Econ 301

Intermediate Microeconomics

Prof. Marek Weretka

Midterm 2 (Group A)

You have 70 minutes to complete the exam. The midterm consists of 4 questions (40,20,20 and 20 points)+Just For Fun question.

Problem 1. (40p) (Edgeworth box and intertemporal choice)

Consider economy with two periods (interpreted as "when young" and "when old" periods) and two consumers, Gillian Murphy and Jeremy Krawczyk. Gillian Murphy is one of the top ballet dancers with lifetime income given by $\omega^G = (50, 20)$. Jeremy Krawczyk is an econ Ph.D. student with income $\omega^J = (20, 50)$. Gillian and Jeremy have identical utility functions given by

$$U(x_1, x_2) = \ln x_1 + \frac{1}{2} \ln x_2$$

a) Plot an Edgeworth box and mark the initial endowment point.

b) Write down the general definition of Pareto efficient allocation (one sentence) and give the equivalent condition in terms of MRS (give formula, you do not need to prove equivalence).

c) Derive the contract curve (write down conditions and solve for the curve) and depict it in the Edgeworth box.

d) Suppose Gillian and Jeremy can "trade" consumption in both periods at prices p_1, p_2. Find competitive equilibrium (six numbers) and depict it in the Edgeworth box.

e) Using MRS condition verify that the equilibrium allocation obtained in point d) is Pareto efficient.

f) Find equilibrium interest rate corresponding to competitive prices obtained in point d) (one number).

g) Very Hard: Suppose Gillian's utility function is $U^G(x_1, x_2) = x_1 + 2x_2$ and Jeremy's utility is $U^J(x_1, x_2) = 2x_1 + x_2$. Without any calculations find (geometrically, in the Edgeworth box) the corresponding contract curve. Hint: do not use the "MRS" conditon, argue from the definition of Pareto efficiency.

Problem 2. (20p) (Uncertainty)

A lottery pays \$9 in a "winning" state and \$1 otherwise. The probability of winning is $\frac{1}{2}$.

a) Find the expected value of lottery $(1, 9)$.

b) Assume Bernoulli utility $u(c) = \sqrt{c}$. Write down the formula for Von Neumann-Morgenstern (expected) utility function, $U(c_1, c_2)$. Without any calculations sketch the indifference curve map.

c) Find the *expected utility* from lottery $(1, 9)$ (one number) and the utility derived from the *expected value of the lottery* for sure (one number). Which of the two numbers is bigger and why? (one sentence)

d) Give answer to point c) assuming Bernouli utility $u(c) = c^2$.

e) Find the certainty equivalent of lottery $(1, 9)$ for two Bernouli utility function: $u(c) = \sqrt{c}$ and $u(c) = c^2$ (two numbers).

Problem 3. (20p) (Producers)

A producer has the following technology

$$y = 6K^{\frac{1}{2}}L^{\frac{1}{2}}$$

a) Prove formally that the production function exhibits constant returns to scale (use "λ" argument).

b) Find analytically MPL and MPK. Are the two marginal products increasing, decreasing or constant?

c) Short run: Given stock of capital $\bar{K} = 4$ find labor demand (formula) of a competitive firm. Find equilibrium real wage rate if labor supply is given by $L^s = 9$ (one number). Find the unemployment rate if the minimal real wage is $w^{min}/p = 3$ (one number+graph).

d) Long run: find cost functions given prices of inputs $w_K = 1$ and $w_L = 4$ (formula). Plot the cost function in the graph.

e) Hard: Write down the two conditions for the profit maximization in long run. Demonstrate that these conditions can be reduced to the condition for the cost minimization.

Problem 4 (20p). (Individual Supply and Entry)

Assume fixed cost $F = 3$ and (variable) cost function $c(y) = 3y^2$.

a) Plot ATC curve. Explain why average cost becomes "infinite" when the level of production is close to zero and when it is very large (two sentences).

b) Find y^{MES} and ATC^{MES} (give two numbers).

c) Give the condition that determines optimal level of output for any level of price and provide the economic interpretation (one sentence). Argue that with fixed cost the condition has to be suplemented with adadditional (inequality) condition.

d) Write down supply function $y(p)$. Plot the supply function, marking a price threshold for non-zero production.

e) Determine the number of firms operating in the industry if demand is $D(p) = 10 - p$, assuming free entry. (one number)

Just For Fun

Argue that with constant results to scale if for some K, L profit is strictly positive, then the profit can be made arbitrarily large by appropriately scaling up the level of production.

Econ 301

Intermediate Microeconomics

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Midterm 2 (Group B)

You have 70 minutes to complete the exam. The midterm consists of 4 questions (40,20,20 and 20 points)+Just For Fun question.

Problem 1. (40p) (Edgeworth box and intertemporal choice)

Consider economy with two periods (interpreted as "when young" and "when old" periods) and two consumers, Gillian Murphy and Jeremy Krawczyk. Gillian Murphy is one of the top ballet dancers with lifetime income given by $\omega^G = (60, 30)$. Jeremy Krawczyk is an econ Ph.D. student with income $\omega^J = (30, 60)$. Gillian and Jeremy have identical utility functions given by

$$U(x_1, x_2) = \ln x_1 + \frac{1}{2} \ln x_2$$

a) Plot an Edgeworth box and mark the initial endowment point.

b) Write down the general definition of Pareto efficient allocation (one sentence) and give the equivalent condition in terms of MRS (give formula, you do not need to prove equivalence).

c) Derive the contract curve (write down conditions and solve for the curve) and depict it in the Edgeworth box.

d) Suppose Gillian and Jeremy can "trade" consumption in both periods at prices p_1, p_2. Find competitive equilibrium (six numbers) and depict it in the Edgeworth box.

e) Using MRS condition verify that the equilibrium allocation obtained in point d) is Pareto efficient.

f) Find equilibrium interest rate corresponding to competitive prices obtained in point d) (one number).

g) Very Hard: Suppose Gillian's utility function is $U^G(x_1, x_2) = 2x_1 + x_2$ and Jeremy's utility is $U^J(x_1, x_2) = x_1 + 2x_2$. Without any calculations find (geometrically, in the Edgeworth box) the corresponding contract curve. Hint: do not use the "MRS" conditon, argue from the definition of Pareto efficiency.

Problem 2. (20p) (Uncertainty)

A lottery pays \$4 in a "winning" state and \$1 otherwise. The probability of winning is $\frac{1}{2}$.

a) Find the expected value of lottery $(1, 4)$.

b) Assume Bernoulli utility $u(c) = \sqrt{c}$. Write down the formula for Von Neumann-Morgenstern (expected) utility function, $U(c_1, c_2)$. Without any calculations sketch the indifference curve map.

c) Find the *expected utility* from lottery $(1, 4)$ (one number) and the utility derived from the *expected value of the lottery* for sure (one number). Which of the two numbers is bigger and why? (one sentence)

d) Give answer to point c) assuming Bernouli utility $u(c) = c^2$.

e) Find the certainty equivalent of lottery $(1, 4)$ for two Bernouli utility function: $u(c) = \sqrt{c}$ and $u(c) = c^2$ (two numbers).

Problem 3. (20p) (Producers)

A producer has the following technology

$$y = 4K^{\frac{1}{2}}L^{\frac{1}{2}}$$

a) Prove formally that the production function exhibits constant returns to scale (use "λ" argument).

b) Find analytically MPL and MPK. Are the two marginal products increasing, decreasing or constant?

c) Short run: Given stock of capital $\bar{K} = 9$ find labor demand (formula) of a competitive firm. Find equilibrium real wage rate if labor supply is given by $L^s = 9$ (one number). Find the unemployment rate if the minimal real wage is $w^{min}/p = 3$ (one number+graph).

d) Long run: find cost functions given prices of inputs $w_K = 4$ and $w_L = 2$ (formula). Plot the cost function in the graph.

e) Hard: Write down the two conditions for the profit maximization in long run. Demonstrate that these conditions can be reduced to the condition for the cost minimization.

Problem 4 (20p). (Individual Supply and Entry)

Assume fixed cost $F = 4$ and (variable) cost function $c(y) = y^2$.

a) Plot ATC curve. Explain why average cost becomes "infinite" when the level of production is close to zero and when it is very large (two sentences).

b) Find y^{MES} and ATC^{MES} (give two numbers).

c) Give the condition that determines optimal level of output for any level of price and provide the economic interpretation (one sentence). Argue that with fixed cost the condition has to be suplemented with adadditional (inequality) condition.

d) Write down supply function $y(p)$. Plot the supply function, marking a price threshold for non-zero production.

e) Determine the number of firms operating in the industry if demand is $D(p) = 16 - p$, assuming free entry. (one number)

Just For Fun

Argue that with constant results to scale if for some K, L profit is strictly positive, then the profit can be made arbitrarily large by appropriately scaling up the level of production.

Econ 301

Intermediate Microeconomics

Prof. Marek Weretka

Midterm 2 (Group C)

You have 70 minutes to complete the exam. The midterm consists of 4 questions (40,20,20 and 20 points)+Just For Fun question.

Problem 1. (40p) (Edgeworth box and intertemporal choice)

Consider economy with two periods (interpreted as "when young" and "when old" periods) and two consumers, Gillian Murphy and Jeremy Krawczyk. Gillian Murphy is one of the top ballet dancers with lifetime income given by $\omega^G = (15, 30)$. Jeremy Krawczyk is an econ Ph.D. student with income $\omega^J = (30, 15)$. Gillian and Jeremy have identical utility functions given by

$$U(x_1, x_2) = \ln x_1 + \frac{1}{2} \ln x_2$$

a) Plot an Edgeworth box and mark the initial endowment point.

b) Write down the general definition of Pareto efficient allocation (one sentence) and give the equivalent condition in terms of MRS (give formula, you do not need to prove equivalence).

c) Derive the contract curve (write down conditions and solve for the curve) and depict it in the Edgeworth box.

d) Suppose Gillian and Jeremy can "trade" consumption in both periods at prices p_1, p_2. Find competitive equilibrium (six numbers) and depict it in the Edgeworth box.

e) Using MRS condition verify that the equilibrium allocation obtained in point d) is Pareto efficient.

f) Find equilibrium interest rate corresponding to competitive prices obtained in point d) (one number).

g) Very Hard: Suppose Gillian's utility function is $U^G(x_1, x_2) = 7x_1 + x_2$ and Jeremy's utility is $U^J(x_1, x_2) = x_1 + 7x_2$. Without any calculations find (geometrically, in the Edgeworth box) the corresponding contract curve. Hint: do not use the "MRS" conditon, argue from the definition of Pareto efficiency.

Problem 2. (20p) (Uncertainty)

A lottery pays \$9 in a "winning" state and \$4 otherwise. The probability of winning is $\frac{1}{2}$.

a) Find the expected value of lottery $(4, 9)$.

b) Assume Bernoulli utility $u(c) = \sqrt{c}$. Write down the formula for Von Neumann-Morgenstern (expected) utility function, $U(c_1, c_2)$. Without any calculations sketch the indifference curve map.

c) Find the *expected utility* from lottery $(4, 9)$ (one number) and the utility derived from the *expected value of the lottery* for sure (one number). Which of the two numbers is bigger and why? (one sentence)

d) Give answer to point c) assuming Bernouli utility $u(c) = c^2$.

e) Find the certainty equivalent of lottery $(4, 9)$ for two Bernouli utility function: $u(c) = \sqrt{c}$ and $u(c) = c^2$ (two numbers).

Problem 3. (20p) (Producers)

A producer has the following technology

$$y = 2K^{\frac{1}{2}}L^{\frac{1}{2}}$$

a) Prove formally that the production function exhibits constant returns to scale (use "λ" argument).

b) Find analytically MPL and MPK. Are the two marginal products increasing, decreasing or constant?

c) Short run: Given stock of capital $\bar{K} = 16$ find labor demand (formula) of a competitive firm. Find equilibrium real wage rate if labor supply is given by $L^s = 16$ (one number). Find the unemployment rate if the minimal real wage is $w^{min}/p = 2$ (one number+graph).

d) Long run: find cost functions given prices of inputs $w_K = 4$ and $w_L = 2$ (formula). Plot the cost function in the graph.

e) Hard: Write down the two conditions for the profit maximization in long run. Demonstrate that these conditions can be reduced to the condition for the cost minimization.

Problem 4 (20p). (Individual Supply and Entry)

Assume fixed cost $F = 4$ and (variable) cost function $c(y) = 4y^2$.

a) Plot ATC curve. Explain why average cost becomes "infinite" when the level of production is close to zero and when it is very large (two sentences).

b) Find y^{MES} and ATC^{MES} (give two numbers).

c) Give the condition that determines optimal level of output for any level of price and provide the economic interpretation (one sentence). Argue that with fixed cost the condition has to be suplemented with adadditional (inequality) condition.

d) Write down supply function $y(p)$. Plot the supply function, marking a price threshold for non-zero production.

e) Determine the number of firms operating in the industry if demand is $D(p) = 20 - p$, assuming free entry. (one number)

Just For Fun

Argue that with constant results to scale if for some K, L profit is strictly positive, then the profit can be made arbitrarily large by appropriately scaling up the level of production.

Econ 301
Intermediate Microeconomics
Prof. Marek Weretka

Midterm 2 (Group D)

You have 70 minutes to complete the exam. The midterm consists of 4 questions (40,20,20 and 20 points)+Just For Fun question.

Problem 1. (40p) (Edgeworth box and intertemporal choice)

Consider economy with two periods (interpreted as "when young" and "when old" periods) and two consumers, Gillian Murphy and Jeremy Krawczyk. Gillian Murphy is one of the top ballet dancers with lifetime income given by $\omega^G = (100, 40)$. Jeremy Krawczyk is an econ Ph.D. student with income $\omega^J = (40, 100)$. Gillian and Jeremy have identical utility functions given by

$$U(x_1, x_2) = \ln x_1 + \frac{1}{2} \ln x_2$$

a) Plot an Edgeworth box and mark the initial endowment point.

b) Write down the general definition of Pareto efficient allocation (one sentence) and give the equivalent condition in terms of MRS (give formula, you do not need to prove equivalence).

c) Derive the contract curve (write down conditions and solve for the curve) and depict it in the Edgeworth box.

d) Suppose Gillian and Jeremy can "trade" consumption in both periods at prices p_1, p_2. Find competitive equilibrium (six numbers) and depict it in the Edgeworth box.

e) Using MRS condition verify that the equilibrium allocation obtained in point d) is Pareto efficient.

f) Find equilibrium interest rate corresponding to competitive prices obtained in point d) (one number).

g) Very Hard: Suppose Gillian's utility function is $U^G(x_1, x_2) = 7x_1 + x_2$ and Jeremy's utility is $U^J(x_1, x_2) = x_1 + 7x_2$. Without any calculations find (geometrically, in the Edgeworth box) the corresponding contract curve. Hint: do not use the "MRS" conditon, argue from the definition of Pareto efficiency.

Problem 2. (20p) (Uncertainty)

A lottery pays \$4 in a "winning" state and \$1 otherwise. The probability of winning is $\frac{1}{2}$.

a) Find the expected value of lottery $(1, 4)$.

b) Assume Bernoulli utility $u(c) = \sqrt{c}$. Write down the formula for Von Neumann-Morgenstern (expected) utility function, $U(c_1, c_2)$. Without any calculations sketch the indifference curve map.

c) Find the *expected utility* from lottery $(1, 4)$ (one number) and the utility derived from the *expected value of the lottery* for sure (one number). Which of the two numbers is bigger and why? (one sentence)

d) Give answer to point c) assuming Bernouli utility $u(c) = c^2$.

e) Find the certainty equivalent of lottery $(1, 4)$ for two Bernouli utility function: $u(c) = \sqrt{c}$ and $u(c) = c^2$ (two numbers).

Problem 3. (20p) (Producers)

A producer has the following technology

$$y = 2K^{\frac{1}{2}}L^{\frac{1}{2}}$$

a) Prove formally that the production function exhibits constant returns to scale (use "λ" argument).

b) Find analytically MPL and MPK. Are the two marginal products increasing, decreasing or constant?

c) Short run: Given stock of capital $\bar{K} = 9$ find labor demand (formula) of a competitive firm. Find equilibrium real wage rate if labor supply is given by $L^s = 9$ (one number). Find the unemployment rate if the minimal real wage is $w^{min}/p = 2$ (one number+graph).

d) Long run: find cost functions given prices of inputs $w_K = 1$ and $w_L = 2$ (formula). Plot the cost function in the graph.

e) Hard: Write down the two conditions for the profit maximization in long run. Demonstrate that these conditions can be reduced to the condition for the cost minimization.

Problem 4 (20p). (Individual Supply and Entry)

Assume fixed cost $F = 4$ and (variable) cost function $c(y) = y^2$.

a) Plot ATC curve. Explain why average cost becomes "infinite" when the level of production is close to zero and when it is very large (two sentences).

b) Find y^{MES} and ATC^{MES} (give two numbers).

c) Give the condition that determines optimal level of output for any level of price and provide the economic interpretation (one sentence). Argue that with fixed cost the condition has to be suplemented with adadditional (inequality) condition.

d) Write down supply function $y(p)$. Plot the supply function, marking a price threshold for non-zero production.

e) Determine the number of firms operating in the industry if demand is $D(p) = 20 - p$, assuming free entry. (one number)

Just For Fun

Argue that with constant results to scale if for some K, L profit is strictly positive, then the profit can be made arbitrarily large by appropriately scaling up the level of production.

Econ 301 Midterm 2 Solutions

Group A

All graphs on last page of key.

Problem 1 (40p) Edgeworth Box and Intertemporal Choice

a) [6] See Figure 1 at end (ignore extra line in corners).

b) [8] An allocation is Pareto efficient if there is no other allocation that makes any individual better off without making at least one individual worse off.

For agents A and B, the MRS condition is that $MRS^G = MRS^J$.

c) [8] Contract curve conditions:

1) No waste: $x_1^J + x_1^J = \omega_1^G + \omega_1^J = 70$ and $x_2^G + x_2^J = \omega_2^G + \omega_2^J = 70$

2) $MRS^J = MRS^G$

$MRS^G = MRS^J \iff \frac{1/x_1^G}{.5/x_2^G} = \frac{1/x_1^J}{.5/x_2^J} \iff \frac{2x_2^G}{x_1^G} = \frac{2x_2^J}{x_1^J}$. Use condition (1) to find $\frac{2x_2^G}{x_1^G} = \frac{2x_2^J}{x_1^J} = \frac{2(70-x_2^G)}{(70-x_1^G)}$. Then by algebra, $x_2^G = x_1^G$.

d) [8] Normalize $p_2 = 1$. Then use an individual's optimization condition $-MRS^G = \frac{p_1}{p_2} = p_1$. By the contract curve, $-MRS^G = \frac{2x_2^G}{x_1^G} = \frac{2x_1^G}{x_1^G} = 2$, so $p_1 = 2$.

Use these prices in the Cobb-Douglas magic formulas to find:

$$x_1^G = (\frac{a}{a+b})(\frac{p_1\omega_1^G + p_2\omega_2^G}{p_1}) = (\frac{1}{3/2})(\frac{2*50+20}{2}) = 40 = x_2^G$$

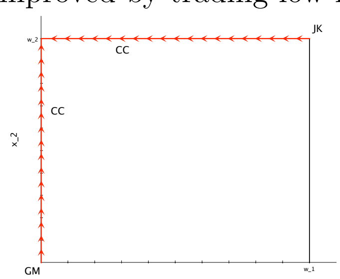
$$x_1^J = (\frac{a}{a+b})(\frac{p_1\omega_1^J + p_2\omega_2^J}{p_1}) = (\frac{1}{3/2})(\frac{2*20+50}{2}) = 30 = x_2^J$$

Alternatively, magic formulas as functions of p_1 sum to endowments.

e) [6] $MRS^G = \frac{-2*40}{40}$ equals $MRS^J = \frac{-2*30}{30}$.

f) [4] Use $\frac{p1}{p2} = 1 + r$, so $2 = 1 + r \iff r = 1$.

g) [0] Contract curve will be on the two edges of the box where agents consume the higher marginal utility good. Since they prefer different goods, interior allocations can always be improved by trading low for high MU goods.



Problem 2 (20p) Uncertainty

a) [2] $EV = \frac{1}{2}1 + \frac{1}{2}9 = 5$

b) [4] $U(C_L, C_W) = \frac{1}{2}\sqrt{C_L} + \frac{1}{2}\sqrt{C_W}$ See Figure 2 below.

c) [5] $U(1, 9) = .5\sqrt{1} + .5\sqrt{9} = 2$

$u(EV) = \sqrt{EV} = \sqrt{5}$

The utility of the expected value is larger than the expected utility because the bernoulli utility function $\sqrt{c}$ is concave, making the agent risk averse.

d) [5] $U(1, 9) = .51^2 + .59^2 = 41$

$u(EV) = EV^2 = 5^2 = 25$

The utility of the expected value is smaller than the expected utility because the bernoulli utility function c^2 is convex, making the agent risk loving.

e) [4] The certainty equivalent of a lottery is the payment delivered with certainty that yields the same utility as the expected utility of the lottery.

1) $u(CE) = \sqrt{CE} = 2$ so $CE = 2^2 = 4$
2) $u(CE) = CE^2 = 41$ so $CE = \sqrt{41}$

Problem 3 (20p) Producers

a) [3] $F(\lambda K, \lambda L) = 6(\lambda K)^{1/2}(\lambda L)^{1/2} = 6\lambda^{1/2}K^{1/2}\lambda^{1/2}L^{1/2} = \lambda 6K^{1/2}L^{1/2} = \lambda F(K, L) \rightarrow CRS$

b) [3] $MPL = \frac{6}{2}K^{1/2}L^{-1/2}$ so $\frac{\partial MPL}{\partial L} = 3\frac{-1}{2}K^{1/2}L^{-3/2} < 0 \rightarrow$ MPL decreases in L

$MPK = \frac{6}{2}K^{-1/2}L^{1/2}$ so $\frac{\partial MPK}{\partial L} = 3\frac{-1}{2}K^{-3/2}L^{1/2} < 0 \rightarrow$ MPK decreases in K

c) [6] Now $y = 12L^{1/2}$. To find labor demand, use the condition that MPL equals the real wage, $6L^{-1/2} = \frac{w_L}{p}$. Solve for L to find $L^D = 36(\frac{w_L}{p})^{-2}$.

To find the equilibrium real wage, note that in equilibrium $L^S = L^D$, so $9 = 36(\frac{w_L}{p})^{-2}$. Solve for $\frac{w_l}{p} = 2$.

Now the minimum wage is set above the equilibrium real wage, so we expect unemployment. Find the number of workers hired by plugging the minimum wage in to the labor demand function to find $L^D(\frac{w_l}{p} = 3) = 4$. Then the unemployment rate is $\dfrac{L^S - L^D}{L^S} = \dfrac{9-4}{9} = 5/9$. Graph in Figure 3.

d) [5] Need to eliminate K and L from typical equation $TC = w_K K + w_L L$

1) Solve for the optimal allocation across inputs using $\frac{MPK}{MPL} = \frac{w_K}{w_L} = 1/4$. Gives allocation $K^* = 4L^*$.

2) Using the allocation from (1), eliminate one input in production function and solve for remaining input $L = y/12$

3) Using the allocation from (1), eliminate one input from $TC = w_L L + w_K K$ and then substitute in y using the expression from (2). This yields $TC = \frac{2}{3}y$. Graph is a line from origin of y-\$ plane with slope 2/3.

e) [3] See lecture notes 16 slide 9. Profit Maximizing Conditions:

1) $MPK = \frac{w_K}{p}$
2) $MPL = \frac{w_L}{p}$

Take the ratio of conditions (1) and (2) to find the cost minimizing condition $\dfrac{MPK}{MPL} = \dfrac{w_K/p}{w_L/p} = \dfrac{w_K}{w_L}$.

Problem 4 (20p) Individual Supply and Entry

a) [4] $ATC = 3/y + 3y$. Graph in Figure 4.

When y is close to zero, the FC/y term dominates and gets arbitrarily large because fixed costs are being averaged over a small level of output. When y is large, the FC/y term goes to goes to zero and the expression converges to the line $cost = 3y$, which is increasing in y because costs per unit increase due to diminishing returns.

b) [4] Use either $\partial ATC/\partial y = 0$ or $ATC = MC$ to find $y^{mes} = 1$ and $ATC^{mes} = ATC(y^{mes}) = 6$

c) [4] The firm wants to set y so that $p = MC$ because for price-takers this maximizes the profit function by producing goods as long as $MR \geq MC$. With a fixed cost, the firm only wants to operate if it is at least breaking even, so it only produces $y > 0$ if $p^* \geq p^{BE} = p^{mes} = ATC^{mes}$.

d) [4] Produce so that $p = MC = 6y$ or $y = p/6$ if producing at all. Supply function is overlapping red line in Figure 4.

$$p(y) = \begin{cases} p/6 & \text{if } p \geq 6 \\ 0 & \text{if } p < 6 \end{cases}$$

e) [4] With free entry we know profits will go to zero, so firms produce $y^{mes} = 1$ and sell at $p^{BE} = 6$. By the demand curve, $Q^D = D(6) = 10 - 6 = 4$. In equilibrium $Q^D = Q^S$, so $Q^S = 4$. Since all firms are identical, $Q^S = Ny^{mes}$, so the number of firms $N = Q^S/y^{mes} = 4$.

Figure 1: #1 (a), (c), (d)

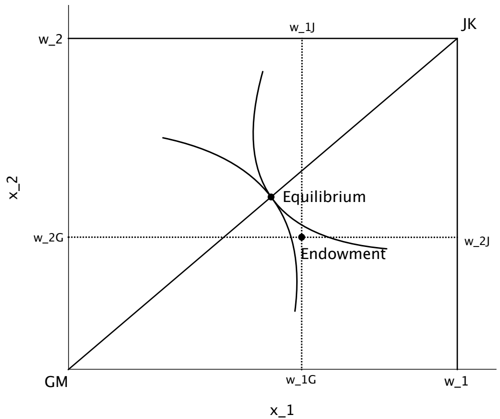


Figure 2: Indifference Curves #2.b

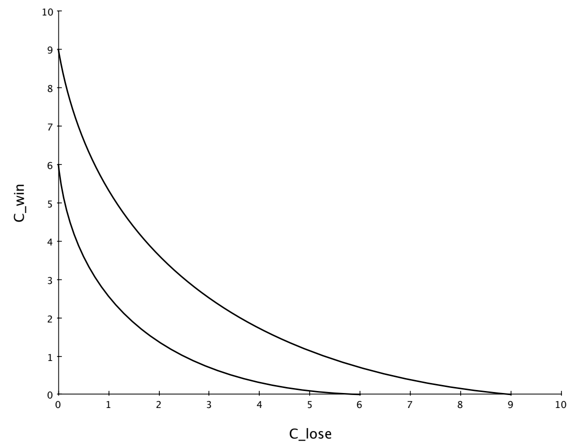


Figure 3: Cost Functions #3 (c) and (d)

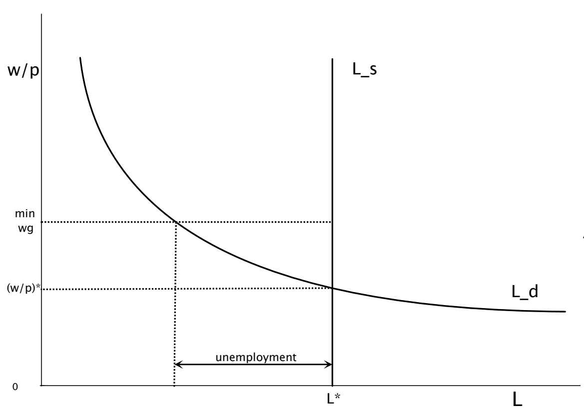
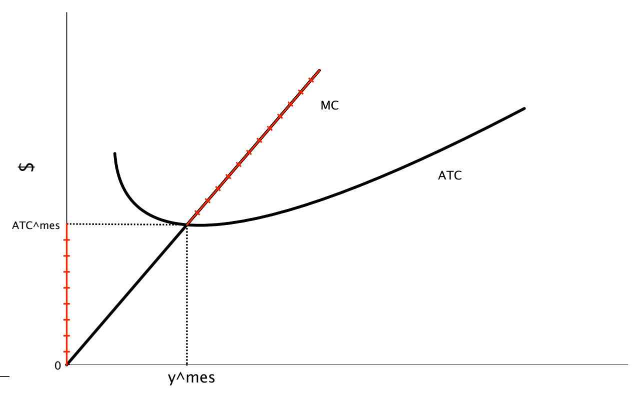


Figure 4: Cost and Supply Curves #4 (c) and (d)



Group B

All graphs on last page of key.

Problem 1 (40p) Edgeworth Box and Intertemporal Choice

a) [6] See Figure 1 at end (ignore extra line in corners).

b) [8] An allocation is Pareto efficient if there is no other allocation that makes any individual better off without making at least one individual worse off.

For agents A and B, the MRS condition is that $MRS^G = MRS^J$.

c) [8] Contract curve conditions:

1) No waste: $x_1^J + x_1^J = \omega_1^G + \omega_1^J = 90$ and $x_2^G + x_2^J = \omega_2^G + \omega_2^J = 90$

2) $MRS^J = MRS^G$

$MRS^G = MRS^J \iff \frac{1/x_1^G}{.5/x_2^G} = \frac{1/x_1^J}{.5/x_2^J} \iff \frac{2x_2^G}{x_1^G} = \frac{2x_2^J}{x_1^J}$. Use condition (1) to find $\frac{2x_2^G}{x_1^G} = \frac{2x_2^J}{x_1^J} = \frac{2(90-x_2^G)}{(90-x_1^G)}$. Then by algebra, $x_2^G = x_1^G$.

d) [8] Normalize $p_2 = 1$. Then use an individual's optimization condition $-MRS^G = \frac{p_1}{p_2} = p_1$. By the contract curve, $-MRS^G = \frac{2x_2^G}{x_1^G} = \frac{2x_1^G}{x_1^G} = 2$, so $p_1 = 2$.

Use these prices in the Cobb-Douglas magic formulas to find:

$$x_1^G = (\frac{a}{a+b})(\frac{p_1\omega_1^G + p_2\omega_2^G}{p_1}) = (\frac{1}{3/2})(\frac{2*60+30}{2}) = 50 = x_2^G$$

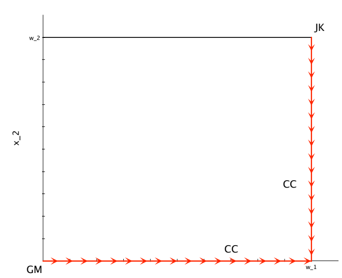
$$x_1^J = (\frac{a}{a+b})(\frac{p_1\omega_1^J + p_2\omega_2^J}{p_1}) = (\frac{1}{3/2})(\frac{2*30+60}{2}) = 40 = x_2^J$$

Alternatively, magic formulas as functions of p_1 sum to endowments.

e) [6] $MRS^G = \frac{-2*50}{50}$ equals $MRS^J = \frac{-2*40}{40}$.

f) [4] Use $\frac{p1}{p2} = 1 + r$, so $2 = 1 + r \iff r = 1$.

g) [0] Contract curve will be on the two edges of the box where agents consume the higher marginal utility good. Since they prefer different goods, interior allocations can always be improved by trading low for high MU goods.



Problem 2 (20p) Uncertainty

a) [2] $EV = \frac{1}{2}1 + \frac{1}{2}4 = 2.5$

b) [4] $U(C_L, C_W) = \frac{1}{2}\sqrt{C_L} + \frac{1}{2}\sqrt{C_W}$ See Figure 2 below.

c) [5] $U(Q,Q) = .5\sqrt{1} + .5\sqrt{4} = 1.5$

$u(EV) = \sqrt{EV} = \sqrt{2.5}$

The utility of the expected value is larger than the expected utility because the bernoulli utility function $\sqrt{c}$ is concave, making the agent risk averse.

d) [5] $U(Q,Q) = .51^2 + .54^2 = 8.5$

$u(EV) = EV^2 = 2.5^2 = 6.25$

The utility of the expected value is smaller than the expected utility because the bernoulli utility function c^2 is convex, making the agent risk loving.

e) [4] The certainty equivalent of a lottery is the payment delivered with certainty that yields the same utility as the expected utility of the lottery.

1) $u(CE) = \sqrt{CE} = 1.5$ so $CE = 1.5^2 = 2.25$
2) $u(CE) = CE^2 = 8.5$ so $CE = \sqrt{8.5}$

Problem 3 (20p) Producers

a) [3] $F(\lambda K, \lambda L) = 4(\lambda K)^{1/2}(\lambda L)^{1/2} = 4\lambda^{1/2}K^{1/2}\lambda^{1/2}L^{1/2} = \lambda 4K^{1/2}L^{1/2} = \lambda F(K,L) \rightarrow CRS$

b) [3] $MPL = \frac{4}{2}K^{1/2}L^{-1/2}$ so $\frac{\partial MPL}{\partial L} = 2\frac{-1}{2}K^{1/2}L^{-3/2} < 0 \rightarrow$ MPL decreases in L

$MPK = \frac{4}{2}K^{-1/2}L^{1/2}$ so $\frac{\partial MPK}{\partial L} = 2\frac{-1}{2}K^{-3/2}L^{1/2} < 0 \rightarrow$ MPK decreases in K

c) [6] Now $y = 12L^{1/2}$. To find labor demand, use the condition that MPL equals the real wage, $6L^{-1/2} = \frac{w_L}{p}$. Solve for L to find $L^D = 36(\frac{w_L}{p})^{-2}$.

To find the equilibrium real wage, note that in equilibrium $L^S = L^D$, so $9 = 36(\frac{w_L}{p})^{-2}$. Solve for $\frac{w_l}{p} = 2$.

Here the minimum wage is set above the equilibrium real wage, so we expect unemployment. Find the number of workers hired by plugging the minimum wage in to the labor demand function to find $L^D(\frac{w_l}{p} = 3) = 4$. Then the unemployment rate is $\dfrac{L^S - L^D}{L^S} = \dfrac{9-4}{9} = 5/9$. Graph in Figure 3.

d) [5] Need to eliminate K and L from typical equation $TC = w_K K + w_L L$

1) Solve for the optimal allocation across inputs using $\frac{MPK}{MPL} = \frac{w_K}{w_L} = 2$. Gives allocation $2K^* = L^*$.

2) Using the allocation from (1), eliminate one input in production function and solve for remaining input $K = y/4\sqrt{2}$

3) Using the allocation from (1), eliminate one input from $TC = w_L L + w_K K$ and then substitute in y using the expression from (2). This yields $TC = \frac{2}{\sqrt{2}}y$. Graph is a line from origin of y-\$ plane with slope $\frac{2}{\sqrt{2}}$.

e) [3] See lecture notes 16 slide 9. Profit Maximizing Conditions:

1) $MPK = \frac{w_K}{p}$

2) $MPL = \frac{w_L}{p}$

Take the ratio of conditions (1) and (2) to find the cost minimizing condition $\dfrac{MPK}{MPL} = \dfrac{w_K/p}{w_L/p} = \dfrac{w_K}{w_L}$.

Problem 4 (20p) Individual Supply and Entry

a) [4] $ATC = 4/y + y$. Graph in Figure 4.

When y is close to zero, the $4/y$ term dominates and gets arbitrarily large because fixed costs are being averaged over a small level of output. When y is large, the FC/y term goes to goes to zero and the expression converges to the line $cost = y$, which is increasing in y because costs per unit increase due to diminishing returns.

b) [4] Use either $\partial ATC/\partial y = 0$ or $ATC = MC$ to find $y^{mes} = 2$ and $ATC^{mes} = ATC(y^{mes}) = 4$

c) [4] The firm wants to set y so that $p = MC$ because for price-takers this maximizes the profit function by producing goods as long as $MR \geq MC$. With a fixed cost, the firm only wants to operate if it is at least breaking even, so it only produces $y > 0$ if $p^* \geq p^{BE} = p^{mes} = ATC^{mes}$.

d) [4] Produce so that $p = MC = 2y$ or $y = p/2$ if producing at all. Supply function is overlapping red line in Figure 4.

$$p(y) = \begin{cases} p/2 & \text{if } p \geq 4 \\ 0 & \text{if } p < 4 \end{cases}$$

e) [4] With free entry we know profits will go to zero, so firms produce $y^{mes} = 2$ and sell at $p^{BE} = 4$. By the demand curve, $Q^D = D(4) = 16 - 4 = 12$. In equilibrium $Q^D = Q^S$, so $Q^S = 12$. Since all firms are identical, $Q^S = Ny^{mes}$, so the number of firms $N = 12/2 = 6$.

Figure 1: #1 (a), (c), (d)

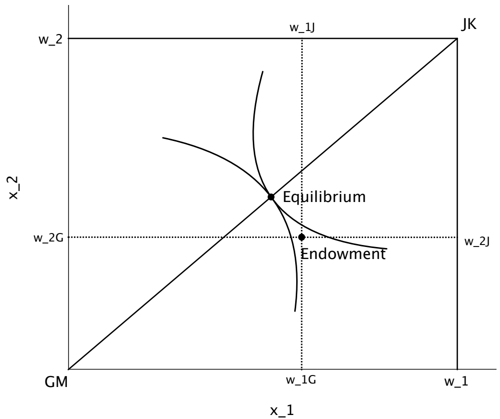


Figure 2: Indifference Curves #2.b

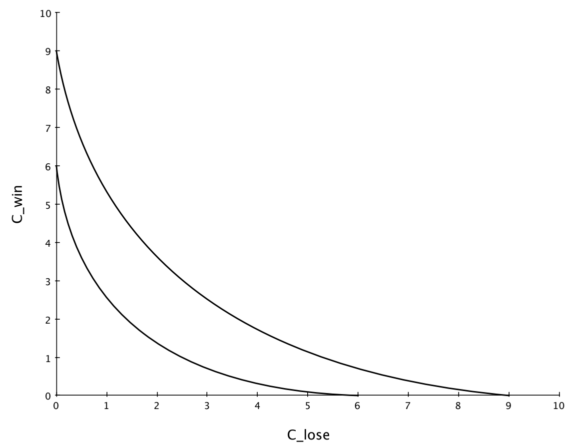


Figure 3: Cost Functions #3 (c) and (d)

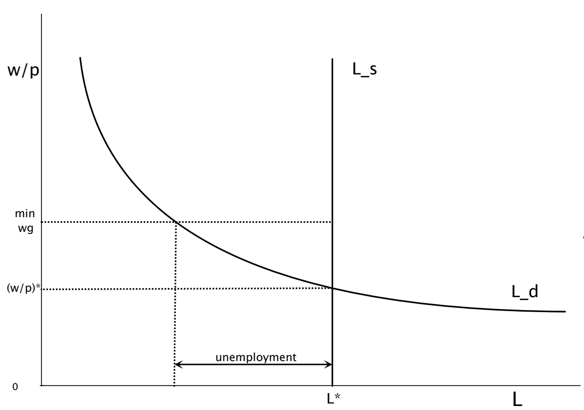
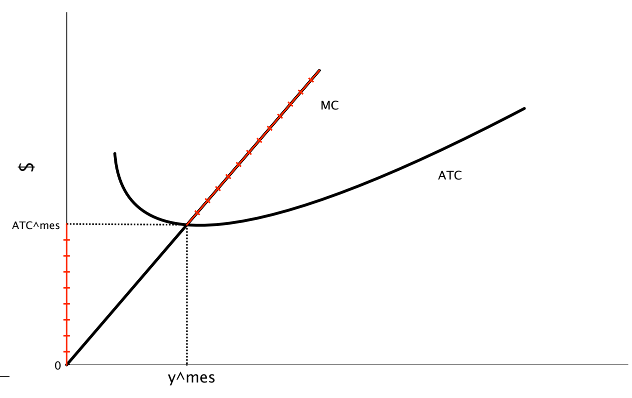


Figure 4: Cost and Supply Curves #4 (c) and (d)



Group C

All graphs on last page of key.

Problem 1 (40p) Edgeworth Box and Intertemporal Choice

a) [6] See Figure 1 at end (ignore extra line in corners).

b) [8] An allocation is Pareto efficient if there is no other allocation that makes any individual better off without making at least one individual worse off.

For agents A and B, the MRS condition is that $MRS^G = MRS^J$.

c) [8] Contract curve conditions:

1) No waste: $x_1^J + x_1^J = \omega_1^G + \omega_1^J = 45$ and $x_2^G + x_2^J = \omega_2^G + \omega_2^J = 45$

2) $MRS^J = MRS^G$

$MRS^G = MRS^J \iff \frac{1/x_1^G}{.5/x_2^G} = \frac{1/x_1^J}{.5/x_2^J} \iff \frac{2x_2^G}{x_1^G} = \frac{2x_2^J}{x_1^J}$. Use condition (1) to find $\frac{2x_2^G}{x_1^G} = \frac{2x_2^J}{x_1^J} = \frac{2(45-x_2^G)}{(45-x_1^G)}$. Then by algebra, $x_2^G = x_1^G$.

d) [8] Normalize $p_2 = 1$. Then use an individual's optimization condition $-MRS^G = \frac{p_1}{p_2} = p_1$. By the contract curve, $-MRS^G = \frac{2x_2^G}{x_1^G} = \frac{2x_1^G}{x_1^G} = 2$, so $p_1 = 2$.

Use these prices in the Cobb-Douglas magic formulas to find:

$$x_1^G = (\frac{a}{a+b})(\frac{p_1\omega_1^G + p_2\omega_2^G}{p_1}) = (\frac{1}{3/2})(\frac{2*15+30}{2}) = 20 = x_2^G$$

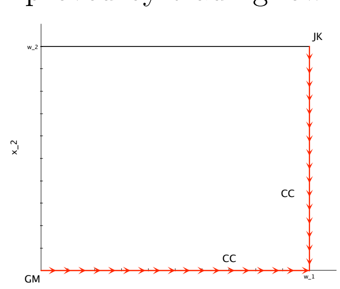
$$x_1^J = (\frac{a}{a+b})(\frac{p_1\omega_1^J + p_2\omega_2^J}{p_1}) = (\frac{1}{3/2})(\frac{2*30+15}{2}) = 25 = x_2^J$$

Alternatively, magic formulas as functions of p_1 sum to endowments.

e) [6] $MRS^G = \frac{-2*20}{20}$ equals $MRS^J = \frac{-2*25}{25}$.

f) [4] Use $\frac{p1}{p2} = 1 + r$, so $2 = 1 + r \iff r = 1$.

g) [0] Contract curve will be on the two edges of the box where agents consume the higher marginal utility good. Since they prefer different goods, interior allocations can always be improved by trading low for high MU goods.



Problem 2 (20p) Uncertainty

a) [2] $EV = \frac{1}{2}4 + \frac{1}{2}9 = 6.5$

b) [4] $U(C_L, C_W) = \frac{1}{2}\sqrt{C_L} + \frac{1}{2}\sqrt{C_W}$ See Figure 2 below.

c) [5] $U(Q, Q) = .5\sqrt{4} + .5\sqrt{9} = 2.5$

$u(EV) = \sqrt{EV} = \sqrt{6.5}$

The utility of the expected value is larger than the expected utility because the bernoulli utility function $\sqrt{c}$ is concave, making the agent risk averse.

d) [5] $U(Q, Q) = .54^2 + .59^2 = 97/2 = 48.5$

$u(EV) = EV^2 = 6.5^2 = (13/2)^2 = 169/4$

The utility of the expected value is smaller than the expected utility because the bernoulli utility function c^2 is convex, making the agent risk loving.

e) [4] The certainty equivalent of a lottery is the payment delivered with certainty that yields the same utility as the expected utility of the lottery.

1) $u(CE) = \sqrt{CE} = 2.5$ so $CE = 2.5^2 = 6.25$
2) $u(CE) = CE^2 = 97/2$ so $CE = \sqrt{97/2}$

Problem 3 (20p) Producers

a) [3] $F(\lambda K, \lambda L) = 2(\lambda K)^{1/2}(\lambda L)^{1/2} = 2\lambda^{1/2}K^{1/2}\lambda^{1/2}L^{1/2} = \lambda 2K^{1/2}L^{1/2} = \lambda F(K, L) \rightarrow CRS$

b) [3] $MPL = \frac{2}{2}K^{1/2}L^{-1/2}$ so $\frac{\partial MPL}{\partial L} = 1\frac{-1}{2}K^{1/2}L^{-3/2} < 0 \rightarrow$ MPL decreases in L

$MPK = \frac{2}{2}K^{-1/2}L^{1/2}$ so $\frac{\partial MPK}{\partial L} = 1\frac{-1}{2}K^{-3/2}L^{1/2} < 0 \rightarrow$ MPK decreases in K

c) [6] Now $y = 8L^{1/2}$. To find labor demand, use the condition that MPL equals the real wage, $4L^{-1/2} = \frac{4}{2} = 2$. Solve for L to find $L^D = 16(\frac{w_L}{p})^{-2}$.

To find the equilibrium real wage, note that in equilibrium $L^S = L^D$, so $16 = 16(\frac{w_L}{p})^{-2}$. Solve for $\frac{w_l}{p} = 1$.

Now the minimum wage is set above the equilibrium real wage, so we expect unemployment. Find the number of workers hired by plugging the minimum wage in to the labor demand function to find $L^D(\frac{w_l}{p} = 2) = 4$. Then the unemployment rate is $\dfrac{L^S - L^D}{L^S} = \dfrac{16-4}{16} = 3/4$. Graph in Figure 3.

d) [5] Need to eliminate K and L from typical equation $TC = w_K K + w_L L$

1) Solve for the optimal allocation across inputs using $\frac{MPK}{MPL} = \frac{w_K}{w_L} = 2$. Gives allocation $2K^* = L^*$.

2) Using the allocation from (1), eliminate one input in production function and solve for remaining input $K = y/(2\sqrt{2})$

3) Using the allocation from (1), eliminate one input from $TC = w_L L + w_K K$ and then substitute in y using the expression from (2). This yields $TC = 8K = \frac{4}{\sqrt{2}}y = 2\sqrt{2}$. Graph is a line from origin of y-\$ plane with slope $2\sqrt{2}$.

e) [3] See lecture notes 16 slide 9. Profit Maximizing Conditions:

1) $MPK = \frac{w_K}{p}$

2) $MPL = \frac{w_L}{p}$

Take the ratio of conditions (1) and (2) to find the cost minimizing condition $\dfrac{MPK}{MPL} = \dfrac{w_K/p}{w_L/p} = \dfrac{w_K}{w_L}$. See lecture notes 16 slide 9.

Problem 4 (20p) Individual Supply and Entry

a) [4] $ATC = 4/y + 4y$. Graph in Figure 4.

When y is close to zero, the FC/y term dominates and gets arbitrarily large because fixed costs are being averaged over a small level of output. When y is large, the FC/y term goes to goes to zero and the expression converges to the line $cost = 4y$, which is increasing in y because costs per unit increase due to diminishing returns.

b) [4] Use either $\partial ATC/\partial y = 0$ or $ATC = MC$ to find $y^{mes} = 1$ and $ATC^{mes} = ATC(y^{mes}) = 8$

c) [4] The firm wants to set y so that $p = MC$ because for price-takers this maximizes the profit function by producing goods as long as $MR \geq MC$. With a fixed cost, the firm only wants to operate if it is at least breaking even, so it only produces $y > 0$ if $p^* \geq p^{BE} = p^{mes} = ATC^{mes}$.

d) [4] Produce so that $p = MC = 8$ or $y = p/8$ if producing at all. Supply function is overlapping red line in Figure 4.

$$p(y) = \begin{cases} p/8 & \text{if } p \geq 8 \\ 0 & \text{if } p < 8 \end{cases}$$

e) [4] With free entry we know profits will go to zero, so firms produce $y^{mes} = 1$ and sell at $p^{BE} = 8$. By the demand curve, $Q^D = D(4) = 20 - 8 = 12$. In equilibrium $Q^D = Q^S$, so $Q^S = 12$. Since all firms are identical, $Q^S = Ny^{mes}$, so the number of firms $N = 12/1 = 12$.

Figure 1: #1 (a), (c), (d)

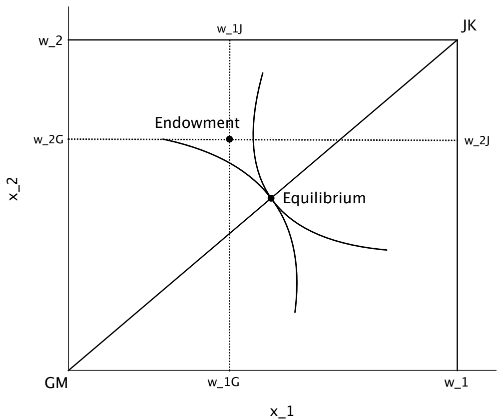


Figure 2: Indifference Curves #2.b

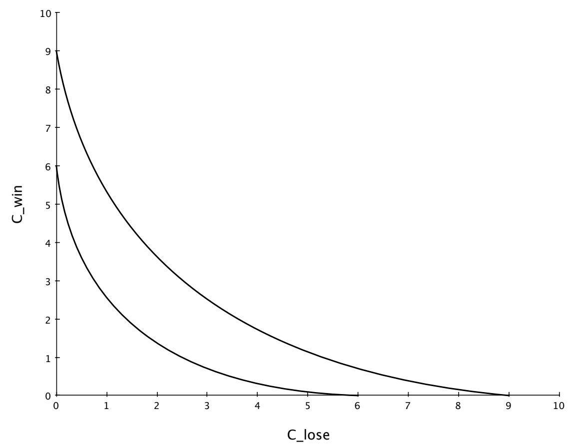


Figure 3: Cost Functions #3 (c) and (d)

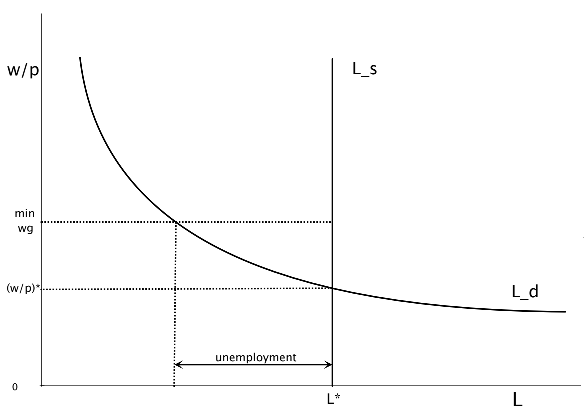
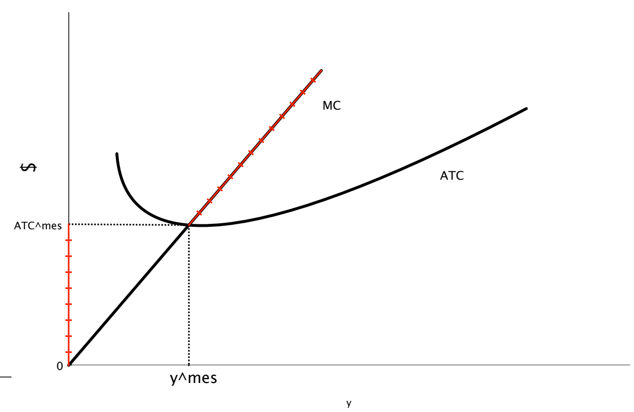


Figure 4: Cost and Supply Curves #4 (c) and (d)



Group D

All graphs on last page of key.

Problem 1 (40p) Edgeworth Box and Intertemporal Choice

a) [6] See Figure 1 at end (ignore extra line in corners).

b) [8] An allocation is Pareto efficient if there is no other allocation that makes any individual better off without making at least one individual worse off.

For agents A and B, the MRS condition is that $MRS^G = MRS^J$.

c) [8] Contract curve conditions:

1) No waste: $x_1^J + x_1^J = \omega_1^G + \omega_1^J = 140$ and $x_2^G + x_2^J = \omega_2^G + \omega_2^J = 140$

2) $MRS^J = MRS^G$

$MRS^G = MRS^J \iff \frac{1/x_1^G}{.5/x_2^G} = \frac{1/x_1^J}{.5/x_2^J} \iff \frac{2x_2^G}{x_1^G} = \frac{2x_2^J}{x_1^J}$. Use condition (1) to find $\frac{2x_2^G}{x_1^G} = \frac{2x_2^J}{x_1^J} = \frac{2(140-x_2^G)}{(140-x_1^G)}$. Then by algebra, $x_2^G = x_1^G$.

d) [8] Normalize $p_2 = 1$. Then use an individual's optimization condition $-MRS^G = \frac{p_1}{p_2} = p_1$. By the contract curve, $-MRS^G = \frac{2x_2^G}{x_1^G} = \frac{2x_1^G}{x_1^G} = 2$, so $p_1 = 2$.

Use these prices in the Cobb-Douglas magic formulas to find:

$$x_1^G = (\frac{a}{a+b})(\frac{p_1\omega_1^G + p_2\omega_2^G}{p_1}) = (\frac{1}{3/2})(\frac{2*100+40}{2}) = 80 = x_2^G$$

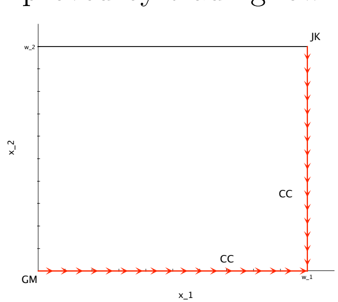
$$x_1^J = (\frac{a}{a+b})(\frac{p_1\omega_1^J + p_2\omega_2^J}{p_1}) = (\frac{1}{3/2})(\frac{2*40+100}{2}) = 60 = x_2^J$$

Alternatively, magic formulas as functions of p_1 sum to endowments.

e) [6] $MRS^G = \frac{-2*80}{80}$ equals $MRS^J = \frac{-2*60}{60}$.

f) [4] Use $\frac{p1}{p2} = 1 + r$, so $2 = 1 + r \iff r = 1$.

g) [0] Contract curve will be on the two edges of the box where agents consume the higher marginal utility good. Since they prefer different goods, interior allocations can always be improved by trading low for high MU goods.



Problem 2 (20p) Uncertainty

a) [2] $EV = \frac{1}{2}4 + \frac{1}{2}9 = 6.5$

b) [4] $U(C_L, C_W) = \frac{1}{2}\sqrt{C_L} + \frac{1}{2}\sqrt{C_W}$ See Figure 2 below.

c) [5] $U(Q, Q) = .5\sqrt{4} + .5\sqrt{9} = 2.5$

$u(EV) = \sqrt{EV} = \sqrt{6.5}$

The utility of the expected value is larger than the expected utility because the bernoulli utility function $\sqrt{c}$ is concave, making the agent risk averse.

d) [5] $U(Q, Q) = .54^2 + .59^2 = 97/2 = 48.5$

$u(EV) = EV^2 = 6.5^2 = (13/2)^2 = 169/4$

The utility of the expected value is smaller than the expected utility because the bernoulli utility function c^2 is convex, making the agent risk loving.

e) [4] The certainty equivalent of a lottery is the payment delivered with certainty that yields the same utility as the expected utility of the lottery.

1) $u(CE) = \sqrt{CE} = 1.5$ so $CE = 1.5^2 = 2.25$
2) $u(CE) = CE^2 = 8.5$ so $CE = \sqrt{8.5}$

Problem 3 (20p) Producers

a) [3] $F(\lambda K, \lambda L) = 2(\lambda K)^{1/2}(\lambda L)^{1/2} = 2\lambda^{1/2}K^{1/2}\lambda^{1/2}L^{1/2} = \lambda Q K^{1/2}L^{1/2} = \lambda F(K, L) \rightarrow CRS$

b) [3] $MPL = \frac{2}{2}K^{1/2}L^{-1/2}$ so $\frac{\partial MPL}{\partial L} = 1\frac{-1}{2}K^{1/2}L^{-3/2} < 0 \rightarrow$ MPL decreases in L

$MPK = \frac{2}{2}K^{-1/2}L^{1/2}$ so $\frac{\partial MPK}{\partial L} = 1\frac{-1}{2}K^{-3/2}L^{1/2} < 0 \rightarrow$ MPK decreases in K

c) [6] Now $y = 6L^{1/2}$. To find labor demand, use the condition that MPL equals the real wage, $3L^{-1/2} = \frac{w_L}{p}$. Solve for L to find $L^D = 9(\frac{w_L}{p})^{-2}$.

To find the equilibrium real wage, note that in equilibrium $L^S = L^D$, so $9 = 9(\frac{w_L}{p})^{-2}$. Solve for $\frac{w_l}{p} = 1$.

Here the minimum wage is set above the equilibrium real wage, so we expect unemployment. Find the number of workers hired by plugging the minimum wage in to the labor demand function to find $L^D(\frac{w_L}{p} = 2) = 9/4$. Then the unemployment rate is $\dfrac{L^S - L^D}{L^S} = \dfrac{9 - 9/4}{9} = 3/4$. Graph in Figure 3.

d) [5] Need to eliminate K and L from typical equation $TC = w_K K + w_L L$

1) Solve for the optimal allocation across inputs using $\frac{MPK}{MPL} = \frac{w_K}{w_L} = 1/2$. Gives allocation $K^* = 2L^*$.

2) Using the allocation from (1), eliminate one input in production function and solve for remaining input $L = y/(2\sqrt{2})$

3) Using the allocation from (1), eliminate one input from $TC = w_L L + w_K K$ and then substitute in y using the expression from (2). This yields $TC = 4L = \frac{2}{\sqrt{2}} y = \sqrt{2}y$. Graph is a line from origin of y-\$ plane with slope $\sqrt{2}$.

e) [3] See lecture notes 16 slide 9. Profit Maximizing Conditions:

1) $MPK = \frac{w_K}{p}$

2) $MPL = \frac{w_L}{p}$

Take the ratio of conditions (1) and (2) to find the cost minimizing condition $\dfrac{MPK}{MPL} = \dfrac{w_K/p}{w_L/p} = \dfrac{w_K}{w_L}$.

Problem 4 (20p) Individual Supply and Entry

a) [4] $ATC = 4/y + y$. Graph in Figure 4.

When y is close to zero, the $4/y$ term dominates and gets arbitrarily large because fixed costs are being averaged over a small level of output. When y is large, the FC/y term goes to goes to zero and the expression converges to the line $cost = y$, which is increasing in y because costs per unit increase due to diminishing returns.

b) [4] Use either $\partial ATC/\partial y = 0$ or $ATC = MC$ to find $y^{mes} = 2$ and $ATC^{mes} = ATC(y^{mes}) = 4$

c) [4] The firm wants to set y so that $p = MC$ because for price-takers this maximizes the profit function by producing goods as long as $MR \geq MC$. With a fixed cost, the firm only wants to operate if it is at least breaking even, so it only produces $y > 0$ if $p^* \geq p^{BE} = p^{mes} = ATC^{mes}$.

d) [4] Produce so that $p = MC = 2y$ or $y = p/2$ if producing at all. Supply function is overlapping red line in Figure 4.

$$p(y) = \begin{cases} p/2 & \text{if } p \geq 4 \\ 0 & \text{if } p < 4 \end{cases}$$

e) [4] With free entry we know profits will go to zero, so firms produce $y^{mes} = 2$ and sell at $p^{BE} = 4$. By the demand curve, $Q^D = D(p^{BE}QQ) = 20 - 4 = 16$. In equilibrium $Q^D = Q^S$, so $Q^S = 16$. Since all firms are identical, $Q^S = Ny^{mes}$, so the number of firms $N = 16/2 = 8$.

Figure 1: #1 (a), (c), (d)

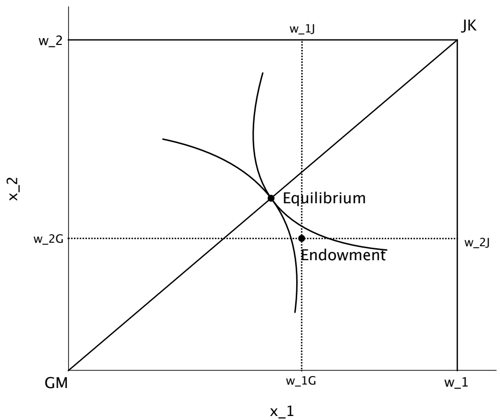


Figure 2: Indifference Curves #2.b

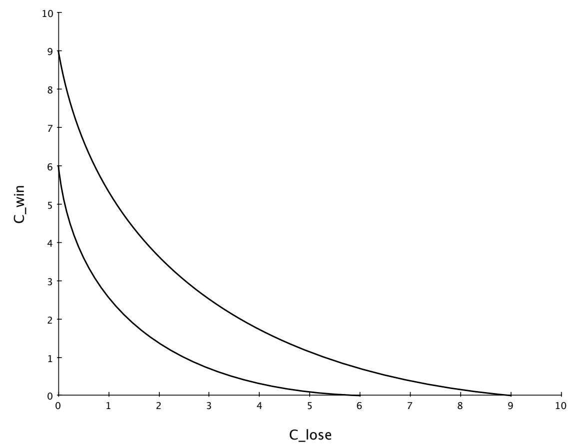


Figure 3: Cost Functions #3 (c) and (d)

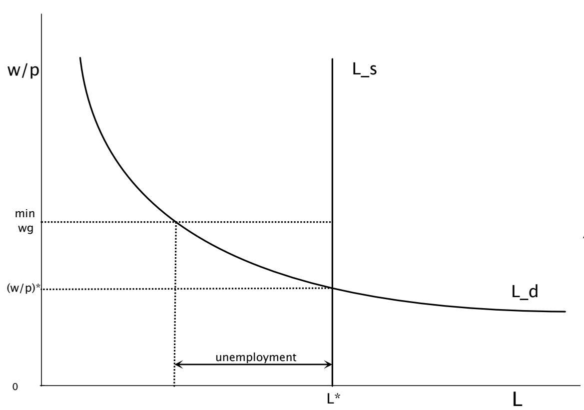
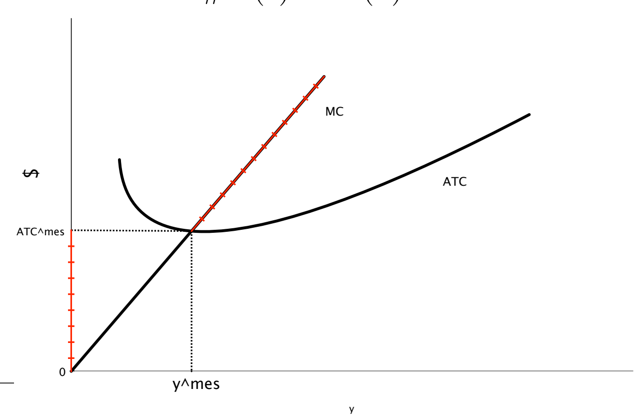


Figure 4: Cost and Supply Curves #4 (c) and (d)



Group E

All graphs on last page of key.

Problem 1 (40p) Edgeworth Box and Intertemporal Choice

a) [6] See Figure 1 at end (ignore extra line in corners).

b) [8] An allocation is Pareto efficient if there is no other allocation that makes any individual better off without making at least one individual worse off.

For agents A and B, the MRS condition is that $MRS^G = MRS^J$.

c) [8] Contract curve conditions:

1) No waste: $x_1^J + x_1^J = \omega_1^G + \omega_1^J = 480$ and $x_2^G + x_2^J = \omega_2^G + \omega_2^J = 280$

2) $MRS^J = MRS^G$

$MRS^G = MRS^J \iff \frac{1/x_1^G}{.5/x_2^G} = \frac{1/x_1^J}{.5/x_2^J} \iff \frac{2x_2^G}{x_1^G} = \frac{2x_2^J}{x_1^J}$. Use condition (1) to find $\frac{2x_2^G}{x_1^G} = \frac{2x_2^J}{x_1^J} = \frac{2(280-x_2^G)}{(480-x_1^G)}$. Then by algebra, $x_2^G = \frac{7}{12}x_1^G$.

d) [8] Normalize $p_2 = 1$. Then use an individual's optimization condition $-MRS^G = \frac{p_1}{p_2} = p_1$. By the contract curve, $-MRS^G = \frac{2x_2^G}{x_1^G} = \frac{2\frac{7}{12}x_1^G}{x_1^G} = \frac{7}{6}$, so $p_1 = 7/6$.

Use these prices in the Cobb-Douglas magic formulas to find:

$$x_1^G = (\frac{a}{a+b})(\frac{p_1\omega_1^G + p_2\omega_2^G}{p_1}) = (\frac{1}{3/2})(\frac{\frac{7}{6}*400+80}{\frac{7}{6}}) = \frac{6560}{21} \text{ and } x_2^G = \frac{7}{12}*\frac{6560}{21}$$

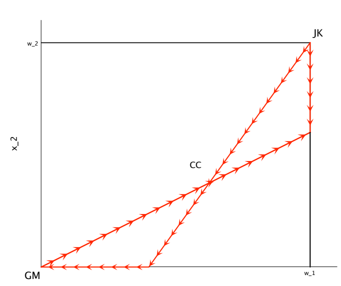
$$x_1^J = (\frac{a}{a+b})(\frac{p_1\omega_1^J + p_2\omega_2^J}{p_1}) = (\frac{1}{3/2})(\frac{\frac{7}{6}*80+200}{\frac{7}{6}}) = \frac{3520}{21} \text{ and } x_2^J = \frac{7}{12}*\frac{3520}{21}$$

Alternatively, magic formulas as functions of p_1 sum to endowments.

e) [6] $MRS^G = \frac{-2*\frac{6560}{21}}{\frac{7}{12}*\frac{6560}{21}}$ equals $MRS^J = \frac{-2*\frac{3520}{21}}{\frac{7}{12}*\frac{3520}{21}}$.

f) [4] Use $\frac{p1}{p2} = 1 + r$, so $r = 1/6$.

g) [0] Contract curve will be the two optimal proportion lines and the outside edges near them. Graph below not to scale.



Problem 2 (20p) Uncertainty

a) [2] $EV = \frac{1}{2}1 + \frac{1}{2}9 = 5$

b) [4] $U(C_L, C_W) = \frac{1}{2}\sqrt{C_L} + \frac{1}{2}\sqrt{C_W}$ See Figure 2 below.

c) [5] $U(1, 9) = .5\sqrt{1} + .5\sqrt{9} = 2$

$u(EV) = \sqrt{EV} = \sqrt{5}$

The utility of the expected value is larger than the expected utility because the bernoulli utility function $\sqrt{c}$ is concave, making the agent risk averse.

d) [5] $U(1, 9) = .51^2 + .59^2 = 41$

$u(EV) = EV^2 = 5^2 = 25$

The utility of the expected value is smaller than the expected utility because the bernoulli utility function c^2 is convex, making the agent risk loving.

e) [4] The certainty equivalent of a lottery is the payment delivered with certainty that yields the same utility as the expected utility of the lottery.

1) $u(CE) = \sqrt{CE} = 2$ so $CE = 2^2 = 4$
2) $u(CE) = CE^2 = 41$ so $CE = \sqrt{41}$

Problem 3 (20p) Producers

a) [3] $F(\lambda K, \lambda L) = 12(\lambda K)^{1/2}(\lambda L)^{1/2} = 12\lambda^{1/2}K^{1/2}\lambda^{1/2}L^{1/2} = \lambda 12K^{1/2}L^{1/2} = \lambda F(K, L) \rightarrow CRS$

b) [3] $MPL = \frac{12}{2}K^{1/2}L^{-1/2}$ so $\frac{\partial MPL}{\partial L} = 6\frac{-1}{2}K^{1/2}L^{-3/2} < 0 \rightarrow$ MPL decreases in L

$MPK = \frac{12}{2}K^{-1/2}L^{1/2}$ so $\frac{\partial MPK}{\partial L} = 6\frac{-1}{2}K^{-3/2}L^{1/2} < 0 \rightarrow$ MPK decreases in K

c) [6] Now $y = 24L^{1/2}$. To find labor demand, use the condition that MPL equals the real wage, $12L^{-1/2} = \frac{w_L}{p}$. Solve for L to find $L^D = 144(\frac{w_L}{p})^{-2}$.

To find the equilibrium real wage, note that in equilibrium $L^S = L^D$, so $9 = 144(\frac{w_L}{p})^{-2}$. Solve for $\frac{w_l}{p} = 4$.

Now the minimum wage is set below the equilibrium real wage, so it has no bite and we do not expect unemployment. Then the unemployment rate is 0. Graph in Figure 3.

d) [5] Need to eliminate K and L from typical equation $TC = w_K K + w_L L$

1) Solve for the optimal allocation across inputs using $\frac{MPK}{MPL} = \frac{w_K}{w_L} = 1/4$. Gives allocation $K^* = 4L^*$.

2) Using the allocation from (1), eliminate one input in production function and solve for remaining input $L = y/24$

3) Using the allocation from (1), eliminate one input from $TC = w_L L + w_K K$ and then substitute in y using the expression from (2). This yields $TC = \frac{1}{3}y$. Graph is a line from origin of y-\$ plane with slope 1/3.

e) [3] See lecture notes 16 slide 9. Profit Maximizing Conditions:

1) $MPK = \frac{w_K}{p}$
2) $MPL = \frac{w_L}{p}$

Take the ratio of conditions (1) and (2) to find the cost minimizing condition $\dfrac{MPK}{MPL} = \dfrac{w_K/p}{w_L/p} = \dfrac{w_K}{w_L}$.

Problem 4 (20p) Individual Supply and Entry

a) [4] $ATC = 5/y + 5y$. Graph in Figure 4.

When y is close to zero, the FC/y term dominates and gets arbitrarily large because fixed costs are being averaged over a small level of output. When y is large, the FC/y term goes to goes to zero and the expression converges to the line $cost = 5y$, which is increasing in y because costs per unit increase due to diminishing returns.

b) [4] Use either $\partial ATC/\partial y = 0$ or $ATC = MC$ to find $y^{mes} = 1$ and $ATC^{mes} = ATC(y^{mes}) = 10$

c) [4] The firm wants to set y so that $p = MC$ because for price-takers this maximizes the profit function by producing goods as long as $MR \geq MC$. With a fixed cost, the firm only wants to operate if it is at least breaking even, so it only produces $y > 0$ if $p^* \geq p^{BE} = p^{mes} = ATC^{mes}$.

d) [4] Produce so that $p = MC = 10y$ or $y = p/10$ if producing at all. Supply function is overlapping red line in Figure 4.

$$p(y) = \begin{cases} p/10 & \text{if } p \geq 10 \\ 0 & \text{if } p < 10 \end{cases}$$

e) [4] With free entry we know profits will go to zero, so firms produce $y^{mes} = 1$ and sell at $p^{BE} = 10$. By the demand curve, $Q^D = D(10) = 20 - 10 = 10$. In equilibrium $Q^D = Q^S$, so $Q^S = 10$. Since all firms are identical, $Q^S = Ny^{mes}$, so the number of firms $N = Q^S/y^{mes} = 10$.

Figure 1: #1 (a), (c), (d)

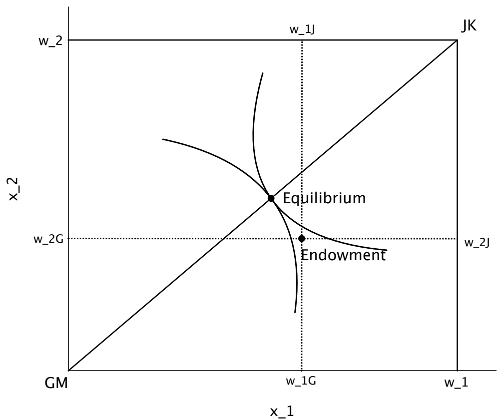


Figure 2: Indifference Curves #2.b

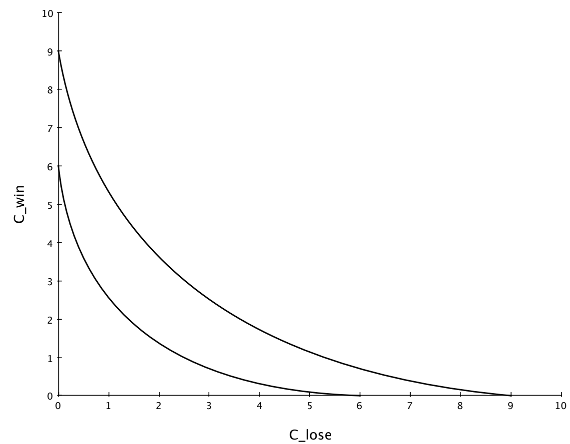


Figure 3: Cost Functions #3 (c) and (d)

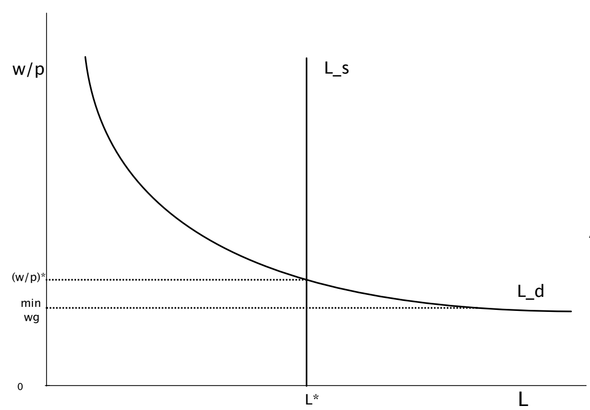


Figure 4: Cost and Supply Curves #4 (c) and (d)