Econ 301
Intermediate Microeconomics
Prof. Marek Weretka

Midterm 1 (A)

You have 70 minutes to complete the exam. The midterm consists of 4 questions (40+15+20+25=100 points) + bonus (just for fun). Make sure you answer the first four questions before working on the bonus one!

Problem 1 (40p) (Well-behaved preferences)

Lila spends her income on two goods: food, x_1, and clothing, x_2.

a) The price food is $p_1 = 2$ and one piece of clothing costs $p_2 = 4$. Show geometrically Lila's budget set if her income is $m = 30$. Find the relative price food in terms of clothing (one number)? Where can the relative price be seen in the graph of a budget set? (one sentence)

b) Lila's preferences are represented by utility function

$$U(x_1, x_2) = (x_1)^2 (x_2)^1 .$$

We know that her preferences can be alternatively represented by function

$$V(x_1, x_2) = 2 \ln x_1 + \ln x_2.$$

Explain the idea behind "monotonic transformation" (one sentence) and derive function V from U.

c) Assuming utility function $V(x_1, x_2) = 2 \ln x_1 + \ln x_2$

-Find marginal rate of substitution (MRS) for all bundles (derive formula). For bundle $(8, 8)$ find the value of MRS (one number). Give economic interpretation of MRS (one sentence). Which of the goods is more valuable given consumption $(8, 8)$?

-Write down two secrets of happiness that determine optimal choice given parameters p_1, p_2 and m. Explain economic intuition behind the two conditions (two sentences for each).

-Using secrets of happiness derive optimal consumption x_1,x_2, given values of $p_1 = 2, p_2 = 4$ and $m = 30$. Is the solution corner or interior (chose one)

d) (Harder) Using magic formulas for Cobb-Douglass preferences argue that the two commodities are 1) ordinary; 2) normal; and 3) neither gross complements nor gross substitutes (one sentence for each property).

Problem 2 (15p) (Perfect substitutes)

You are planing a budget for the state of Wisconsin. The two major budget positions include education, x_1 and health care, x_2. Your preferences over the two are represented by function

$$U(x_1, x_2) = 3x_1 + x_2.$$

a) Find marginal rate of substitution (give number).

b) Find optimal consumption of x_1 and x_2, given prices $p_1 = 2$ and $p_2 = 1$ and available funds $m = 50$ (two numbers).

c) Is solution interior? (yes-no answer). Is marginal utility of a dollar equalized? (give two numbers and yes-no answer)

Problem 3 (20p) (Intertemporal choice)

Zoe is a professional Olympic skier. Her income when "young" is high ($m_1 = 30$) but her future (period two) is not so bright ($m_2 = 20$)

a) Depict Zoe's budget set, assuming that she can borrow and save at the interest rate $r = 100\%$. Partition her budget set into three regions: the area that involves saving, borrowing and none of the two.

b) Find PV and FV of Zoe's lifetime income (two numbers) and show the two values in the graph. Interpret economically PV.

c) Zoe's utility function is $U(C_1, C_2) = \ln C_1 + \frac{1}{1+\delta} \ln C_2$ where discount rate is $\delta = 2$. Using magic formulas, find optimal consumption plan (C_1, C_2) (two numbers) and the corresponding level of savings/borrowing, S.

d) Is Zoe tilting her consumption over time? (yes-no answer).

Problem 4 (25p) (Short questions)

a) Given utility function $U(C, R) = \min(2C, R)$, daily endowment of time $24h$, price and wage $p_c = w = 2$, find optimal choice of C, relaxation time R and labor supply L. (three numbers, use secrets of happiness for perfect complements).

b) Find optimal choice given quasilinear preferences $U(x_1, x_2) = x_1 + 100 \ln x_2$, prices $p_1 = \$8, p_2 = \$1$ and income $m = \$60$. Is your solution corner or interior?

c) Assume Interest rate $r = 10\%$. Choose one of the two: a consol (a type of British government bond) that pays annually $\$1000$ forever, starting next year or cash $\$12,000$ now (compare PV).

d) Your annual income when working (age 21-60) is $\$60,000$ and then you are are going to live for the next 30 years. Write down equation that determines constant (maximal) level of consumption during your lifetime. Assume annual interest rate $r = 2\%$.

Bonus question (Just for fun)

a) Derive magic formulas for perfect complements $U(x_1, x_2) = \min(ax_1, bx_2)$ that give optimal choices x_1, x_2 as a function of a, b, p_1, p_2, m.

b) Provide economic intuition for the magic formula for perfect complements (economic interpretation for the numerator and denominator).

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Midterm 1 (B)

You have 70 minutes to complete the exam. The midterm consists of 4 questions (40+15+20+25=100 points) + bonus (just for fun). Make sure you answer the first four questions before working on the bonus one!

Problem 1 (40p) (Well-behaved preferences)

Lila spends her income on two goods: food, x_1, and clothing, x_2.

a) The price food is $p_1 = 2$ and one piece of clothing costs $p_2 = 4$. Show geometrically Lila's budget set if her income is $m = 60$. Find the relative price food in terms of clothing (one number)? Where can the relative price be seen in the graph of a budget set? (one sentence)

b) Lila's preferences are represented by utility function

$$U(x_1, x_2) = (x_1)^2 (x_2)^1 .$$

We know that her preferences can be alternatively represented by function

$$V(x_1, x_2) = 2 \ln x_1 + \ln x_2.$$

Explain the idea behind "monotonic transformation" (one sentence) and derive function V from U.

c) Assuming utility function $V(x_1, x_2) = 2 \ln x_1 + \ln x_2$

-Find marginal rate of substitution (MRS) for all bundles (derive formula). For bundle $(2, 2)$ find the value of MRS (one number). Give economic interpretation of MRS (one sentence). Which of the goods is more valuable given consumption $(2, 2)$?

-Write down two secrets of happiness that determine optimal choice given parameters p_1, p_2 and m. Explain economic intuition behind the two conditions (two sentences for each).

-Using secrets of happiness derive optimal consumption x_1,x_2, given values of $p_1 = 2, p_2 = 4$ and $m = 60$. Is the solution corner or interior (chose one)

d) (Harder) Using magic formulas for Cobb-Douglass preferences argue that the two commodities are 1) ordinary; 2) normal; and 3) neither gross complements nor gross substitutes (one sentence for each property).

Problem 2 (15p) (Perfect substitutes)

You are planing a budget for the state of Wisconsin. The two major budget positions include education, x_1 and health care, x_2. Your preferences over the two are represented by function

$$U(x_1, x_2) = x_1 + x_2.$$

a) Find marginal rate of substitution (give number).

b) Find optimal consumption of x_1 and x_2, given prices $p_1 = 2$ and $p_2 = 1$ and available funds $m = 50$ (two numbers).

c) Is solution interior? (yes-no answer). Is marginal utility of a dollar equalized? (give two numbers and yes-no answer)

Problem 3 (20p) (Intertemporal choice)

Zoe is a professional Olympic skier. Her income when "young" is high ($m_1 = 40$) but her future (period two) is not so bright ($m_2 = 20$)

a) Depict Zoe's budget set, assuming that she can borrow and save at the interest rate $r = 100\%$. Partition her budget set into three regions: the area that involves saving, borrowing and none of the two.

b) Find PV and FV of Zoe's lifetime income (two numbers) and show the two values in the graph. Interpret economically PV.

c) Zoe's utility function is $U(C_1, C_2) = \ln C_1 + \frac{1}{1+\delta} \ln C_2$ where discount rate is $\delta = 3$. Using magic formulas, find optimal consumption plan (C_1, C_2) (two numbers) and the corresponding level of savings/borrowing, S.

d) Is Zoe tilting her consumption over time? (yes-no answer).

Problem 4 (25p) (Short questions)

a) Given utility function $U(C, R) = \min(2C, R)$, daily endowment of time $24h$, price and wage $p_c = w = 1$, find optimal choice of C, relaxation time R and labor supply L. (three numbers, use secrets of happiness for perfect complements).

b) Find optimal choice given quasilinear preferences $U(x_1, x_2) = x_1 + 100 \ln x_2$, prices $p_1 = \$8, p_2 = \$1$ and income $m = \$80$. Is your solution corner or interior?

c) Assume Interest rate $r = 10\%$. Choose one of the two: a consol (a type of British government bond) that pays annually $\$1000$, starting next year or cash $\$8,000$ now (compare PV).

d) Your annual income when working (age 21-60) is $\$70,000$ and then you are are going to live for the next 35 years. Write down equation that determines constant (maximal) level of consumption during your lifetime. Assume annual interest rate $r = 10\%$.

Bonus question (Just for fun)

a) Derive magic formulas for perfect complements $U(x_1, x_2) = \min(ax_1, bx_2)$ that give optimal choices x_1, x_2 as a function of a, b, p_1, p_2, m.

b) Provide economic intuition for the magic formula for perfect complements (economic interpretation for the numerator and denominator).

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Midterm 1 (C)

You have 70 minutes to complete the exam. The midterm consists of 4 questions (40+15+20+25=100 points) + bonus (just for fun). Make sure you answer the first four questions before working on the bonus one!

Problem 1 (40p) (Well-behaved preferences)

Lila spends her income on two goods: food, x_1, and clothing, x_2.

a) The price food is $p_1 = 2$ and one piece of clothing costs $p_2 = 4$. Show geometrically Lila's budget set if her income is $m = 60$. Find the relative price food in terms of clothing (one number)? Where can the relative price be seen in the graph of a budget set? (one sentence)

b) Lila's preferences are represented by utility function

$$U(x_1, x_2) = (x_1)^1 (x_2)^2 . \tag{1}$$

We know that her preferences can be alternatively represented by function

$$V(x_1, x_2) = \ln x_1 + 2 \ln x_2. \tag{2}$$

Explain the idea behind "monotonic transformation" (one sentence) and derive function V from U.

c) Assuming utility function $V(x_1, x_2) = \ln x_1 + 2 \ln x_2$

-Find marginal rate of substitution (MRS) for all bundles (derive formula). For bundle $(2, 2)$ find the value of MRS (one number). Give economic interpretation of MRS (one sentence). Which of the goods is more valuable given consumption $(2, 2)$?

-Write down two secrets of happiness that determine optimal choice given parameters p_1, p_2 and m. Explain economic intuition behind the two conditions (two sentences for each).

-Using secrets of happiness derive optimal consumption x_1,x_2, given values of $p_1 = 2, p_2 = 4$ and $m = 60$. Is the solution corner or interior (chose one)

d) (Harder) Using magic formulas for Cobb-Douglass preferences argue that the two commodities are 1) ordinary; 2) normal; and 3) neither gross complements nor gross substitutes (one sentence for each property).

Problem 2 (15p) (Perfect substitutes)

You are planing a budget for the state of Wisconsin. The two major budget positions include education, x_1 and health care, x_2. Your preferences over the two are represented by function

$$U(x_1, x_2) = 2x_1 + 2x_2.$$

a) Find marginal rate of substitution (give number).

b) Find optimal consumption of x_1 and x_2, given prices $p_1 = 2$ and $p_2 = 1$ and available funds $m = 100$ (two numbers).

c) Is solution interior? (yes-no answer). Is marginal utility of a dollar equalized? (give two numbers and yes-no answer)

Problem 3 (20p) (Intertemporal choice)

Zoe is a professional Olympic skier. Her income when "young" is high ($m_1 = 80$) but her future (period two) is not so bright ($m_2 = 40$)

a) Depict Zoe's budget set, assuming that she can borrow and save at the interest rate $r = 100\%$. Partition her budget set into three regions: the area that involves saving, borrowing and none of the two.

b) Find PV and FV of Zoe's lifetime income (two numbers) and show the two values in the graph. Interpret economically PV.

c) Zoe's utility function is $U(C_1, C_2) = \ln C_1 + \frac{1}{1+\delta} \ln C_2$ where discount rate is $\delta = 3$. Using magic formulas, find optimal consumption plan (C_1, C_2) (two numbers) and the corresponding level of savings/borrowing, S.

d) Is Zoe tilting her consumption over time? (yes-no answer).

Problem 4 (25p) (Short questions)

a) Given utility function $U(C, R) = \min(C, 2R)$, daily endowment of time $24h$, price and wage $p_c = w = 1$, find optimal choice of C, relaxation time R and labor supply L. (three numbers, use secrets of happiness for perfect complements).

b) Find optimal choice given quasilinear preferences $U(x_1, x_2) = x_1 + 50 \ln x_2$, prices $p_1 = \$8, p_2 = \$1$ and income $m = \$40$. Is your solution corner or interior?

c) Assume Interest rate $r = 10\%$. Choose one of the two: a consol (a type of British government bond) that pays annually $\$500$ starting next year or cash $\$4,000$ now (compare PV).

d) Your annual income when working (age 21-60) is $\$70,000$ and then you are are going to live for the next 35 years. Write down equation that determines constant (maximal) level of consumption during your lifetime. Assume annual interest rate $r = 10\%$.

Bonus question (Just for fun)

a) Derive magic formulas for perfect complements $U(x_1, x_2) = \min(ax_1, bx_2)$ that give optimal choices x_1, x_2 as a function of a, b, p_1, p_2, m.

b) Provide economic intuition for the magic formula for perfect complements (economic interpretation for the numerator and denominator).

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Midterm 1 (D)

You have 70 minutes to complete the exam. The midterm consists of 4 questions (40+15+20+25=100 points) + bonus (just for fun). Make sure you answer the first four questions before working on the bonus one!

Problem 1 (40p) (Well-behaved preferences)

Lila spends her income on two goods: food, x_1, and clothing, x_2.

a) The price food is $p_1 = 4$ and one piece of clothing costs $p_2 = 8$. Show geometrically Lila's budget set if her income is $m = 120$. Find the relative price food in terms of clothing (one number)? Where can the relative price be seen in the graph of a budget set? (one sentence)

b) Lila's preferences are represented by utility function

$$U(x_1, x_2) = (x_1)^1 (x_2)^2 . \tag{3}$$

We know that her preferences can be alternatively represented by function

$$V(x_1, x_2) = \ln x_1 + 2 \ln x_2. \tag{4}$$

Explain the idea behind "monotonic transformation" (one sentence) and derive function V from U.

c) Assuming utility function $V(x_1, x_2) = \ln x_1 + 2 \ln x_2$

-Find marginal rate of substitution (MRS) for all bundles (derive formula). For bundle $(2, 2)$ find the value of MRS (one number). Give economic interpretation of MRS (one sentence). Which of the goods is more valuable given consumption $(2, 2)$?

-Write down two secrets of happiness that determine optimal choice given parameters p_1, p_2 and m. Explain economic intuition behind the two conditions (two sentences for each).

-Using secrets of happiness derive optimal consumption x_1,x_2, given values of $p_1 = 4, p_2 = 8$ and $m = 120$. Is the solution corner or interior (chose one)

d) (Harder) Using magic formulas for Cobb-Douglass preferences argue that the two commodities are 1) ordinary; 2) normal; and 3) neither gross complements nor gross substitutes (one sentence for each property).

Problem 2 (15p) (Perfect substitutes)

You are planing a budget for the state of Wisconsin. The two major budget positions include education, x_1 and health care, x_2. Your preferences over the two are represented by function

$$U(x_1, x_2) = 3x_1 + 3x_2.$$

a) Find marginal rate of substitution (give number).

b) Find optimal consumption of x_1 and x_2, given prices $p_1 = 2$ and $p_2 = 1$ and available funds $m = 100$ (two numbers).

c) Is solution interior? (yes-no answer). Is marginal utility of a dollar equalized? (give two numbers and yes-no answer)

Problem 3 (20p) (Intertemporal choice)

Zoe is a professional Olympic skier. Her income when "young" is high ($m_1 = 80$) but her future (period two) is not so bright ($m_2 = 40$)

a) Depict Zoe's budget set, assuming that she can borrow and save at the interest rate $r = 100\%$. Partition her budget set into three regions: the area that involves saving, borrowing and none of the two.

b) Find PV and FV of Zoe's lifetime income (two numbers) and show the two values in the graph. Interpret economically PV.

c) Zoe's utility function is $U(C_1, C_2) = \ln C_1 + \frac{1}{1+\delta} \ln C_2$ where discount rate is $\delta = 4$. Using magic formulas, find optimal consumption plan (C_1, C_2) (two numbers) and the corresponding level of savings/borrowing, S.

d) Is Zoe tilting her consumption over time? (yes-no answer).

Problem 4 (25p) (Short questions)

a) Given utility function $U(C, R) = \min(C, 2R)$, daily endowment of time $24h$, price and wage $p_c = w = 2$, find optimal choice of C, relaxation time R and labor supply L. (three numbers, use secrets of happiness for perfect complements).

b) Find optimal choice given quasilinear preferences $U(x_1, x_2) = x_1 + 50 \ln x_2$, prices $p_1 = \$8, p_2 = \$1$ and income $m = \$60$. Is your solution corner or interior?

c) Assume Interest rate $r = 10\%$. Choose one of the two: Choose one of the two: a consol (a type of British government bond) that pays annually $\$100$, starting next year or $\$1,200$ now (compare PV).

d) Your annual income when working (age 21-60) is $\$70,000$ and then you are are going to live for the next 35 years. Write down equation that determines constant (maximal) level of consumption during your lifetime. Assume annual interest rate $r = 10\%$.

Bonus question (Just for fun)

a) Derive magic formulas for perfect complements $U(x_1, x_2) = \min(ax_1, bx_2)$ that give optimal choices x_1, x_2 as a function of a, b, p_1, p_2, m.

b) Provide economic intuition for the magic formula for perfect complements (economic interpretation for the numerator and denominator).

Midterm 1 Solution for Version A

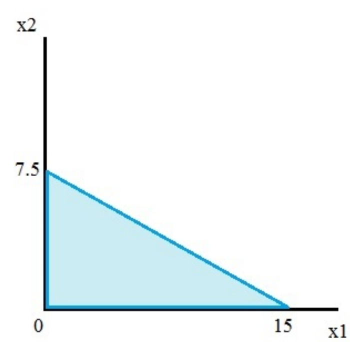
[Econ 301] Spring 2014

Problem 1 (40p) (Well-behaved preferences)

Lila spends her income on two goods: food, x1, and clothing, x2.

a) 6pts The price food is $p_1 = 2$ and one piece of clothing costs $p_2 = 4$. Show geometrically Lila's budget set if her income is $m = 30$. Find the relative price food in terms of clothing (one number)? Where can the relative price be seen in the graph of a budget set? (one sentence)

Ans) The relative price food in terms of clothing is $\frac{p_1}{p_2} = \frac{2}{4} = \frac{1}{2}$. Absolute value of slope of budget line is the relative price.



b) 7pts Lila's preferences are represented by utility function

$$U(x_1, x_2) = (x_1)^2(x_2)^1$$

We know that her preferences can be alternatively represented by function

$$V(x_1, x_2) = 2\ln x_1 + \ln x_2$$

Explain the idea behind "monotonic transformation"(one sentence) and derive function V from U.

Ans) Taking logarithm of U becomes V. Logarithm is a positive monotone transformation, and monotonic transformation of utility function does not distort original preference order. That means, whenever $U(a, b) > U(c, d)$, it is true that $V(a, b) > V(c, d)$ and vice versa.

$$V(x_1, x_2) = 2\ln x_1 + \ln x_2 = \ln x_1^2 + \ln x_2 = \ln (x_1)^2(x_2)^1 = \ln U(x_1, x_2)$$

c) 15pts Assuming utility function $V(x_1, x_2) = 2\ln x_1 + \ln x_2$

- Find marginal rate of substitution (MRS) for all bundles (derive formula). For bundle $(8, 8)$ find the value of MRS (one number). Give economic interpretation of MRS (one sentence). Which of the goods is more valuable given consumption (8, 8)?

Ans) $MRS_{12} = -\frac{MU_1}{MU_2} = -\frac{2x_2}{x_1}$. At $(8, 8)$, $MRS = -\frac{16}{8} = -2$. MRS_{12} means the amount of x_2 which consumer is willing to give up in order to get additional unit of x_1 while holding same utility level. x_1 is more (twice) valuable than x_2.

- Write down two secrets of happiness that determine optimal choice given parameters p_1,p_2 and m. Explain economic intuition behind the two conditions (two sentences for each).

Ans)

$$
\begin{aligned}
x_1p_1 + x_2p_2 &= m \\
\frac{MU_1}{P_1} = \frac{MU_2}{P_2} &\qquad or \quad MRS_{12} = -\frac{P_1}{P_2}
\end{aligned}
$$

The first equation is a budget constraint which requires consumers to spend all of their income on the consumption of two goods, x_1 and x_2. The second condition is about efficient combination of two goods. Mathematically, the slope of the indifference curve passing through the optimal consumption bundle to be tangent to the price ratio. Economically, marginal utility per dollar of each good should become equal at the optimal consumption bundle, or consumer's relative evaluation about two goods in terms of marginal utility should coincide market's relative evaluation in terms of price.

- Using secrets of happiness derive optimal consumption x_1,x_2, given values of $p_1 = 2$, $p_2 = 4$ and $m = 30$. Is the solution corner or interior (chose one)

Ans) Since utility function is Cobb-Douglas, optimal consumption is following.

$$
\begin{aligned}
x_1 &= \frac{a}{a+b}\frac{m}{p_1} = \frac{2}{3}\frac{30}{2} = 10 \\
x_2 &= \frac{b}{a+b}\frac{m}{p_2} = \frac{1}{3}\frac{30}{4} = 2.5
\end{aligned}
$$

Both consumptions are positive, so it is interior solution.

d) 12pts (Harder) Using magic formulas for Cobb-Douglass preferences argue that the two commodities are 1) ordinary, 2) normal, and 3) neither gross complements nor gross substitutes (one sentence for each property).

Ans) When p_1 increases x_1 decreases, and when p_2 increases x_2 decreases: $\frac{\partial x_1}{\partial p_1} = -\frac{a}{a+b}\frac{m}{p_1^2} < 0$, $\frac{\partial x_2}{\partial p_2} = -\frac{b}{a+b}\frac{m}{p_2^2} < 0$, so both are ordinary. When m increases, both x_1 and x_2 increases: $\frac{\partial x_1}{\partial m} = \frac{a}{(a+b)p_1} > 0$, $\frac{\partial x_2}{\partial m} = \frac{b}{(a+b)p_2} > 0$. Therefore, both are normal. When p_1 increases x_2 doesn't change, and when p_2 increases x_1 doesn't change: $\frac{\partial x_1}{\partial p_2} = 0$, $\frac{\partial x_2}{\partial p_2} = 0$. Therefore, x_1 and x_2 are gross neutral (neither gross complements nor gross substitutes).

Problem 2 (15p) (Perfect substitutes)

You are planing a budget for the state of Wisconsin. The two major budget positions include education,x_1 and health care, x_2. Your preferences over the two are represented by function

$$U(x_1, x_2) = 3x_1 + x_2$$

a) 3pts Find marginal rate of substitution (give number).

Ans) $MRS_{12} = -\frac{MU_1}{MU_2} = -\frac{3}{1} = -3$.

b) 4pts Find optimal consumption of x_1 and x_2, given prices $p_1 = 2$ and $p_2 = 1$ and available funds $m = 50$ (two numbers).

Ans) Compare marginal utility per dollars. Since $\frac{MU_1}{P_1} = \frac{3}{2} = 1.5$ is larger than $\frac{MU_2}{P_2} = \frac{1}{1} = 1$, optimal choice is consuming only x_1. $x_1 = \frac{m}{p_1} = \frac{50}{2} = 25$ and $x_2 = 0$.

c) 8pts Is solution interior? (yes-no answer). Is marginal utility of a dollar equalized? (give two numbers and yes-no answer)

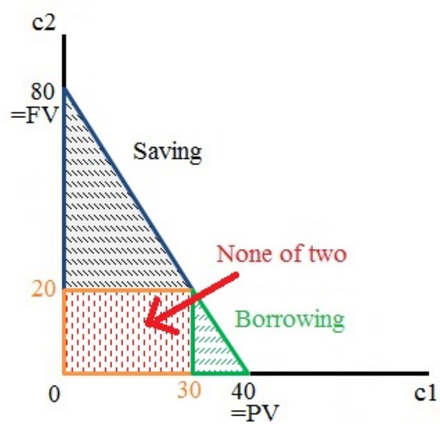
Ans) No, it is a corner solution since I consume zero x_2. No, $\frac{MU_1}{P_1} = \frac{3}{2} = 1.5 \neq \frac{MU_2}{P_2} = \frac{1}{1} = 1$.

Problem 3 (20p) (Intertemporal choice)

Zoe is a professional Olympic skier. Her income when "young" is high ($m_1 = 30$) but her future (period two) is not so bright ($m_2 = 20$).

a) 4pts Depict Zoe's budget set, assuming that she can borrow and save at the interest rate $r = 100\%$. Partition her budget set into three regions: the area that involves saving, borrowing and none of the two.

Ans) For $c_1 \leq 30, c_2 \leq 20$, neither borrowing nor saving is required. $c_1 < 30, c_2 > 20$ means saving and $c_1 > 30, c_2 < 20$ means borrowing.



b) 6pts Find PV and FV of Zoe's lifetime income (two numbers) and show the two values in the graph. Interpret economically PV.

Ans) With $r = 1$, $PV = m_1 + \frac{m_2}{1+r} = 30 + 20/2 = 40$, $FV = (1+r)m_1 + m_2 = 2 * 30 + 20 = 80$. PV is x-intercept of budget line and FV is y-intercept of budget line. PV means Zoe's life time income evaluated at time 1 and it represents the maximum amount of c_1 Zoe can afford by borrowing $\frac{m_2}{1+r} = 10$.

c) 6pts Zoe's utility function is $U(C_1, C_2) = \ln C_1 + \frac{1}{1+\delta} \ln C_2$ where discount rate is $\delta = 2$. Using magic formulas, find optimal consumption plan (C_1, C_2) (two numbers) and the corresponding level of savings/borrowing, S.

Ans) Given utility function represents same preference of $V(C_1, C_2) = C_1^{(1+\delta)} C_2 = C_1^3 C_2$. Define $P_1 = 1$, $P_2 = 1/(1+r) = 1/2$, and $m = PV = 40$. Magic formulas of Cobb-Douglas tell us to consume $C_1 = 30$ and $C_2 = 20$. There is no needs for borrowing or saving, so $S = 0$.

Alternative approach $PV(m) = PV(c)$ and $MU_1 = (1+r)MU_2$ are

two conditions for optimal choice.

$$\begin{aligned} m_1 + \frac{m_2}{1+r} &= C_1 + \frac{C_2}{1+r} \\ \frac{1}{C_1} = \frac{1+r}{(1+\delta)C_2} &\Leftrightarrow C_1 = \frac{(1+\delta)C_2}{1+r} \end{aligned}$$

Plug in numbers, then we get $C_1 = \frac{3}{2}C_2$ and $40 = C_1 + C_2/2$. Therefore $C_1 = 30$ and $C_2 = 20$. $S = m_1 - C_1 = 0$

d) 4pts Is Zoe tilting her consumption over time? (yes-no answer).

Ans) Yes, since time preference parameter $\delta = 2$ whereas interest rate $r = 1$, i.e. since $\delta > r$, it is optimal to consume more today ($C_1 = 30$) and to consume less tomorrow ($C_2 = 20$). Therefore, $C_1 \neq C_2$, and it means she tilts her consumption over time.

Problem 4 (25p) (Short questions)

a) 6pts Given utility function $U(C, R) = \min(2C, R)$, daily endowment of time $24h$, price and wage $p_c = w = 2$, find optimal choice of C, relaxation time R and labor supply L. (three numbers, use secrets of happiness for perfect complements).

Ans) Two formulas are $ax_1 = bx_2$ and $p_1x_1 + p_2x_2 = m$. Here, $2C = R$ and $2C + 2R = 48$ are optimal conditions. $C = 8$, $R = 16$ and $L = 24 - R = 8$.

b) 7pts Find optimal choice given quasilinear preferences $U(x_1, x_2) = x_1 + 100 \ln x_2$, prices $p_1 = \$8$, $p_2 = \$1$ and income $m = \$60$. Is your solution corner or interior?

Ans) $\frac{MU_1}{P_1} = \frac{MU_2}{P_2}$ gives me threshold of non-linear good (x_2), and budget constraint $p_1x_1 + p_2x_2 = m$ need to be satisfied.

$$\frac{MU_1}{P_1} = \frac{MU_2}{P_2} \Leftrightarrow \frac{1}{8} = \frac{100}{x_2} \Leftrightarrow x_2 = 800$$

I want to buy 800 units of x_2 first, and then want to buy x_1 as much as I can afford. However, I have only \$60, so I cannot afford both goods (800 units of x_2 costs \$800). Optimal choice is buying only x_2 good. $x_1 = 0$, $x_2 = \frac{m}{p_2} = \frac{60}{1} = 60$. My solution is corner because I don't buy any x_1 good.

c) 6pts Assume Interest rate $r = 10\%$. Choose one of the two: a consol (a type of British government bond)that pays annually $\$1,000$, starting next year or cash $\$12,000$ now (compare PV).

Ans) Consol bond pays coupon $\$1,000$ forever. $r = 0.1$. The cash flow of consol bond is same with perpetuity. Recall that the present value of perpetuity is

$$\sum_{n=1}^{\infty} \frac{x}{(1+r)^n} = \frac{\frac{x}{1+r}}{1-\frac{1}{1+r}} = \frac{x}{r}.$$

$$\begin{aligned} PV(\texttt{consol}) &= \frac{x}{r} = \frac{1,000}{0.1} = 10,000 \\ PV(\texttt{cash}) &= 12,000 \end{aligned}$$

Current cash doesn't require discount. Taking cash is better: $12,000 > 10,000$.

d) 6pts Your annual income when working (age 21-60) is $\$60,000$ and then you are going to live for the next 30 years. Write down equation that determines constant (maximal) level of consumption during your lifetime. Assume annual interest rate $r = 2\%$.

Ans) For first 40 years, I will earn $\$60,000$ annually, and I will consume constant level c annually for 70 years. Recall that the present value of annuity cash flow is

$$\sum_{n=1}^{T} \frac{x}{(1+r)^n} = \frac{\frac{x}{1+r}\left[1-(\frac{1}{1+r})^T\right]}{1-\frac{1}{1+r}} = \frac{x}{r}\left(1-\frac{1}{(1+r)^T}\right),$$

and $r = 0.02$.

$$\begin{aligned} PV(\texttt{income}) &= \frac{60,000}{0.02}\left(1-\left[\frac{1}{1.02}\right]^{40}\right) \\ PV(\texttt{consumption}) &= \frac{c}{0.02}\left(1-\left[\frac{1}{1.02}\right]^{70}\right) \\ \therefore \quad \frac{60,000}{0.02}\left(1-\left[\frac{1}{1.02}\right]^{40}\right) &= \frac{c}{0.02}\left(1-\left[\frac{1}{1.02}\right]^{70}\right) \end{aligned}$$

Midterm 1 Solution for Version B

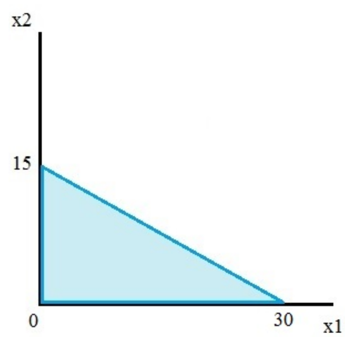
[Econ 301] Spring 2014

Problem 1 (40p) (Well-behaved preferences)

Lila spends her income on two goods: food, x1, and clothing, x2.

a) 6pts The price food is $p_1 = 2$ and one piece of clothing costs $p_2 = 4$. Show geometrically Lila's budget set if her income is $m = 60$. Find the relative price food in terms of clothing (one number)? Where can the relative price be seen in the graph of a budget set? (one sentence)

Ans) The relative price food in terms of clothing is $\frac{p_1}{p_2} = \frac{2}{4} = \frac{1}{2}$. Absolute value of slope of budget line is the relative price.



b) 7pts Lila's preferences are represented by utility function

$$U(x_1, x_2) = (x_1)^2(x_2)^1$$

We know that her preferences can be alternatively represented by function

$$V(x_1, x_2) = 2\ln x_1 + \ln x_2$$

Explain the idea behind "monotonic transformation"(one sentence) and derive function V from U.

Ans) Taking logarithm of U becomes V. Logarithm is a positive monotone transformation, and monotonic transformation of utility function does not distort original preference order. That means, whenever $U(a, b) > U(c, d)$, it is true that $V(a, b) > V(c, d)$ and vice versa.

$$V(x_1, x_2) = 2\ln x_1 + \ln x_2 = \ln x_1^2 + \ln x_2 = \ln (x_1)^2(x_2)^1 = \ln U(x_1, x_2)$$

c) 15pts Assuming utility function $V(x_1, x_2) = 2\ln x_1 + \ln x_2$

- Find marginal rate of substitution (MRS) for all bundles (derive formula). For bundle $(2, 2)$ find the value of MRS (one number). Give economic interpretation of MRS (one sentence). Which of the goods is more valuable given consumption $(2, 2)$?

Ans) $MRS_{12} = -\frac{MU_1}{MU_2} = -\frac{2x_2}{x_1}$. At $(2, 2)$, $MRS = -\frac{4}{2} = -2$. MRS_{12} means the amount of x_2 which consumer is willing to give up in order to get additional unit of x_1 while holding same utility level. x_1 is more (twice) valuable than x_2.

- Write down two secrets of happiness that determine optimal choice given parameters p_1,p_2 and m. Explain economic intuition behind the two conditions (two sentences for each).

Ans)

$$\begin{aligned} x_1p_1 + x_2p_2 &= m \\ \frac{MU_1}{P_1} = \frac{MU_2}{P_2} & \qquad or \quad MRS_{12} = -\frac{P_1}{P_2} \end{aligned}$$

The first equation is a budget constraint which requires consumers to spend all of their income on the consumption of two goods, x_1 and x_2. The second condition is about efficient combination of two goods. Mathematically, the slope of the indifference curve passing through the optimal consumption bundle to be tangent to the price ratio. Economically, marginal utility per dollar of each good should become equal at the optimal consumption bundle, or consumer's relative evaluation about two goods in terms of marginal utility should coincide market's relative evaluation in terms of price.

- Using secrets of happiness derive optimal consumption x_1,x_2, given values of $p_1 = 2$, $p_2 = 4$ and $m = 60$. Is the solution corner or interior (chose one)

Ans) Since utility function is Cobb-Douglas, optimal consumption is following.

$$\begin{aligned} x_1 &= \frac{a}{a+b}\frac{m}{p_1} = \frac{2}{3}\frac{60}{2} = 20 \\ x_2 &= \frac{b}{a+b}\frac{m}{p_2} = \frac{1}{3}\frac{60}{4} = 5 \end{aligned}$$

Both consumptions are positive, so it is interior solution.

d) 12pts (Harder) Using magic formulas for Cobb-Douglass preferences argue that the two commodities are 1) ordinary, 2) normal, and 3) neither gross complements nor gross substitutes (one sentence for each property).

Ans) When p_1 increases x_1 decreases, and when p_2 increases x_2 decreases: $\frac{\partial x_1}{\partial p_1} = -\frac{a}{a+b}\frac{m}{p_1^2} < 0$, $\frac{\partial x_2}{\partial p_2} = -\frac{b}{a+b}\frac{m}{p_2^2} < 0$, so both are ordinary. When m increases, both x_1 and x_2 increases: $\frac{\partial x_1}{\partial m} = \frac{a}{(a+b)p_1} > 0$, $\frac{\partial x_2}{\partial m} = \frac{b}{(a+b)p_2} > 0$. Therefore, both are normal. When p_1 increases x_2 doesn't change, and when p_2 increases x_1 doesn't change: $\frac{\partial x_1}{\partial p_2} = 0$, $\frac{\partial x_2}{\partial p_2} = 0$. Therefore, x_1 and x_2 are gross neutral (neither gross complements nor gross substitutes).

Problem 2 (15p) (Perfect substitutes)

You are planing a budget for the state of Wisconsin. The two major budget positions include education,x_1 and health care, x_2. Your preferences over the two are represented by function

$$U(x_1, x_2) = x_1 + x_2$$

a) 3pts Find marginal rate of substitution (give number).

Ans) $MRS_{12} = -\frac{MU_1}{MU_2} = -\frac{1}{1} = -1$.

b) 4pts Find optimal consumption of x_1 and x_2, given prices $p_1 = 2$ and $p_2 = 1$ and available funds $m = 50$ (two numbers).

Ans) Compare marginal utility per dollars. Since $\frac{MU_1}{P_1} = \frac{1}{2} = 0.5$ is smaller than $\frac{MU_2}{P_2} = \frac{1}{1} = 1$, optimal choice is consuming only x_2. $x_1 = 0$ and $x_2 = \frac{m}{p_2} = \frac{50}{1} = 50$.

c) 8pts Is solution interior? (yes-no answer). Is marginal utility of a dollar equalized? (give two numbers and yes-no answer)

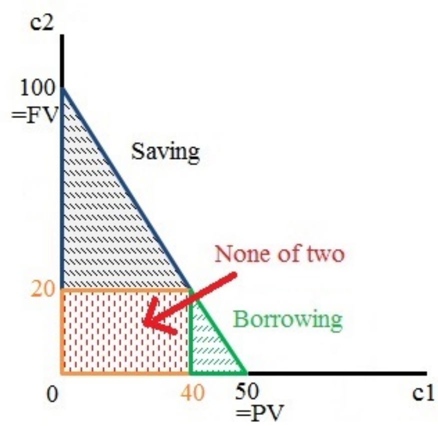
Ans) No, it is corner solution since I consume zero x_1. No, $\frac{MU_1}{P_1} = \frac{1}{2} = 0.5 \neq \frac{MU_2}{P_2} = \frac{1}{1} = 1$.

Problem 3 (20p) (Intertemporal choice)

Zoe is a professional Olympic skier. Her income when "young"is high ($m_1 = 40$) but her future (period two) is not so bright ($m_2 = 20$).

a) 4pts Depict Zoe's budget set, assuming that she can borrow and save at the interest rate $r = 100\%$. Partition her budget set into three regions: the area that involves saving, borrowing and none of the two.

Ans) For $c_1 \le 40, c_2 \le 20$, neither borrowing nor saving is required. $c_1 < 40, c_2 > 20$ means saving and $c_1 > 40, c_2 < 20$ means borrowing.



b) 6pts Find PV and FV of Zoe's lifetime income (two numbers) and show the two values in the graph. Interpret economically PV.

Ans) With $r = 1$, $PV = m_1 + \frac{m_2}{1+r} = 40 + 20/2 = 50$, $FV = (1+r)m_1 + m_2 = 2 * 40 + 20 = 100$. PV is x-intercept of budget line and FV is y-intercept of budget line. PV means Zoe's life time income evaluated at time 1 and it represents the maximum amount of c_1 Zoe can afford by borrowing $\frac{m_2}{1+r} = 10$.

c) 6pts Zoe's utility function is $U(C_1, C_2) = \ln C_1 + \frac{1}{1+\delta} \ln C_2$ where discount rate is $\delta = 3$. Using magic formulas, find optimal consumption plan (C_1, C_2) (two numbers) and the corresponding level of savings/borrowing, S.

Ans) Given utility function represents same preference of $V(C_1, C_2) = C_1^{(1+\delta)} C_2 = C_1^4 C_2$. Define $P_1 = 1$, $P_2 = 1/(1+r) = 1/2$, and $m = PV = 50$. Magic formulas of Cobb-Douglas tell us to consume $C_1 = 40$ and $C_2 = 20$. There is no needs for borrowing or saving, so $S = 0$.

Alternative approach $PV(m) = PV(c)$ and $MU_1 = (1+r)MU_2$ are

two conditions for optimal choice.

$$\begin{aligned} m_1 + \frac{m_2}{1+r} &= C_1 + \frac{C_2}{1+r} \\ \frac{1}{C_1} = \frac{1+r}{(1+\delta)C_2} &\Leftrightarrow C_1 = \frac{(1+\delta)C_2}{1+r} \end{aligned}$$

Plug in numbers, then we get $C_1 = \frac{4}{2}C_2$ and $50 = C_1 + C_2/2$. Therefore $C_1 = 40$ and $C_2 = 20$. $S = m_1 - C_1 = 0$.

d) 4pts Is Zoe tilting her consumption over time? (yes-no answer).

Ans) Yes, since time preference parameter $\delta = 3$ whereas interest rate $r = 1$, i.e. since $\delta > r$, it is optimal to consume more today ($C_1 = 40$) and to consume less tomorrow ($C_2 = 20$). Therefore, $C_1 \neq C_2$, and it means she tilts her consumption over time.

Problem 4 (25p) (Short questions)

a) 6pts Given utility function $U(C, R) = \min(2C, R)$, daily endowment of time $24h$, price and wage $p_c = w = 1$, find optimal choice of C, relaxation time R and labor supply L. (three numbers, use secrets of happiness for perfect complements).

Ans) Two formulas are $ax_1 = bx_2$ and $p_1x_1 + p_2x_2 = m$. Here, $2C = R$ and $C + R = 24$ are optimal conditions. $C = 8$, $R = 16$ and $L = 24 - R = 8$.

b) 7pts Find optimal choice given quasilinear preferences $U(x_1, x_2) = x_1 + 100 \ln x_2$, prices $p_1 = \$8$, $p_2 = \$1$ and income $m = \$80$. Is your solution corner or interior?

Ans) $\frac{MU_1}{P_1} = \frac{MU_2}{P_2}$ gives me threshold of non-linear good (x_2), and budget constraint $p_1x_1 + p_2x_2 = m$ need to be satisfied.

$$\frac{MU_1}{P_1} = \frac{MU_2}{P_2} \Leftrightarrow \frac{1}{8} = \frac{100}{x_2} \Leftrightarrow x_2 = 800$$

I want to buy 800 units of x_2 first, and then want to buy x_1 as much as I can afford. However, I have only \$80, so I cannot afford both goods (800 units of x_2 costs \$800). Optimal choice is buying only x_2 good. $x_1 = 0$, $x_2 = \frac{m}{p_2} = \frac{80}{1} = 80$. My solution is corner because I don't buy any x_1 good.

c) 6pts Assume Interest rate $r = 10\%$. Choose one of the two: a consol (a type of British government bond)that pays annually $\$1,000$, starting next year or cash $\$8,000$ now (compare PV).

Ans) A consol bond pays coupon $\$1,000$ forever. $r = 0.1$. The cash flow of a consol bond is same with perpetuity. Recall that the present value of perpetuity is

$$\sum_{n=1}^{\infty} \frac{x}{(1+r)^n} = \frac{\frac{x}{1+r}}{1 - \frac{1}{1+r}} = \frac{x}{r}.$$

$$\begin{aligned} PV(\texttt{consol}) &= \frac{x}{r} = \frac{1,000}{0.1} = 10,000 \\ PV(\texttt{cash}) &= 8,000 \end{aligned}$$

Current cash doesn't require discount. Taking a consol is better: $10,000 > 8,000$.

d) 6pts Your annual income when working (age 21-60) is $\$70,000$ and then you are going to live for the next 35 years. Write down equation that determines constant (maximal) level of consumption during your lifetime. Assume annual interest rate $r = 10\%$.

Ans) For first 40 years, I will earn $\$70,000$ annually, and I will consume constant level c annually for 75 years. Recall that the present value of annuity cash flow is

$$\sum_{n=1}^{T} \frac{x}{(1+r)^n} = \frac{\frac{x}{1+r}\left[1 - (\frac{1}{1+r})^T\right]}{1 - \frac{1}{1+r}} = \frac{x}{r}\left(1 - \frac{1}{(1+r)^T}\right),$$

and $r = 0.1$.

$$\begin{aligned} PV(\texttt{income}) &= \frac{70,000}{0.1}\left(1 - \left[\frac{1}{1.1}\right]^{40}\right) \\ PV(\texttt{consumption}) &= \frac{c}{0.1}\left(1 - \left[\frac{1}{1.1}\right]^{75}\right) \\ \therefore \quad \frac{70,000}{0.1}\left(1 - \left[\frac{1}{1.1}\right]^{40}\right) &= \frac{c}{0.1}\left(1 - \left[\frac{1}{1.1}\right]^{75}\right) \end{aligned}$$

Midterm 1 Solution for Version C

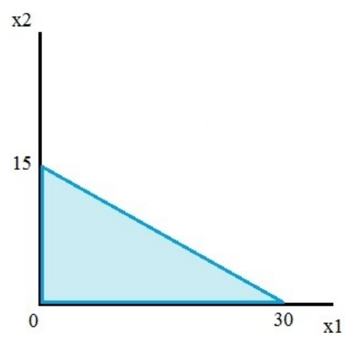
[Econ 301] Spring 2014

Problem 1 (40p) (Well-behaved preferences)

Lila spends her income on two goods: food, x1, and clothing, x2.

a) 6pts The price food is $p_1 = 2$ and one piece of clothing costs $p_2 = 4$. Show geometrically Lila's budget set if her income is $m = 60$. Find the relative price food in terms of clothing (one number)? Where can the relative price be seen in the graph of a budget set? (one sentence)

Ans) The relative price food in terms of clothing is $\frac{p_1}{p_2} = \frac{2}{4} = \frac{1}{2}$. Absolute value of slope of budget line is the relative price.



b) 7pts Lila's preferences are represented by utility function

$$U(x_1, x_2) = (x_1)^1(x_2)^2$$

We know that her preferences can be alternatively represented by function

$$V(x_1, x_2) = \ln x_1 + 2\ln x_2$$

Explain the idea behind "monotonic transformation"(one sentence) and derive function V from U.

Ans) Taking logarithm of U becomes V. Logarithm is a positive monotone transformation, and monotonic transformation of utility function does not distort original preference order. That means, whenever $U(a, b) > U(c, d)$, it is true that $V(a, b) > V(c, d)$ and vice versa.

$$V(x_1, x_2) = \ln x_1 + 2\ln x_2 = \ln x_1 + \ln x_2^2 = \ln (x_1)^1(x_2)^2 = \ln U(x_1, x_2)$$

c) 15pts Assuming utility function $V(x_1, x_2) = \ln x_1 + 2 \ln x_2$

- Find marginal rate of substitution (MRS) for all bundles (derive formula). For bundle $(2, 2)$ find the value of MRS (one number). Give economic interpretation of MRS (one sentence). Which of the goods is more valuable given consumption $(2, 2)$?

Ans) $MRS_{12} = -\frac{MU_1}{MU_2} = -\frac{x_2}{2x_1}$. At $(2, 2)$, $MRS = -\frac{2}{4} = -0.5$. MRS_{12} means the amount of x_2 which consumer is willing to give up in order to get additional unit of x_1 while holding same utility level. x_2 is more (twice) valuable than x_1.

- Write down two secrets of happiness that determine optimal choice given parameters p_1,p_2 and m. Explain economic intuition behind the two conditions (two sentences for each).

Ans)

$$\begin{aligned} x_1p_1 + x_2p_2 &= m \\ \frac{MU_1}{P_1} = \frac{MU_2}{P_2} & \qquad or \quad MRS_{12} = -\frac{P_1}{P_2} \end{aligned}$$

The first equation is a budget constraint which requires consumers to spend all of their income on the consumption of two goods, x_1 and x_2. The second condition is about efficient combination of two goods. Mathematically, the slope of the indifference curve passing through the optimal consumption bundle to be tangent to the price ratio. Economically, marginal utility per dollar of each good should become equal at the optimal consumption bundle, or consumer's relative evaluation about two goods in terms of marginal utility should coincide market's relative evaluation in terms of price.

- Using secrets of happiness derive optimal consumption x_1,x_2, given values of $p_1 = 2$, $p_2 = 4$ and $m = 60$. Is the solution corner or interior (chose one)

Ans) Since utility function is Cobb-Douglas, optimal consumption is following.

$$\begin{aligned} x_1 &= \frac{a}{a+b}\frac{m}{p_1} = \frac{1}{3}\frac{60}{2} = 10 \\ x_2 &= \frac{b}{a+b}\frac{m}{p_2} = \frac{2}{3}\frac{60}{4} = 10 \end{aligned}$$

Both consumptions are positive, so it is interior solution.

d) 12pts (Harder) Using magic formulas for Cobb-Douglass preferences argue that the two commodities are 1) ordinary, 2) normal, and 3) neither gross complements nor gross substitutes (one sentence for each property).

Ans) When p_1 increases x_1 decreases, and when p_2 increases x_2 decreases: $\frac{\partial x_1}{\partial p_1} = -\frac{a}{a+b}\frac{m}{p_1^2} < 0$, $\frac{\partial x_2}{\partial p_2} = -\frac{b}{a+b}\frac{m}{p_2^2} < 0$, so both are ordinary. When m increases, both x_1 and x_2 increases: $\frac{\partial x_1}{\partial m} = \frac{a}{(a+b)p_1} > 0$, $\frac{\partial x_2}{\partial m} = \frac{b}{(a+b)p_2} > 0$. Therefore, both are normal. When p_1 increases x_2 doesn't change, and when p_2 increases x_1 doesn't change: $\frac{\partial x_1}{\partial p_2} = 0$, $\frac{\partial x_2}{\partial p_2} = 0$. Therefore, x_1 and x_2 are gross neutral (neither gross complements nor gross substitutes).

Problem 2 (15p) (Perfect substitutes)

You are planing a budget for the state of Wisconsin. The two major budget positions include education,x_1 and health care, x_2. Your preferences over the two are represented by function

$$U(x_1, x_2) = 2x_1 + 2x_2$$

a) 3pts Find marginal rate of substitution (give number).

Ans) $MRS_{12} = -\frac{MU_1}{MU_2} = -\frac{2}{2} = -1$.

b) 4pts Find optimal consumption of x_1 and x_2, given prices $p_1 = 2$ and $p_2 = 1$ and available funds $m = 100$ (two numbers).

Ans) Compare marginal utility per dollars. Since $\frac{MU_1}{P_1} = \frac{2}{2} = 1$ is smaller than $\frac{MU_2}{P_2} = \frac{2}{1} = 2$, optimal choice is consuming only x_2. $x_1 = 0$ and $x_2 = \frac{m}{p_2} = \frac{100}{1} = 100$.

c) 8pts Is solution interior? (yes-no answer). Is marginal utility of a dollar equalized? (give two numbers and yes-no answer)

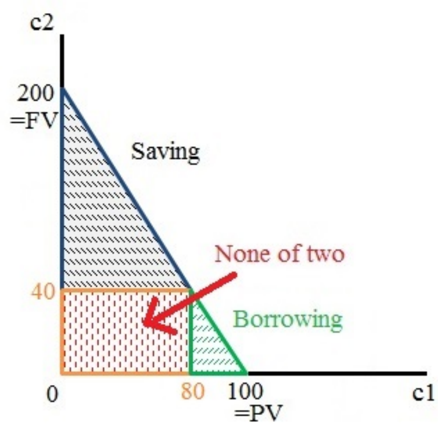
Ans) No, it is corner solution since I consume zero x_1. No, $\frac{MU_1}{P_1} = \frac{2}{2} = 1 \neq \frac{MU_2}{P_2} = \frac{2}{1} = 2$.

Problem 3 (20p) (Intertemporal choice)

Zoe is a professional Olympic skier. Her income when "young" is high ($m_1 = 80$) but her future (period two) is not so bright ($m_2 = 40$).

a) 4pts Depict Zoe's budget set, assuming that she can borrow and save at the interest rate $r = 100\%$. Partition her budget set into three regions: the area that involves saving, borrowing and none of the two.

Ans) For $c_1 \leq 80, c_2 \leq 40$, neither borrowing nor saving is required. $c_1 < 80, c_2 > 40$ means saving and $c_1 > 80, c_2 < 40$ means borrowing.



b) 6pts Find PV and FV of Zoe's lifetime income (two numbers) and show the two values in the graph. Interpret economically PV.

Ans) With $r = 1$, $PV = m_1 + \frac{m_2}{1+r} = 80 + 40/2 = 100$, $FV = (1+r)m_1 + m_2 = 2 * 80 + 40 = 200$. PV is x-intercept of budget line and FV is y-intercept of budget line. PV means Zoe's life time income evaluated at time 1 and it represents the maximum amount of c_1 Zoe can afford by borrowing $\frac{m_2}{1+r} = 20$.

c) 6pts Zoe's utility function is $U(C_1, C_2) = \ln C_1 + \frac{1}{1+\delta} \ln C_2$ where discount rate is $\delta = 3$. Using magic formulas, find optimal consumption plan (C_1, C_2) (two numbers) and the corresponding level of savings/borrowing, S.

Ans) Given utility function represents same preference of $V(C_1, C_2) = C_1^{(1+\delta)} C_2 = C_1^4 C_2$. Define $P_1 = 1$, $P_2 = 1/(1+r) = 1/2$, and $m = PV = 100$. Magic formulas of Cobb-Douglas tell us to consume $C_1 = 80$ and $C_2 = 40$. There is no needs for borrowing or saving, so $S = 0$.

Alternative approach $PV(m) = PV(c)$ and $MU_1 = (1+r)MU_2$ are

two conditions for optimal choice.

$$\begin{aligned} m_1 + \frac{m_2}{1+r} &= C_1 + \frac{C_2}{1+r} \\ \frac{1}{C_1} = \frac{1+r}{(1+\delta)C_2} &\Leftrightarrow C_1 = \frac{(1+\delta)C_2}{1+r} \end{aligned}$$

Plug in numbers, then we get $C_1 = \frac{4}{2}C_2$ and $100 = C_1 + C_2/2$. Therefore $C_1 = 80$ and $C_2 = 40$. $S = m_1 - C_1 = 0$.

d) 4pts Is Zoe tilting her consumption over time? (yes-no answer).

Ans) Yes, since time preference parameter $\delta = 3$ whereas interest rate $r = 1$, i.e. since $\delta > r$, it is optimal to consume more today ($C_1 = 80$) and to consume less tomorrow ($C_2 = 40$). Therefore, $C_1 \neq C_2$, and it means she tilts her consumption over time.

Problem 4 (25p) (Short questions)

a) 6pts Given utility function $U(C, R) = \min(C, 2R)$, daily endowment of time $24h$, price and wage $p_c = w = 1$, find optimal choice of C, relaxation time R and labor supply L. (three numbers, use secrets of happiness for perfect complements).

Ans) Two formulas are $ax_1 = bx_2$ and $p_1x_1 + p_2x_2 = m$. Here, $C = 2R$ and $C + R = 24$ are optimal conditions. $C = 16$, $R = 8$ and $L = 24 - R = 16$.

b) 7pts Find optimal choice given quasilinear preferences $U(x_1, x_2) = x_1 + 50 \ln x_2$, prices $p_1 = \$8$, $p_2 = \$1$ and income $m = \$40$. Is your solution corner or interior?

Ans) $\frac{MU_1}{P_1} = \frac{MU_2}{P_2}$ gives me threshold of non-linear good (x_2), and budget constraint $p_1x_1 + p_2x_2 = m$ need to be satisfied.

$$\frac{MU_1}{P_1} = \frac{MU_2}{P_2} \Leftrightarrow \frac{1}{8} = \frac{50}{x_2} \Leftrightarrow x_2 = 400$$

I want to buy 400 units of x_2 first, and then want to buy x_1 as much as I can afford. However, I have only \$40, so I cannot afford both goods (400 units of x_2 costs \$400). Optimal choice is buying only x_2 good. $x_1 = 0$, $x_2 = \frac{m}{p_2} = \frac{40}{1} = 40$. My solution is corner because I don't buy any x_1 good.

c) 6pts Assume Interest rate $r = 10\%$. Choose one of the two: a consol (a type of British government bond)that pays annually \$500, starting next year or cash \$4,000 now (compare PV).

Ans) A consol bond pays coupon \$500 forever and $r = 0.1$. The cash flow of a consol bond is same with perpetuity. Recall that the present value of perpetuity is

$$\sum_{n=1}^{\infty} \frac{x}{(1+r)^n} = \frac{\frac{x}{1+r}}{1 - \frac{1}{1+r}} = \frac{x}{r}.$$

$$\begin{aligned} PV(\texttt{consol}) &= \frac{x}{r} = \frac{500}{0.1} = 5,000 \\ PV(\texttt{cash}) &= 4,000 \end{aligned}$$

Current cash doesn't require discount. Taking a consol is better: $5,000 > 4,000$.

d) 6pts Your annual income when working (age 21-60) is \$70,000 and then you are going to live for the next 35 years. Write down equation that determines constant (maximal) level of consumption during your lifetime. Assume annual interest rate $r = 10\%$.

Ans) For first 40 years, I will earn \$70,000 annually, and I will consume constant level c annually for 75 years. Recall that the present value of annuity cash flow is

$$\sum_{n=1}^{T} \frac{x}{(1+r)^n} = \frac{\frac{x}{1+r}\left[1 - (\frac{1}{1+r})^T\right]}{1 - \frac{1}{1+r}} = \frac{x}{r}\left(1 - \frac{1}{(1+r)^T}\right),$$

and $r = 0.1$.

$$\begin{aligned} PV(\texttt{income}) &= \frac{70,000}{0.1}\left(1 - \left[\frac{1}{1.1}\right]^{40}\right) \\ PV(\texttt{consumption}) &= \frac{c}{0.1}\left(1 - \left[\frac{1}{1.1}\right]^{75}\right) \\ \therefore \quad \frac{70,000}{0.1}\left(1 - \left[\frac{1}{1.1}\right]^{40}\right) &= \frac{c}{0.1}\left(1 - \left[\frac{1}{1.1}\right]^{75}\right) \end{aligned}$$

Midterm 1 Solution for Version D

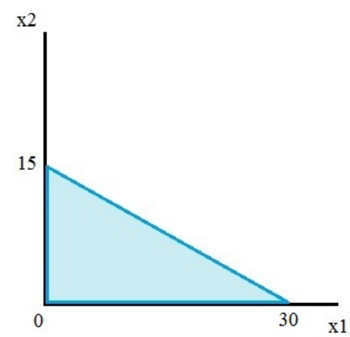
[Econ 301] Spring 2014

Problem 1 (40p) (Well-behaved preferences)

Lila spends her income on two goods: food, x1, and clothing, x2.

a) 6pts The price food is $p_1 = 4$ and one piece of clothing costs $p_2 = 8$. Show geometrically Lila's budget set if her income is $m = 120$. Find the relative price food in terms of clothing (one number)? Where can the relative price be seen in the graph of a budget set? (one sentence)

Ans) The relative price food in terms of clothing is $\frac{p_1}{p_2} = \frac{4}{8} = \frac{1}{2}$. Absolute value of slope of budget line is the relative price.



b) 7pts Lila's preferences are represented by utility function

$$U(x_1, x_2) = (x_1)^1 (x_2)^2$$

We know that her preferences can be alternatively represented by function

$$V(x_1, x_2) = \ln x_1 + 2 \ln x_2$$

Explain the idea behind "monotonic transformation"(one sentence) and derive function V from U.

Ans) Taking logarithm of U becomes V. Logarithm is a positive monotone transformation, and monotonic transformation of utility function does not distort original preference order. That means, whenever $U(a, b) > U(c, d)$, it is true that $V(a, b) > V(c, d)$ and vice versa.

$$V(x_1, x_2) = \ln x_1 + 2 \ln x_2 = \ln x_1 + \ln x_2^2 = \ln (x_1)^1 (x_2)^2 = \ln U(x_1, x_2)$$

c) 15pts Assuming utility function $V(x_1, x_2) = \ln x_1 + 2\ln x_2$

- Find marginal rate of substitution (MRS) for all bundles (derive formula). For bundle $(2,2)$ find the value of MRS (one number). Give economic interpretation of MRS (one sentence). Which of the goods is more valuable given consumption $(2,2)$?

Ans) $MRS_{12} = -\frac{MU_1}{MU_2} = -\frac{x_2}{2x_1}$. At $(2,2)$, $MRS = -\frac{2}{4} = -0.5$. MRS_{12} means the amount of x_2 which consumer is willing to give up in order to get additional unit of x_1 while holding same utility level. x_2 is more (twice) valuable than x_1.

- Write down two secrets of happiness that determine optimal choice given parameters p_1,p_2 and m. Explain economic intuition behind the two conditions (two sentences for each).

Ans)

$$\begin{aligned} x_1p_1 + x_2p_2 &= m \\ \frac{MU_1}{P_1} = \frac{MU_2}{P_2} & \qquad or \quad MRS_{12} = -\frac{P_1}{P_2} \end{aligned}$$

The first equation is a budget constraint which requires consumers to spend all of their income on the consumption of two goods, x_1 and x_2. The second condition is about efficient combination of two goods. Mathematically, the slope of the indifference curve passing through the optimal consumption bundle to be tangent to the price ratio. Economically, marginal utility per dollar of each good should become equal at the optimal consumption bundle, or consumer's relative evaluation about two goods in terms of marginal utility should coincide market's relative evaluation in terms of price.

- Using secrets of happiness derive optimal consumption x_1,x_2, given values of $p_1 = 4$, $p_2 = 8$ and $m = 120$. Is the solution corner or interior (chose one)

Ans) Since utility function is Cobb-Douglas, optimal consumption is following.

$$\begin{aligned} x_1 &= \frac{a}{a+b}\frac{m}{p_1} = \frac{1}{3}\frac{120}{4} = 10 \\ x_2 &= \frac{b}{a+b}\frac{m}{p_2} = \frac{2}{3}\frac{120}{8} = 10 \end{aligned}$$

Both consumptions are positive, so it is interior solution.

d) 12pts (Harder) Using magic formulas for Cobb-Douglass preferences argue that the two commodities are 1) ordinary, 2) normal, and 3) neither gross complements nor gross substitutes (one sentence for each property).

Ans) When p_1 increases x_1 decreases, and when p_2 increases x_2 decreases: $\frac{\partial x_1}{\partial p_1} = -\frac{a}{a+b}\frac{m}{p_1^2} < 0$, $\frac{\partial x_2}{\partial p_2} = -\frac{b}{a+b}\frac{m}{p_2^2} < 0$, so both are ordinary. When m increases, both x_1 and x_2 increases: $\frac{\partial x_1}{\partial m} = \frac{a}{(a+b)p_1} > 0$, $\frac{\partial x_2}{\partial m} = \frac{b}{(a+b)p_2} > 0$. Therefore, both are normal. When p_1 increases x_2 doesn't change, and when p_2 increases x_1 doesn't change: $\frac{\partial x_1}{\partial p_2} = 0$, $\frac{\partial x_2}{\partial p_2} = 0$. Therefore, x_1 and x_2 are gross neutral (neither gross complements nor gross substitutes).

Problem 2 (15p) (Perfect substitutes)

You are planing a budget for the state of Wisconsin. The two major budget positions include education,x_1 and health care, x_2. Your preferences over the two are represented by function

$$U(x_1, x_2) = 3x_1 + 3x_2$$

a) 3pts Find marginal rate of substitution (give number).

Ans) $MRS_{12} = -\frac{MU_1}{MU_2} = -\frac{3}{3} = -1$.

b) 4pts Find optimal consumption of x_1 and x_2, given prices $p_1 = 2$ and $p_2 = 1$ and available funds $m = 100$ (two numbers).

Ans) Compare marginal utility per dollars. Since $\frac{MU_1}{P_1} = \frac{3}{2} = 1.5$ is smaller than $\frac{MU_2}{P_2} = \frac{3}{1} = 3$, optimal choice is consuming only x_2. $x_1 = 0$ and $x_2 = \frac{m}{p_2} = \frac{100}{1} = 100$.

c) 8pts Is solution interior? (yes-no answer). Is marginal utility of a dollar equalized? (give two numbers and yes-no answer)

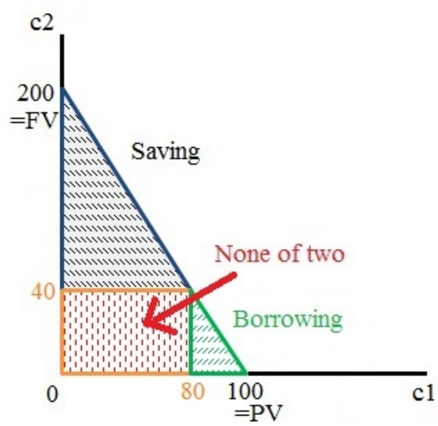
Ans) No, it is corner solution since I consume zero x_1. No, $\frac{MU_1}{P_1} = \frac{3}{2} = 1.5 \neq \frac{MU_2}{P_2} = \frac{3}{1} = 3$.

Problem 3 (20p) (Intertemporal choice)

Zoe is a professional Olympic skier. Her income when "young" is high ($m_1 = 80$) but her future (period two) is not so bright ($m_2 = 40$).

a) 4pts Depict Zoe's budget set, assuming that she can borrow and save at the interest rate $r = 100\%$. Partition her budget set into three regions: the area that involves saving, borrowing and none of the two.

Ans) For $c_1 \leq 80, c_2 \leq 40$, neither borrowing nor saving is required. $c_1 < 80, c_2 > 40$ means saving and $c_1 > 80, c_2 < 40$ means borrowing.



b) 6pts Find PV and FV of Zoe's lifetime income (two numbers) and show the two values in the graph. Interpret economically PV.

Ans) With $r = 1$, $PV = m_1 + \frac{m_2}{1+r} = 80 + 40/2 = 100$, $FV = (1+r)m_1 + m_2 = 2 * 80 + 40 = 200$. PV is x-intercept of budget line and FV is y-intercept of budget line. PV means Zoe's life time income evaluated at time 1 and it represents the maximum amount of c_1 Zoe can afford by borrowing $\frac{m_2}{1+r} = 20$.

c) 6pts Zoe's utility function is $U(C_1, C_2) = \ln C_1 + \frac{1}{1+\delta} \ln C_2$ where discount rate is $\delta = 4$. Using magic formulas, find optimal consumption plan (C_1, C_2) (two numbers) and the corresponding level of savings/borrowing, S.

Ans) Given utility function represents same preference of $V(C_1, C_2) = C_1^{(1+\delta)} C_2 = C_1^5 C_2$. Define $P_1 = 1$, $P_2 = 1/(1+r) = 1/2$, and $m = PV = 100$. Magic formulas of Cobb-Douglas tell us to consume $C_1 = \frac{500}{6} = \frac{250}{3} \approx 83.33$ and $C_2 = \frac{100}{3} \approx 33.33$. She needs to borrow $B = 10/3$ (because $c_1 > m_1$), or her saving is $S = -\frac{10}{3}$.

Alternative approach $PV(m) = PV(c)$ and $MU_1 = (1+r)MU_2$ are

two conditions for optimal choice.

$$m_1 + \frac{m_2}{1+r} \quad = \quad C_1 + \frac{C_2}{1+r}$$
$$\frac{1}{C_1} = \frac{1+r}{(1+\delta)C_2} \quad \Leftrightarrow \quad C_1 = \frac{(1+\delta)C_2}{1+r}$$

Plug in numbers, then we get $C_1 = \frac{5}{2}C_2$ and $100 = C_1 + C_2/2$. Therefore $C_1 = \frac{250}{3}$ and $C_2 = \frac{100}{3}$. Saving is $S = m_1 - C_1 = -\frac{10}{3}$ (or borrowing $B = C_1 - m_1 = \frac{10}{3}$).

d) 4pts Is Zoe tilting her consumption over time? (yes-no answer).

Ans) Yes, since time preference parameter $\delta = 4$ whereas interest rate $r = 1$, i.e. since $\delta > r$, it is optimal to consume more today ($C_1 \approx 83.33$) and to consume less tomorrow ($C_2 \approx 33.33$). Therefore, $C_1 \neq C_2$, and it means she tilts her consumption over time.

Problem 4 (25p) (Short questions)

a) 6pts Given utility function $U(C, R) = \min(C, 2R)$, daily endowment of time $24h$, price and wage $p_c = w = 2$, find optimal choice of C, relaxation time R and labor supply L. (three numbers, use secrets of happiness for perfect complements).

Ans) Two formulas are $ax_1 = bx_2$ and $p_1x_1 + p_2x_2 = m$. Here, $C = 2R$ and $2C+2R = 48$ are optimal conditions. $C = 16$, $R = 8$ and $L = 24-R = 16$.

b) 7pts Find optimal choice given quasilinear preferences $U(x_1, x_2) = x_1+50\ln x_2$, prices $p_1 = \$8$, $p_2 = \$1$ and income $m = \$60$. Is your solution corner or interior?

Ans) $\frac{MU_1}{P_1} = \frac{MU_2}{P_2}$ gives me threshold of non-linear good (x_2), and budget constraint $p_1x_1 + p_2x_2 = m$ need to be satisfied.

$$\frac{MU_1}{P_1} = \frac{MU_2}{P_2} \Leftrightarrow \frac{1}{8} = \frac{50}{x_2} \Leftrightarrow x_2 = 400$$

I want to buy 400 units of x_2 first, and then want to buy x_1 as much as I can afford. However, I have only \$60, so I cannot afford both goods (400 units of x_2 costs \$400). Optimal choice is buying only x_2 good. $x_1 = 0$, $x_2 = \frac{m}{p_2} = \frac{60}{1} = 60$. My solution is corner because I don't buy any x_1 good.

c) 6pts Assume Interest rate $r = 10\%$. Choose one of the two: a consol (a type of British government bond)that pays annually \$100, starting next year or cash \$1,200 now (compare PV).

Ans) A consol bond pays coupon \$100 forever and $r = 0.1$. The cash flow of a consol bond is same with perpetuity. Recall that the present value of perpetuity is

$$\sum_{n=1}^{\infty} \frac{x}{(1+r)^n} = \frac{\frac{x}{1+r}}{1 - \frac{1}{1+r}} = \frac{x}{r}.$$

$$\begin{aligned} PV(\texttt{consol}) &= \frac{x}{r} = \frac{100}{0.1} = 1,000 \\ PV(\texttt{cash}) &= 1,200 \end{aligned}$$

Current cash doesn't require discount. Taking cash is better since $1,200 > 1,000$.

d) 6pts Your annual income when working (age 21-60) is \$70,000 and then you are going to live for the next 35 years. Write down equation that determines constant (maximal) level of consumption during your lifetime. Assume annual interest rate $r = 10\%$.

Ans) For first 40 years, I will earn \$70,000 annually, and I will consume constant level c annually for 75 years. Recall that the present value of annuity cash flow is

$$\sum_{n=1}^{T} \frac{x}{(1+r)^n} = \frac{\frac{x}{1+r}\left[1 - (\frac{1}{1+r})^T\right]}{1 - \frac{1}{1+r}} = \frac{x}{r}\left(1 - \frac{1}{(1+r)^T}\right),$$

and $r = 0.1$.

$$\begin{aligned} PV(\texttt{income}) &= \frac{70,000}{0.1}\left(1 - \left[\frac{1}{1.1}\right]^{40}\right) \\ PV(\texttt{consumption}) &= \frac{c}{0.1}\left(1 - \left[\frac{1}{1.1}\right]^{75}\right) \\ \therefore \quad \frac{70,000}{0.1}\left(1 - \left[\frac{1}{1.1}\right]^{40}\right) &= \frac{c}{0.1}\left(1 - \left[\frac{1}{1.1}\right]^{75}\right) \end{aligned}$$