Problem Set 6

Due Mon 12/12/2011

1. Consider two sequences of random variables $\{Y_n\}_{n\geq 1}$ and $\{Z_n\}_{n\geq 1}$ such that $Y_n = O_p(1)$ and $X_n = o_p(1)$. Use the definition of bounded in probability and convergence in probability to show that $Z_n := X_n Y_n = o_p(Y_n) = O_p(X_n) = o_p(1)$. (Remark: because this is true, often people write $Z_n = O_p(1) \cdot o_p(1) = o_p(1)$ or $Z_n = O_p(1) \cdot X_n = o_p(X_n)$, etc.)

2. Derive the asymptotic distribution of the estimator for the projection coefficient in Example 2 of Lecture 16.

3. 6.2.2 HMC

4. Let Y be a Poisson random variable with parameter λ. Let ε be a Bernoulli random variable with success rate p, and ε is independent of Y. Both λ and p are unknown parameters. Suppose that $X = \varepsilon Y$. Then X is said to follow a zero-inflated Poisson distribution – it take the value 0 more often than a Poisson distribution.

   (a) Derive the pmf for X, $p_X(\cdot)$.

   (b) Suppose you have a random n-sample from $p_X(\cdot)$: $(X_1, ..., X_n)$, write down the log-likelihood function.

   (c) Derive the Cramer-Rao lower bound for unbiased estimators of (λ, p).

   (d) Find the maximum likelihood estimator for (λ, p) and show that it is consistent and asymptotically efficient.

   (e) Suppose that we believe the arrival rate λ and zero-inflating probability p are heterogenous among our sample. Assume that they depend on some other random variables W and Z. That is, we assume that X conditional on (W, Z) follows a zero-inflated Poisson distribution with parameters $\lambda(W, \theta)$ and $p(Z, \beta)$ for some known functions $\lambda(w, \theta)$ and $p(z, \beta)$ and unknown parameters θ and β. Suppose we observe a random n-sample $((X_i, W_i, Z_i))_{i=1}^n$. Write down the conditional log-likelihood function and the score functions of the log-likelihood.

5. 5.4.11 HMC

6. 5.4.25 HMC

7. 5.5.4 HMC

8. 8.1.7 HMC

9. 8.3.6 HMC