collateral is around three percent of total collateral value. Last but not least, I find that liquidity shock has a relatively large variance, although agency friction is still the more important source of inefficiency, accounting for more than seventy percent of observed defaults. In absence of agency friction, total firm output would be fourteen percent higher.

If counterfactually banks are able to make and commit to long-term contracts, how and to what degree do firms benefit? Using estimates of the baseline model, I conduct a counterfactual exercise where banks compete by offering long-term contingent contracts to firms. The long-term contingent contract is essentially a schedule of contract terms as a function of future states, where states encode information about the number of years into the contract, and firm's performances in each year of the contract. Such contract can be seen as an approximation of real-life loan commitments. For example, in US, it is well expected that credit limit increases over the time and good performances on a line of credit. And a large percentage of loans in US includes performance-pricing provisions, which explicitly makes the interest as a function of the borrowers financial condition (Asquith et al., 2005; Manso et al., 2010; Berg et al., 2016). Such state contingent terms is the key component of long-term contracts in my counterfactual model.

I find that optimal long-term contracts improve efficiencies through two channels. First, the *intertemporal* structure of the contract is such that lending terms become increasingly favorable to firms over time, with declining prices and growing capital flow. Firms entering such contracts are incentivized to stick to it and reap the benefits from good terms at the later periods. Thus defaults incentives are reduced since default triggers a severance from the contract. Second, under uncertainty about firm's future profitability prospect, the long-term pricing scheme specifies a price discount when firm's financial situation deteriorates. This *intratemporal* structure serves like an insurance against adverse realizations, which not only enhances the overall value of the contract to risk-averse firms, but also decrease default incentives of firms with poor performance. In other words, it is a cross-subsidization from good-performing firms to others. Ex ante, such cross-subsidization is favored by firms.

The two channels combined reduces cumulative default rates by 17.15%, increases firm size in the last period by 1.23%, and expand total outputs during the seven-year period by 2.63%.[6] It recovers 31.46% of total welfare loss from agency friction and incomplete information. To further see how the insurance channel contributes to the welfare improvement compared with the intertemporal channel, I look at another counterfactual scenario where insurance is removed from long-term contracts. I find that the reduction in defaults, increase in eventual firm size, and expansion of total outputs are only slightly higher than the previous case, suggesting that the intratemporal structure is less important than the intertemporal structure in term of curbing default rates, promoting firm growth, and boosting overall firm outputs.

Related Literature This paper touches on three different fields, relating to the finance literature on optimal long-term financing contract, the IO literature on limited long-term commitment, and the macroeconomic literature on the relationship between financial friction and firm dynamics/growth. To my best knowledge this is the first study to look at the role of limited long-term commitment in the dynamic financing contracts, and the consequences for firm growth. In terms of methodology, my model is novel in that it examines corporate lending transactions with rich specifications for firm heterogeneity, which makes it suitable for micro-level data.

There has been a long discussion in finance about the optimal long-term financing contracts in which credit constraints emerge endogenously as a feature of the optimal contract design (Gertler, 1992; Thomas and Worrall, 1994; Albuquerque and Hopenhayn, 2004; Quadrini, 2004; Clementi and Hopenhayn, 2006; DeMarzo and Sannikov, 2006; DeMarzo and Fishman, 2007; Biais et al., 2010; Boualam, 2018). In particular, Boualam (2018) apply model of optimal long-term financing contracts to the US commercial loan market, where he argues long-term contracts are available at a relatively low cost. My paper instead looks at countries where long-term contracting is not easily accessible, and quantify the value of having long-term financing contracts.

The discussion of limited long-term commitment in IO has been studied in the context of life or health insurance (Cochrane, 1995; Hendel and Lizzeri, 2003; Atal, 2019). In Hendel and Lizzeri (2003), the supply side (i.e., insurance companies) is able to commit but the consumers cannot (i.e., they can switch), which leads to front-loading premiums in long-term contracts. This paper finds that, when the supply side (i.e., banks) can commit, optimal long-term contracts are also front-loaded. But the reason behind front-loading is completely different from Hendel and Lizzeri (2003): Here front-loading is employed to alleviate agency friction arising from the

[6] Here I measure firm size by outputs, which is increasing in loan size. Cumulative default rates refers to the fraction of firms ever defaulted in the sample period.

borrower's incentive to avoid repayment, while in Hendel and Lizzeri (2003) front-loading is a result of a lack of consumer commitment. This paper thus complements the discussion of the effect of a lack of long-term economic arrangements by examining a market characterized by agency frictions.

This paper pertains to empirical studies of credit markets with agency frictions, such as Adams, Einav, and Levin (2009), Einav et al. (2012), Crawford, Pavanini, and Schivardi (2018), Xin (2020). In particular, Adams, Einav, and Levin (2009) and Crawford, Pavanini, and Schivardi (2018) interpret a positive correlation of default and loan liability (conditional on observables) as moral hazard in credit market. The agency friction in my paper fits into this (broad) definition of moral hazard. It is akin to the type of "moral hazard" in Einav et al. (2012), i.e., willful default after income/revenue is realized, as apposed to the kind of *ex ante* moral hazard in Xin (2020). Compared to Einav et al. (2012) and Crawford et al. (2018), my model is different in that it is in the setting of *repeated* lending transactions and estimated using a panel data on lending transactions, while Einav et al. (2012) and Crawford et al. (2018) focus on one-time loan transactions.[7]

This paper is also related to structural estimation of learning models (Ackerberg, 2003; Crawford and Shum, 2005; Pastorino, 2019). Ackerberg (2003) and Crawford and Shum (2005) assume the "signals" through which learning takes place is unobserved, while Pastorino (2019) treats observed panel data on performance as a direct evidence of signals. My approach is similar to Pastorino (2019) in that I also utilize observed measures of performance to pin down the belief updating process. These papers are in the context of either single-agent dynamic problem or dynamic problem with no agency frictions, I contribute to this slew of studies by estimating the learning process in an environment with agency friction.

Finally, there is a vast literature on the relationship between financial frictions and entrepreneurship (Cooley and Quadrini, 2001; Huynh and Petrunia, 2010; Buera et al., 2011; Arellano et al., 2012; Midrigan and Xu, 2014; Buera et al., 2015), and this paper contributes by providing micro-level evidence of how the friction in writing long-term contracts can affect the growth of small firms through inefficient default choice and inefficient use of inputs.

The rest of the paper is organized as follows. Section 2 provides institutional backgrounds, and describes key features and patterns in the data. Section 3 describes the model, Section 4 discuss identification concerns, specifies the empirical model, and estimates the model primitives.

[7] This is not saying that previous literature in this realm do not use dynamic models. Einav et al. (2012) and many papers on residential mortgage loans analyze subsequent repayment behavior as dynamic programming problem. But the loan transaction between a lender and a borrower is one-time, which is where my paper differs.

Section 5 conducts the counterfactual analysis. Section 6 concludes.

2 Data and Institutional Background

2.1 China's Banking Industry

The banking sector in China originated from a centralized system in 1949 when the People's Bank of China (PBC), as China's central bank, governed both commercial bank businesses (e.g., deposits, lending, and foreign exchange) and central bank functions. Along with economic opening policies being instituted by Deng Xiaoping in 1978, the banking system entered a period of reform. In 1983, the PBC began to focus on national macroeconomic policy, monetary stability, and economic development. At the same time, the big four commercial banks (i.e., the ICBC, ABC, BOC and, CCB) started to take over commercial bank businesses, and each was specialized in a specific area. The Bank of Communications' experience in reform and development has paved the way for the development of shareholding commercial banks in China and exemplified banking reforms in China (Gao et al., 2019a).

Between 1988 and 2005, twelve joint equity banks were established, mostly as SOEs or institutions transformed from local financial companies. Although joint equity banks are also national banks, unlike big five commercial banks, they usually focus on local business and operate on a much smaller scale. By the end of the year 2013, as reported by China Banking Regulatory Commission (CBRC)'s annual reports, the big five commercial banks dominate the market and control for approximately 43.3% of the market share. On the other hand, joint equity banks are much smaller and control for about 17.8% of the market share. The rest of the financial institutions belong to the third tier such as municipal commercial banks.

2.2 Deregulation of Credit Controls and Interest Rates

The first step in deregulation of credit control is taken was 1998. Until then, the central bank had controlled the lending of Chinese banks through binding credit quotas. This binding credit plan system was formally abolished in 1998, replaced with an indicative non-binding credit target. In other words, commercial banks in China are no longer required to provide loans in compliance with state directives or policy targets. Instead, they are encouraged to allocate funds "on the basis of proper credit assessments" and lend based on economic and commercial considerations. This change in policy has been hailed by Chinese monetary authorities as an important initial step in transforming the credit culture of Chinese banks (Xu et al., 2016).[8]

Lending rates in China have been substantially more liberalized than deposit rates throughout the path of interest rate deregulation, starting with the interbank offered rate in the capital market to be fully market-priced in 1996. From 1998 onward, the People's Bank of China (PBC) started to widen the floating band on banks' interest rates. In 2004, the deposit rate floor and the lending rate ceiling were eliminated for the major banks. The remaining lending rate floor was gradually widened and eventually completely removed.[9] In practice, the lending rate floor was largely non-binding even before it was removed (Xu et al., 2016). Deposit ceiling was binding (He and Wang, 2012), and it was not removed until October 23, 2015.

The day following October 23, 2015 marked the last change of benchmark lending rates until this day. The benchmark lending rate refers to the official reference for lending rate published by PBC. It served as a non-binding "guidance" on lending rates in the market, and had been an active policy instruments. Prior to October 2015, changes in the benchmark lending rates had been made by PBC[10] at random dates, typically seven or eight times a year. After October 24, 2015, however, it seems that the benchmark lending rate has ceased to function as an active policy instrument since no adjustment has been made so far; instead, the focus of PBC is increasing on short-term money market rates, namely the 7-day interbank pledged repo rate (DR007).[11] This move is generally seen as part of interest rate liberalization that allowed PBC to improve its policy framework (McMahon et al., 2018).

2.3 Data Description

This paper utilizes three sources of data from a Chinese bank: (1) loan contract and loan outcome data, (2) (anonymized) corporate borrower data, and (3) internal rating data. The loan contract data contains detailed information on loan contracts with corporate borrowers, including a loan identifier, a firm identifier, date of origination, contract interest rates, loan size, loan maturity, repayment frequency, types of collateral, value of each type of collateral, branch

[8]There are still signs of quantitative controls on bank credit, as the central bank employs an array of quantitative instruments aimed at controlling credit growth, such as yearly aggregate target levels for new loans and the use of so-called window guidance which can be described as a form of moral persuasion aimed at controlling the sectoral direction of lending (Okazaki, 2007).

[9]The lending rate floor was reduced to 0.9 times the benchmark official lending rate in October 2004, 0.8 times the benchmark lending rate in June 2012, 0.7 times in July 2012, followed by a complete removal in July 2013.

[10]These changes need to be approved by the State Council (China's equivalent to a government cabinet) as well (McMahon et al., 2018).

[11]In the 2016, third-quarter Monetary Policy Executive Report, the PBC stated that "DR007 moves around the open market operation 7-day reverse repo rate. The DR007 can better reflect the liquidity condition in the banking system and has an active role to cultivate the market base rate".

and sub-branch identifier. Loan outcome is an indicator of whether the loan is classified as a nonperforming asset (NPA) as of June 2018. NPAs are listed on the balance sheet of the bank after a prolonged period of non-payment and evidence of extremely low repayment probability. They are typically viewed as loans that are in default.

The corporate borrower data used in this paper include the the borrowing firm's industry code, location (up to district level), size category, ownership type, date of incorporation, initial capital, and an initial assessment. The initial assessment is conducted by the bank when the firm borrows for the first time. It is a categorical variable that summarizes all soft and hard information that bank knows about the perceived quality of the firm at the beginning.

I also draw on data on bank's internal ratings for loans. For each loan, banks in China are required to report a rating to CBRC on a monthly bases until the loan is off the bank's balance sheet. These ratings are assigned according to a five-category loan classification system, in which there are five levels: 1 is the highest rating for the "normal" loans, 2 is for the "special mentioned", 3 is for the "substandard", 4 is for the "doubtful" and 5 is for the "loss". In this paper, internal rating is defined as a dummy variable for whether the loan rating is normal or poor (i.e., ratings from 2 to 5), as in Gao et al. (2019b). This method of classification is mainly based on borrowers' *repayment ability*, that is, their *actual* ability to repay principal and interests. Assessing this ability entails, for example, monitoring and analyzing the borrower's changes in revenue and profits, cash flow, financial position, management efficiency, etc. One bank can request records of a borrowing firm's past rating history from CBRC at low monetary costs. Although banks report ratings every month until a loan is paid off, I can only observe one snapshot of the monthly ratings taken at the month of December in that corresponding year. I refer to this as the *internal rating* on that loan.

The loan data is merged with firm data, which is then merged with the internal rating data with loan identifier. The data includes the date of the first loan of each firm, thus enabling me to identify the true beginning of the lending relationship. The end of my sample period is the end of 2017.[12]

Sample Restriction For the purpose of this paper I restrict the sample to only small and medium-sized young firms that first borrow from the bank only after the beginning of the sample period, Jan 2, 2010. Specifically, the firm-level data has a categorical variable indicating firm size—

[12] I can observe data on contract terms up to the middle of 2018, but the outcomes on loans originated in 2018 has not been recorded. So my final sample does not include those loans.

large, medium, small, which is defined according to a national criteria[13]. I define young firm is firms whose age is less or equal to five at the first time of borrowing. In total, there are 6358 such firms in my sample.

Feature 1: Loan contract terms are negotiated annually. This is based on three observations of the loan-level data. First, all of the loans are short-term loans: 80.63% of loan terms are one-year, and the rest is between six-month to one-year. Second, on average 78.6% of firms borrow only once in a given year, and among those who borrow more than once, the interest rates on multiple loans originated in the same year are similar, in contrast with the relatively large year-to-year variations on lending rates within a firm.[14] Lastly, nearly all of the loans has one-time repayment due at the end of the loan maturity.

Based on this feature, I aggregate the loan-level data to firm-year level. Specifically, for each firm, I take the average interest rates and value of collateral, the sum of loan size, and the outcome of the last loans within each year, and get a firm-level annual panel data on loans. In total, there are 17518 observations on firm-year level. Summary Statistics of year one to seven is shown in Table 14.

Feature 2: Repeated lending is common. To find out the longitudinal properties of the panel data, I define the duration of the relationship as the difference between the last time it borrows from the bank and the first time. Figure 2 shows the distribution of relationship duration. Three quarters of firms borrow more than once, and the median duration is three years. For firms who repay their last year's loan, 70.09% of them come back and take a new loan. Firms who default exit the sample.Among the firms who never default, 90.6% has been borrowing annually without a gap, and 7.45% has had only one gap year in their borrowing history, with only 1.95% firms have a two-year gap or longer.[15]

Feature 3: Default is a prominent source of risk despite common usage of collateral. There are three types of collateral: securities, fixed-assets, and third-party guarantee, which are ordered in the perceived recoverability. In the sample, all of the loans are secured by at least one type of collateral: 35.5% of loans only have third-party guarantee, 28.4% only pledge fixed-assets, 23.97% pledge both fixed-assets and third-party guarantee, and the remaining 12% uses

[13] See the administrative rule at `http://www.gov.cn/zwgk/2011-07/04/content_1898747.htm`. This classification rule defines the range of number of employees and annual revenues to qualify for a small-sized company, which varies by industry. For example, in the retail industry, a company with employment 5 to 20 and annual revenue 10 to 50 million RMB (1.4 to 7.2 million USD) is classified as a small-sized company.

[14] On average the coefficient of variation of interest rates on multiple loans within the same year for the same firm is 0.07.

[15] I cannot identify reasons of leaving for a firm who has not defaulted previously due to limitations of the data.

securities as collateral. In spite of ubiquitous collateral usage, the bank still faces significant losses in the event of default. This is seen from the set of recovery rates on each type of collateral, which is used by the bank to project risks. The recovery rate on third-party guarantee is around 0.16, which can vary with the credibility of the guarantor. The recovery rate on fixed assets is around half, varying with the specific type of assets. Securities has the highest recovery rate due to its high liquidability, around eighty to ninety percent. I calculate the recoverable value of collateral using the set of recovery rates provided by the bank and calculate the *collateral coverage* ratio as the ratio between recoverable value of collateral and the loan amount. The collateral coverage ratio is on average around 55%, which means that the bank's expected loss from default is around fifty cents per dollar of the loan. There is also considerable default risks: around 33% of firms in my sample ends up with default over the period from 2010 to 2017.

Feature 4: For firms with good rating history, contract terms improve over the course of the relationship. I define firms with good rating history as firms who do not have a single poor rating in their entire rating history. In my sample, 76% of all firms have a good rating history. Among those firms, I first regress contract terms (lending spread, loan size, collateral coverage) on observed firm characteristics (industry code, location, size category, ownership type, date of incorporation, initial capital, and an initial assessment), cohort fixed effects (i.e., fixed effects of the first year of borrowing), monthly time fixed effects, and branch fixed effects, in order to get the residuals.[16] I then plot the residualized contract terms (lending spread, loan size, collateral coverage) over year in relationship, as shown in Figure 3. Overall, the residualized lending spread and collateral coverage decreases with the year in relationship, while the residualized loan size increases with the year in relationship. Alternatively, we can check the intertemporal trend of contract terms within each firm. Utilizing the panel structure of the data, I run a regression of contract terms (y_{it} = lending spread, loan size, collateral coverage) on the year in relationship t controlling for firm fixed effects (α_i) and time fixed effects (ξ_t):

$$y_{it} = \beta_0 + \beta_1 t + \alpha_i + \xi_t + e_{it} \tag{1}$$

Results are reported in the column (1) to (3) of Table 1 (where t stands for year in relationship). On average, as relationship proceeds, lending spread drops by 2.83 bps per year, loan size increases by 162,000 CNY ($\approx$ 24 thousand dollars) per year, and collateral coverage ratio drops by 0.35 % per year. This suggest that contract terms in general become more favorable to firms over the course of the relationship, in cases where the firm is never rated poorly.

[16]Lending spread is the difference between interest rates and deposit rate.

Table 1: Regression of Contract Terms on Past Performance and Year of Relationship

	Firms w. good rating history			Firms w. poor rating history		
	(1) Lending Spread	(2) Loan Size	(3) Collateral Coverage	(4) Lending Spread	(5) Loan Size	(6) Collateral Coverage
t	-0.0282	0.162	-0.00350	-0.0120	0.0387	0.00836
	(-3.72)	(8.81)	(-1.99)	(-1.53)	(0.84)	(1.07)
$PoorRating_{t-1}$				0.0399	-0.144	0.0162
				(3.15)	(-2.38)	(1.68)
$t \times PoorRating_{t-1}$				-0.00655	0.0267	-0.00797
				(-2.53)	(2.51)	(-2.17)
Observations	8328	8328	8328	2344	2344	2344

t statistics in parentheses. Other controls: firm fixed-effects, quarterly time fixed-effects.

Feature 5: A poor rating is associated with an adverse change in contract terms next period, and such change attenuates over the course of the relationship. Let period t denote the year in relationship. Suppose in period t a firm is rated poorly for the first time, then how will contract terms respond in the next period? And how will such responses change with t? To see this, I regress contract terms (y_{it} = lending spread, loan size, collateral coverage) on a dummy variable indicating whether last period's rating is poor ($PoorRating_{t-1}$), year in relationship t, and their interaction term $t \times PoorRating_{t-1}$, while controlling for firm fixed effects (α_i) and time fixed effects (ξ_t). The regression is run on firms who have at least one poor rating in their history, and for each firm i, I keep observation $t <= t_i^* + 1$ where t_i^* is the first time firm i is rated poorly.

$$y_{it} = \beta_0 + \beta_1 t + \beta_2 PoorRating_{i,t-1} + \beta_3 t \times PoorRating_{i,t-1} + \alpha_i + \xi_t + e_{it} \tag{2}$$

Results of the regressions are shown in column (4) to (6) of Table 1. For a firm who is rated poorly in the first period, then on average its contract terms next period became tougher: lending spread increases by around three bps, loan size go down by around one hundred and twenty thousand CNY, collateral coverage ratio increases by around one percent. However, if a firm's first poor rating happens only in period $t > 1$, then the adverse changes on its contract terms in the following period are not as large in magnitude as the previous case. In other words, new information about the firm's performance that arrives later in the relationship worsens contract terms to a lesser degree.

In sum, Feature 1 suggests that observed loan contracts are short-term with annual negotiations. Feature 2 suggests that there can be meaningful dynamics in the repeated lending contracts. Feature 3 suggests that default risks are important considerations in this environment, since default rates are relatively high and collateral is commonly used, which is typically seen as an disincentive to defaults. Feature 4 and 5 together is consistent with a Bayesian learning process, where poor ratings lead to more pessimistic beliefs about the firm's profitability and higher expected default risks, and it is reflected in tougher contract terms. A feature of Bayesian learning process is that new information happens later in the process updates beliefs less, which can explain why contractual responses to poor ratings attenuate over time.

3 Model

Inspired by the patterns in the data, I develop a dynamic structural model of firms (borrowers) and banks (lenders), in an environment featuring limited commitment and symmetric learning. Limited commitment means the firm cannot pre-commit to repay; instead, it has the option to choose to repay or default after observing revenue realizations. Symmetric learning means banks and the firm are equally uninformed about the unobserved firm type, and they share the same initial prior beliefs. In subsequent periods, they all update beliefs based on publicly observed firm performances, so they share the same belief at any point in time.

The assumption about symmetric learning is admittedly a strong assumption, but perhaps more justifiable in my case of young firms. These firms, or entrepreneurs, are just starting out themselves, and do not necessarily accumulate too much private information to hide from banks. On the other hand, banks have funded similar projects in the past and do not necessarily know less about the future prospect than the firm. This assumption helps keep the model tractable while allowing us to see how informational frictions arising from incomplete information can affect the dynamics of contract terms as well as firm outcomes.

I solve for equilibrium short-term contracts, assuming banks do not have access to long-term contracting because of high contractual costs. This assumption about the lack of long-term contracting is based on the institutional settings in my observed market. Firms and banks do not have formal binding long-term agreements that would commit the firm to future borrowing or hold the bank accountable for its promises on future contract terms. They negotiate contract terms on a yearly basis, and the observed contracts are mostly one-year term loans.

3.1 Model Setup

Time is discrete with infinite horizon. There are two types of infinitely lived agents: firms , and banks. Agents share the same discount factor $\beta \in (0,1)$.

Firms

Banks are risk neutral. Firms, or entrepreneurs, on the other hand, are risk averse. They operate their firms in order to maximize their expected utility from consumption within each period, where consumption is total output net any payment. Firms are assumed to be hand-to-mouth without access to a storage technology, although this model can be easily generalized to allow accumulation of capital.The flow utility $u : \mathbb{R} \to \mathbb{R}$ satisfies standard regularity conditions: $u' > 0$, $u'' < 0$, $\lim_{x\to 0} u'(x) = \infty$, and $\lim_{x\to\infty} u'(x) = 0$. Although many papers on dynamic debt contracting assumes risk neutrality (with exceptions like Thomas and Worrall (1994), Boualam (2018)), this paper adopts risk aversion to better represent young or distressed firms who lack ability to diversify firm-specific risk.

Each firm has access to a production technology, but it is cashless initially and has to seek out external financing in order to start production. When funded with capital k, a firm of *quality type* $\theta \in \Theta$ and *characteristics* $s \in \mathscr{S}$ can generate output

$$y = af(k), \quad a \sim G(\cdot|\theta, s)$$

where the function f is differentiable, strictly increasing and strictly concave, and satisfies $f'(k) > 0$, $f''(k) < 0$, $f(0) = 0$, $\lim_{k\to 0} f_k(k) = +\infty$, and $\lim_{k\to\infty} f_k(k) = 0$.[17] Productivity a of a firm with type θ and characteristics s in each period t is a random draw from the probability distribution $G(\cdot|\theta, s)$, with the associated probability density function (or probability mass function) denoted as $g(\cdot|\theta, s)$.

Both quality type and firm characteristics s are related with the distribution of a firm's productivity.[18] Specifically, conditional on the same s, a higher value of θ means higher productivity in the first-order stochastic dominance sense. Formally, if $\theta' > \theta$, then $G(a|\theta', s) \geq G(a|\theta, s), \forall a$. In other words, higher θ indicating higher quality.

[17]The production function abstracts from labor and can be viewed as a profit function that already accounts for the optional choice of labor input and associated wages.

[18]In empirical specification, I let quality type θ and characteristics s play different roles in productivity distribution G. Conditional on the same θ, I assume s only moves the distribution G horizontally without changing its shape.(Formally, that means for $s' \neq s$, there exists a scalar b such that $G(a;\theta, s') = G(a + b;\theta, s'), \forall a$). On the other hand, θ can change the shape of distribution G conditional on the same s. High θ means a more right-skewed distribution G and thus higher expectation of productivity.

Firms have a choice of whether to repay the bank loan in each period. Following production, an i.i.d utility shock ϵ associated with repaying the bank loan is realized. Realization of the repayment shock ϵ is observed by the firm, based on which the repayment decision is made. Repayment shocks ϵ are assumed to be independent of quality θ. The mean of ϵ is zero, and the variance of ϵ is denoted σ. Repayment shocks ϵ can be interpreted, for example, as liquidity shocks that makes loan repayment harder for the borrowing firm.

Note that this setup nests the standard optimal long-term debt contract model with endogenous borrowing constraints (Albuquerque and Hopenhayn, 2004; Boualam, 2018) by setting σ to zero. In this case there is no randomness in default behavior and it is possible to completely prevent defaults by designing the contract subject to default-free constraint. In general, a positive σ introduces some randomness in firm's default behavior, which allows positive default rates in equilibrium.

Loan Supply and Information Structure

There are two banks in a competitive bank loan market.[19] Each bank acts as an intermediary channeling funds from depositors to firms at a funding cost c, and the funding cost is the same for every bank. Abstraction from bank heterogeneity allows me to focus on how agency friction and learning shapes the dynamics of contract terms. Funding cost $c \in \mathbb{C}$ follows a stochastic process with transition probability $\Gamma_c : \mathbb{C} \times \mathbb{C} \to [0,1]$.

Information is symmetric among agents in this model. In the beginning, all agents, including bank and the firm itself, do not know the firm's quality type θ, but can observe firm characteristics s. Their initial prior belief is given by the distribution of θ conditional on firm characteristics s, i.e., agents have rational expectations. In subsequent periods, the realizations of firm's outputs, or productivity, are commonly observed, based on which they update their belief via Bayes rule.

Formally, denote $p_t(\theta)$ the belief that a given firm is of type θ at the beginning of period t (before production takes place in this period). Suppose $\Lambda(\theta|s)$ is the true distribution of quality types conditional on firm characteristics s. Then the initial prior $p_1 : \Theta \to [0,1]$ is:

$$p_1(\theta) = \Lambda(\theta|s).$$

Given period t belief p_t, and this period's productivity realization, a_t, we can find the updated beliefs in $t+1$, $p_{t+1} : \Theta \to [0,1]$, by Bayes rule:

[19]We can think of the two banks as *my bank* (the bank in my data) and the competing bank.

$$p_{t+1}(\theta) = \frac{g(a_t|\theta, s)p_t(\theta)}{\int_{\theta'} g(a_t|\theta', s)p_t(\theta')d\theta'} \tag{3}$$

This learning process where all agents share the same belief at any point in time is also known as *symmetric learning*. Such deviation from the asymmetric information literature is partly driven by my focus on young firms, where the entrepreneur does not have extensive experiences of running this business and thus has fewer private information to hide from the bank. The bank, as argued in (Manove et al., 2001), has funded many similar projects in the industry, and does not necessarily have fewer knowledge about the prospect than the entrepreneur. Extensive initial screening and close monitoring are also in place to garner comprehensive information about her ability to produce profits in the process of running a firm.

Timing

Figure 1 describes the timeline of the model, which consists of two stages. Period 0 is the origination stage where banks compete by offering short-term contracts, and the firm chooses one bank. The the firm-bank pair proceeds to the dynamic contracting stage (period $t \geq 1$). In each period, the order of events are as follows: First, capital is advanced and production takes place. Then productivity is observed and used to update belief. Then repayment shock realize, after which the firm makes repayment decision. The contract continues only when the firm repays; otherwise the contract ends.

Lending Contracts

Contract offers are built from standard single-period lending contracts. A single-period lending contract in period t consists of interest rate (plus one) $r_t \in \mathbb{R}^+$, loan size $k_t \in \mathbb{R}^+$, and collateral coverage $z_t \in [0,1]$. It stipulates the firm put up an asset worth zk as collateral when taking out the loan. At the end of the period, either the firm repays rk to the bank, or, in the event of default, the firm loses possession of this collateral to the bank.[20]

The use of collateral generally involves a variety of costs, which might include necessary legal documentation, monitoring and/or insurance for the asset to maintain the collateral's value at the agreed level, as well as implicit costs for the borrower in being forced to relinquish discre-

[20]Here we think of collateral as coming from the firm's illiquid assets, which is different from working capital. I do not consider constraints on the total pledgeable assets that a firm might face, or how past bank loans might lead to accumulation of total pledgeable assets. These are interesting theory extensions, but extra data on firm's assets is needed to identify related parameters.

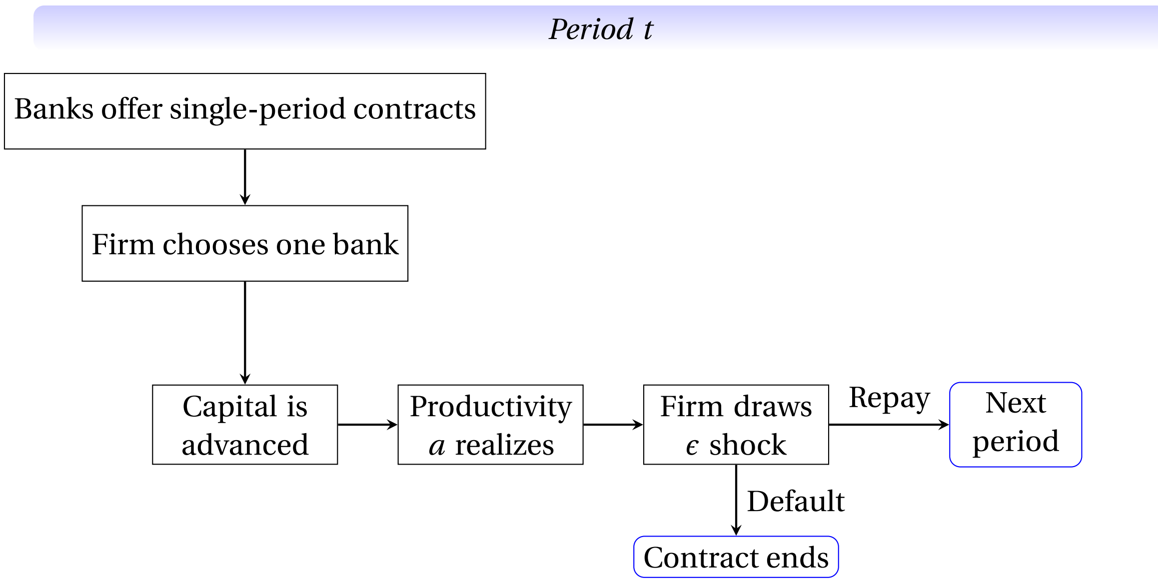


Figure 1: Timing

tionary use of the asset (Chan and Kanatas, 1985). Regardless of their origin, these costs can be assumed to be a strictly increasing function of the value of the collateral. Without loss of generality, I assume the costs is a fraction γ of the total value of collateral zk. Furthermore, I assume the firms pay transaction cost.[21] Note that transaction costs accrue when firms take out the loan at the beginning of each period, regardless of whether they ended up defaulting or not.

What then happens to a defaulting firm depends on the larger economic institutional systems in place, and here I summarize the various costs related with defaults in a reduced-form way by assuming that a firm would lose a fraction $1-\delta$ of firm value. Put it differently, a defaulted firm has salvage value that equals δ times the future firm value if the firm has not defaulted. An extreme case of $\delta = 1$ is where default event does not make any dent on the a firm's future value and it can continue accessing the credit market seamlessly. And the case of $\delta = 0$ is the opposite situation where defaulting firms are excluded from credit market forever and completely cease to produce.

In general an environment with higher δ gives the firms higher incentive to default since the salvage value is higher. (Or equivalently, the costs of defaults are lower.) Thus, the parameter δ determines the level of agency friction in place.[22]

[21] According to Chan and Kanatas (1985), in a competitive credit market with costly use of collateral, if lender and borrower can negotiate how to divide the transaction costs of collateral between them, the optimal solution is the borrower pays all transaction costs. Therefore I use this as an assumption to simplify the model.

[22] Parameter δ, γ, and σ can be different for firms with different characteristics s, as in the estimated specification.

3.2 Equilibrium Short-Term Contract

I consider an environment where long-term contracting is unaccessible. Alternatively, we can think of this case as lacking long-term commitments, i.e., a firms can unilaterally leave the long-term contract after period-t loan is resolved and borrow from the spot market instead,[23] and a bank might renege its previous promise on this period's contract terms. In other words, banks cannot make credible promises on future contract terms (i.e., promises on future terms cannot be *enforced* in this institutional environment[24]), and firms cannot make credible promises on future borrowing. They only choose contract terms for the current period.

An equilibrium short-term contract is a single-period contract that maximize the firm's value while subject to zero expected bank profits. This is a direct result from bank competition on three dimensions: price (i.e., interest rates), loan size, and collateral coverage. To see this, first consider the case where a contract brings positive expected profits to a bank (and to all banks since banks are homogeneous). Another bank can undercut by lowering prices by a small amount and win all the firms (which is the standard Bertrand competition logic), so it cannot be the equilibrium contract. The contract cannot bring negative expected bank profit either, by the bank rationality requirement. So the equilibrium contract must bring zero expected bank profit. There can be many contracts that satisfy zero-profit constraint, and it turns out only the one that maximize the firm value can be the equilibrium contract. If a bank offers a zero-profit contract that does not bring the highest possible firm value, then another bank could optimize the contract structure and achiever higher firm value while still earning infinitesimal positive profit per contract, which attracts all firms. Thus, the equilibrium contract must bring the highest possible firm value while subject the zero expected bank profit.

Let $W(p_t, c_t)$ and $\Pi(p_t, c_t)$ denote the expected firm value and expected bank profits when the belief of the firm's unobserved type is p_t and fund cost is c_t in period t. To express bank profits and firm value, we can first find the firm's value of choosing to repay and default, as well as the associated repayment probability (or, equivalently, the firm's default probability) on a given short-term contract (r_t, k_t, z_t).

Given period t's realization of productivity a_t and repayment shock, ϵ_t, the value of choosing to repay for a firm with characteristics s is

$$u\left(y_t - \gamma z_t k_t - r_t k_t\right) - \epsilon_t + \beta E_t W(p_{t+1}, c_{t+1})$$

[23] Walking away before resolving this period's loan is just defaulting, which would lead to loss of collateral.

[24] It is worth noting that the lack of enforcement is on *future* transactions, as well as default behaviors. But conditional on default, the transfer of collateral can be enforced.

where $E_t W(p_{t+1}, c_{t+1})$ is short for $E_{c_{t+1}}\left[W(p_{t+1}, c_{t+1})|p_t, c_t, a_t\right]$ with next period's belief p_{t+1} completely determined by p_t and a_t through Equation (3). And $\gamma z_t k_t$ is the transaction cost associated with collateral, which accrues to the firm at the beginning of this period.

If the firm chooses to default, it looses the value of collateral $z_t k_t$, so the firm's value of choosing to default is

$$u\left(y_t - (1+\gamma) z_t k_t\right) + \beta\delta E_t W(p_{t+1}, c_{t+1})$$

where $\delta E_t W(p_{t+1}, c_{t+1})$ is the salvage value for a defaulted firm.

Thus, the probability that a firm repays the loan viewed at the beginning of period t can be expressed as:

$$\Phi_t(a_t) \equiv \Pr\left(\epsilon_t \leq u\left(y_t - \gamma z_t k_z - r_t k_t\right) - u\left(y_t - (1+\gamma) z_t k_t\right) + \beta(1-\delta) E_t W(p_{t+1}, c_{t+1}) \middle| a_t\right) \tag{4}$$

Note that the repayment probability is contingent on realized a_t, since default decision is made after productivity realizations. Specifically it determines the next period's belief p_{t+1}.

Using this notation of repayment probability, we can express the bank's expected profits at the beginning of period t as:

$$\Pi(p_t, c_t) = -c_t k_t + E_{a_t}\left[\Phi_t(a_t) r_t k_t + (1 - \Phi_t(a_t))\, z_t k_t \middle| p_t\right] \tag{5}$$

Finally, we can define the equilibrium firm value as $W(\cdot)$ that satisfies:

$$\begin{aligned} W(p_t, c_t) = \max_{r,k,z} E_{a_t,\epsilon_t}\Big[& \max\left\{u\left(y_t - \gamma z_t k_t - r_t k_t\right) - \epsilon_t + \beta E_t W(p_{t+1}, c_{t+1}),\right. \\ & \left. u\left(y_t - (1+\gamma) z_t k_t\right) + \beta\delta E_t W(p_{t+1}, c_{t+1})\right\} \Big| p_t \Big] && (6) \\ \text{s.t. } 0 = \Pi(p_t, c_t) & && (7) \end{aligned}$$

and the associated policy function $r(p_t, c_t), k(p_t, c_t), z(p_t, c_t)$ constitutes the equilibrium contract terms. Note that the expectation is over a_t and ϵ_t, both of which realize after the loan origination. Productivity realization a_t determines next period's belief p_{t+1}. ϵ_t determines the repayment/default choice (which are inside of the maximization operator).

4 Empirical Analysis

We now move to the empirical aspects of the model, stating with a more detailed discussion of assumptions maintained in estimation, identification of the model, and the estimated specification. Since I only observe firms borrowing from *the* sample bank, from now on, I denote by $t = 1$ the first year that a firm borrows from the bank. Thus, t denotes the year of the firm-bank lending relationship.

4.1 Preliminaries

I assume there are two quality types: High type (θ^H) and Low type (θ^L).[25] This means that the belief can be summarized by only one scalar p, which is the perceived probability that a firm is of, say, high type. It greatly simplifies the model while still maintain the essence of learning. The true fraction of high type conditional on s is given by λ_s, which also constitutes the initial prior belief for firms with characteristic s by assuming rational expectation.

The distribution of productivity conditional on s and θ is assumed to be a two-point distribution: $a \in \{\overline{a}_s, \underline{a}_s\}$ with $\overline{a}_s$ strictly larger than $\underline{a}_s$. Note that the support of a does not change with the unobserved type θ. Otherwise, agents would immediately infer the firm's type just after one period. The event of realizing $\underline{a}_s$ given s is called a *poor performance*; and the event of realizing $\overline{a}_s$ given s is called a *good performance*. Firms' performance outcomes are observed by the econometrician.

Conditional on the same s, firms with different quality type θ have different probability of realizing $\overline{a}_s$ (and thus $\underline{a}_s$). Specifically, the high type θ^H has a strictly lower probability of poor performance (i.e., realizing $\underline{a}_s$): $g(\underline{a}_s|\theta^H, s) < g(\underline{a}_s|\theta^L, s)$.[26](Or equivalently, $g(\overline{a}_s|\theta^H, s) > g(\overline{a}_s|\theta^L, s)$.)

I allow firm characteristics s, which are known to banks and firms, to be unobserved by the econometrician. It is, however, correlated with a set of firm-level variables X that is observed by the econometrician. Specifically, I assume that firm characteristics falls into S discrete groups: $s = 1, 2, ..., S$ (where S is known), and the distribution of s conditional on X is given by a known function $H(X;\eta)$. So η is the set of parameters that determines the distribution of s.

In addition, the firm-level production function $f(k)$ is assumed to be of Cob-Douglas form $f(k) = k^\alpha$ with decreasing returns to scale parameter α. And the period utility of entrepreneurs is CRRA with coefficient of relative risk aversion ρ. I fix risk aversion parameter and discount

[25]The binary assumptions on unobserved types is also seen in Pastorino (2019), Xin (2020).

[26]This is implied by the first-order stochastic dominance assumption (mentioned in the model section) in the context of binary type and two-point distribution.

factor, so the remaining structural parameters in the econometric model includes:

1. the parameters of conditional distribution of θ types, $\{\lambda_s\}_{s=1}^S$, and the parameters of the marginal distribution of characteristic groups s: η;
2. the parameters of productivity distribution conditional on s groups and θ types, $\{\overline{a}_s, \underline{a}_s, g(\underline{a}_s|\theta^H, s), g(\underline{a}_s|\theta^L, s)\}_{s=1}^S$;
3. the parameters of variance of repayment shocks $\{\sigma_s\}_{s=1}^S$, the parameter of the default costs δ, and the parameter of transaction costs associated with collateral: γ;
4. the parameters of returns to scale α;

4.2 Identification

I start the discussion by considering how the structural parameters are identified *conditional on s*. Then I discuss how the distribution of s is identified conditional on variables X that are observed in data. Here I mostly provide heuristic identification arguments.

1. The observed response of contract terms to past performances helps determine how different the high and low types are in terms of performances; or, more specifically, the ratio of $g(\underline{a}|\theta^L)$ to $g(\underline{a}|\theta^H)$.[27] Consider an extreme case where high type will never realize a poor performance and low type always realize poor performance (i.e., the ratio $g(\underline{a}|\theta^L)/g(\underline{a}|\theta^H)$ approaches ∞), then types are completely learned after the first loan, which means that we would see contract terms change from period one to two, but stay constant afterwards (conditional on funding costs). As the other extreme case, the high type has almost the same probability of realizing a poor performance as the low type (i.e., the ratio is near 1), then each observation of performance does not update beliefs very much, and learning takes place slowly and over a long period of time.[28] This means that in the data, we would see contract terms' responses to performances have very slow rate of change over time.

The rate of change of contractual responses to performances is captured in the coefficient for the interaction term, β_3, of regression (2). Consider a group of firms who has at least on poor

[27] s is omitted here since we are conditional on the same s throughout this part. The ratio $g(\underline{a}|\theta^L)/g(\underline{a}|\theta^H)$ lies in $(1,\infty)$ by definition.

[28] This can be seen mathematically from the Bayesian rule:

$$p_t = \frac{p_{t-1}}{p_{t-1} + (1-p_{t-1})g(\underline{a}|\theta^L)/g(\underline{a}|\theta^H)}$$

When $g(\underline{a}|\theta^L)/g(\underline{a}|\theta^H) \to \infty$, belief shrinks to 0 immediately; when $g(\underline{a}|\theta^L)/g(\underline{a}|\theta^H) \to 1$, belief hardly updates.

rating in their history, with some firms' poor performance happens in the early years, and others happened at the later periods of their relationships. If learning is fast, beliefs in later periods can be hardly changed by any news, then this new performance information that arrived in the later periods has way smaller effects on contract terms than if it arrived in the earlier periods, which means the sign of β_3 is opposite from β_2 and the magnitude of β_3 is large compared to that of β_2. On the contrary, when learning is slow, new information arrived in later periods can be almost as important as in early periods, so we would expect β_3 to be small in magnitude (but still has the opposite sign than β_2 as long as learning exits). From Table 1, we find that β_3 does have the opposite sign than β_2 in all of the three regressions, suggesting that learning does seem to take place. The magnitudes of β_3 are relatively small compared to that of β_2, indicating that learning is still happening at the later half the relationship.

2. Once the ratio of $g(\underline{a}|\theta^L)$ to $g(\underline{a}|\theta^H)$ is determined, the separate values of $g(\underline{a}|\theta^H)$ and $g(\underline{a}|\theta^L)$, as well as the parameter of θ-type distribution, λ, can then be identified from the first two period's performance data. Intuitively, this is because when there is only one type, the average firm performance in period-two should be similar to period-one, assuming sample attrition for reasons other than defaults are exogenous. When the type distribution is, say, half high and half low, since low types are more likely to perform poorly, and default probability conditional on poor performance are higher, more low types drop out from the cohort than high types, resulting in a positive *selection*. As a consequence, the second-period's average firm performance is better than the first-period, and the magnitude of improvement is informative of type distribution. In addition, the levels of average firm performance helps pin down performance distributions for each type.

We can also see this in a more formal way. Note that the mean of poor performance in period-one is (conditional on s):

$$E[PoorPerf_{i1}] = \lambda g(\underline{a}|\theta^H) + (1-\lambda)g(\underline{a}|\theta^L) \tag{8}$$

Suppose D_1 is the fraction of defaulted firms at the end of the first period, which can be observed from the data. Then the proportion of low types within these defaulted firms is given by

$$\eta_1 = \frac{(1-\lambda)g(\underline{a}|\theta^L)\underline{d}_1}{\lambda g(\underline{a}|\theta^H)\underline{d}_1 + (1-\lambda)g(\underline{a}|\theta^L)\underline{d}_1} \tag{9}$$

where $\underline{d}_1$ is the probability of default conditional on poor performance (or low realization of a) in the first period. Importantly, this probability is only determined by the *belief* held by the firm after its first performance, *not* by the true type of the firm, since we assume the firm do not observe its true type. In other words, if a high-type firm and low-type firm both realize a poor performance in the first period, they will have the same belief when they make default decision on the first loan, and thus they have the same default probability (conditional on s). Thus the same period-one default probability conditional on poor performance appears on both the numerator and denominator of Equation (9), so it can be canceled out, leaving η_1 a function of only $g(\underline{a}|\theta^H), g(\underline{a}|\theta^L), \lambda$.

Given η_1 and D_1, we can find the proportion of low type at the beginning of period-two, $1-\lambda-D_1\eta_1$, and the proportion of high type $\lambda+D_1\eta_1$. So the mean of poor performance in period-two is (conditional on s):

$$E[PoorPerf_{i2}] = (\lambda + D_1\eta_1)g(\underline{a}|\theta^H) + (1-\lambda - D_1\eta_1)g(\underline{a}|\theta^L) \tag{10}$$

Given data on firm performances, we can measure the right-hand side of Equation (8) and (10) using data, and the left-hand side of the two equations can be re-written in terms of $g(\underline{a}|\theta^L)/g(\underline{a}|\theta^H)$, $g(\underline{a}|\theta^H)$, and λ, in which only the later two objects are unknown. So they can be jointly determined by the two equations.

3. The parameter of default costs δ and the variance of repayment shock σ can be pinned down by the levels of default rates and the response of default rates to funding costs. Intuitively, both parameters can control the observed level of default rates, but σ also determines how "exogenous" the default events are.

Consider an extreme case of σ close to infinity, then defaults becomes almost irrelevant with the comparison of firms values associated with repayment and default, i.e., it is close to a random event that happens exogenously to firms. On the other extreme, when σ is close to zero, then default is almost deterministic, and it almost surely will not happen if the firm's value of repaying is strictly larger than the value of defaults. It follows that σ should determine how sensitive defaults are with respect to exogenous variations in firm's value of repay, say, variations in funding costs which affect interest rates and repayment. Thus, co-movements of default rates and funding costs can identify σ. Once σ is pinned down, the observed level of default rates can then pin down the parameter of default cost δ since it controls the default incentive of firms.

4. The parameter of collateral costs can be identified by the observed usage of collateral after pinning down δ and σ. The role of collateral in this model is to provide a disincentive for the firm to default. Once the levels of default incentives are pinned down by δ and σ, the most

important factor affecting the usage of collateral is the associated transaction cost, which is indicated by γ.

5. The returns to scale parameter α can be identified from the joint distribution of initial loan size and funding costs. This is because conditional on initial beliefs (which is given by the true distribution of types), higher α means higher sensitivity of loan size with respect to funding costs.

6. The high and low realizations of productivity $\{\overline{a}, \underline{a}\}$ can be identified from the joint distribution of loan sizes and performance history at the end of their relationships. Once we identified the learning parameters, we can pin down the difference in beliefs for firms with different performance histories at the end of the sample period, and how the difference in beliefs translates into difference in loan sizes depend largely on the levels of productivity $\{\overline{a}, \underline{a}\}$, and returns to scale parameter α. Once α is identified from the previous step, we can then identify $\{\overline{a}, \underline{a}\}$.

So far, the identification argument is conditional on firm's characteristic group s. The (parametric) distribution of group s conditional on observed firm variables X can be pinned down by the distribution of initial contract terms conditional on firms' X, since initial contract terms are completely determined by a firm's s group. In other words, the pattern of how firms with certain value of X tend to share similar initial contract terms helps identify the link between s group and firm variables X, which is summarized in the set of parameter η.

4.3 Functional Forms

Repayment Shocks I assume the repayment shock ϵ_t has a normal distribution with mean zero and variance σ_s.[29]

Firm Characteristics In estimation, I let the number of firm characteristic groups $S = 3$, which means that a firm belongs to one of the three characteristic groups $s = 1,2,3$. I use ordered probit model to model the distribution of s conditional on X. Specifically, $s = j \Leftrightarrow \eta_j^c \leq X'\eta^f + e \leq \eta_{j+1}^c$, for $j = 1,2,3$. Here e is a standard normal random variable, η^f is a set of linear coefficients associated with X, and $\{\eta_j^c\}_{j=1}^4$ is the set of cut points with $\eta_1^c = -\infty$ and $\eta_4^c = +\infty$. In other words, the distribution of characteristic groups conditional on X in given by $P(s = j|X) = \Phi(\eta_{j+1}^c - X'\eta^f) - \Phi(\eta_j^c - X'\eta^f)$, where $\Phi(\cdot)$ is the standard normal distribution function.

To construct X, I consider the following six firm-level attributes: industry, firm size, region,

[29] The normal distribution assumption aids calculation of the term $E[\epsilon|repay]$, which is a truncated mean. Since the expectation of truncated normal distribution has a simple analytical form, normal distribution is assumed for ϵ.

registered capital, cohort, initial assessment. All variables except for the registered capital are categorical, and registered capital is continuous. Thus I generate dummy variables for each categorical variables, which add up to 20 dummy variables. In total, I have 20+1 variables for X.

Distribution of Productivity Conditional on Firm Type. Conditional on the same θ, firms with different characteristics s have different supports of productivity distribution. I make two simplifying assumptions with regard to how s affects the productivity distribution: First, firms of the same quality type all share the same probability of success, regardless of their characteristics s. In other words, $g(\overline{a}_s|\theta, s)$ is the same across s, and thus I will drop s from the parenthesis and denote it as $g(\overline{a}_s|\theta)$. Second, I assume the difference between $\overline{a}_s$ and $\underline{a}_s$ does not vary with s, and I define this difference as $\Delta a \equiv \overline{a}_s - \underline{a}_s$. In other words, fixing θ, characteristics s only horizontally shifts the distribution of productivity without changing its shape.

Measurement of Firm Performances The outcome of performances (whether good or poor) drives the learning process. Formally, I define a variable $PoorPerf$ to reflect this binary outcome for a given firm i in a given period t: $PoorPerf_{it} = 1\{a_{it} = \underline{a}\}$.

I use the internal ratings to measure firm performances. As mentioned in Section 2.3, banks in China are required to compile monthly *internal* ratings on every loan that stands out on the bank's balance sheet. These dynamic internal ratings, by definition, directly reflect bank's assessments about the firm's repayment ability. They are different from the actual loan outcomes: banks often downgrade a firm's internal rating before actual delinquency happens as a result of close monitoring (Gao et al., 2019b).

Data used in this paper contains one snapshot of the monthly internal ratings on every loan. Specifically, that snapshot is taken in December, which means that for loans that are due in, say, next March, this rating is recorded three months ahead of the due date. I use observations whose due date is at least three months ahead of December, in order to ensure that the internal rating does not merely reflect the loan outcome; it contains valuable information that shapes the bank's belief about the firm's quality. This way we have an internal rating on each firm i in each period t. A questionable internal rating on a firm at period t is taken as an observation of an $PoorPerf_{it} = 1$.

Lending Cost I use the benchmark one-year deposit rates as the empirical counterpart of c. Using data on the deposit rates from 2009 to 2018, I discretize the data into four bins: 1.75% ,2.25%, 2.75%, 3.25%, and estimate the transition matrix, which is then transformed to the transition matrix for yearly frequency.

4.4 Estimation

The estimation is based on simulated method of moments. The set of parameters to be estimated is listed in Table 2, which is collected in vector Ξ. Following Boualam (2018), discount factor β is set to 0.9542, and the relative risk-aversion is set to 0.6.

Table 2: List of Parameters

Parameters to be Estimated	Definition
$\{\bar{a}_s\}_{s=1}^3$	High productivity realization for each characteristic group s
Δa	Difference between high and low productivity realization
$\{\lambda_s\}_{s=1}^3$	Fraction of high type within each characteristic group s
$\eta^f, \eta_2^c, \eta_3^c$	Coefficients and cut points in determining s
$g(\bar{a}\vert\theta^h)$	Prob. of high type realize high productivity
$g(\bar{a}\vert\theta^l)$	Prob. of low type realize high productivity
$\{\delta_s\}_{s=1}^3$	Default costs as a fraction of firm value for each s
$\{\gamma_s\}_{s=1}^3$	Transaction cost of collateral for firms in each s
$\{\sigma\}_{s=1}^3$	Variance of liquidity shock for firms in each s
α	Returns to scale parameter
mc	Bank's marginal cost of lending (in addition to funding cost)

Given a vector of primitives Ξ, for each $s \in \{1,2,3\}$, I solve for the value function $W^s(p,c)$, associated policy functions $r^s(p,c)$, $k^s(p,c)$, $z^s(p,c)$, as well as repayment probabilities conditional on each performance outcomes.

Then for each firm $i = 1$ to $N = 6358$,

1. I draw its quality type θ_i, based on which I draw a panel of performance outcomes $\{perf_{it}\}$ for $t = 1$ to $T = 7$.

2. Based on the initial state of funding cost c_{i1}, I simulate a path of funding cost for T period ahead: $\{c_{it}\}$ for $t = 1, ..., T$.

3. I also draw the characteristic s that it belongs to based on its X. The s characteristics determines the initial prior belief p_{i1}, which, combined with the panel of performance outcomes, completely determines the path of beliefs $\{p_{it}\}$.

4. Now we have funding costs and belief (p_{it}, c_{it}) for $t = 1, ..., T$, so we can apply policy functions $r^s(p,c)$, $k^s(p,c)$, $z^s(p,c)$ to find the contract terms $\{r_{it}, k_{it}, z_{it}\}$ in each period.

5. The default probabilities can also be found using both (p_t, c_t) and the simulated firm performance outcomes, based on which the default outcomes $\{d_{it}\}$ are drawn. Defaulted firms are then removed from the simulated data following its default.

I estimate the primitive using method of moments estimator by minimizing the differences between model prediction and the data counterpart of the following moments conditional on exogenous firm variables X: (1) contract terms and defaults $o_{it} = \{r_{it}, k_{it}, z_{it}, d_{it}\}$; (2) firm performance outcomes $perf_{it}$; (3) covariance between firm performance and contract terms and default $perf_{it} \times o_{it}$; (4) covariance between funding costs and contract terms and default $c_i \times o_{it}$. Formally, let $m_{it}(\Xi) = \left\{o_{it}, perf_{it}, perf_{it} \times o_{it}, c_i \times o_{it}\right\}$, and its data counterpart $\hat{m} = \left\{\hat{o}_{it}, \hat{perf}_{it}, \hat{perf}_{it} \times \hat{o}_{it}, \hat{c}_{it} \times \hat{o}_{it}\right\}$, then the moment restrictions used in the estimation are (detailed lists are in Table 15):

$$g_t(\Xi) = \mathbb{E}_{\{i\}}\left[m_{it}(\Xi) - \hat{m}_{it}|X_i\right], \forall t = 1, ..., T \tag{11}$$

I implement two-step generalized method of moments where I estimate the model using identity weighting matrix and obtain an estimator $\hat{\Xi}_1$. Using the estimator, I obtain the optimal weighting matrix, $\hat{\Gamma}$, and solve for $\hat{\Xi}$ by

$$\hat{\Xi} = \arg\min_{\Xi} g(\Xi)\hat{\Gamma} g(\Xi)$$

where $g(\Xi) = \{g_t(\Xi)\}_{t=1}^{T}$. In total, I have 21 covariate vectors and $21 \times (28 + 7 + 28 + 28) = 1911$ moment restrictions to estimate 35 parameters.

5 Estimation Results

In this section, I first discuss my parameter estimates. I then provide several pieces of evidence to evaluate the model fit. I report the parameter estimates in Table 3. Panel A of Table 3 shows estimates for parameters that vary across characteristic group s, which includes the high realization of productivity $\bar{a}_s$, the fraction of firm value that can be salvaged after a firm defaults, δ_s, the fraction of high type firms conditional on s, λ_s, and the variance of liquidity shock σ_s. It implies that there is considerable firm heterogeneity. The relatively large range in $\bar{a}_s$ translate into large range in initial loan sizes: ranging from 2.43 to 8.93 (in millions RMB). The equilibrium default probabilities also have a wide range (1.3% to 12.03%), largely due to the relatively large variations in δ_s and σ_s across s.

Panel B of Table 3 shows estimates for parameters that do not vary across characteristic group

Table 3: Parameter Estimates

Panel A: Parameters varying across characteristics s

	$\bar{a}_s$	δ_s	λ_s	σ_s
$s=1$	2.679	0.920	0.399	3.055
$s=2$	3.261	0.937	0.486	4.062
$s=3$	4.187	0.945	0.471	2.210

Panel B: Parameters that do not vary with s

Δa	$g(\theta^h)$	$g(\theta^l)$	α	γ	mc
0.702	0.974	0.867	0.607	0.032	0.015

s, which includes the difference between high and low realizations of productivity δa, the high-quality-type's probability of realizing high productivity, $g(\theta^h)$, and the low-type's probability of realizing high productivity, $g(\theta^l)$, the return-to-scale parameter, α, the transaction cost associated with collateral, γ, and the constant marginal cost of lending mc. Reading from $g(\theta^h)$ and $g(\theta^l)$, the high-type has a 2.6% change of poor performance, while it is 13.3% for the low-type. The transaction cot associated with collateral is around 3% of the collateral value, which is close to what credit guarantee companies charge on an annual basis.[30] The marginal cost of lending is estimated to be 1.5%, which is lower than the estimated "profit rate" 2.2% in Porter et al. (2009) since Porter et al. (2009) does not take into account expected default costs.

5.1 Model Fit

The model predicts five observed variables (interest rates, loan sizes, collateral requirement, firm performances, and default) over a period of seven years. I first check how do these variables track the ones observed in the data over the seven years across all firms, and then break down by different firm characteristics. The results are shown in Table.

Another important aspect of the model of the co-movement of firm performance and contract terms. Specifically, how contract terms respond to new information of firms performance, and how these responses change over time, speak directly to the speed of learning process. To summarize this information succinctly, I use firm fixed effect model to regress contract terms on years of relationship t, indicator of poor performance in the previous period, $PoorRating_{t-1}$, and the interaction term $t \times PoorRating_{t-1}$, as in Equation (2). I conduct this regression for firms who never had a single poor performance in their history (in which case the variable

[30]See Li and Lin (2017) for details on credit guarantee companies in China.

Table 4: Model Fit: Contract Terms, Performance, and Loan Outcomes

t	r		k		z		Performance		Default	
	Data	Model	Data	Model	Data	Model	Data	Model	Data	Model
1	7.043	7.236	4.482	4.822	0.573	0.556	0.081	0.085	0.040	0.056
2	7.051	6.986	5.009	4.902	0.573	0.554	0.085	0.088	0.066	0.060
3	6.826	6.728	5.371	4.991	0.570	0.551	0.084	0.089	0.081	0.063
4	6.577	6.511	5.839	5.085	0.568	0.548	0.082	0.082	0.051	0.051
5	6.420	6.366	6.485	5.165	0.565	0.546	0.089	0.085	0.079	0.047
6	6.165	6.246	7.120	5.237	0.560	0.544	0.083	0.087	0.050	0.051
7	6.004	6.136	7.572	5.314	0.535	0.542	0.089	0.087	0.080	0.049
Average	6.863	6.643	5.195	5.055	0.567	0.553	0.085	0.086	0.058	0.054

Table 5: Regressions of Contract Terms on Past Performance: Using Simulated Data

	Firms w. good rating history			Firms w. poor rating history		
	(1) Lending Spread	(2) Loan Size	(3) Collateral Coverage	(4) Lending Spread	(5) Loan Size	(6) Collateral Coverage
t	-0.00531	0.0119	-0.000192	-0.00508	0.0126	-0.000201
	(-64.31)	(64.30)	(-114.01)	(-20.10)	(29.28)	(-19.46)
$PoorRating_{t-1}$				0.0677	-0.120	0.00198
				(27.92)	(-28.94)	(20.01)
$t \times PoorRating_{t-1}$				-0.000220	-0.00621	0.000156
				(-0.45)	(-7.42)	(7.77)
Observations	18195	18195	18195	7613	7613	7613

t statistics in parentheses. Other controls: firm fixed-effects, funding costs fixed-effects.

$PoorPerf_{t-1}$ and its interaction term with t will be omitted, and we will only compare the rate of change of contract terms over time), and for firms who have at least one poor performance in their history (again for these firms I keep observation $t <= t_i^* + 1$ where t_i^* is the first time firm i is rated poorly). Results are shown in Table 5.

The signs of coefficients in the regression using the simulated data are in line with empirical results in Table 1, where we see a decline of lending spreads and collateral coverage, as well as a gradual growth of loan size over the course of relationship for the firms who has never had a poor rating (which are overwhelmingly high type). As for firms who performed poorly in period t, the interest rate and collateral coverage next period is likely to go up, and loan size is likely to go down in the next period.

5.2 Inefficiencies from Agency Friction and Incomplete Information

In the estimated model, there are two sources of inefficiency: incomplete information and agency friction (or limited commitment). Incomplete information (or uncertainty) refers to the fact that banks and the firm do not know the true θ type of the firm, which leads to a mismatch of contract terms and firms: firms are assigned contract terms that are not optimal for their true types. Although learning could reduce uncertainty and alleviate the implications from incomplete information in subsequent period, the reduction might not be complete. To see the inefficiencies cause by incomplete information, I consider the counterfactual case with full information, where firm types are fully known to both the bank and the firm from the beginning. In this case, the firm's value function is type-specific, and dynamics in contract terms are not driven by learning.

Agency friction comes from the fact that firms cannot commit to repay; instead, firms have a choice of whether to repay or default based on realized revenue and liquidity shocks. This can also be viewed as a form of *ex post* moral hazard (Xin, 2020). To see how contract terms and default rates are different if there was no agency friction, I consider the planner's problem, where the repayment behavior can be enforced by the contract. This means that, in the planer's contracting problem, choice variables not only include (r, k, z), but also repayment policy (repay or default). For example, in the case with full information and no agency friction, the contract would be given be solution to the following problem of maximizing the joint value between firm and bank subject the the bank earning zero profit:

$$V_\theta = \max_{r,k,z} E \max\left\{ \underbrace{u\left(y - \gamma z k - r k\right) + r k + \beta V_\theta}_{v_{\text{repay}}} - \epsilon, \underbrace{u\left(y - (1+\gamma) z k\right) + z k + \beta \delta V_\theta}_{v_{\text{default}}} \right\} - c k$$

$$\text{s.t. } 0 = -c k + r k - D(r - z) k$$

where default probability $D \equiv 1 - \Phi\left(\frac{v_{\text{repay}} - v_{\text{default}}}{\sigma}\right)$.

I calculate the (counterfactual) short-term contracts in four scenarios: (1) no agency friction and full information; (2) no agency friction and incomplete information; (3) with agency friction and full information; (4) with agency friction and incomplete information (the estimated baseline model). For each firm, I find the associated cumulative default probability over the seven-year period, total amount of capital and output taken over the seven-year period, the firm size measured by output in the last period, and the initial firm value in the four equilibrium. Summary of these statistics by firm quality type is show in Table 6.

Comparing column (3) with column (4), we can see that incomplete information leads to misal-

Table 6: Comparisons Across Baseline Model, Models with One or Neither Source of Inefficiency

Scenario	Benchmarks			Baseline
	(1) No AF Full Info.	(2) No AF Inc. Info.	(3) AF Full Info.	(4) AF Inc. Info.
	High-Type Firms			
Cumulative Default Prob.	.0877069	.0968651	.3237055	.3325114
Total Capital	37.72948	36.20978	32.05138	30.71789
Total Output	65.62923	63.9358	56.94183	55.4055
Final Size	5.571345	5.380186	5.261407	5.071709
Firm Value	88.68119	87.42284	87.55135	86.39585
	Low-Type Firms			
Cumulative Default Prob.	.0934925	.0863882	.316283	.3117363
Total Capital	33.13112	34.22402	28.31473	29.27848
Total Output	58.12727	59.32193	50.78448	51.87864
Final Size	4.898396	5.039846	4.612279	4.751198
Firm Value	84.28747	85.30146	83.33981	84.25609
	All Firms			
Cumulative Default Prob.	.0909091	.0910664	.3195974	.3210129
Total Capital	35.1844	35.11071	29.98323	29.92121
Total Output	61.47708	61.38213	53.53389	53.45347
Final Size	5.198884	5.191816	4.90213	4.894314
Firm Value	86.24937	86.24871	85.22037	85.21154

location of capital: high-types in baseline case are assigned lower capital than the full information case as in column (3), whereas low types in baseline case are assigned higher capital than the full information case. This mismatch of capital and firm productivity leads to lower overall outputs. In addition, there is an issue of mispricing due to incomplete information, where high-types in baseline case are charged with higher interest rates than the full information case, and low-types are charged with lower interest rates than the full information case. Such mispricing of default risks and misallocation of capital leads to negative impacts on outcomes of high-type firms, as shown in Figure 4. Learning, however, can reduced the negative impacts over the course of the relationship to some degree. But at the end of the period, we still see around half mispricing and misallocation compared to the initial period, as shown in Figure 5 and Figure 6.[31]

Comparing column (2) with column (4), we can see that in the benchmark scenario with no agency friction (i.e., in the planner's problem), default would be extremely rare, and capital flow would be significantly enhanced. Specifically, the number of defaulted firms would be 71.63% fewer than the baseline case, and total output would be 14.83% higher. In other words, the borrowing constrains in the baseline case is largely due to agency friction. In total, agency friction accounts for the vast majority of total welfare loss from both agency friction and incomplete information.

6 Long-Term Financing Contracts

In this section, I consider the counterfactual scenario where long-term contracting is available, and banks compete by offer long-term contracts. I start from a model with two-period contingent contracts to show features of the optimal long-term contracts and explain my solution method. Then I describe the multi-period contingent contracts and the results from my counterfactual simulation exercises, like how firm outcomes are different under long-term contracts compared to the short-term contracts.

6.1 Two-Period Contingent Contracts

To better illustrate the how long-term contracts work, I begin with the simplest case of long-term contingent contract — two-period contract. Suppose banks offer two-period contingent

[31]Here I measure mispricing by $(r_i^{CI}/\hat{r}_i - 1) \times 100\%$ where r^{CI} is interest rate in the counterfactual scenario of complete information and$\hat{r}$ is interest rate in the baseline model. I measure misallocation of capital by $(k_i^{CI}/\hat{k}_i - 1) \times 100\%$, where k^{CI} is loan size in the counterfactual scenario of complete information, and $\hat{k}$ is loan size in the baseline model. Mispricing in the initial period is $\approx 1.7\%$, and misallocation of capital in the beginning is $\approx 4.8\%$.

contracts to *new firms* at the beginning of period one, and such contracts stipulate lending terms in the first period, as well as the state-contingent lending terms of the second period. After the two-period contract ends, a firm enters the infinite-horizon short-term lending market, where competition takes place as described in the previous section. In other words, the thought experiment I engage here is about providing long-term contracts at the *beginning* of a firm's life, and see how it affects the firm outcomes compared to the baseline model where firms are born into a short-term lending market.

In this model, the banks compete at the beginning of period one by offering a two-period contingent contract. Let l_t denote the lending terms in period t, i.e., $l_t = (r_t, k_t, z_t)$. Then the two-period contract can be written as $\{l_1, l_2(p_2, c_2)\}$ where the second-period terms l_2 is a function of state variable (p_2, c_2).

I consider two-period contracts that are are *binding* conditional on repayment, meaning that as long as the first-period loan is repaid by the borrower, then second-period borrowing activity must take place according to the lending terms pre-determined in the contract. Specifically, it implies that the lender cannot renege on the second-period terms and the borrower cannot switch, if the first-period loan is repaid. If the borrower defaults on the first-period loan, then the contractual relationship breaks and the firm receives scrap value that equals to a fraction δ of its equilibrium firm value in the short-term lending market.

By the same logic as before, Bertrand equilibrium in this model results in maximization of firm's value subject to the zero profit condition. However, zero profit condition in this case means that a bank expects the overall profit *over the course of the long-term contract* is zero, as apposed to zero profit expected within each period. In other words, it is possible that the first-period expected profits are positive (negative), and the ex-ante second-period profits are negative (positive). This is because the binding long-term contract implies that there is no competition from outside banks at the beginning of the second period, thus the second-period expected profit in each possible state is no longer constrained to zero.

To succinctly state the problem solving for the equilibrium long-term contract, I first define a few objects. Let $U(l_t; a_t)$ and $U^F(l_t; a_t)$ denote the firm's flow utility in period t for repayment and default, respectively. They are functions of this period's lending terms l_t as well as the realization of productivity a_t which determines the level of outputs.

$$U(l_t; a_t) = u\left(y_t(a_t) - \gamma z_t k_t - r_t k_t\right)$$
$$U^F(l_t; a_t) = u\left(y_t(a_t) - (1+\gamma) z_t k_t\right)$$

Using $U(l_t;a_t)$ and $U^F(l_t;a_t)$, we can express the firm's utility at the beginning of $t=2$ as a function of the second-period lending terms l_2 and state variables p_2, c_2:

$$w_2(l_2,p_2,c_2) = E_{a_2,\epsilon_2}\Big[\max\{U(l_2;a_2)-\epsilon_2+\beta E\left[W(p_3,c_3)|a_2\right], \\ U^F(l_2;a_2)+\beta\delta E\left[W(p_3,c_3)|a_2\right]\}\Big|p_2,c_2\Big] \tag{12}$$

where $W(\cdot,\cdot)$ is firm's value function in the short-term lending equilibrium. Here we maintain the same assumption as the baseline model with regards to defaults: defaulting firms receive scrap value that equals to a fraction δ of the firm value W.

It follows that default probability in the second period conditional on realization of a_2 can be written as

$$\Phi_2(l_2,p_2,c_2;a_2) = \Pr\Big(\epsilon_2 \le U(l_2,a_2)-U^F(l_2,a_2)+\beta(1-\delta)E\left[W(p_3,c_3)|a_2,p_2,c_2\right]\Big) \tag{13}$$

Using the default probability function, we can find the expected bank profit for the second-period:

$$\pi_2(l_2,p_2,c_2) = -(c_2-z_2)k_2+E_{a_2}\left[\Phi_2(l_2,a_2,p_2,c_2)(r_2-z_2)k_2\big|p_2\right] \tag{14}$$

At the beginning of the first period, for any given lending policy in period 1 and period 2, (l_1,l_2), we can express firm's expected utility as a function of l_1, l_2 as well as state variables (p_1,c_1):

$$w_1\left(l_1,l_2,p_1,c_1\right) = E_{a_1,\epsilon_1}\Big[\max\{U(l_1;a_1)-\epsilon_1+\beta E\left[w_2\left(l_2,p_2,c_2\right)|a_1\right], \\ U^F(l_1;a_1)+\beta\delta E\left[W(p_2,c_2)|a_1\right]\}\Big|p_1,c_1\Big] \tag{15}$$

The difference in future values for the choice of repayment and default comes from the assumption that defaults break the long-term contract and leave firm with a scrap value that equals to a fraction δ of the firm value W. This leads to the following expression for the default probability in the first period conditional on a_1:

$$\Phi_1(l_1,l_2,p_1,c_1;a_1) \equiv \Pr\Big(\epsilon_1 \le U(l_1,a_1)-U^F(l_1,a_1)+\beta E\left[w_2\left(l_2,p_2,c_2\right)-\delta W(p_2,c_2)|a_1,p_1,c_1\right]\Big) \tag{16}$$

Thus the bank's expected profit over the two periods is

$$\pi_1(l_1, l_2, p_1, c_1) = -(c_1 - z_1)k_1 + E_{a_1}\left[\Phi_1(l_1, l_2, p_1, c_1; a_1)(r_1 - z_1)k_1 + \beta E\left[\pi_2(l_2, p_2, c_2)|a_1\right] \middle| p_1, c_1\right] \tag{17}$$

6.1.1 Original Problem

The original problem that a bank solves to obtain lending terms of the two-period contract in the equilibrium is the following:

$$\begin{aligned} &\max_{l_1, l_2(p_2,c_2)} w_1\left(l_1, l_2(p_2, c_2), p_1, c_1\right) \\ &\text{s.t.} \quad 0 = \pi_1\left(l_1, l_2(p_2, c_2), p_1, c_1\right) \end{aligned} \tag{18}$$

In other words, the bank chooses lending terms of the first period, l_1, as well as second-period terms contingent on state (p_2, c_2), that maximizes firm's utility subject to zero expected profits over the course of the two periods.

In the case where productivity a_t has a two-point distribution, then conditional on p_1, next period's belief p_2 has two possible values, $p_2\left(p_1|a_1 = \bar{a}\right)$, and $p_2\left(p_1|a_1 = \underline{a}\right)$. Continuing the assumption that c_t takes four values, then there are $2 \times 4 = 8$ states for the second period. Since lending terms l include three variables, r, k, z, in total there will be $3 + 8 \times 3 = 27$ choice variables in this problem. And if we later extend the two-period contracts to multiple periods, the number of choice variables will explode. Thus, in this paper I consider an alternative approach to solve for equilibrium long-term contracts.

6.1.2 Transformed Problem

Inspired by Albuquerque and Hopenhayn (2004), I transform the problem into one where the lender chooses a summary statistics for future contract instead of fully spelling out each lending term. This also helps forming a recursive representation of the original problem (18).

Note that given each state in period two, (p_2, c_2), for a certain level of expected bank profits $\bar{\pi}$, there is a unique solution to the constrained maximization problem of firm value subject to the bank's expected profit equal to $\bar{\pi}$, and the maximum firm value is defined as $W_2(\bar{\pi}, p_2, c_2)$:

$$W_2(\bar{\pi}, p_2, c_2) \equiv \max_{l_2} w_2(l_2, p_2, c_2) \text{ s.t. } \bar{\pi} = \pi_2(l_2, p_2, c_2) \tag{19}$$

Using $\bar{\pi}$ as an "index" for second-period contract contract contingent on the state (p, c), I rewrite the period-one objects (15)-(17) using $\bar{\pi}$ instead of l_2: (I suppressed the parenthesis of $\bar{\pi}$ in (15') and (16'))

$$w_1(l_1, \bar{\pi}, p_1, c_1) = E\Big[\max\{U(l_1; a_1) - \epsilon_1 + \beta E\left[W_2\left(\bar{\pi}(p_2), p_2, c_2\right) | a_1\right], \\ U^F(l_1; a_1) + \beta\delta E\left[W(p_2, c_2)|a_1\right]\}\Big| p_1, c_1\Big] \tag{15'}$$

$$\Phi_1\left(l_1, \bar{\pi}, p_1, c_1; a_1\right) = \Pr\Big(\epsilon_1 \leq U\left(l_1, a_1\right) - U^F\left(l_1, a_1\right) + \beta E\left[W_2\left(\bar{\pi}, p_2, c_2\right) - \delta W(p_2, c_2)|a_1, p_1, c_1\right]\Big) \tag{16'}$$

$$\pi_1(l_1, \bar{\pi}, p_1, c_1) = -(c_1 - z_1)k_1 + E_{a_1}\left[\Phi_1\left(l_1, \bar{\pi}, p_1, c_1; a_1\right)(r_1 - z_1)k_1 + \beta E\left[\bar{\pi}(p_2, c_2)|a_1\right] \big| p_1, c_1\right] \tag{17'}$$

Thus, at the beginning of period one when banks make contract, instead of choosing three lending terms for each state of period two, they choose an index $\bar{\pi}$ for each state of period two:

$$\max_{l_1, \bar{\pi}(p_2, c_2)} w_1\left(l_1, \bar{\pi}(p_2, c_2), p_1, c_1\right) \\ \text{s.t. } 0 = \pi_1\left(l_1, \bar{\pi}(p_2, c_2), p_1, c_1\right) \tag{20}$$

The optimal choice of $\bar{\pi}$ corresponds to a set of lending terms in the second period as described in (19). In this problem, the number of choice variable is reduced to $3 + 2 \times 4 = 11$, which is less than half of that in the original problem (18).

Still, this is not a small number of choice variables by any measure, so to further simplify the problem, I restrict myself to the case where the choice of $\bar{\pi}$ is a function of only p_2, not c_2. This is *not* to say next period's interest rates do not vary with next period's funding costs; rather, this is akin to keeping bank's markups same across next period's funding costs. To put it another way, second-period interest rates still vary with cost realizations, but in a way that is not about insuring firms against those shocks, but merely to treat costs as an anchor. By doing so, I focus on whether and how contract terms insure against individual firm risks, not aggregate cost shocks. In the end, the version of the problem I solve in the two-period setting is the following:

$$\max_{l_1, \bar{\pi}(p_2)} w_1\left(l_1, \bar{\pi}(p_2), p_1, c_1\right) \\ \text{s.t. } 0 = \pi_1\left(l_1, \bar{\pi}(p_2), p_1, c_1\right) \tag{21}$$

The number of choice variables in this problem is $3 + 2 = 5$, making extension to multiple periods computationally feasible. In the numerical exercise, I first solve the second-period problem (19) under each combination of $\bar{\pi}$, p and c, where $\bar{\pi}$, p are discretized on a grid and c takes four discrete values.[32] I interpolate in between grids of $\bar{\pi}$ to obtain the function $W_2(\cdot, p, c)$ for each combination of p and c.[33] In the first period, for any p_1 on the grid of p and for each of the four values of c_1, I find the two possible next period belief p_2 conditional on a high and low realization of a_1, denoted as $p_2^H = p_2\left(p_1 | a_1 = \bar{a}\right)$ and $p_2^L = p_2\left(p_1 | a_1 = \underline{a}\right)$. Then I plug functions $W_2(\cdot, p_2^H, c_2)$ and $W_2(\cdot, p_2^L, c_2)$ for each of the four values of c_2 into the first-period problem (21), which is solved to obtain the optimal first period lending terms l_1, as well as indices $\bar{\pi}$ following a high and low realizations: $\bar{\pi}(p_2^H)$ and $\bar{\pi}(p_2^L)$. Finally, using the chosen indices $\bar{\pi}(p_2^H)$ and $\bar{\pi}(p_2^L)$, I can back out the associated optimal second period lending terms under each values of c_2, i.e., $l_2(p_2^H, c_2)$ and $l_2(p_2^L, c_2)$. Thus, for each initial state (p_1, c_1), I obtain period-one terms $l_1(p_1, c_1)$ and state-contingent period-two terms $l_2\left(p_2(p_1|a_1), c_2\right)$ for eight possible states, and I also derive equilibrium default probabilities in each period as a function of (p_1, c_1).

6.1.3 Numerical Results

Case 1: with complete type information. To begin with, I abstract from the learning dynamics and consider the case with complete information about type. In other words, my analysis first focuses on the case with $p_1 = 0$ (low type) and $p_1 = 1$ (high type). For each type (high/low), each characteristic group s and each c_1, I calculate the first-period lending terms (interest rates, loan size, and collateral coverage), and expected second-period lending terms.[34] Table (7) shows results for lending terms by each characteristic group compared with the short-term model, where the first period funding cost is fixed at $c_1 = 1.03$. Interest rates are shown in terms of Annualized Percentage Rate, i.e., it equals $(r - 1) \times 100\%$.

The intertemporal pattern of interest rate is similar cross all characteristics group and productivity type: interest rates are higher than the short-term rates in the first period, but lower than the short-term rates in the second period. Note that the short-term rates have a slight decrease from the first to the second period because of transition of funding costs. But this decrease is more dramatic for the model with the short-term contracts, which is almost a fifty percent decrease from period one to period two. Meanwhile loan size increase and collateral coverage decrease from the first to the second period. And the period-two collateral coverage in the long-term contracts is always lower than in the short-term contracts. This pattern translates into a

[32] The grid of p is the same as in baseline estimation, from 0 to 1 with step length 0.01. The grid of $\bar{\pi}$ is -1 to 1 with step length 0.01, and this grid is large enough in the sense that chosen values of $\bar{\pi}$ in the equilibrium do not hit the bounds of the grid.

[33] I use cubic spline interpolation to ensure smoothness and facilitate the first-period maximization problem.

[34] The expectation is taken over c_2 conditional on c_1.

negative expected bank profit for the second period.[35] For the high type, the average second-period expected bank profit across s group is -0.0988; for low type it is -0.0874.

To compare firm outcomes, I find the associated default probabilities over the two periods, from which I calculate the cumulative default probability, defined as the probability that a firm defaults in either of the first two periods.[36] Lastly I obtain the expected firm value in the beginning of the first period. I average the cumulative default probability and firm value across c_1 and s using empirical distribution obtained from the estimation, and compare those statistics with the short-term model. Results are shown in Table (8). Cumulative default probabilities decrease for both types, but magnitude is small. For the high type, the cumulative default probability with two-period long-term contract is 10.665%, which is a 0.244 percent decrease from the short-term contract model, and the expected firm value at the beginning increases by 0.024 percent. For the low type, the cumulative default probability with two-period long-term contract is 11.402%, which is a 0.018 percent decrease from the short-term contract model, and the expected firm value at the beginning increases by 0.002 percent. Overall, the high type benefit more from the long-term contracts than the low type.

Case 2: with incomplete type information. Now I examine the case where initial prior belief $p_1 \in (0,1)$. For each s, I fix p_1 at the estimated value of λ_s, which is the proportion of high type in the population conditional on s. And c_1 is fixed at 1.03 as in the previous case. I find the first-period lending terms, as well as expected second-period lending terms following a high and low realization of a_1. (As in the previous case, I take expectation over c_2 using transition matrix of c). Results are shown in Table 9. First, the intertemporal pricing pattern is still a front-loading one, where prices in the first period is much higher than the second period. This echoes the findings in the complete information case. Second, by comparing the second-period terms across different states (i.e., in the good state where the first period productive realization is high ($a_1 = \bar{a}$) and belief is updated in a favorable way ($p_2 = p_2^H$), and in the bleak state where the first period productive realization is low ($a_1 = \underline{a}$) and belief is updated in an adverse way($p_2 = p_2^L$)), we find that interest rates are lower in the bleak state than in the good state, so do collateral requirements. For loan size though, it is smaller in the bleak state than the good state. This is because the firm's expected productivity is tuned down according to the updated belief, which translate into a smaller loan size. Still, the loan size for bleak state under long-term contracts is slightly larger than that in the short-term contracts, so conditional on a bad realization of a_1, firms are strictly better off under long-term contracts than short-term contracts. This speaks to the insurance role of the long-term contract.

[35]Note that in the complete information case $\bar{\pi}$ is no longer contingent on realization of a_1.

[36]That is to say, one minus cumulative default probability is the probability of surviving the first two periods.

Table 7: Lending Terms in Model with Two-Period Long-Term Contracts vs. Short-Term Contracts

Type	Period	Interest Rate (%)		Loan Size		Collateral Coverage	
		LT	ST	LT	ST	LT	ST
		Characteristic Group $s = 1$					
High	1	9.1419	7.258	2.7016	2.7019	0.86738	0.85783
	2	5.1543	7.1211	2.7115	2.7109	0.81941	0.85697
Low	1	9.2978	7.5127	2.3948	2.3992	0.87777	0.86663
	2	5.57	7.3753	2.4115	2.4072	0.83137	0.86577
		Characteristic Group $s = 2$					
High	1	12.8213	9.8274	4.2589	4.2516	1	0.98152
	2	6.4374	9.6849	4.2601	4.2659	0.92143	0.98053
Low	1	13.0551	10.169	3.8861	3.8856	0.99913	0.97816
	2	6.8497	10.0259	3.8997	3.8987	0.91933	0.97717
		Characteristic Group $s = 3$					
High	1	6.7322	5.2464	8.8922	8.8923	0.74596	0.74719
	2	3.5596	5.1143	8.9213	8.9218	0.72034	0.74634
Low	1	6.7078	5.2677	8.3683	8.372	0.76982	0.76798
	2	3.6624	5.1355	8.4019	8.3997	0.74261	0.7671

[a] LT stands for model with two-period long-term contracts; SL stands for model with short-term contracts.

[b] Column Interest Rate is transformed from the variable r via $(r-1) \times 100\%$.

[c] The results are calculated under first period funding cost $c1 = 1.03$.

Another angle to look at this is through $\bar{\pi}(p_2)$, which is bank's expected profits in the second period when updated belief is p_2. For each group s, I tabulate $\bar{\pi}(p_2)$ for $p_2 = p_2^H$ and for $p_2 = p_2^L$ in the last two rows of Table 10. I also find the expected profit for the first-period *only*, which is shown in the first row of Table 10. The weighted sum of the three items in each column should be zero, where the weight for second-period profits include discount factor β, probability of each realization of a_1, and the probability of repaying the first loan conditional on a_1. Results show a clear pattern of front-loading, where banks expect to earn positive profits on the first-period loan and "give back" to firms in the second period in the form of low interest rates. Furthermore, banks give back more in when firms are hit by negative cost shock and thus hold a more pessimistic outlook for future prospect. This echoes the insurance role of long-term contracts shown in Table 9, which benefits risk-averse firms.

Table 8: Firm Outcomes in Model with Two-Period Long-Term Contracts vs. Short-Term Contracts

	High Type			Low Type		
	LT	ST	$\Delta \times 100\%$	LT	ST	$\Delta \times 100\%$
Cum. Def. Prob. (%)	10.6647	10.6791	-0.24454	11.4016	11.4028	-0.018184
Firm Value	86.4002	86.3805	0.023763	84.2922	84.2905	0.0020495

[a] LT stands for model with two-period long-term contracts; SL stands for model with short-term contracts; $\Delta \times 100\%$ is $(LT - ST)/ST \times 100\%$.
[b] Cum. Def. Prob. stands for cumulative default probability, calculated as the probability of default in either period one or period two. It is represented in terms of percent.
[c] Items in this table is averaged across c_1 and s using empirical distributions.

6.2 Multi-period Contingent Contracts

The model with two-period contracts can be naturally extended to multiple periods using the transformation approach introduced in Section 6.1.2. Suppose the duration of the long-term contingent contract is T period, $T \geq 2$. Note that the horizon of the model, though, is still infinite as with the short-term model, which makes firms outcomes comparable between the long-term and short-term model. Specifically, new born firms take T-period contingent contracts, and after T periods they enter the infinite-horizon short-term lending market. Again we are looking at the effects of long-term contracts at the beginning of a firm's life.

Assumptions on defaults are same as in Section 6.1, where default triggers the severance of the lending relationship between the firm and the current lending bank, with defaulted firm's salvage value equal a fraction $\delta \in [0,1]$ of the equilibrium firm value in the spot market. Another way to interpret this is that defaulted firms only regain access to the spot market with a constant probability $\delta \in [0,1]$. The spot market is filled with short-term contracts only, which are equivalent to the contracts in the no-commitment case, so the firm value in this market is given by $W(p,c)$, where W is defined in Equation (6).

Following notations introduced in Section 6.1.2, I start with the last period's problem. As in (19), I define the last period's firm value as a function of index $\bar{\pi}$, p_T and c_T

$$W_T(\bar{\pi}, p_T, c_T) \equiv \max_{l_T} w_T(l_T, p_T, c_T) \text{ s.t. } \bar{\pi} = \pi_T(l_T, p_T, c_T) \tag{22}$$

Table 9: Lending Terms in Model with Two-Period Long-Term Contracts vs. Short-Term Contracts with Incomplete Type Information

Period	Contingency	Interest Rate (%)		Loan Size		Collateral Coverage	
		LT	ST	LT	ST	LT	ST
		Characteristic Group $s = 1$					
1		9.2046	7.4128	2.5057	2.5092	0.87225	0.86272
2	$a_1 = \bar{a}$	5.5453	7.268	2.5296	2.5264	0.82875	0.8616
	$a_1 = \underline{a}$	2.9986	7.3481	2.4479	2.4389	0.78225	0.865
		Characteristic Group $s = 2$					
1		12.9132	10.004	4.0586	4.0553	0.99983	0.9795
2	$a_1 = \bar{a}$	6.8118	9.8511	4.0785	4.0799	0.9233	0.97862
	$a_1 = \underline{a}$	4.6683	9.9744	3.9526	3.9524	0.88098	0.97758
		Characteristic Group $s = 3$					
1		12.9132	10.004	4.0586	4.0553	0.99983	0.9795
2	$a_1 = \bar{a}$	6.8118	9.8511	4.0785	4.0799	0.9233	0.97862
	$a_1 = \underline{a}$	4.6683	9.9744	3.9526	3.9524	0.88098	0.97758

[a] LT stands for model with two-period long-term contracts; SL stands for model with short-term contracts.
[b] Column Interest Rate is transformed from the variable r via $(r-1)\times 100\%$.
[c] The results are calculated under first period funding cost $c1 = 1.03$.
[d] The initial prior belief p_1 for $s = 1,2,3$ is set at 0.399, 0.486, and 0.471, respectively. (From Table 3)

where w_T is firm's utility as a function of lending terms l_T and states (p_T, c_T):

$$w_T(l_T, p_T, c_T) = E_{a_T,\epsilon_T}\Big[\max\{U(l_T; a_T) - \epsilon_T + \beta E\left[W(p_{T+1}, c_{T+1})|a_T\right], \\ U^F(l_T; a_T) + \beta\delta E\left[W(p_{T+1}, c_{T+1})|a_T\right]\}\Big| p_T, c_T\Big] \tag{23}$$

It follows that default probability in the second period conditional on realization of a_T can be written as

$$\Phi_T(l_T, p_T, c_T; a_T) = \Pr\Big(\epsilon_T \leq U(l_T, a_T) - U^F(l_T, a_T) + \beta(1-\delta) E\left[W(p_{T+1}, c_{T+1})|a_T, p_T, c_T\right]\Big) \tag{24}$$

Using the default probability function, we can find the expected bank profit in period T:

$$\pi_T(l_T, p_T, c_T) = -(c_T - z_T)k_2 + E_{a_T}\left[\Phi_2(l_T, a_T, p_T, c_T)(r_T - z_T)k_2 \middle| p_T\right] \tag{25}$$

The only difference between the multi-period case and the two-period case is the addition of

Table 10: Expected Bank Profits in Each Period

Period	Contingency	Charateristic Group s		
		$s=1$	$s=2$	$s=3$
1		0.046506	0.12135	0.13054
2	$a_1=\bar{a}$	-0.043113	-0.12937	-0.11749
	$a_1=\underline{a}$	-0.11142	-0.22231	-0.29787

[a] Items in this table shows expected bank profits for the given period only. In period 1, it is calculated under first period funding cost $c1 = 1.03$ and initial prior belief $p_1 = 0.399, 0.486, 0.471$ for $s = 1,2,3$ respectively. In period 2, it is calculated under updated beliefs following a high/low realization of a_1, and averaged across different funding costs c_2.
[b] The unit of profits is in million RMB (approximately 150 thousands dollars).

interim period t where $t < T$ and $t > 1$. For an interim period, we need to derive the firm's value function as a function of the *current* period's index $\bar{\pi}$ and states (p_t, c_t) by maximizing over this period's lending terms l_t and *next periods*' state-contingent indices $\bar{\pi}'(p_{t+1})$.

$$W_t(\bar{\pi}, p_t, c_t) \equiv \max_{l_t, \bar{\pi}'(p_{t+1})} w_t\left(l_t, \bar{\pi}'(p_{t+1}), p_t, c_t\right)$$
$$\text{s.t. } \bar{\pi} = \pi_t\left(l_t, \bar{\pi}'(p_{t+1}), p_t, c_t\right) \tag{26}$$

where w_t is different from w_T in its future values:

$$w_t\left(l_t, \bar{\pi}', p_t, c_t\right) = E_{a_t,\epsilon_t}\Big[\max\big\{U(l_t; a_t) - \epsilon_t + \beta E\left[W_{t+1}\left(\bar{\pi}', p_{t+1}, c_{t+1}\right)|a_t\right], \\ U^F(l_t; a_t) + \beta\delta E\left[W(p_{t+1}, c_{t+1})|a_t\right]\big\}\Big| p_t, c_t\Big]$$

and the bank's expected profit function π_t is

$$\pi_t(l_t, \bar{\pi}', p_t, c_t) = -(c_t - z_t)k_t + E_{a_t}\left[\Phi_t(l_t, \bar{\pi}', p_t, c_t; a_t)(r_t - z_t)k_t + \beta E\left[\bar{\pi}'(p_{t+1})|a_t\right]\Big| p_t, c_t\right]$$

with the conditional default probability function in period t defined as

$$\Phi_t(l_t, \bar{\pi}', p_t, c_t; a_t) \equiv \Pr\Big(\epsilon_t \le U(l_t, a_t) - U^F(l_t, a_t) \\ + \beta E\left[W_{t+1}\left(\bar{\pi}', p_{t+1}, c_{t+1}\right) - \delta W(p_{t+1}, c_{t+1})|a_t, p_t, c_t\right]\Big)$$

Since a has two possible realizations, so do p_{t+1}, that means $\bar{\pi}'(p_{t+1})$ is a two-dimensional vector. In total there are five choice variables in this constrained optimization problem.

The first period's problem is the same as in (21). In terms of solution method, I use backward induction, starting from the last period. Using estimated parameters and firm's value function in the short-term model, I first calculate $W_T(\cdot,\cdot,\cdot)$ from (22) under each combination of $\bar{\pi}$, p and c, where $\bar{\pi}$, p are discretized on a grid and c takes four discrete values.[37] I interpolate in between grids of $\bar{\pi}$ to obtain the function $W_T(\cdot, p, c)$ for each combination of p and c.[38]

For any $t = T-1, ..., 1$, given the function $W_{t+1}(\cdot, p, c)$, for any $\bar{\pi}_t$ on the grid of $\bar{\pi}$, any p_t on the grid of p and each of the four values of c_t, I find the two possible next period belief p_{t+1} conditional on a high and low realization of a_t, denoted as $p^H_{t+1} = p_{t+1}\left(p_t | a_t = \bar{a}\right)$ and $p^L_{t+1} = p_{t+1}\left(p_t | a_t = \underline{a}\right)$. Then I plug functions $W_{t+1}(\cdot, p^H_{t+1}, c_{t+1})$ and $W_{t+1}(\cdot, p^L_{t+1}, c_{t+1})$ into the period t problem (26) where $\bar{\pi}_t$ serves in the constraint. By solving this problem I obtain the optimal first period lending terms l_t, as well as indices $\bar{\pi}$ following a high and low realizations: $\bar{\pi}(p^H_{t+1})$ and $\bar{\pi}(p^L_{t+1})$. And I interpolate in between grids of $\bar{\pi}$ to obtain the function $W_t(\cdot, p, c)$.

I repeat the process above until I reach the first period. The only difference between $t = 1$ an $t > 1$ is that I do not need to solve the first-period problem under various values of $\bar{\pi}_1$; instead, the equilibrium condition imposes $\bar{\pi}_1 = 0$, i.e., the bank's expected profit at the beginning of the first period should be zero.

After getting the firm's value function $W_t(\cdot, p, c)$ and the policy functions $l_t(\cdot,\cdot,\cdot)$ and $\bar{\pi}'(\cdot)$, I simulated a panel of lending terms, state variables, productivity realizations, and default outcomes, for each firm in the observed data, where parameters are based on estimates of the baseline model. The results shown in the next section are under $T = 20$. (But only the first seven year's statistics are shown because the baseline model is only seven period).

6.3 Simulation Results

Optimal Long-Term Contract

Optimal long-term contracts specify a schedule of future contract terms based on both the length of the contractual relationship and performances in each year of the relationship. I later refer to it as a *relationship and performance-based schedule*. It shows different dynamics than the repeated short-term contracts. As a first pass, we can look at Table 11 which shows mean r,

[37] The grid of p is the same as in baseline estimation, from 0 to 1 with step length 0.01. The grid of $\bar{\pi}$ is -1 to 1 with step length 0.01, and this grid is large enough in the sense that chosen values of $\bar{\pi}$ in the equilibrium do not hit the bounds of the grid.

[38] I use cubic spline interpolation to ensure smoothness and facilitate the first-period maximization problem.

k, z and default rates from the baseline model and counterfactual model in each period, by different firm characteristic groups. Initial interest rates in the counterfactual long-term contract are much higher than in the short-term contract, but later on interest rates in the long-term contract drastically decline, and in most cases even lower than interest rates than the short-term contract in the last several periods. This *front-loading* pricing scheme locks firms in by charging high initial prices, and gradually let price go down (even lower than the spot-market price) in later periods.

Why this front-loading pricing structure is optimal? This is because the promise of lower prices in the future can incentivize firms to hold on and not default, which can alleviate agency friction to some degree. The reduced agency friction also means less constrained capital input, i.e., loan size in later periods of the long-term contracts are larger than the short-term contracts. Another consequence of reduced agency friction is the less collateral coverage in long-term contracts, which is beneficial to firms because it saves transaction costs associated with collateral. Overall we see default rates in each single period go down, and lower defaults means more firms can survive.

Long-term contracts also differ from repeated short-term contracts in the contractual response to firm performances. Recall that in Table 5, the short-term contracts in the baseline model features a jump in interest rates for firms whose performance is poor in the last period. Does the optimal long-term contract still have this feature of raising prices following poor performance? It turns out it is the opposite: the long-term contract actually specify a lower price if the firm is hit by a negative productivity shock. We can see this from running the same regression as in Equation (2), where I use firm fixed effect model to regress contract terms on years of relationship t, indicator of poor performance in the previous period, $PoorRating_{t-1}$, and the interaction term $t \times PoorRating_{t-1}$. I conduct this regression for firms who never had a single poor performance in their history (in which case the variable $PoorPerf_{t-1}$ and its interaction term with t will be omitted, and we will only compare the rate of change of contract terms over time), and for firms who have at least one poor performance in their history (again for these firms I keep observation $t <= t_i^* + 1$ where t_i^* is the first time firm i is rated poorly). Results are shown in Table 12.

Comparing column (1) to (3) of Table 12 with column (1) to (3) of Table 5, we find that signs of coefficients in the two tables are same, but the magnitudes of coefficients in the counterfactual long-term contracts are much bigger, meaning that long-term contract has a more significant time trend of declining lending spread and collateral coverage, and growing loan size. Focusing on column (4), we find that coefficients of $PoorRating$ and the interaction term in the coun-

Table 11: Comparison of Long-Term Counterfactual Model vs. Short-Term Baseline Model by Firm Characteristics

	Interest Rate		Loan Size		Collateral Coverage		Default	
	Baseline	C.F.	Baseline	C.F.	Baseline	C.F.	Baseline	C.F.
	Characteristic Group $s = 1$							
1	7.276	20.891	2.518	2.485	0.546	0.561	0.053	0.043
2	7.105	18.885	2.529	2.511	0.545	0.421	0.053	0.041
3	6.906	16.788	2.542	2.536	0.544	0.287	0.063	0.048
4	6.775	14.483	2.550	2.558	0.544	0.160	0.055	0.040
5	6.677	12.390	2.557	2.578	0.544	0.056	0.049	0.037
6	6.622	9.997	2.561	2.611	0.543	0.000	0.061	0.046
7	6.567	7.764	2.564	2.655	0.543	0.000	0.054	0.033
	Characteristic Group $s = 2$							
1	9.830	23.422	4.072	4.094	0.619	0.633	0.110	0.088
2	9.647	19.872	4.090	4.157	0.618	0.548	0.136	0.117
3	9.469	16.669	4.106	4.190	0.618	0.432	0.134	0.102
4	9.325	13.605	4.120	4.216	0.617	0.332	0.111	0.095
5	9.254	10.574	4.128	4.235	0.617	0.248	0.114	0.086
6	9.188	6.782	4.134	4.248	0.617	0.179	0.101	0.073
7	9.105	3.889	4.144	4.261	0.617	0.142	0.119	0.097
	Characteristic Group $s = 3$							
1	5.088	15.366	8.644	8.631	0.480	0.418	0.017	0.013
2	4.882	12.650	8.688	8.677	0.479	0.299	0.013	0.012
3	4.746	9.603	8.718	8.712	0.479	0.201	0.017	0.015
4	4.602	6.358	8.748	8.746	0.478	0.133	0.012	0.012
5	4.535	3.454	8.763	8.766	0.478	0.097	0.012	0.011
6	4.482	3.270	8.774	8.777	0.478	0.096	0.018	0.019
7	4.433	3.223	8.785	8.788	0.477	0.096	0.013	0.013

Table 12: Regressions of Contract Terms on Past Performance: Using Counterfactual Long-Term Contract

	Firms w. good rating history			Firms w. poor rating history		
	(1) Lending Spread	(2) Loan Size	(3) Collateral Coverage	(4) Lending Spread	(5) Loan Size	(6) Collateral Coverage
t	-2.223	0.0225	-0.112	-2.263	0.0224	-0.130
	(-400.19)	(134.16)	(-274.70)	(-192.97)	(49.02)	(-144.29)
$PoorRating_{t-1}$				-4.152	-0.137	-0.460
				(-36.67)	(-30.93)	(-52.80)
$t \times PoorRating_{t-1}$				0.491	-0.000174	0.0824
				(21.47)	(-0.20)	(46.87)
Observations	18859	18859	18859	7899	7899	7899

t statistics in parentheses. Other controls: firm fixed-effects, funding costs fixed-effects.

terfactual model has the opposite sign than those in the baseline model: In Table 12, lending spreads on average decrease by nearly four percentage point for firms whose previous performance is poor, whereas in 5 it increases by around seven basis point. This reflects the contingent "insurance" function of long-term contracts, which smooth consumption across different states and protect firms against risks from uncertainty about their productivity status.

How important the insurance structure is in terms of welfare improvement compared to the intertemporal structure? To see this, I also consider another type of long-term contract that does not have the insurance structure, i.e., the relationship-based schedule.

Relationship-Based Schedule

Long-term contracts with relationship-based schedule comes from solving (**??**) with π' not contingent on a_t. In other words, future contract terms do not directly depend on this period's performance. For example, firms that have a poor performance in the first period do not receive lower second-period interest rates than those with a good first-period performance, whereas In the precious case, firms with a poor performance in the first period receive on average 1.59 percentage point lower second-period interest rates than those with a good first-period performance

At a first pass, we can check how effective it is at reducing default rates compared with the relationship and performance-based schedule (i.e., optimal long-term contracts) by Figure 7. We find that among the high-type firms, default rates is lower in the relationship-based schedule than in the relationship and performance-based schedule, whereas for low-type firms, default rates is lower in the relationship and performance-based schedule than in the relationship-based schedule. This is because the relationship and performance-based schedule provides price discounts for firms with poor performance, which are more likely to be the low-type firms, and such price discounts lead to lower default rates. Essentially the relationship and performance-based schedule is employing a transfer from high-type to low-type.

To see the overall welfare implications of the two schedules, I calculate the associated cumulative default probability over the seven-year period, total amount of capital and output taken over the seven-year period, the firm size measured by output in the last period, and the initial firm value in the baseline model with short-term contract, counterfactual model with relationship-based long-term contract, and counterfactual model with relationship and performance-based long-term contract. Summary of these statistics by firm quality type is show in Table 13.

Comparing column (3) with column (1), we find that with long-term contracts with fully flexible schedule, cumulative default probability over the seven-year period drops by 17.15%, with high-type firm's cumulative default probability drops by 16.86% and low-type 17.41%. Total firm outputs over the seven-year period increases by 2.63%, where high-type firm's total output increases by 2.84% (due to a 2.88% increase in total capital input), and low-type firm's total outputs increase by 2.45% (due to a 2.58% increase in total capital input). Comparing column (2) with column (3), we find that the relationship-based schedule does a good job of capturing the vast majority of the efficiency improvement without using complex contingency contracting.

Table 13: Comparison Across Different Contracts

Scenario	Baseline	Counterfactual Long-Term Contracts	
	(1)	(2)	(3)
	Short-Term	Relationship-Based	Relationship and Performance-Based
	High-Type Firms		
Cumulative Default Prob.	.3325114	.2736879	.2765058
Total Capital	30.71789	31.64557	31.62901
Total Output	55.4055	57.00239	56.97651
Final Size	5.071709	5.130478	5.128197
Firm Value	86.39585	86.72219	86.7231
	Low-Type Firms		
Cumulative Default Prob.	.3117363	.2585962	.2574595
Total Capital	29.27848	30.05414	30.03602
Total Output	51.87864	53.19209	53.15497
Final Size	4.751198	4.806808	4.809732
Firm Value	84.25609	84.58102	84.58202
	All Firms		
Cumulative Default Prob.	.3210129	.265335	.2659641
Total Capital	29.92121	30.76476	30.74733
Total Output	53.45347	54.89348	54.86138
Final Size	4.894314	4.951335	4.951934
Firm Value	85.21154	85.5371	85.53806

I compare firm welfare in model with long-term contract with fully flexible schedules with the efficient benchmark (with no agency friction and full information, as in column (1) of Table 6), and I find that long-term contracts recover 31.46% of total welfare loss from both sources of inefficiency (i.e., agency friction and incomplete information). And the average Compensating Variation (CV) across firms is around $92,106.

7 Conclusion

Studies that focus on US commercial loan market highlight the long-term contractual relationships between firms and banks. However, in developing countries with relatively underdeveloped financial market, long-term contracting can be too costly to implement. So how much do firms in those markets miss out because of a lack of long-term contracting? This is the question that this paper aims to answer.

This paper utilizes data from a Chinese bank where long-term contracting is hard to access, and document several stylized facts in this novel dataset. I find that observed contracts are short-term, with banks and firms negotiating contract terms on a annual basis. I find evidence of learning by examining the dynamics of contract terms, the contractual response to measures of firm performance, and how these responses change over time.

This paper provides a framework to analyze welfare impacts of contract structure. I develop a dynamic model of lending markets to analysis bank's choice of contract terms and firm's choice of repay or default. Inspired by features in the data, I assume that banks are unable to commit to long-term financing contracts and can only offer short-term contracts, and that firms are heterogeneous in unobserved types, with types unknown to banks and firms in the beginning but can be learned over time. I estimate the model using the panel data, and identification comes mainly from the over-time variations of contract terms and the its correlation with firm performance and funding costs.

In the model there are two sources of inefficiency: agency friction (i.e., repayment behavior cannot be enforced by contracts) and incomplete information (i.e., firm's quality is unknown in the beginning). Estimates show that agency friction is the more important source of inefficiency in this market. By conducting counterfactual analysis of enabling banks to commit to long-term contracts, I find that long-term contracting can effectively alleviate agency friction through its use of both intertemporal structure and intratemporal structure. The intertemporal structure front-loads prices, which allows contract terms become increasingly favorable to firms over time and thus disincentivize firm defaults. The intratemporal structure is like an insurance, protecting firms against risks of negative productivity shocks. I find that the majority of welfare improvement comes from the intertemporal structure of long-term contracts, and a long-term contract with only relationship-based schedule is almost as effective as the more complex schedule that are contingent on performance history.

The main contribution of this paper is two fold. First, this paper provides an empirical framework for the corporate lending market that incorporate both observed and unobserved firm heterogeneity. Compared to previous models in macro-finance literature, my model can accommodate assumptions on different contract structures, and utilize collateral information, firm performance and loan outcome data to form a more accurate estimate on the level of agency frictions in place. Second, this paper quantifies the value of long-term contracting for small and medium-sized young firms in China. There has long been discussion about long-term financing support for small and medium-sized firms in China, and this paper can provide a measure of potential benefits that long-term financing arrangement can bring about.

The model can also be used for various policy experiments. For example, we can calculate how does a reduction in bank's funding costs translate into changes in interest rates, loan size, and collateral coverage, and the associated default rates. As another example, we can simulate how does a loan guarantee program (like Small Business Administration in US) change interest rates, loan size and the associated default rates. We can further compare the effectiveness of different policy interventions in terms of default prevention and enhancement in capital flow.

The model can be extended in the following aspects: (1) Learning by doing. We can allow firms to "learn" through past production, i.e., more capital taken in the past can lead to higher productivity on average. Then part of the observed growth in capital could be attributed to learning-by-doing (In fact, observed capital growth is larger than the predicted capital growth from the current model, and the unexplained part can be accounted for by a learning-by-doing process). This can still be identified because we do see the firm performance history, which pins down the belief updating process. So capital growth that cannot be explained by the belief updating process will identify the learning-by-doing process. This is similar to the identification argument in Pastorino (2019). (2) Aggregate productivity shocks. In the current model firm's productivity shocks are i.i.d. However it is possible that macro-economic conditions can cause firm's productivity realizations to be correlated (say firms within the same industry have correlations in their productivity). It is possible to allow correlation structures among firms with similar observed characteristics.

The model is better suited to study young firms, since I abstract away from asymmetric information, which could be a more severe issue in establish firms. Another limitation is that model does not take into account *ex ante* model hazard problem, where firms choose "effort" levels in the beginning of each period that are unobserved by banks but could affect payoffs. In this case the insurance-like structure of the long-term contract with relationship and performance-based schedule might be problematic, but the relationship-based schedule can still use intertemporal pricing structure as an incentive devise.

References

D. A. Ackerberg. Advertising, learning, and consumer choice in experience good markets: an empirical examination. *International Economic Review*, 44(3):1007–1040, 2003.

W. Adams, L. Einav, and J. Levin. Liquidity constraints and imperfect information in subprime lending. *American Economic Review*, 99(1):49–84, 2009.

R. Albuquerque and H. A. Hopenhayn. Optimal lending contracts and firm dynamics. *The Review of Economic Studies*, 71(2):285–315, 2004.

C. Arellano, Y. Bai, and J. Zhang. Firm dynamics and financial development. *Journal of Monetary Economics*, 59(6):533–549, 2012.

P. Asquith, A. Beatty, and J. Weber. Performance pricing in bank debt contracts. *Journal of Accounting and Economics*, 40(1-3):101–128, 2005.

J. P. Atal. Lock-in in dynamic health insurance contracts: Evidence from chile. PIER Working Paper No. 5694, 2019.

T. Berg, A. Saunders, and S. Steffen. The total cost of corporate borrowing in the loan market: Don't ignore the fees. *The Journal of Finance*, 71(3):1357–1392, 2016.

B. Biais, T. Mariotti, J.-C. Rochet, and S. Villeneuve. Large risks, limited liability, and dynamic moral hazard. *Econometrica*, 78(1):73–118, 2010.

Y. Boualam. Credit markets and relationship capital. Working paper, University of North Carolina, 2018.

F. J. Buera, J. P. Kaboski, and Y. Shin. Finance and development: A tale of two sectors. *American Economic Review*, 101(5):1964–2002, 2011.

F. J. Buera, J. P. Kaboski, and Y. Shin. Entrepreneurship and financial frictions: A macrodevelopment perspective. *Annual Review of Economics*, 7(1):409–436, 2015.

Y.-S. Chan and G. Kanatas. Asymmetric valuations and the role of collateral in loan agreements. *Journal of money, credit and banking*, 17(1):84–95, 1985.

G. L. Clementi and H. A. Hopenhayn. A theory of financing constraints and firm dynamics. *The Quarterly Journal of Economics*, 121(1):229–265, 2006.

J. H. Cochrane. Time-consistent health insurance. *Journal of Political Economy*, 103(3):445–473, 1995.

T. F. Cooley and V. Quadrini. Financial markets and firm dynamics. *American economic review*, 91(5):1286–1310, 2001.

B. G. S. Crawford, N. Pavanini, and F. Schivardi. Asymmetric Information and Imperfect Competition in Lending Markets. 108(7):1659–1701, 2018.

G. S. Crawford and M. Shum. Uncertainty and learning in pharmaceutical demand. *econometrica*, 73(4):1137–1173, 2005.

P. M. DeMarzo and M. J. Fishman. Optimal long-term financial contracting. *The Review of Financial Studies*, 20(6):2079–2128, 2007.

P. M. DeMarzo and Y. Sannikov. Optimal security design and dynamic capital structure in a continuous-time agency model. *The Journal of Finance*, 61(6):2681–2724, 2006.

L. Einav, M. Jenkins, and J. Levin. Contract pricing in consumer credit markets. *Econometrica*, 80(4):1387–1432, 2012.

H. Gao, H. Ru, R. M. Townsend, and X. Yang. Rise of bank competition: Evidence from banking deregulation in china. *NBER working paper*, (w25795), 2019a.

H. Gao, H. Ru, and X. Yang. What do a billion observations say about distance and relationship lending? *Available at SSRN 3195616*, 2019b.

M. Gertler. Financial capacity and output fluctuations in an economy with multi-period financial relationships. *The Review of Economic Studies*, 59(3):455–472, 1992.

D. He and H. Wang. Dual-track interest rates and the conduct of monetary policy in china. *China Economic Review*, 23(4):928–947, 2012.

I. Hendel and A. Lizzeri. The role of commitment in dynamic contracts: Evidence from life insurance. *The Quarterly Journal of Economics*, 118(1):299–328, 2003.

C.-T. Hsieh and P. J. Klenow. Misallocation and manufacturing tfp in china and india. *The Quarterly journal of economics*, 124(4):1403–1448, 2009.

C.-T. Hsieh and P. J. Klenow. The life cycle of plants in india and mexico. *The Quarterly Journal of Economics*, 129(3):1035–1084, 2014.

K. P. Huynh and R. J. Petrunia. Age effects, leverage and firm growth. *Journal of Economic*

Dynamics and Control, 34(5):1003–1013, 2010.

R. G. King and R. Levine. Finance, entrepreneurship and growth. *Journal of Monetary economics*, 32(3):513–542, 1993.

J. Li and X. Lin. Assessing credit guarantee companies in china: Applying a new framework. *China Economic Review*, 44:98–111, 2017.

M. Manove, A. J. Padilla, and M. Pagano. Collateral versus project screening: A model of lazy banks. *Rand journal of economics*, pages 726–744, 2001.

G. Manso, B. Strulovici, and A. Tchistyi. Performance-sensitive debt. *The Review of Financial Studies*, 23(5):1819–1854, 2010.

M. McMahon, M. A. Schipke, and X. Li. *China's monetary policy communication: Frameworks, impact, and recommendations.* International Monetary Fund, 2018.

V. Midrigan and D. Y. Xu. Finance and misallocation: Evidence from plant-level data. *American economic review*, 104(2):422–58, 2014.

K. Okazaki. Banking system reform in china: The challenges of moving toward a market-oriented economy. Technical report, Rand Corp Arlington Va National Security Research Div, 2007.

E. Pastorino. Careers in firms: the role of learning and human capital. Technical report, Technical report, mimeo, 2019.

N. Porter, E. Takáts, and T. Feyzioglu. Interest rate liberalization in china. *IMF Working Papers*, pages 1–28, 2009.

V. Quadrini. Investment and liquidation in renegotiation-proof contracts with moral hazard. *Journal of Monetary Economics*, 51(4):713–751, 2004.

S. A. Sharpe. Credit rationing, concessionary lending, and debt maturity. *Journal of Banking & Finance*, 15(3):581–604, 1991.

J. Thomas and T. Worrall. Foreign direct investment and the risk of expropriation. *The Review of Economic Studies*, 61(1):81–108, 1994.

Y. Xin. Asymmetric information, reputation, and welfare in online credit markets. *Available at SSRN 3580468*, 2020.

B. Xu, A. van Rixtel, and M. Van Leuvensteijn. Measuring bank competition under binding interest rate regulation: the case of china. *Applied Economics*, 48(49):4699–4718, 2016.

8 Additional Tables

Table 14: Summary Statistics

Year in Relationship t	Interest Rate r	Loan Size k	Collateral Coverage z	PoorRating	Default
1	7.043	4.482	0.573	0.081	0.040
2	7.051	5.009	0.573	0.085	0.066
3	6.826	5.371	0.570	0.084	0.081
4	6.577	5.839	0.568	0.082	0.051
5	6.420	6.485	0.565	0.089	0.079
6	6.165	7.120	0.560	0.083	0.050
7	6.004	7.572	0.535	0.089	0.080
Average	6.863	5.195	0.567	0.085	0.058

Table 15: Moment Restrictions

Meaning	Moment Restrictions	Dimension
Decisions on contract term and default	$\mathbb{E}_{\{i\}}\left[O_i - \hat{O}_i \| X_i\right]$	21×28
Firm performances	$\mathbb{E}_{\{i\}}\left[P_i - \hat{P}_i \| X_i\right]$	21×7
Covariance between firm performance and decisions	$\mathbb{E}_{\{i\}}\left[P_i O_i - \hat{P}_i \hat{O}_i \| X_i\right]$	21×28
Covariance between cost of fund and decisions	$\mathbb{E}_{\{i\}}\left[C_i O_i - \hat{C}_i \hat{O}_i \| X_i\right]$	21×28

9 Additional Graphs

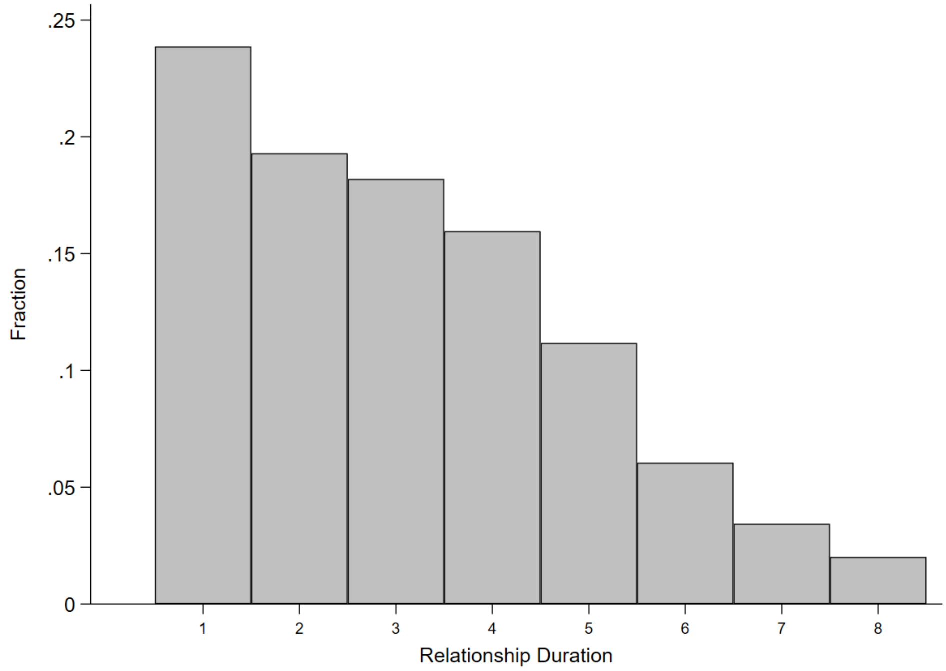
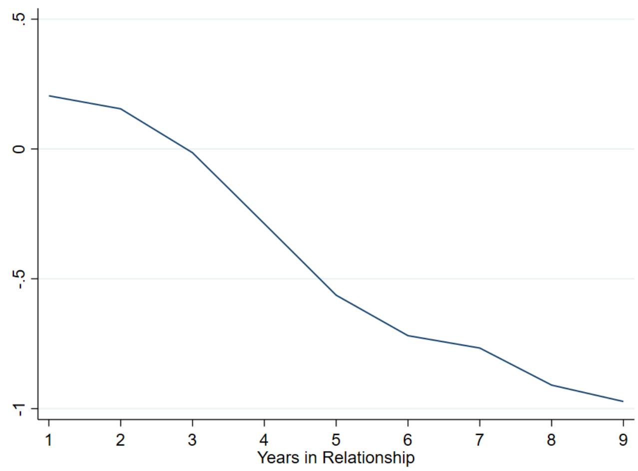
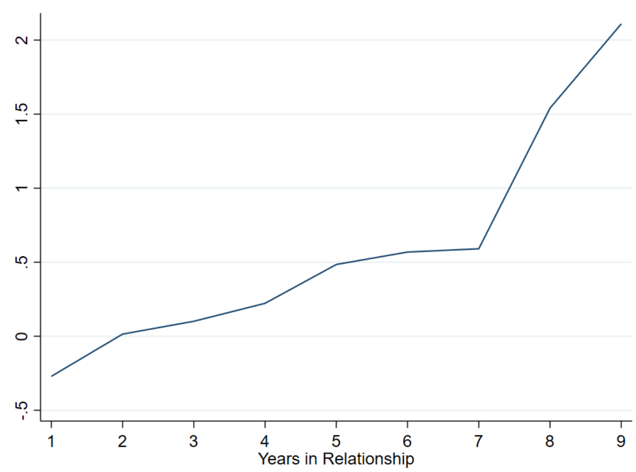


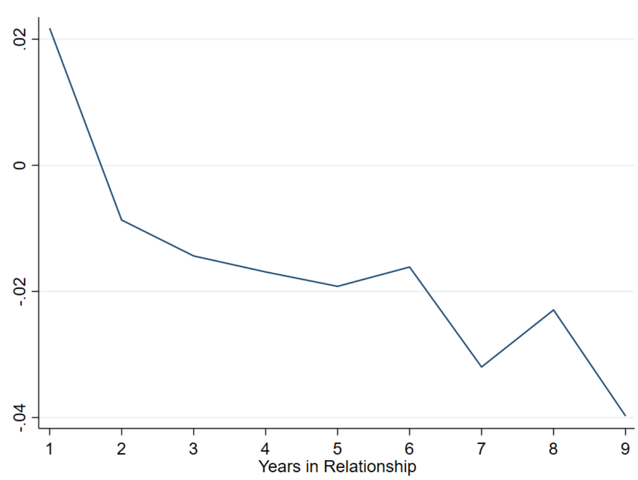
Figure 2: Distribution of Relationship Duration



(a) Residualized Lending Spread

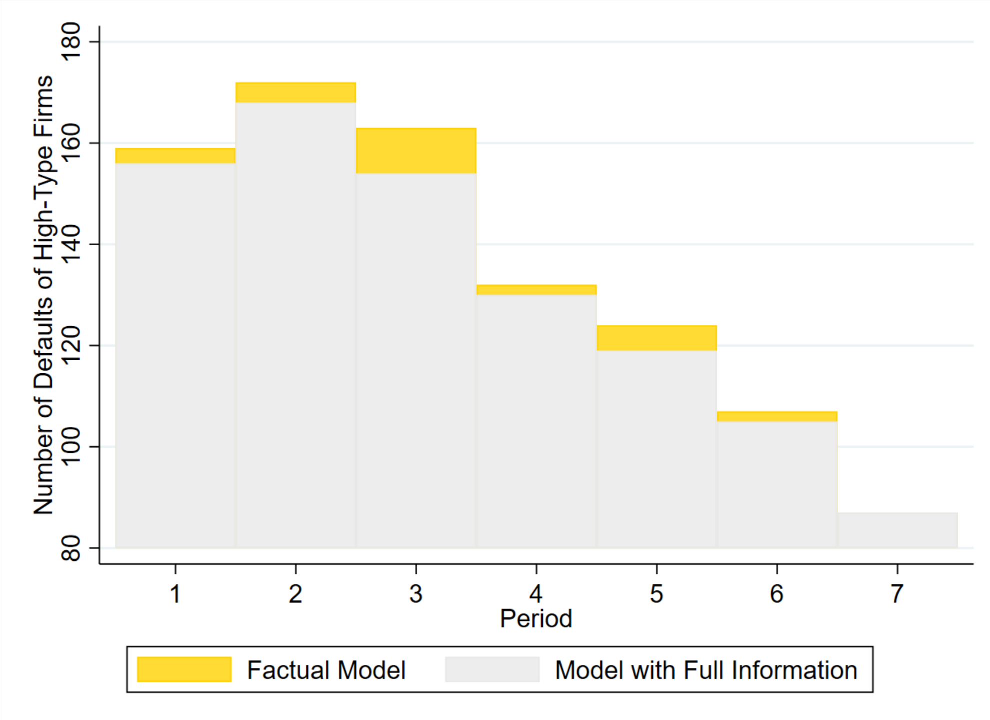


(b) Residualized Loan Size

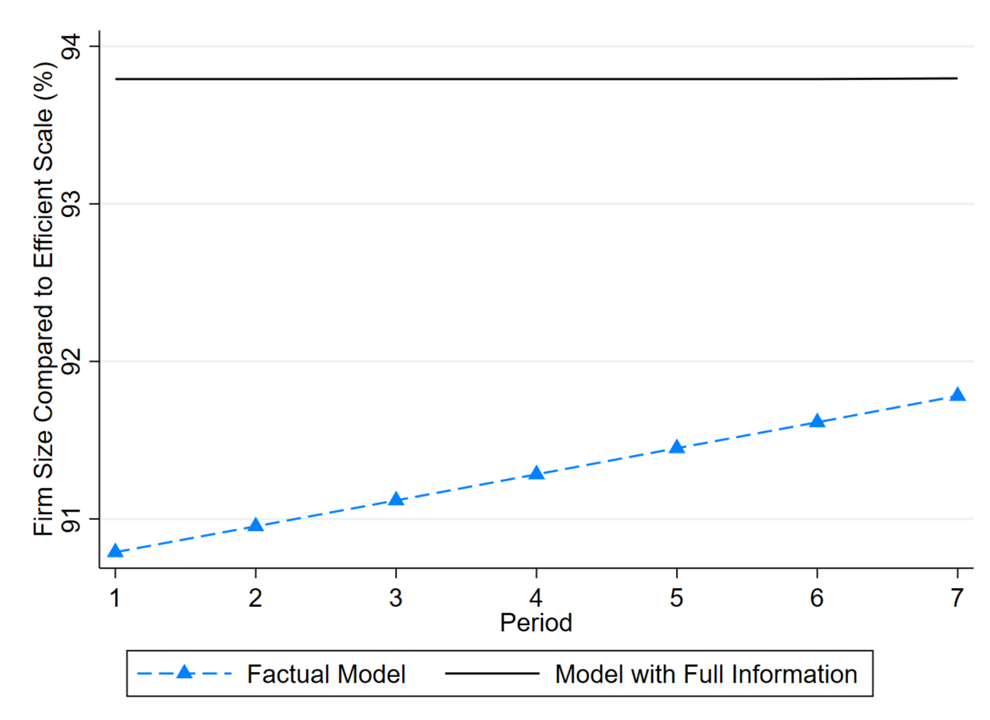


(c) Residualized Collateral Coverage

Figure 3: Time Profile of Residualized Contract Terms

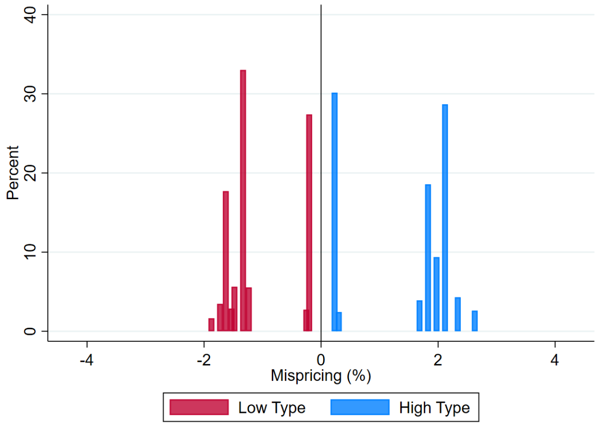


(a) Defaults over the Course of the Relationship

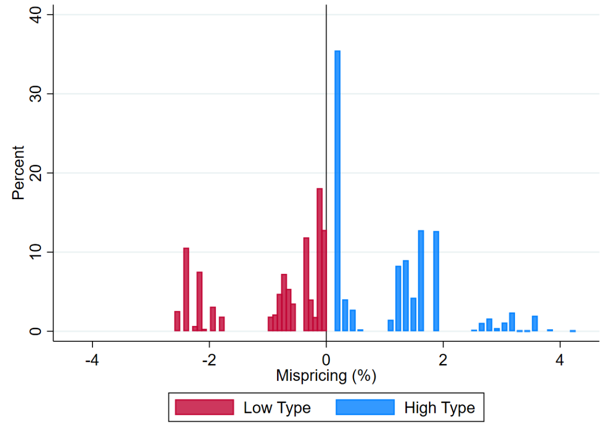


(b) Firm Size Compared to Efficient Size

Figure 4: Implications of Incomplete Information on Outcomes of High-Type Firms

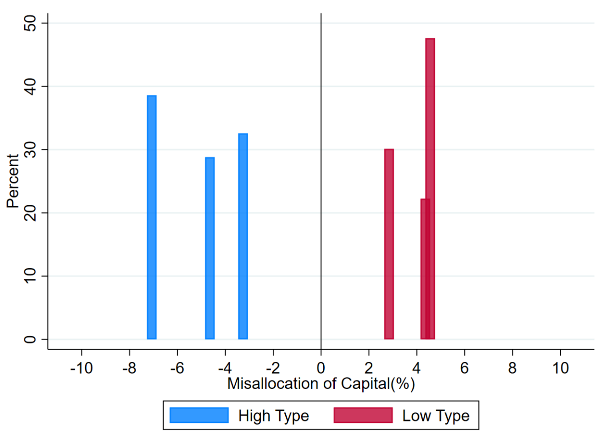


(a) Mispricing in the First Period

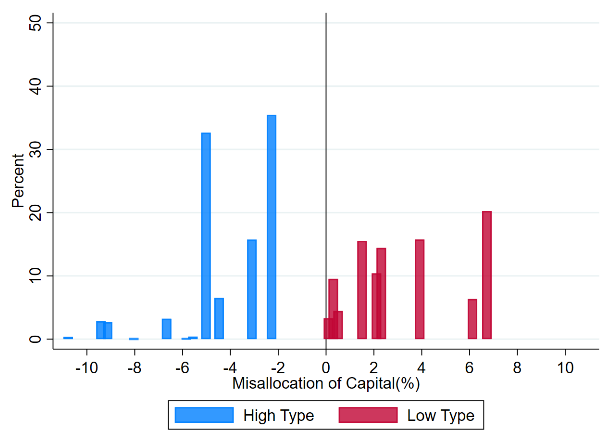


(b) Mispricing in the Last Period

Figure 5: Mispricing over the Course of the Relationship



(a) Misallocation of Capital in the First Period



(b) Misallocation of Capital in the Last Period

Figure 6: Misallocation of Capital over the Course of the Relationship

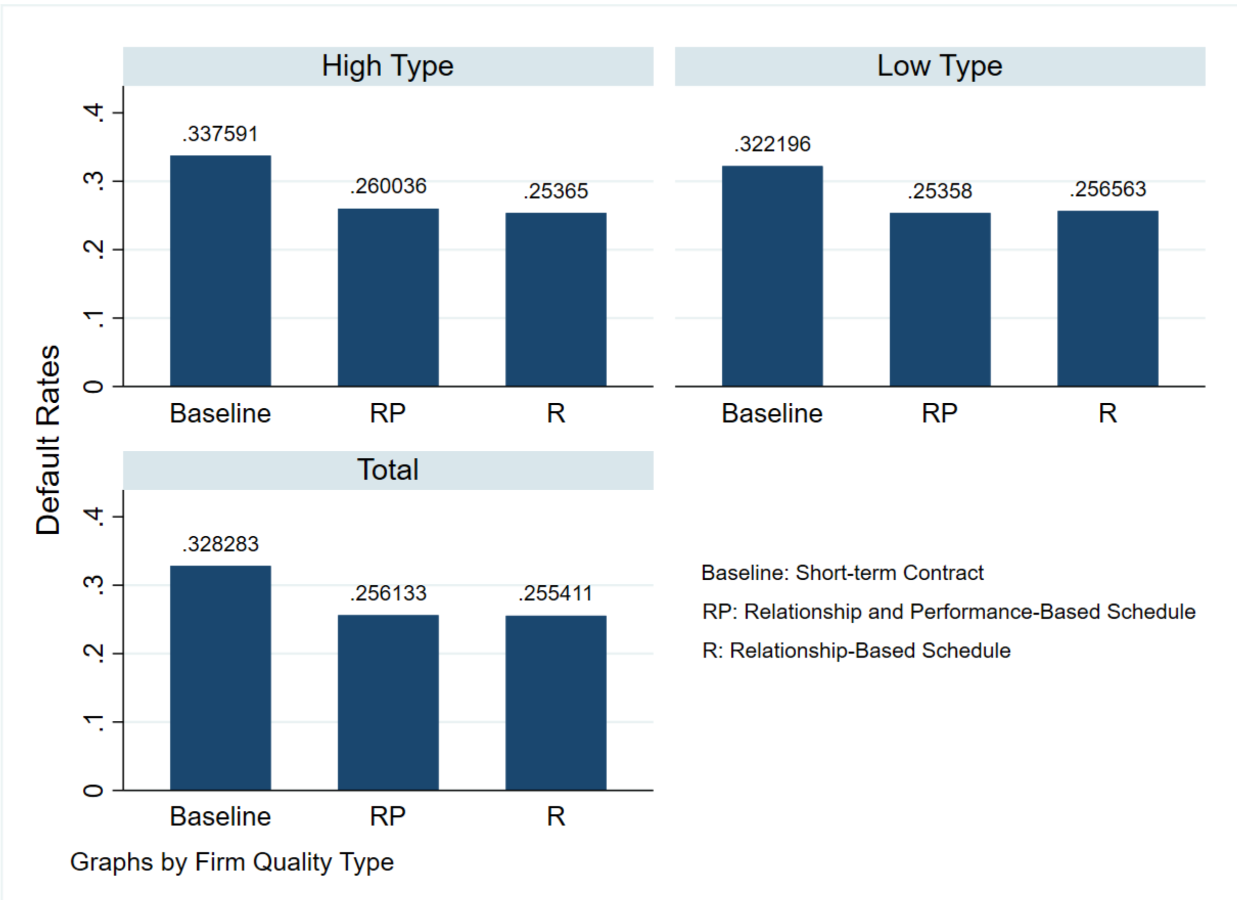


Figure 7: Comparison of Default Rates Across Different Contracts