Econ 301
Intermediate Microeconomics
Prof. Marek Weretka

Final Exam (Group A)

You have 2h to complete the exam. The final consists of 6 questions (25+20+20+15+10+10=100).

Problem 1. (Quasilinaer income effect)

Mirabella consumes chocolate candy bars x_1 and fruits x_2. The prices of the two goods are $p_1 = 4, p_2 = 4$, respectively and Mirabella's income is $m = 20$. Her utility function is

$$U(x_1, x_2) = 2 \ln x_1 + x_2$$

a) In the commodity space plot Mirabella's budget set. Find the slope of budget line (one number). Provide the economic interpretation of the slope (one sentence).

b) Find analytically formula that gives Mirabella's MRS for any bundle (x_1, x_2) (a function). Give the economic and the geometric interpretation of MRS (two sentences). Find the value of MRS at bundle $(x_1, x_2) = (4, 4)$ (one number). At this bundle, which of the two commodities is (locally) more valuable? (chose one)

c) Write down two secrets of happiness that determine Mirabellas's optimal choice (two equation). Provide the geometric interpretation of the conditions in the commodity space.

d) Find Mirabella's optimal choice (two numbers). Is solution interior (yes-no answer).

e) Suppose the price of a chocolate candy bar goes down to $p_1 = 2$, while other price $p_2 = 4$ and income $m = 20$ are unchanged. Find the new optimal choice (two numbers). Is a chocolate candy bar an ordinary or Giffen good (pick one)?

f) Decompose the change in demand for x_1 in points d) and e) into a substitution and income effect.

Problem 2. (Equilibrium)

Consider an economy with two consumers, Adalia and Briana and two goods: bicycles x_1 and flowers x_2. Adalia initial endowment of the commodities is $\omega^A = (40, 60)$ and Briana endowment is $\omega^B = (60, 40)$. Adalia and Briana utility functions are given by, $i = A, B$

$$U^i(x_1, x_2) = 4 \ln x_1 + 4 \ln x_2$$

a) Plot an Edgeworth box and mark the point that corresponds to initial endowments.

b) Give a definition of a Pareto efficient allocation (one sentence).

c) Give a (general) equivalent condition for Pareto efficiency in terms of MRS. Provide geometric arguments that demonstrate the necessity and sufficiency of MRS condition for Pareto efficiency.

d) Find competitive equilibrium (six numbers). Depict the obtained equilibrium in the Edgeworth box. Using MRS condition verify that the equilibrium is Pareto efficient.

e) Using (one of) the secrets of happiness prove that a competitive equilibrium is Pareto efficient in any economy.

Problem 3. (Short questions)

a) Using λ argument prove that Cobb-Douglass production function $y = 2KL$ exhibits increasing returns to scale. Without any calculations, sketch total cost function $c(y)$ corresponding to the production function.

b) Now consider a firm (different from point a)) with variable cost $c(y) = 2y^2$ and fixed cost $F = 2$. Find ATC^{MES} and y^{MES} (two numbers). In a long-run equilibrium with free entry how many firms should be expect in the industry if inverse demand is $D(p) = 10 - p$?

c) Suppose a Bernoulli utility function is $u(x) = x^2$ and two states are equally likely (probability $\frac{1}{2}$). Write down the corresponding von Neuman-Morgenstern utility function. Find the certainty equivalent and the expected value of lottery $(0, 2)$ (two numbers). Which of the two is bigger and why? (two numbers and one sentence.)

d) Find Herfindahl–Hirschman Index (HHI) for industry with $N = 50$ identical firms (one number). Is the industry concentrated?

e) Derive formula for the present value of perpetuity.

Problem 4. (Market Power)

Consider an industry with inverse demand $p(y) = 8 - y$, and a monopoly with cost function $TC(y) = 0$ who cannot discriminate.

a) What are the total gains-to-trade (or potential total surplus) in this industry? (give one number)

b) Write down monopoly's profit function. Derive the condition on MR and MC that gives profit maximizing level of production. Provide economic interpretation of this condition.

c) Find the level of production, the price, the deadweight loss and the elasticity of the demand at optimum (four numbers). Illustrate the choice in a graph.

d) Assuming the same demand function find the individual and the aggregate level of production and the price in the Cournot-Nash equilibrium with $N = 3$ identical firms (give three numbers). Show the deadweight loss in the graph.

Problem 5.(Externality)

Lucy is addicted to nicotine. Her utility from smoking c cigarettes (net of their cost) is given by

$$U^L(c) = 2\ln c - c$$

Her sister Taja prefers healthy lifestyle, her favorite commodity is orange juice, j. The two sisters live together and Taja is exposed to second-hand smoke and hence her utility is adversely affected by Lucy consumption of cigarettes c. In particular, her utility function (net of cost of orange juice) is given by

$$U^T(j, c) = \ln(j - c) - j.$$

a) Market outcome: Find consumption of cigarettes c that maximizes the utility of Lucy and the amount of orange juice chosen by Taja (assuming c is optimal for Lucy) (two numbers)

b) Find the Pareto efficient level of c and j. Is the value of c higher or smaller than in a)? Why? (two numbers + one sentence) Hint: Derivative of $\ln(j - c)$ with respect to c is $-\frac{1}{j-c}$.

Problem 6. (Asymmetric information)

In Shorewood Hills area there are two types of homes: lemons (bad quality homes) and plums (good quality ones). The fraction of lemons is equal to $\frac{1}{2}$. The value of a home for the two parties depends on its type and is given by

	Lemon	Plum
Seller	0	12
Buyer	10	18

Both parties agree on the price that is in between the value of a buyer and a seller.

a) Buyers and sellers can perfectly determine the quality of a house before transaction takes place What is expected total, buyers and sellers surplus (three numbers)

b) Now assume that the buyers are not able to determine quality of a house. Find the price of a house, and the expected buyers and sellers surplus (three numbers). Is a pooling equilibrium sustainable, or will this market result in a separating equilibrium? Is outcome Pareto efficient (why or why not)?

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Final Exam (Group B)

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Problem 1. (Quasilinaer income effect)

Mirabella consumes chocolate candy bars x_1 and fruits x_2. The prices of the two goods are $p_1 = 2, p_2 = 2$, respectively and Mirabella's income is $m = 20$. Her utility function is

$$U(x_1, x_2) = 2\ln x_1 + x_2$$

a) In the commodity space plot Mirabella's budget set. Find the slope of budget line (one number). Provide the economic interpretation of the slope (one sentence).

b) Find analytically formula that gives Mirabella's MRS for any bundle (x_1, x_2) (a function). Give the economic and the geometric interpretation of MRS (two sentences). Find the value of MRS at bundle $(x_1, x_2) = (8, 8)$ (one number). At this bundle, which of the two commodities is (locally) more valuable? (chose one)

c) Write down two secrets of happiness that determine Mirabellas's optimal choice (two equation). Provide the geometric interpretation of the conditions in the commodity space.

d) Find Mirabella's optimal choice (two numbers). Is solution interior (yes-no answer).

e) Suppose the price of a chocolate candy bar goes down to $p_1 = 1$, while other price $p_2 = 2$ and income $m = 20$ are unchanged. Find the new optimal choice (two numbers). Is a chocolate candy bar an ordinary or Giffen good (pick one)?

f) Decompose the change in demand for x_1 in points d) and e) into a substitution and income effect.

Problem 2. (Equilibrium)

Consider an economy with two consumers, Adalia and Briana and two goods: bicycles x_1 and flowers x_2. Adalia initial endowment of the commodities is $\omega^A = (50, 100)$ and Briana endowment is $\omega^B = (100, 50)$. Adalia and Briana utility functions are given by, $i = A, B$

$$U^i(x_1, x_2) = 2\ln x_1 + 2\ln x_2$$

a) Plot an Edgeworth box and mark the point that corresponds to initial endowments.

b) Give a definition of a Pareto efficient allocation (one sentence).

c) Give a (general) equivalent condition for Pareto efficiency in terms of MRS. Provide geometric arguments that demonstrate the necessity and sufficiency of MRS condition for Pareto efficiency.

d) Find competitive equilibrium (six numbers). Depict the obtained equilibrium in the Edgeworth box. Using MRS condition verify that the equilibrium is Pareto efficient.

e) Using (one of) the secrets of happiness prove that a competitive equilibrium is Pareto efficient in any economy.

Problem 3. (Short questions)

a) Using λ argument prove that Cobb-Douglass production function $y = 2KL$ exhibits increasing returns to scale. Without any calculations, sketch total cost function $c(y)$ corresponding to the production function.

b) Now consider a firm (different from point a)) with variable cost $c(y) = 4y^2$ and fixed cost $F = 4$. Find ATC^{MES} and y^{MES} (two numbers). In a long-run equilibrium with free entry how many firms should be expect in the industry if inverse demand is $D(p) = 16 - p$?

c) Suppose a Bernoulli utility function is $u(x) = x^2$ and two states are equally likely (probability $\frac{1}{2}$). Write down the corresponding von Neuman-Morgenstern utility function. Find the certainty equivalent and the expected value of lottery $(0, 2)$ (two numbers). Which of the two is bigger and why? (two numbers and one sentence.)

d) Find Herfindahl–Hirschman Index (HHI) for industry with $N = 100$ identical firms (one number). Is the industry concentrated?

e) Derive formula for the present value of perpetuity

Problem 4. (Market Power)

Consider an industry with inverse demand $p(y) = 12 - y$, and a monopoly with cost function $TC(y) = 0$ who cannot discriminate.

a) What are the total gains-to-trade (or potential total surplus) in this industry? (give one number)

b) Write down monopoly's profit function. Derive the condition on MR and MC that gives profit maximizing level of production. Provide economic interpretation of this condition.

c) Find the level of production, the price, the deadweight loss and the elasticity of the demand at optimum (four numbers). Illustrate the choice in a graph.

d) Assuming the same demand function, find the individual and the aggregate level of production and the price in the Cournot-Nash equilibrium with $N = 3$ identical firms (give three numbers). Show the deadweight loss in the graph.

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$$U^T(j, c) = \ln(j - c) - j.$$

a) Market outcome: Find consumption of cigarettes c that maximizes the utility of Lucy and the amount of orange juice chosen by Taja (assuming c is optimal for Lucy) (two numbers)

b) Find the Pareto efficient level of c and j. Is the value of c higher or smaller than in a)? Why? (two numbers + one sentence) Hint: Derivative of $\ln(j - c)$ with respect to c is $-\frac{1}{j-c}$.

Problem 6. (Asymmetric information)

In Shorewood Hills area there are two types of homes: lemons (bad quality homes) and plums (good quality ones). The fraction of lemons is equal to $\frac{1}{2}$. The value of a home for the two parties depends on its type and is given by

	Lemon	Plum
Seller	6	14
Buyer	10	22

Both parties agree on the price that is in between the value of a buyer and a seller.

a) Buyers and sellers can perfectly determine the quality of a house before transaction takes place What is expected total, buyers and sellers surplus (three numbers)

b) Now assume that the buyers are not able to determine quality of a house. Find the price of a house, and the expected buyers and sellers surplus (three numbers). Is a pooling equilibrium sustainable, or will this market result in a separating equilibrium? Is outcome Pareto efficient (why or why not)?