Econ 301

Intermediate Microeconomics

Prof. Marek Weretka

Midterm 2 (Group A)

You have 70 minutes to complete the exam. The midterm consists of 4 questions (40,20,20 and 20 points)+Just For Fun question.

Problem 1. (40p) (Edgeworth box and intertemporal choice)

Consider economy with two periods (interpreted as "when young" and "when old" periods) and two consumers, Gillian Murphy and Jeremy Krawczyk. Gillian Murphy is one of the top ballet dancers with lifetime income given by $\omega^G = (50, 0)$. Jeremy Krawczyk is an econ Ph.D. student with income $\omega^J = (0, 50)$. Gillian and Jeremy have identical utility functions given by

$$U(x_1, x_2) = \ln x_1 + \frac{1}{2} \ln x_2$$

a) Plot an Edgeworth box and mark the initial endowment point.

b) Write down a definition of Pareto efficiency (one sentence) and give the equivalent condition in terms of MRS (give formula, you do not need to prove equivalence).

c) Derive the contract curve (write down conditions and solve for the curve) and depict it in the Edgeworth box.

d) Suppose Gillian and Jeremy can "trade" consumption in both periods at prices p_1, p_2. Find competitive equilibrium (six numbers) and depict it in the Edgeworth box.

e) Using MRS condition verify that the equilibrium allocation obtained in point d) is Pareto efficient.

f) Find equilibrium interest rate corresponding to competitive prices obtained in point d) (one number).

g) Hard: Suppose Gillian's utility function is $U^G(x_1, x_2) = x_1 + 2x_2$ and Jeremy's utility is $U^J(x_1, x_2) = 2x_1 + x_2$. Without any calculations find (geometrically, in the Edgeworth box) the corresponding contract curve.

Problem 1. (20p) (Uncertainty)

A lottery pays \$4 in a "winning" state and \$0 otherwise. The probability of winning is $\frac{1}{2}$.

a) Find the expected value of lottery $(4, 0)$.

b) Assume Bernoulli utility $u(c) = \sqrt{c}$. Write down the formula for Von Neumann-Morgenstern (expected) utility function, $U(c_1, c_2)$. Plot the indifference curve map.

c) Find the *expected utility* from lottery $(4, 0)$ (one number) and the utility derived from the *expected value of lottery* for sure (one number). Which of the two numbers is bigger and why? (one sentence)

d) Give answer to point c) assuming Bernouli utility $u(c) = c^2$.

e) Find the certainty equivalent of lottery $(4, 0)$ for two Bernouli utility function: $u(c) = \sqrt{c}$ and $u(c) = c^2$ (two numbers).

Problem 3. (20p) (Producers)

A producer has the following technology

$$y = K^{\frac{1}{2}} L^{\frac{1}{2}}$$

a) Prove formally that the production function exhibits constant returns to scale (use "λ" argument).

b) Find analytically MPL and MPK. Are the two marginal products increasing, decreasing or constant?

c) Short run: Given stock of capital $\bar{K} = 1$ find labor demand (formula) of a competitive firm. Find equilibrium real wage rate if labor supply is given by $L^s = 4$ (one number). Find the unemployment rate if the minimal real wage is $w^{min}/p = 2$ (one number+graph).

d) Long run: find cost functions given prices of inputs $w_K = 1$ and $w_L = 2$ (formula). Plot the cost function in the graph.

e) Hard: Write down the two conditions for profit maximization in long run. Demonstrate that these conditions can be reduced to the analogous condition for cost minimization.

Problem 4 (20p). (Individual Supply and Entry)

Assume fixed cost $F = 2$ and (variable) cost function $c(y) = y^2$.

a) Plot ATC curve. Explain why average cost becomes "infinite" when the level of production is close to zero and when it is very large? (two sentences)

b) Find y^{MES} and ATC^{MES} (give two numbers).

c) Give two conditions that determine optimal level of output for any level of price. Provide economic interpretation for each of them (one sentence for each)

d) Write down supply function $y(p)$. Plot the supply function, marking a price threshold for non-zero production.

e) Determine the number of firms operating in the industry if demand is $D(p) = 40 - p$, assuming free entry. (one number)

Just For Fun

Argue that with constant results to scale if for some K, L profit is strictly positive, then the profit can be made arbitrarily large by appropriately scaling up the level of production.