Problem Set 2 (Due 10/3/2011 in class)

September 28, 2011

1. Provide simulation evidence for the Monty Hall problem. Any programming language is acceptable. Hand in code with clear description of every line and a (or more) paragraph at the beginning describing the program and summarize the steps in clear English. Make sure that a reader NOT familiar with the programming language you are using can understand what it does easily.

2. You and your friend both have three dice. Both of you throw your own dice on the table and cover them with a bowl. You can see your own dice but not your friend's and vice versa.

   (a) Suppose that you have no 6. What is your best guess of the total number of 6's on the table? What if you have one? or two? or three? (Hint: best guess means the number with the highest probability of being exactly right.)

   (b) The game that you are playing is like this: one of you start by making a first guess of the total number of 6's on the table. The other person can either challenge you or make a bigger guess. If she challenges you, both of you uncover the bowl and count the total number of 6's. If the total number is greater than or equal to your guess, the challenger loses \$1, otherwise, the guesser loses \$1. If she makes a bigger guess, then it is your turn to either challenge her or make a bigger guess. The game ends only when there is a challenge.
   Suppose that your friend knows that you always start the game by making your best guess. Then given your initial guess, what is her best guess if she has one 6? What if she has none? or two? or three?

   (c) Suppose that both of you follow the following strategy: make your best guess (given all present information) if you are the first one to guess or your best guess is bigger than the previous guess, challenge if your best guess is smaller than the previous guess, and if your best guess is the same as the previous guess, challenge with probability 1/2 and add one to the previous guess with probability 1/2. Suppose that it is common knowledge that both are following this same strategy. Is there an advantage of being the first one to guess?

(d) (Optional) you can verify your answer using simulation.

3. Consider a medical test that produces two results: negative (not having the disease) and positive (having the disease). Suppose there are 4 types of people, those without the disease, those at an early stage of the disease, those at an intermediate stage of the disease and those at a late stage of the disease, each with probability 60%, 30%, 9.9% and 0.1%. Suppose that the medical test makes mistakes 5% of the time when testing a person without the disease, 50% when testing a person at the early stage of the disease, 5% of the time when testing a person at the intermediate stage of the disease and makes no mistake when testing a person at the late stage of the disease.

   (a) What is the probability that a person actually is healthy given that the test returns negative?

   (b) Give one prior probability distribution of the 4 types of people, under which a person can be 95% confident that he is in fact healthy when tested negative. (Hint: there are many such distributions. Please just give one.)

4. Consider a random variable $X : (\Omega, \mathcal{F}) \to (\Omega_X, \mathcal{B}(\Omega_X))$ and a random variable $Y : (\Omega, \mathcal{F}) \to (\Omega_Y, \mathcal{B}(\Omega_Y))$. Let P be a probability distribution on $(\Omega, \mathcal{F})$. The random variables are called **independent** under P if for every $A \in \mathcal{B}(\Omega_X)$ and $B \in \mathcal{B}(\Omega_Y)$, $X^{-1}(A)$ and $Y^{-1}(B)$ are independent events on the probability space $(\Omega, \mathcal{F}, P)$. Let $g : \Omega_X \to \Omega_g$ and $f : \Omega_Y \to \Omega_f$ be measurable functions.
   Show that $g(X)$ and $f(Y)$ are independent random variables.

5. 1.5.1 of HCM

6. 1.5.3 of HCM

7. 1.5.8 of HCM