Problem Set 1 (Due 9/19/2011 in class)

September 11, 2011

1. Isaac Newton and Samuel Pepys debated the following question: is the probability of tossing at least one six in six tosses with a fair die smaller than, equal to, or larger than the probability of tossing at least two sixes in twelve tosses?

2. Prove the distributive law of set operation.

3. 1.3.20 of HCM.

4. 1.3.23 of HCM.

5. About 2% of North American wolves are white. On a hunting trip, a friend of yours encountered a pack of 6 wolves.

    (a) Before he tells you their colors, use the probability theory to derive an upper bound and a lower bound for the probability that at least one of the 6 wolves is white. (Hint: no dependence structure is assumed between the wolves in the pack.)

    (b) Under what assumption is the lower bound achieved?

    (c) Under what assumption is the upper bound achieved?

    (d) If you believed that wolves randomly form packs, what would you say about the above probability?

    (This is not a wolf question – you will see such calculations in the multiple hypotheses testing literature. The "white wolf" part of the question is motivated by the bestseller "A Song of Ice and Fire", a fantastic book to read in your spare time.)

6. 1.3.19 of HCM.