

Information and Market Power in Rational Expectations Equilibrium*

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Early and Preliminary Version

Abstract

We study demand function competition in small markets. We follow the payoff structure found in Vives (2011), we consider N agents with quadratic payoffs and uncertainty over their marginal valuation of a divisible good. We study how the information structure affects the outcome of the Bayes Nash equilibrium in which agents compete in demand functions. We study the effects of the information structure along two dimensions. First, we analyze how the endogenous information in prices affects the set of feasible outcomes. We do this by comparing how the set of feasible outcomes under demand function competition compares with the set of feasible outcomes under Cournot competition. Second, we study how the information structure changes the market power agents have in equilibrium. Methodologically speaking, we define a solution concept which we call correlated REE. This is the natural extension of Bayes Correlated equilibrium, as it is defined by Bergemann and Morris (2013), but to accommodate demand function competition.

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1 Introduction

In the present paper we study rational expectations equilibrium (henceforth, REE) in an economy with finite number of agents. We study the case in which there is an exogenous supply of a good that is traded, and the profits of agents is quadratic in the quantity traded. We assume there is incomplete information over the marginal valuation of the agents (henceforth, the agent's type) and this is symmetric and normally distributed with some arbitrary interdependence across agents. We study the equilibrium in a game in which agents submit demand functions, and a Walrasian auctioneer sets a price that clears the market.

The objective of the paper is to analyze how does the set of feasible outcomes of the equilibrium in demand function vary with the information agents have. There are two dimensions over which we analyze how the information structure affects the outcomes. First, we analyze how the information structure affects the market power agents have in equilibrium. Second, we analyze the joint distribution of quantities traded in equilibrium and the marginal valuation of agents. We show that both of these dimensions of an equilibrium are strongly affected by the assumed information structure.

To describe the set of feasible outcomes we define a solution concept which we call correlated REE. This is the natural extension of Bayes Correlated equilibrium, as it is defined by Bergemann and Morris (2013), but to accommodate demand function competition. The description of a correlated equilibrium is done in terms of a parameter denote market power and a joint distribution of quantities traded and realized types, subject to a restriction. This restriction is interpreted as a first order condition and corresponds to the best response in the case in which agents choose the amount they want to trade, knowing the equilibrium price and the action they take, but also anticipating their price impact. The fact that agents know the equilibrium price when they take an action has a well established interpretation in a REE with a continuum of players, agents know the equilibrium price and condition on this, even when this is affected by the actions of all other players. Assuming agents know the quantity they demand, and nothing else, has the common interpretation of Bayes correlated equilibrium, in which the action of an agents provides a sufficient statistic of the information structure an agent has. In our environment the action taken by an agent provides a sufficient statistic of his private information. There is a second aspect to the solution concept, which is a novel way to accommodate the fact that agents have a positive price impact when the economy has only a finite number of agents. This is accommodated by assuming agents anticipating what their price impact will be, and taking this as given when choosing their demanded quantity. In equilibrium they correctly anticipate their price impact, and this can be taken as a constant in their best response.

Our first result is to show that a correlated REE describes the same set of outcomes as the set of outcomes in competition in demand functions. Although methodologically it is very similar to the work found in Bergemann and Morris (2013), there are some additional conceptual aspects one needs to ensure work out correctly. In a equilibrium in demand functions the empirically relevant outcome is not only the realized price, realized traded quantity and realized type, but also the demand function submitted by each agent. The realized traded quantity, price and type of agents is described in a correlated REE by a joint distribution of variables. On the other hand, the slope of the demand each agent submits is mapped into the one dimensional parameter which we call the market power. This is precisely the quantity that described the impact that agents anticipate they will have on the equilibrium price, and

this can be empirically inferred from the slope of the demand function submitted by agents.

As a by-product of proving the equivalence between both solution concepts, we can provide a simple canonical information structure that allows us to decentralize the set of feasible outcomes under demand function competition. This constitutes our first characterization of the set of feasible outcomes of a REE. The canonical information structure consists of a one dimensional noisy signal. The noise in the signal may be correlated across agents, and the common and idiosyncratic part of an agents type do not necessarily have equal weights on the signal. Thus, the signal is described by three parameters. The variance of the noise, the correlation across agents in the noise term and the relative weights of the idiosyncratic and common part of an agents type. These three quantities get mapped into a single parameter, which completely summarizes the equilibrium market power an agent will have in equilibrium. Given a signal structure, and a associated REE in demand functions, one can find out the ex-ante distribution of types and traded quantities. This joint distribution plus the market power parameter provides a equivalent description of the equilibrium, but with the interpretation of being a correlated REE.

Our second characterization of the set of feasible outcomes of a REE is done in statistical terms. We provide a sharp characterization of the first and second moments of the joint distribution of types and traded quantities of any correlated REE, and thus joint distribution of variables feasible under a equilibrium in demand functions for some information structure. The market power has the characteristic of scaling the amounts traded in equilibrium, and thus scaling the mean and variance of the quantities traded. The second moments on the other hand are characterized by three correlations. The correlation between quantities traded by agents, correlation between the quantity traded by an agent and his type and the correlation between the quantity traded by an agent and the average type. The set of feasible correlations is described by two inequalities and one equality. The inequalities come from the statistical restriction that a valid variance/covariance matrix is positive semidefinite. The equality comes from the restriction that agents can condition on the price when they take an action, and thus they know the equilibrium average action when they take their own individual action. We use these two characterizations to understand market power in a REE in demand functions, and the informational constraints imposed by agents conditioning on prices in a REE.

The equilibrium market power can range from $-1/2$ to infinity, independent of the joint distribution of types. The market power depends on the information structure agents have, but the set of possible parameters for the market power is independent of the number of agents. The interpretation of the driving forces behind the market power of agents are interpreted. There are several noteworthy intuitions and results which are worth emphasizing concerning the equilibrium market power.

First, the more information is aggregated by prices, the bigger is the market power. That is, as agents can perfectly predict the equilibrium price with their private signal, the market power goes to 0. If the private information of agents is very informative on the average type, then agents will have very little market power in equilibrium. On the other hand, if the private signal of agents is very informative of the idiosyncratic part of their type, then the market power is very high, and the price level approaches the collusive market power. Since prices are always informative on the average type, this provides a clear trade-off between the role of prices in aggregating information and the losses in welfare due to market power.

Second, the market power strongly depends on the private information structure agents have, and not necessarily on the uncertainty agents have on the realization of the payoff types ex-post. For example, there are several

information structures in which, in equilibrium, agents using the equilibrium prices and private information can infer perfectly the realization of the average payoff type and the idiosyncratic part of their type, thus leading to no uncertainty over the payoff relevant parameters ex-post. Yet, each of these information structures may lead to very different equilibrium market power. As an example we show a simple information structure and analyze the effects of a public disclosure of information. We show that depending on the form of the information disclosure, the equilibrium market power may increase or decrease.

Third, the equilibrium market power that maximizes the profits of agents is lower than the one that imposes the optimal price level, but higher than the competitive one (which is equal to 0). The intuition is simple. A high market power maximizes the profits agents get from the variations in the average type. That is, it maximizes the profits agents can get from the exogenous supply of asset. Nevertheless, if the market power is too high the trade between agents is diminished. Thus, the optimal market power provides the exact tradeoff between the benefits that agents get from trading between each other and the profits agents get from the exogenous supply of assets. We provide a sharp characterization of the set of correlation in agents type and slope in the exogenous supply of asset for which the optimal market power is equal to the complete information market power. Generically the optimal market power is different than the complete information market power, yet this can be lower or higher depending on the value of the payoff relevant parameters. The interpretation is the same as before, the complete information market power may be too low if agents would like to increase the profits they get from the exogenous supply of asset, or too high if they want to increase the profits they get from trading between each other.

The second contribution of the paper is to understand how the informational content in prices shapes the set of feasible outcomes in a REE. For this we provide a characterization of the set of feasible REE in terms of the set of outcomes in a Bayes Nash equilibrium when agents compete in quantities. We show that the set of feasible distributions of a correlated REE can also be achieved in an equilibrium in which agents compete in quantities, but with a normalization in the best response. That is, an equilibrium in a demand function competition can be seen as an equilibrium in quantity competition in which the private information of agents is modified and the actions are rescaled. In a model with a continuum of agents, both models are equivalent, except for the fact that in a correlated REE agents use price to update their beliefs on their own type. Thus, with a continuum of agents the set of outcomes of a correlated REE corresponds to a subset of outcomes of Bayes correlated equilibrium, where the restriction is in the information structures considered. Thus, there is a well defined sense in which this is the natural benchmark to understand the impact of the endogenous information contained in price.

We compare how the set of outcomes of a REE compares with an economy in which agents compete a la Cournot. We show that the set of feasible first moments is more restrictive in Cournot competition, as this is completely determined by the payoff relevant parameters of the economy and independent of the information structure. Nevertheless, in demand function competition the first moments are affected by market power, which is driven by the information structure of agents. On the other hand, the feasible second moments is more restrictive in demand function competition. As we mentioned, the set of feasible correlations is only two dimensional when agents compete in demand functions, while this is three dimensional when agents compete in quantities. This just comes from the fact that competition in demand function induces an additional restriction on the information structure agents have. Namely, in equilibrium they know the average action of other players when they take their own action. We use the

characterization of correlated REE to provide bounds on price volatility, and explain how aggregate fluctuations are bounded by aggregated shocks in a correlated REE.

It is worth noting that we depart from a large part of the literature studying REE in the methodology we use to study the impact of the information in prices in the economy. A large part of the literature in REE has focused on finding conditions under which prices are fully or privately revealing, and thus discuss how efficiently prices aggregate information. For example, Vives (2011) show that in a symmetric environment and under a particular information structure, for each agent prices are a sufficient statistic of the aggregate information in the economy. Hence prices are said to be privately revealing. On the other hand, Rostek and Wernetka (2012) show that by keeping the information structure, but allowing for asymmetric distributions of types, prices are no longer privately revealing. In contrast, our interest is in quantifying how does the endogenous information in prices affect the set of possible outcomes. In Section 5.4 we discuss how our results are invariant to the actual information structure that decentralizes a REE, and thus hold irrespective of whether prices are fully revealing or not. Thus, we see our paper as an important complement to the literature analyzing the efficiency of prices in aggregating information.

In terms of the model, our paper is closely related to Vives (2011). We consider the same payoff relevant environment, but enlarge the set of feasible information structures agents may get. Although from a payoff relevant perspective both papers are essentially equivalent, we analyze different aspects of the model. Since we consider a richer set of information structure we can differentiate what is driving the outcomes and market power in equilibrium.

Our paper is also closely related to the discussion on how idiosyncratic uncertainty can lead to aggregate fluctuations in an economy. In a well known contribution, Angeletos and La'O (2013) provide a model of an economy in which idiosyncratic uncertainty leads to aggregate fluctuations. One of the key features of this economy is that the production decision is done prior to the period when markets open and agents exchange. Thus, agents cannot condition on prices when taking their production decisions, and thus there is no information aggregation through prices. In previous work, Bergemann, Heumann, and Morris (2013) show that in a similar economy, when agents cannot condition their decisions on prices, arbitrarily large idiosyncratic uncertainty can lead to arbitrarily large aggregate fluctuations. We can provide a sharp characterization on how prices that provide endogenous information on the average action of all players restricts the possibility of idiosyncratic uncertainty leading to aggregate fluctuations. This does not imply that such mechanisms are implausible, but the fact that agents cannot condition on prices of a centralized market plays an important role.

From a methodological perspective, our work is most closely related to the literature of Correlated Bayes equilibrium, as introduced by Bergemann and Morris (2013) for normal-linear environments. More particularly, we can provide a sharp connection between the literature of Bayes Nash equilibrium, which describes the set of equilibrium when firms compete a la Cournot, and REE, which describes the set of equilibria when firms compete in demand functions. Compared to the results found in Bergemann, Heumann, and Morris (2013), which analyze the case of Cournot competition, we show that correlated REE provides an additional restriction on the set of feasible information structures considered but provides an extra degree of freedom which is the market power. Methodologically speaking, our paper can be seen as a natural extension of the analysis found in Bergemann, Heumann, and Morris (2013) to allow for demand function competition.

Conceptually speaking, our paper is also related to the literature that tries to connect general equilibrium theory

with the strategic component that comes with atomic agents. For example, Weretka (2011) provides a general framework to think of general equilibrium when agents are atomic and thus are not price takers. This solution concept relies in agents correctly anticipating their price impact and taking their price impact as given. Our definition of correlated REE can be seen as a combination of the definition of slope takers found in Weretka (2011) and Bayes correlated equilibrium found in Bergemann and Morris (2013). As we consider a simple framework, we can analyze in detail the characteristics of the equilibrium, although we obviously cannot provide the generality found in Weretka (2011).

The rest of the paper is organized as follows. In Section 2 we present the model and the two solution concepts, correlated REE and REE in demand functions. In Section 3 we provide the equivalence result between both solution concepts. In Section 4 we provide the two aforementioned characterizations of the set of feasible REE outcomes. In Section 5 we analyze how the information contained in prices restrict the set of feasible outcomes. In Section 6 we interpret the drivers of market power, and analyze the impact of market power on profits. Finally, in Section ?? we extend the model to allow for a asymmetric equilibria in an environment with asymmetric type distribution.

2 Model and Equilibrium Concepts

We consider an economy with finite number of agents, indexed by $i \in N = \{1, \dots, N\}$ (as usual we use the letter N to denote the set of agents and the number of agents). There is a divisible good which is traded by agents. The realized profits of an agent that buys an amount a_i of asset at price p is given by,

$$\pi(\theta_i, a_i, p) = \theta_i a_i - \frac{1}{2} a_i^2 - a_i p,$$

where θ_i is a parameter that affects an agents utility, henceforth the agents type. The good is supplied by a seller, who gets a surplus of:

$$S(A) \triangleq -P_0 A - \frac{r}{2} A^2 + A p$$

from supplying an amount A at price p . Thus, throughout the paper we assume that there is an exogenous supply function given by:

$$p = P_0 + r A.$$

We assume a symmetric normal environment. Thus, for any pair of agents $i, j \in N$ their type is distributed according to:

$$\begin{pmatrix} \theta_i \\ \theta_j \end{pmatrix} \sim \mathcal{N} \left(\begin{pmatrix} \mu_\theta \\ \mu_\theta \end{pmatrix}, \begin{pmatrix} \sigma_\theta^2 & \rho_{\theta\theta} \sigma_\theta^2 \\ \rho_{\theta\theta} \sigma_\theta^2 & \sigma_\theta^2 \end{pmatrix} \right).$$

Throughout the paper we will assume that $\rho_{\theta\theta}$ takes any value that is statistically feasible given the symmetry assumption. That is, $\rho_{\theta\theta} \in [\frac{-1}{N-1}, 1]$.

Before we proceed to the equilibrium definitions it is convenient to make some definitions to simplify notation. We define the average type as follows,

$$\bar{\theta} \triangleq \frac{1}{N} \sum_{i \in N} \theta_i$$

and the deviation of an agent's type from the average is given by¹,

$$\Delta\theta_i \triangleq \theta_i - \bar{\theta}.$$

2.1 Rational Expectations Equilibrium in Demand Functions

We begin by studying the case in which agents have private information and compete in demand functions. We assume a symmetric information structure, in which each agent $i \in N$ gets a normal information structure denoted by $\{\mathcal{I}_i\}_{i \in N}$ ². To fix ideas we can think of the information structure as a one dimensional signal of the following form,

$$s_i = \Delta\theta_i + \lambda\bar{\theta} + \varepsilon_i, \quad (1)$$

where $\lambda \in \mathbb{R}$ is a parameter and ε_i is a normally distributed noise terms (that is, independent of θ_i for all $i \in N$) with expectation zero and correlation $\rho_{\varepsilon\varepsilon} \geq -1/n$ across agents. Just to ease the reader into the notation explained in footnote 1 we note that,

$$\bar{\varepsilon} \triangleq \frac{1}{N} \sum_{i \in N} \varepsilon_i ; \Delta\varepsilon_i \triangleq \varepsilon_i - \bar{\varepsilon}$$

The game is as follows. Agents simultaneously submit demand functions $X_i(s_i, p)$. A Walrasian auctioneer sets a price \hat{p} that clears the market. That is,

$$\hat{p} = P_0 + r \sum_{i \in N} X_i(s_i, \hat{p}).$$

Finally, the realized payoff of each agent is given by,

$$\pi(\theta_i, \hat{p}, X_i(s_i, \hat{p})).$$

We study the symmetric linear Bayes Nash Equilibrium equilibrium of this game, which we call a Rational Expectations Equilibrium (henceforth REE) in demand functions.

Definition 1 (Symmetric REE in Demand Functions).

For given normal information structure $\{\mathcal{I}_i\}_{i \in N}$, demand function $X(\mathcal{I}_i, p)$ constitutes a symmetric linear REE in demand functions if,

1. $X(\mathcal{I}_i, p)$ is normally distributed, measurable with respect to p and \mathcal{I}_i , and linear in both of its arguments.
2. Equilibrium price \hat{p} is given by,

$$\hat{p} = P_0 + r \sum_{i \in N} X(\mathcal{I}_i, \hat{p})$$

for all realizations of $\{s_i\}_{i \in N}$.

¹Henceforth we will use a bar above a variable to denote the average over all agents, and a Δ before the variable to denote the deviation of the variable with respect to the average.

²We say a information structure $\{\mathcal{I}_i\}_{i \in N}$ is normal if it can be described by the joint distribution of k random vectors normally distributed.

3. $X(\mathcal{I}_i, p)$ solves the following maximization problem,

$$X(s_i, p) \in \arg \max_{x(\mathcal{I}_i, p) \in \mathbb{R}^{k+2}} \mathbb{E}[\pi(\theta_i, x(\mathcal{I}_i, p), \hat{p}) | \mathcal{I}_i];$$

$$\text{subject to } \hat{p} = P_0 + \sum_{j \neq i} X(s_j, \hat{p}) + x(\mathcal{I}_i, \hat{p}),$$

where k is the cardinality of the vector of random variables in \mathcal{I}_i .

In the literature of REE the discussion frequently concerns the informational property of prices and how does information get into price. We will relegate those discussions for later as we do not seek to discuss these topics in depth. Instead our discussion will focus on understanding the set of outcomes that are feasible under some information structure. Yet, as we want to understand our results in light of the previous literature we go back to these and discuss them in due cause.

2.2 Correlated Rational Expectations Equilibrium

We will now discuss a different solution concept. Although it is not formally necessary to set-up a maximization problem to define a correlated rational expectations equilibrium, we will do so to provide the heuristics on why the definition will be useful. It is important to highlight that all definitions and explanations throughout this subsection will not be formal, and are only meant to provide the heuristic thinking.

We think of each player maximizing the following profit function (note we incur in the abuse of notation that p is the equilibrium price and the informational content of the price),

$$\max_{a_i} \mathbb{E}[\pi(\theta_i, a_i, p) | p, a_i].$$

The price is pin down by the exogenous supply of goods, given by

$$p = P_0 + rN\bar{a},$$

where we recall the reader that we follow the notation explained in footnote 1. The first order condition of agents is given by,

$$\mathbb{E}[\theta_i | a_i, p] - p - a_i - \frac{\partial p}{\partial a_i} a_i = 0$$

$$\Rightarrow a_i = \frac{\mathbb{E}[\theta_i | a_i, p] - p}{1 + \frac{\partial p}{\partial a_i}}.$$

In practice $\frac{\partial p}{\partial a_i}$ is the slope of the linear residual supply that player i faces. In a linear equilibrium we will have that $\frac{\partial p}{\partial a_i}$ is constant, and thus we define,

$$\tau \triangleq \frac{\partial p}{\partial a_i}.$$

Note that by definition τ will be the market power of a individual agent. To ensure concavity of the objective function of agent i we will have to impose,

$$\tau > -1/2.$$

Note that in the previous maximization problem there is an abuse of notation as p is used as a signal but also as the payoff relevant parameter. Moreover, we are assuming that agent maximizing assuming they know the equilibrium price, although this depends on the actions taken by all other players. Although for a continuum of players this is not so troublesome as no agent can affect prices, a priori with finite number of players this becomes more complicated. Thus, we will avoid formally defining such a problem, and leave the previous derivation just as a heuristic argument on which we rely for intuitions. Keeping the previous derivations in mind, we make the following definition,

Definition 2 (Symmetric Correlated REE).

We define a correlated REE as a market power parameter τ , with $\tau > -1/2$, and a joint distribution of normal random variables $(p, a_i, \bar{a}, \theta_i, \bar{\theta})'$,

$$\begin{pmatrix} p \\ a_i \\ \bar{a} \\ \theta_i \\ \bar{\theta} \end{pmatrix} \sim \mathcal{N} \left(\begin{pmatrix} \mu_p \\ \mu_a \\ \mu_a \\ \mu_\theta \\ \mu_\theta \end{pmatrix}, \begin{pmatrix} \sigma_p^2 & \rho_{ap}\sigma_a\sigma_p & \rho_{p\bar{a}}\sigma_{\bar{a}}\sigma_p & \rho_{\theta p}\sigma_\theta\sigma_p & \rho_{p\bar{\theta}}\sigma_{\bar{\theta}}\sigma_p \\ \rho_{ap}\sigma_a\sigma_p & \sigma_a^2 & \rho_{a\bar{a}}\sigma_{\bar{a}}\sigma_a & \rho_{a\theta}\sigma_\theta\sigma_a & \rho_{a\bar{\theta}}\sigma_{\bar{\theta}}\sigma_a \\ \rho_{p\bar{a}}\sigma_{\bar{a}}\sigma_p & \rho_{a\bar{a}}\sigma_{\bar{a}}\sigma_a & \sigma_{\bar{a}}^2 & \rho_{\theta\bar{a}}\sigma_\theta\sigma_{\bar{a}} & \rho_{\bar{a}\bar{\theta}}\sigma_{\bar{\theta}}\sigma_{\bar{a}} \\ \rho_{\theta p}\sigma_\theta\sigma_p & \rho_{a\theta}\sigma_\theta\sigma_a & \rho_{\theta\bar{a}}\sigma_\theta\sigma_{\bar{a}} & \sigma_\theta^2 & \rho_{\theta\bar{\theta}}\sigma_\theta\sigma_{\bar{\theta}} \\ \rho_{p\bar{\theta}}\sigma_{\bar{\theta}}\sigma_p & \rho_{a\bar{\theta}}\sigma_{\bar{\theta}}\sigma_a & \rho_{\bar{a}\bar{\theta}}\sigma_{\bar{\theta}}\sigma_{\bar{a}} & \rho_{\theta\bar{\theta}}\sigma_\theta\sigma_{\bar{\theta}} & \sigma_{\bar{\theta}}^2 \end{pmatrix} \right), \quad (2)$$

such that the following restrictions hold:

$$\mathbb{E}[\theta_i | a_i, p] - p - a_i - \tau a_i = 0; \quad (3)$$

$$p = P_0 + rN\bar{a}. \quad (4)$$

The definition of a correlated REE is just a joint distribution of variables imposing the restriction that the first order condition holds and the equation that determines prices also holds. Since the price is perfectly collinear with the aggregate demand, we will frequently refer to a correlated REE in terms of the variables $(a_i, \bar{a}, \theta_i, \bar{\theta})'$, without making an explicit reference to the price. Nevertheless, it is important to highlight the model is sufficiently flexible to accommodate potential extensions in which the exogenous supply of goods is stochastic.

A correlated REE is the natural extension of Bayes correlated equilibrium in linear environments, developed by Bergemann and Morris (2013) in earlier work, to allow for supply function equilibria. Although we discuss the precise connection between these two solution concepts in the following section, it is worth briefly discussing the different elements of a correlated REE in light of previous work.

A Bayes correlated equilibrium is equivalent to studying the set of feasible outcome distributions when agents compete in quantities (i.e. a la Cournot). In this case, agents try to anticipate the quantities others will submit, but the quantity submitted by any player does not depend directly on the quantity submitted by other players. When agents compete in demand functions there are two elements that change with respect to competing in quantities. First, agents demand a quantity conditional on a price, and thus they can anticipate what will be their demand be conditional on the actions of others. This is a purely informational restriction over Bayes correlated equilibrium. Namely, agents take an action, conditioning on knowing the average action taken by all other players. Although in terms of timing this seems may seem paradoxical, in practice equilibrium in supply functions gives the foundations to achieve such an equilibrium. If we were looking at an economy with a continuum of players, then the definition

of a correlated REE would be exactly as the one previously given, but imposing $\tau = 0$. Thus, the only difference between a correlated REE and a Bayes correlated equilibrium would be that correlated REE imposes an additional restriction on the information agents have when they take actions. Namely, all agents know the average action of all other players.

There is a second difference between demand function competition and competition in quantities. This comes from the fact that an agent anticipates that the demand of all other agents is conditioned on the price. Thus, an agent understands that the change in price by changing the quantity he demands is not only the direct change given by the exogenous supply, but also given by the change the amount demanded by all other players in equilibrium. This interaction component is summarized in the parameter τ . Agents anticipate that their demand will affect the amount demanded by other players in equilibrium, and this is summarized by the parameter τ . In our linear-symmetric the single parameter τ gives a sufficient statistic for the changes one agent has over the others. As we show later, if we relax the symmetry assumption each agent would have a different impact on the price, as the response of agents to prices is heterogenous. Thus, it will be necessary to allow for a vector of parameters $\{\tau_i\}_{i \in \mathbb{N}}$ to fully describe the price impact that the demand of each agent has. Although we do not explore non-linear settings in this paper, one could anticipate that for arbitrary, well behaved equilibria, one would a priori need a function $\tau(p)$ to describe the market power that an agent has.

3 Equivalence Result

We will now provide a result that formally connects both solution concepts. The next proposition shows that all possible set of outcomes in a REE in demand functions can be described by a correlated REE. In a REE in demand functions one takes as primitive the information structure agents have and calculates the Bayesian equilibrium in demand functions. This equilibrium yields a particular joint distribution of realized traded quantities, prices and types. Instead of describing an equilibrium through the process of finding the Bayes Nash equilibrium, one can simply describe the joint distributions of realized traded quantities, prices and types, taking into account the equilibrium market power. This latter description of an equilibrium is precisely a correlated REE.

Proposition 3 (Equivalence of Definitions).

A set of normal information structures $\{\mathcal{I}_i\}_{i \in \mathbb{N}}$ and demand function $X(\mathcal{I}_i, p)$ constitute a symmetric REE in demand functions if and only if there exists market power τ and joint distribution of variables $(p, a_i, \bar{a}, \theta_i, \bar{\theta})'$ that constitute a symmetric correlated REE such that,

$$p = \hat{p} \text{ and } a_i = X_i(s_i, \hat{p}),$$

where \hat{p} is the equilibrium price in the demand functions equilibrium.

Proof. (Only If) We first consider a market power constant τ and joint distribution of variables $(p, a, \bar{a}, \theta_i, \bar{\theta})'$ that constitute a symmetric correlated REE, and show there exists normal signals $\{s_i\}_{i \in \mathbb{N}}$ and demand function $X(s_i, p)$ that constitute a symmetric REE in demand functions such that,

$$p = \hat{p} \text{ and } a_i = X_i(s_i, \hat{p}),$$

where \hat{p} is the equilibrium price in the demand functions equilibrium.

Define constant c as follows,

$$c \triangleq \frac{r - \tau}{\tau r(N - 1)},$$

and suppose players receive signals $s_i = a_i + cp$. We will show that the demand functions

$$X(s_i, p) = s_i - cp \tag{5}$$

constitute a symmetric REE in linear demand functions. If all players submit demand functions as previously defined, then each player will face a residual demand given by,

$$p_i = \frac{1}{1 + r(N - 1)c} (P_i + ra) \tag{6}$$

where

$$P_i \triangleq P_0 + r \sum_{j \neq N} cs_j.$$

Note that by definition, if $a = a_i$, then $p_i = p$.

We now consider the following fictitious game for player i . We assume all players different than i submit demand functions given by (5) first. Then player i observes P_i and decides how much quantity he wants to buy assuming the market clearing price will be given by (6). If we keep the demand functions of players different than i fixed, this fictitious game will obviously yield weakly better profits for agent i than the original game in which he submits demand functions simultaneously with the rest of the players.

In the fictitious game player i solves the following maximization problem:

$$\max_a \mathbb{E}[\theta_i a - \frac{1}{2} a^2 - p_i a | s_i, P_i].$$

The first order condition is given by (where a^* denotes the optimal demand),

$$\mathbb{E}[\theta_i | s_i, P] - a^* - p_i - \frac{\partial p_i}{\partial a} a^* = 0.$$

We can rewrite the first order condition as follows,

$$a^* = \frac{\mathbb{E}[\theta_i | s_i, P] - p_i}{1 + \frac{\partial p_i}{\partial a}}.$$

Also, note that,

$$\frac{\partial p_i}{\partial a} = \frac{r}{1 + r(N - 1)c} = \tau.$$

Moreover, remember that if $a = a_i$ then $p_i = p$. This also implies that P_i is informationally equivalent to p . Thus, we have that if $a^* = a_i$ the first order condition is satisfied. Thus, $a_i^* = a_i$ is a solution to the optimization problem.

Finally, if agent i submits the demand function $X(s_i, p) = s_i - cp = a_i$ he would play in the original game in the same way as in the fictitious game. Thus, he will be playing optimally as well. Thus, the demand function $X(s_i, p)$ is an optimal response given that all other players submit the same demand. Thus, this constitutes a REE in demand functions.

(If) We now consider some information structure $\{\mathcal{I}_i\}_{i \in N}$ and some symmetric linear REE in demand functions given by $X(\mathcal{I}_i, p)$. We first note that we can always find a set of one dimensional signals $\{s_i\}_{i \in N}$ such that there exists demand functions, denoted by $X'(s_i, p)$, that constitute a REE and that are outcome equivalent to the REE given by $X(\mathcal{I}_i, p)$. For this, just define signal s_i as follows,

$$s_i \triangleq X(\mathcal{I}_i, p) - c_p p \text{ where } c_p \triangleq \frac{\partial X(\mathcal{I}_i, p)}{\partial p}.$$

We now define,

$$X'(s_i, p) \triangleq s_i + c_p p = X(\mathcal{I}_i, p).$$

By definition $X'(s_i, p)$ is measurable with respect to (s_i, p) . We now check $X'(s_i, p)$ constitutes a REE. By definition, if all players $j \neq i$ submit demand functions $X'(s_j, p)$, then player i faces exactly the same problem as in the REE when players submit demand functions given by $X(\mathcal{I}_i, p)$, except he has information s_i instead of \mathcal{I}_i . From the way s_i is defined, it is clear that \mathcal{I}_i is weakly more informative than s_i . Thus, if $X(\mathcal{I}_i, p)$ is a best response when player has information \mathcal{I}_i , then $X(\mathcal{I}_i, p)$ would also be a best response when player i has information s_i . Yet, if player submits demand function $X'(s_i, p)$ he will be submitting the same demand function as $X(\mathcal{I}_i, p)$, thus this is a best response. Thus, $X'(s_i, p)$ constitutes a REE that is outcome equivalent to $X(\mathcal{I}_i, p)$.

We now consider some one dimensional signals $\{s_i\}_{i \in N}$ and some symmetric linear REE in demand functions given by $X(s_i, p)$ that constitute a REE in demand functions and show that there exists a correlated REE that is outcome equivalent. We know that we can write $X(s_i, p)$ as follows,

$$X(s_i, p) = c_0 + c_s s_i + c_p p$$

where c_0, c_s, c_p are constant. In the REE in demand functions player i faces a residual demand given by,

$$p = P_i + \frac{r}{1 - (N-1)c_p} a_i,$$

where,

$$P_i = P_0 + r(N-1)c_0 + r \sum_{j \neq i} c_s s_j.$$

In the REE in demand functions player i cannot do better than if he knew what was the residual demand he was facing and he responded to this. In such a case, we would solve,

$$\max_a \mathbb{E}[\theta_i a - \frac{1}{2} a_i^2 - a_i p | P_i, s_i].$$

The best response to the previous maximization problem is given by,

$$\mathbb{E}[\theta_i | P_i, s_i] - a_i^* - (P_i + \frac{r}{1 - (N-1)c_p} a_i^*) - \frac{\partial p}{\partial a_i} = 0.$$

Note that conditioning on the intercept of the residual demand that agent faces is equivalent to conditioning on the equilibrium price

$$p = P_i + \frac{r}{1 - (N-1)c_p} a_i^*.$$

Thus, the first order condition can be written as follows,

$$\mathbb{E}[\theta_i|p, s_i] - a_i^* - p - \frac{\partial p}{\partial a_i} = 0.$$

But, note that agent i can get exactly the same outcome by submitting the demand function,

$$X(s_i, p) = \frac{\mathbb{E}[\theta_i|p, s_i] - p}{1 + \frac{\partial p}{\partial a_i}},$$

thus this must be the submitted demand function in equilibrium. Thus, in any REE the equilibrium realized quantities satisfy the following conditions,

$$\mathbb{E}[\theta_i|p, s_i] - a_i^* - p - \frac{\partial p}{\partial a_i} = 0.$$

Besides the market clearing condition $p = P_0 + rN\bar{a}$ is also obviously satisfied. Since in equilibrium all quantities are normally distributed, we have that $(p, \bar{a}, \Delta a_i, \bar{\theta}, \Delta \theta_i)$ form a correlated REE. ■

Remark 4 (Decentralization of $\tau = 0$).

Before we proceed it is important to remark a particular feature of the decentralization of a correlated REE when $\tau = 0$. Strictly speaking to implement the outcome of a correlated REE with $\tau = 0$ as a Bayes Nash equilibrium of demand competition, it is necessary to allow agents to submit demand correspondence. That is, submit for each price a set of quantities that they are willing to buy. Since this is a knife edge case and it does not relate to our central discussion, we relegate the discussion on how to implement a correlated REE with $\tau = 0$ to Section 6.

We see that Proposition 3 allow us to connect both solution concepts, and show that they describe the same set of outcomes. Note that in a REE in demand functions predicts not only an outcome, but also the demand function submitted by agents. In many cases this demand function can be observed in the data, so it is also empirically relevant. Yet, a correlated REE also contains the information pertaining to the demand functions agents submit. Instead of describing the slope of the demand function an agent submits, a correlated REE specifies the market power each agent has. Yet, there is a bijection between market power and the slope of the demand an agent submits in equilibrium. Thus, by specifying a correlated REE we are not only specifying outcomes in terms of quantities traded, prices and types, but also the slope of the demands agents must have submitted in equilibrium.

4 Two Characterizations of REE Outcomes

We will now provide two different characterizations of the set of feasible outcomes as a REE. The first one is a characterization in terms of a canonical information structure which allow us to decentralize all outcomes. This characterization provides a link between information structure as primitive and outcome, and thus it is more suitable to provide intuitions behind the driving mechanisms behind market power. The second characterization is in statistical terms. We provide a sharp characterization of what are the feasible outcomes under any information structure. This characterization will be more suitable to understand how the inclusion of prices, which are a source of endogenous information, allow us to restrict the set of outcomes with respect to an economy without prices.

4.1 One Dimensional Signals

Almost direct from the proof of Proposition 3 we can get an additional result. We can find a canonical set of one dimensional signals that allow us to decentralize all possible outcomes. This is important as it allow us to describe the set of outcomes in terms of an associated information structure. This will allow us to provide intuitions on what are the drivers of market power in equilibrium. We will first state the result formally.

Lemma 5 (Canonical Information Structure).

For all set of outcomes $(a_i, \bar{a}, \theta_i, \bar{\theta})$ and market power τ that constitute a correlated REE, there exists an outcome equivalent REE in demand functions in which players receive a one dimensional signal of the form,

$$s_i = \Delta\theta_i + \lambda\bar{\theta} + \varepsilon_i. \quad (7)$$

Proof. In the proof of Proposition 3 we already showed that for any information structure \mathcal{I}_i , there exists a one dimensional signal s_i such that the REE in demand functions is outcome equivalent under \mathcal{I}_i and s_i . Yet, any one dimensional signal can be written in the form of (7). One just needs to normalize the signal such that the covariance between $\Delta\theta_i$ and s_i is 1. ■

Lemma 5 provides a simple set of signals that allow to decentralize all possible REE outcomes. Vives (2011) considers the case in which signals players receive are such that $\lambda = 1$ and $\rho_{\varepsilon\varepsilon}$, thus signals of the form signals of form 5 have 2 extra dimensions. In the next subsection we show that in general both of these parameters affect the outcome of the REE, and thus in general the set of REE is three dimensional and can be parametrized by $\lambda, \rho_{\varepsilon\varepsilon}$ and σ_{ε} .

Characterization of the Canonical Signal Structure

We now provide the explicit characterization of the set of equilibria when agents receive a one dimensional signal. As previously explained, this set of signals allows to decentralized all possible outcomes of a REE in demand functions, and thus it can be seen as a canonical set of signals.

Proposition 6 (One Dimensional Signals).

Consider the REE in demand functions when players receive a one dimensional signal of the form:

$$s_i = \Delta\theta_i + \lambda\bar{\theta} + \varepsilon_i. \quad (8)$$

Then,

(i) *In equilibrium the realized demand quantities are given by,*

$$\bar{a} = \frac{\mathbb{E}[\bar{\theta}|\bar{s}]}{1 + \tau + Nr} ; \Delta a_i = \frac{\mathbb{E}[\Delta\theta_i|\Delta s_i]}{1 + \tau}$$

(ii) *The expectations are given by,*

$$\mathbb{E}[\bar{\theta}|\bar{s}] = (B\frac{\bar{s}}{\lambda} + (1 - B)\mu_{\theta}) ; \mathbb{E}[\Delta\theta_i|\Delta s_i] = b\Delta s_i$$

with

$$b \triangleq \frac{\sigma_{\Delta\theta}^2}{\sigma_{\Delta\theta}^2 + \sigma_{\Delta\varepsilon}^2} = \frac{(1 - \rho_{\theta\theta})\sigma_{\theta}^2}{(1 - \rho_{\theta\theta})\sigma_{\theta}^2 + (1 - \rho_{\varepsilon\varepsilon})\sigma_{\varepsilon}^2}; \quad B \triangleq \frac{\sigma_{\theta}^2\lambda^2}{\sigma_{\theta}^2\lambda^2 + \sigma_{\varepsilon}^2} = \frac{(1 + (N-1)\rho_{\theta\theta})\sigma_{\theta}^2\lambda^2}{(1 + (N-1)\rho_{\theta\theta})\sigma_{\theta}^2\lambda^2 + (1 + (N-1)\rho_{\varepsilon\varepsilon})\sigma_{\varepsilon}^2}, \quad (9)$$

(iii) The market power in equilibrium is given by,

$$\tau = \frac{1}{2} \left(-Nr \frac{(N-1)\iota - 1}{(N-1)\iota + 1} + \sqrt{\left(Nr \frac{(N-1)\iota - 1}{(N-1)\iota + 1} \right)^2 + 2Nr + 1 - 1} \right) \quad (10)$$

where

$$\iota \triangleq \frac{b\lambda}{B}.$$

Proof. We assume agents receive a one dimensional signal of the form

$$s_i = \Delta\theta_i + \Delta\varepsilon_i + \bar{\varepsilon} + \lambda\bar{\theta}.$$

We find explicitly the equilibrium in demand functions. We conjecture that agents submit demand functions of the form,

$$X(s_i, p) = c_0 + c_s s_i + c_p p \quad (11)$$

Note that,

$$\frac{1}{N} \sum_{i \in N} X(s_i, p) = c_0 + c_s \bar{s} + c_p p,$$

and thus in equilibrium,

$$\begin{aligned} \hat{p} &= P_0 + r \sum_{i \in N} X(s_i, \hat{p}) = P_0 + Nr(c_0 + c_s \bar{s} + c_p \hat{p}) \\ \Rightarrow \hat{p} &= \frac{1}{1 - Nrc_p} (P_0 + Nr(c_0 + c_s \bar{s})). \end{aligned} \quad (12)$$

Thus,

$$\bar{s} = \frac{(1 - Nrc_p)\hat{p} - P_0 - Nrc_0}{Nrc_s}.$$

Also, note that if all agents submit demand functions of the form (11), then agent $i \in N$ will face a residual demand with a slope given by,

$$\frac{\partial p_i}{\partial a} = \tau = \frac{r}{1 - r(N-1)c_p}. \quad (13)$$

As before, we use the variable p_i for the residual supply that agent i faces.

We now note that,

$$\mathbb{E}[\theta_i | s_i, \hat{p}] = \mathbb{E}[\theta_i | \Delta s_i, \bar{s}] = \mathbb{E}[\Delta\theta_i | \Delta s_i] + \mathbb{E}[\bar{\theta} | \bar{s}].$$

Calculating each of the terms,

$$\begin{aligned} \mathbb{E}[\Delta\theta_i | \Delta s_i] &= \frac{\sigma_{\Delta\theta}^2}{\sigma_{\Delta\theta}^2 + \sigma_{\Delta\varepsilon}^2} \Delta s_i = \frac{\sigma_{\Delta\theta}^2}{\sigma_{\Delta\theta}^2 + \sigma_{\Delta\varepsilon}^2} \left(s_i - \frac{(1 - Nrc_p)\hat{p} - P_0 - Nrc_0}{Nrc_s} \right); \\ \mathbb{E}[\bar{\theta} | \bar{s}] &= \frac{\sigma_{\theta}^2}{\sigma_{\theta}^2 + \sigma_{\varepsilon}^2/\lambda^2} \bar{s} + \frac{\sigma_{\varepsilon}^2}{\sigma_{\theta}^2 + \sigma_{\varepsilon}^2/\lambda^2} \mu_{\theta} = \frac{\sigma_{\theta}^2}{\sigma_{\theta}^2 + \sigma_{\varepsilon}^2/\lambda^2} \frac{(1 - Nrc_p)\hat{p} - P_0 - Nrc_0}{\lambda Nrc_s} + \frac{\sigma_{\varepsilon}^2}{\sigma_{\theta}^2 + \sigma_{\varepsilon}^2/\lambda^2} \mu_{\theta}. \end{aligned}$$

It is convenient to define,

$$b \triangleq \frac{\sigma_{\Delta\theta}^2}{\sigma_{\Delta\theta}^2 + \sigma_{\Delta\varepsilon}^2} = \frac{(1 - \rho_{\theta\theta})\sigma_{\theta}^2}{(1 - \rho_{\theta\theta})\sigma_{\theta}^2 + (1 - \rho_{\varepsilon\varepsilon})\sigma_{\varepsilon}^2}; B \triangleq \frac{\sigma_{\theta}^2}{\sigma_{\theta}^2 + \sigma_{\varepsilon}^2/\lambda^2} = \frac{(1 + (N - 1)\rho_{\theta\theta})\sigma_{\theta}^2}{(1 + (N - 1)\rho_{\theta\theta})\sigma_{\theta}^2 + (1 + (N - 1)\rho_{\varepsilon\varepsilon})\sigma_{\varepsilon}^2/\lambda^2}$$

We conjecture that the following demand functions form an equilibrium:

$$\begin{aligned} X(s_i, p) &= \frac{\mathbb{E}[\theta_i | s, p] - p}{1 + \tau} \\ &= \frac{b(s_i - \frac{(1 - Nrc_p)p - P_0 - Nrc_0}{Nrc_s}) + B \frac{(1 - Nrc_p)p - P_0 - Nrc_0}{\lambda Nrc_s} + (1 - B)\mu_{\theta} - p}{1 + \tau} \end{aligned}$$

We just express the two solutions from matching coefficients,

$$\begin{aligned} &\left\{ \begin{aligned} \text{cs} &\rightarrow \frac{b(\kappa - \sqrt{\kappa^2 + 2nr + 1} - 1)}{\kappa + nr} \\ \tau &\rightarrow \frac{1}{2}(-\kappa - \sqrt{\kappa^2 + 2nr + 1} - 1) \\ \text{c0} &\rightarrow \frac{P_0((n - 2)r - \kappa) - (B - 1)r\mu_{\theta}(-\kappa + \sqrt{\kappa^2 + 2nr + 1} + 1)}{r(-\kappa + n^2r - n(\kappa + 3r))} \\ \text{cp} &\rightarrow \frac{r(-\kappa + \sqrt{\kappa^2 + 2nr + 1} + n - 1) - \kappa}{(n - 1)r(nr - \kappa)} \end{aligned} \right\}; \\ &\left\{ \begin{aligned} \text{cs} &\rightarrow \frac{b(\kappa + \sqrt{\kappa^2 + 2nr + 1} - 1)}{\kappa + nr} \\ \tau &\rightarrow \frac{1}{2}(-\kappa + \sqrt{\kappa^2 + 2nr + 1} - 1) \\ \text{c0} &\rightarrow \frac{(B - 1)r\mu_{\theta}(\kappa + \sqrt{\kappa^2 + 2nr + 1} - 1) + P_0((n - 2)r - \kappa)}{r(-\kappa + n^2r - n(\kappa + 3r))} \\ \text{cp} &\rightarrow -\frac{\kappa + r(\kappa + \sqrt{\kappa^2 + 2nr + 1} - n + 1)}{(n - 1)r(nr - \kappa)} \end{aligned} \right\}, \end{aligned}$$

with

$$\kappa \triangleq nr \frac{b(n - 1)\lambda - B}{b(n - 1)\lambda + B}.$$

Note that for the first solution $\tau \leq -1/2$ and thus this does not constitute a valid equilibrium. On the other hand, for the second solution $\tau \geq -1/2$, and thus this constitutes a valid equilibrium. By rewriting the terms and using the definition of ι we get the result.

■

The characterization of a REE in demand functions when agents get signals of the form (8) has three elements. The first part of the characterization, element i, describes the realized average traded quantity and realized idiosyncratic part of the traded quantity of agents in terms of the realized signal. The description is particularly simple, as it states that the equilibrium can be calculated as if each agent could observe the signal of all other players. The average action is the expectation of the average type with respect to the average signal, correctly normalized taking into account the market power. While the idiosyncratic deviation of an agents action is the expected deviation of the

agents type, also correctly normalized taking into account that the market power each agent has. Note that whenever agents get multidimensional signals, prices need not be fully revealing. Moreover, the general characterization of equilibria for arbitrary information structure is generally cumbersome. Nevertheless, there is always an equivalent one dimensional signal that yields the same outcome.

The second part of the characterization (part ii) just provides the explicit formula for the updating of the agents beliefs. As we previously mentioned, agents update as if they saw the signals of all other players. In a linear-normal environment like ours, the updating just comes from well known formulas for updating of normal random variables.

Finally, the third part of the characterization (part iii) provides an explicit formula for the market power in terms of the informations structure. The intuition on what are the drivers of market power are relegated to Section 6.

4.2 Statistical Characterization

We will now provide a statistical characterization of the set of feasible distributions of quantities and prices in any correlated REE. We begin by proving a simple lemma which will allow us to reduce the number of variables we need to consider in the statistical description of an equilibrium. Lemma 7 will show that we can always write $\rho_{a\bar{\theta}}, \rho_{\bar{a},\theta}$ and $\rho_{\bar{a}\bar{\theta}}$ in terms of $\rho_{aa}, \rho_{a\theta}$ and one extra parameter. Thus, before we proceed it is convenient to define the following parameter:

$$\rho_{a\phi} \triangleq \rho_{\bar{a}\bar{\theta}} \sqrt{\left(\frac{1-\rho_{aa}}{N} + \rho_{aa}\right)\left(\frac{1-\rho_{\theta\theta}}{N} + \rho_{\theta\theta}\right)}.$$

Since we will need a single parameter to describe the correlations $\rho_{a\bar{\theta}}, \rho_{\bar{a},\theta}$ and $\rho_{\bar{a}\bar{\theta}}$, we use the variable $\rho_{a\phi}$ for this, even when technically speaking $\rho_{a\phi}$ does not represent the correlation between any random variables we use.

Lemma 7 (Reduction of Parameters).

Let the normal random variables $(a_1, \dots, a_N, \theta_1, \dots, \theta_N)$ be symmetrically distributed, then the normal random variables $(\theta_i, \bar{\theta}, a_i, \bar{a})$ satisfy,

$$\mu_{\bar{a}} = \mu_a ; \mu_{\bar{\theta}} = \mu_{\theta} ; \sigma_{\bar{a}}^2 = \text{cov}(a_i, \bar{a}) ; \sigma_{\bar{\theta}}^2 = \text{cov}(\theta_i, \bar{\theta}) ; \text{cov}(\bar{a}, \bar{\theta}) = \text{cov}(a_i, \bar{\theta}) = \text{cov}(\bar{a}, \theta_i).$$

Thus, the joint distribution of variables can be written as follows,

$$\begin{pmatrix} a_i \\ \bar{a} \\ \theta_i \\ \bar{\theta} \end{pmatrix} \sim \mathcal{N} \left(\begin{pmatrix} \mu_a \\ \mu_a \\ \mu_{\theta} \\ \mu_{\theta} \end{pmatrix}, \begin{pmatrix} \sigma_a^2 & \left(\frac{1-\rho_{aa}}{N} + \rho_{aa}\right)\sigma_a^2 & \rho_{a\theta}\sigma_{\theta}\sigma_a & \rho_{a\phi}\sigma_{\theta}\sigma_a \\ \left(\frac{1-\rho_{aa}}{N} + \rho_{aa}\right)\sigma_a^2 & \left(\frac{1-\rho_{aa}}{N} + \rho_{aa}\right)\sigma_a^2 & \rho_{a\phi}\sigma_{\theta}\sigma_a & \rho_{a\phi}\sigma_{\theta}\sigma_a \\ \rho_{a\theta}\sigma_{\theta}\sigma_a & \rho_{a\phi}\sigma_{\theta}\sigma_a & \sigma_{\theta}^2 & \left(\frac{1-\rho_{\theta\theta}}{N} + \rho_{\theta\theta}\right)\sigma_{\theta}^2 \\ \rho_{a\phi}\sigma_{\theta}\sigma_a & \rho_{a\phi}\sigma_{\theta}\sigma_a & \left(\frac{1-\rho_{\theta\theta}}{N} + \rho_{\theta\theta}\right)\sigma_{\theta}^2 & \left(\frac{1-\rho_{\theta\theta}}{N} + \rho_{\theta\theta}\right)\sigma_{\theta}^2 \end{pmatrix} \right), \quad (14)$$

Proof. The conditions on the first moments are direct from the symmetry assumption. To be more specific,

$$\bar{\theta} = \frac{1}{N} \sum_{i \in N} \mathbb{E}[\theta_i]$$

Taking expectations of the previous equation,

$$\mu_{\bar{\theta}} = \mathbb{E}[\bar{\theta}] = \frac{1}{N} \sum_{i \in N} \mathbb{E}[\theta_i] = \frac{1}{N} \sum_{i \in n} \mu_{\theta_i} = \mu_{\theta}.$$

The same obviously holds for $\mu_{\bar{a}} = \mu_a$.

To prove the results on the second moments, we first prove that,

$$\sum_{i \in N} \Delta \theta_i = 0$$

For this just note that,

$$\sum_{i \in N} \Delta \theta_i = \sum_{i \in N} (\theta_i - \bar{\theta}) = \sum_{i \in N} \theta_i - N\bar{\theta} = \sum_{i \in N} \theta_i - N \left(\frac{1}{N} \sum_{i \in N} \theta_i \right) = 0$$

Note that in any symmetric equilibrium, we must have that for all $i, j \in N$, $\text{cov}(\theta_i, \bar{\theta}) = \text{cov}(\theta_j, \bar{\theta})$. Thus, we have that,

$$\text{cov}(\bar{\theta}, \theta_i) = \frac{1}{N} \text{cov}(\bar{\theta}, \sum_{i \in N} \theta_i) = \frac{1}{N} \text{cov}(\bar{\theta}, \sum_{i \in N} \bar{\theta} + \Delta \theta_i) = \text{var}(\bar{\theta}) + \frac{1}{N} \underbrace{\text{cov}(\bar{\theta}, \sum_{i \in N} \Delta \theta_i)}_{=0} = \text{var}(\bar{\theta}).$$

For the rest of the moments we obviously just proceed the same way.

■

Using the previous lemma, it is worth highlighting that the joint distribution of variables can be expressed in a simpler way by using a simple change of variables. The same distribution of variables can be expressed as follows,

$$\begin{pmatrix} \Delta a_i \\ \bar{a} \\ \Delta \theta_i \\ \bar{\theta} \end{pmatrix} \sim \mathcal{N} \left(\begin{pmatrix} 0 \\ \mu_a \\ 0 \\ \mu_\theta \end{pmatrix}, \begin{pmatrix} \frac{(N-1)\sigma_a^2(1-\rho_{aa})}{N} & 0 & \rho_{\Delta\Delta}\sigma_a\sigma_\theta & 0 \\ 0 & \frac{\sigma_a^2((N-1)\rho_{aa}+1)}{N} & 0 & \sigma_a\sigma_\theta(\rho_{a\theta} - \rho_{\Delta\Delta}) \\ \rho_{\Delta\Delta}\sigma_a\sigma_\theta & 0 & \frac{(N-1)\sigma_\theta^2(1-\rho_{\theta\theta})}{N} & 0 \\ 0 & \sigma_a\sigma_\theta(\rho_{a\theta} - \rho_{\Delta\Delta}) & 0 & \frac{\sigma_\theta^2((N-1)\rho_{\theta\theta}+1)}{N} \end{pmatrix} \right). \quad (15)$$

It is perhaps worth highlighting that Lemma 7 is a consequence of the assumption of the symmetry of the joint distribution of variables. Thus, this holds independent of the additional constraints imposed in a correlated REE. We now can characterize the set of correlated REE,

Proposition 8. *[Statistical Characterization of correlated REE]*

The normal random variables $(\theta_i, \bar{\theta}, a_i, \bar{a})$ and the market power parameter τ form a symmetric correlated REE if and only if,

1. The joint distribution of variables $(\theta_i, \bar{\theta}, a_i, \bar{a})$ is given by (14)
2. The mean individual action is given by,

$$\mu_a = \frac{\mu_\theta - P_0}{1 + Nr + \tau} \quad (16)$$

3. The variance of the individual action is given by,

$$\sigma_a = \frac{\rho_{a\theta}\sigma_\theta}{1 + \tau + r((N-1)\rho_{aa} + 1)} \quad (17)$$

4. The correlations satisfy the following equality

$$\frac{N\rho_{a\phi}}{(1 + \tau + rN)((1 - \rho_{aa})(N - 1) + 1)} = \frac{\rho_{a\theta}}{1 + \tau + r((N - 1)\rho_{aa} + 1)}, \quad (18)$$

and the following inequalities,

$$\frac{(N - 1)^2}{N^2} (1 - \rho_{aa})(1 - \rho_{\theta\theta}) \geq (\rho_{a\theta} - \rho_{a\phi})^2; \quad \frac{((N - 1)\rho_{aa} + 1)((N - 1)\rho_{\theta\theta} + 1)}{N^2} \geq \rho_{a\phi}^2; \quad (19)$$

$$\rho_{aa} \in \left[\frac{-1}{N - 1}, 1 \right]. \quad (20)$$

Proof. (Only If)

We first prove that if normal random variables $(\theta_i, \bar{\theta}, a_i, \bar{a})$ and the market power parameter τ form a correlated REE then conditions 1-4 hold. Condition 1 is trivial from the fact that the definition of correlated REE imposes normality. If the normal random variables are normally distributed, then their variance/covariance must be positive-semidefinite. But this is equivalent to imposing the that variance/covariance matrix of the random variables as in (15) is positive semidefinite. Yet, it is direct that these implies condition 4.

If normal random variables $(\theta_i, \bar{\theta}, a_i, \bar{a})$ and the market power parameter τ form a CREE then we have that,

$$\mathbb{E}[\theta_i | \bar{a}, a_i] - a_i - p - \tau a_i = 0$$

where we use that p and \bar{a} are informationally equivalent. Taking expectations of the previous equality and using the Law of Iterated Expectations we get condition (16). If we multiply the previous equation by a_i we get,

$$\mathbb{E}[a_i \theta_i | \bar{a}, a_i] - a_i^2 - a_i(P_0 + Nr\bar{a}) - \tau a_i^2 - \underbrace{\mu_a(\mu_\theta - P_0 - \mu_a(1 + \tau + Nr))}_{=0} = 0$$

Grouping up terms, we get:

$$cov(a_i \theta_i) - var(a_i) - Nrcov(a_i, \bar{a}) - \tau var(a_i) = 0.$$

But, just by rewriting the value of the variances and covariances, the previous equality can be written as follows,

$$\sigma_a = \frac{\rho_{a\theta}\sigma_\theta}{1 + \tau + r((N - 1)\rho_{aa} + 1)}.$$

Thus, we get (17). If we repeat the same as before but multiply by \bar{a} instead of a_i we get,

$$cov(\bar{a}, \theta_i) - cov(a_i, \bar{a}) - Nrvar(\bar{a}) - \tau cov(\bar{a}, a_i) = 0.$$

As before, by rewriting the value of the variances and covariances, the previous equality can be written as follows,

$$\sigma_a = \frac{N\rho_{a\phi}\sigma_\theta}{(1 + \tau + rN)((1 - \rho_{aa})(N - 1) + 1)}.$$

Using (17) we get (18).

(If) We now consider normal random variables $(\theta_i, \bar{\theta}, a_i, \bar{a})$ such that conditions 1-4 are satisfied. First, note that condition 4 guarantees that the variance/covariance matrix is positive-semidefinite, and thus a well defined

variance/covariance matrix. Moreover, if condition 1 is satisfied, we can obviously relabel the terms such that we can rewrite the distribution as in 2. We just need to prove that restrictions (3) and (4) of the definition of correlated REE are also satisfied. We will show that the following restriction holds,

$$\mathbb{E}[\theta_i|a_i, p] - (P_0 + rN\bar{a}) - a_i - \tau a_i = 0; \quad (21)$$

Then obviously restriction (4) is just the determination of the price in terms of the average quantity \bar{a} and it will be evidently satisfied by defining the price in this way.

We now show that conditions 2 and 3 imply that equation (21) is satisfied. We define the random variable

$$z \triangleq \mathbb{E}[\theta_i|a_i, p] - (P_0 + rN\bar{a}) - a_i - \tau a_i.$$

Since $(\theta_i, \bar{\theta}, a_i, \bar{a})$ are jointly normal, we have that z is normally distributed. If we calculate the expected value of z we get:

$$\mathbb{E}[z] = \mu_\theta - (P_0 + rN\mu_a) - \mu_a - \tau\mu_a = 0,$$

where the second equality is from condition 2. If we calculate the variance of z we get,

$$\begin{aligned} \text{var}(z) &= \text{var}(\mathbb{E}[\theta_i|a_i, p] - (P_0 + rN\bar{a}) - a_i - \tau a_i) \\ &= \text{cov}(z, \mathbb{E}[\theta_i|a_i, p] - (P_0 + rN\bar{a}) - a_i - \tau a_i) \\ &= \text{cov}(z, \mathbb{E}[\theta_i|a_i, p]) - (1 + \tau)\text{cov}(z, a_i) - rN\text{cov}(z, \bar{a}) - P_0\text{cov}(z, 1). \end{aligned}$$

Note that $\text{cov}(z, 1) = \mathbb{E}[z] = 0$ by condition 2. On the other hand, it is direct that $\text{cov}(z, a_i) = 0$ by (17) and $\text{cov}(z, \bar{a}) = 0$ by (18). On the other hand, we can find constants $\alpha, \beta, \gamma \in \mathbb{R}$ such that,

$$\mathbb{E}[\theta_i|a_i, p] = \mathbb{E}[\theta_i|a_i, \bar{a}] = \alpha a_i + \beta \bar{a} + \gamma.$$

Thus, we have that,

$$\text{cov}(z, \mathbb{E}[\theta_i|a_i, p]) = \alpha\text{cov}(z, a_i) + \beta\text{cov}(z, \bar{a}) + \gamma\text{cov}(z, 1) = 0,$$

by the same argument as before. Thus, we have that $\mathbb{E}[z] = \text{var}(z) = 0$. Since z is normally distributed, this implies that $z = 0$. Thus, (21) is satisfied. Thus, by adequately defining p we have that restrictions (3) and (4) are satisfied. Hence, we get the result. ■

Proposition 8 provides a sharp characterization of the set of distributions that are feasible as a correlated REE. Not only we can provide the set of outcomes in statistical terms, but we can also easily compute the profits that each distribution yields to agents in a correlated REE. In the following lemma we connect the statistical characterization of an outcome with the profits it yields in equilibrium.

Lemma 9 (Profits in Statistical Terms).

Let $(p, a', \bar{a}, \theta_i, \bar{\theta})$ be a correlated REE. Then the expected profits of the agents are given by,

$$\mathbb{E}[\pi(\theta_i, a_i, p)] = \left(\frac{1}{2} + \tau\right)(\mu_a^2 + \sigma_a^2)$$

Proof. The proof just relies in using the first order condition of agents, which is reflected on the restrictions imposed in a REE. By definition,

$$\mathbb{E}[\pi(\theta_i, a_i, p)] = \mathbb{E}[\theta_i a_i - \frac{1}{2} a_i^2 - p a_i] = \mathbb{E}[a_i(\theta_i - a_i - p - \tau a_i)] + (\frac{1}{2} + \tau) \mathbb{E}[a_i^2]$$

By definition we know that $\mathbb{E}[a_i^2] = \sigma_a^2 + \mu_a^2$. On the other hand, we have that,

$$\mathbb{E}[a_i(\theta_i - a_i - p - \tau a_i)] = \mathbb{E}[\mathbb{E}[a_i(\theta_i - a_i - p - \tau a_i)|p, a_i]] = \mathbb{E}[a_i \mathbb{E}[(\theta_i - a_i - p - \tau a_i)|p, a_i]] = 0$$

Where in the first equality we use the law of iterated expectations and on the last equality we use that

$$\mathbb{E}[(\theta_i - a_i - p - \tau a_i)|p, a_i] = 0$$

by definition of a correlated REE. Thus, the expected profits are given by,

$$\mathbb{E}[a_i(\theta_i - a_i - p - \tau a_i)] = (\frac{1}{2} + \tau)(\mu_a^2 + \sigma_a^2)$$

■

Given the characterization provided in Proposition 8 we know that the first moment depends only the market power parameter τ . On the other hand, using (17) we know that,

$$\sigma_a^2 = \frac{\rho_{a\theta} \sigma_a \sigma_\theta - rN(\frac{1-\rho_{aa}}{N} + \rho_{aa}) \sigma_a^2}{1 + \tau}.$$

Thus, we can note that,

$$\sigma_a = \frac{\rho_{a\theta} \sigma_\theta}{1 + \tau + rN(\frac{1-\rho_{aa}}{N} + \rho_{aa})}.$$

Therefore, in any correlated REE the profits of agents is a function of the market power parameter τ and the correlations $(\rho_{aa}, \rho_{a\theta})$.

5 Informational Constraints Imposed by Prices

We now proceed to explain how the endogenous information contained in prices restrict the set of feasible outcomes in a correlated REE. From a technical perspective, we are interested to understand how the fact that agents condition on prices to updated their beliefs on their type affects the set of feasible outcomes. The benchmark we will compare with is the case in which agents compete a la Cournot. This is precisely the model studied by Bergemann, Heumann, and Morris (2013), in which they study the set of outcomes when the information structure of agents is unrestricted. Thus, we now proceed to explain the relation in outcomes between agents competing in quantities (i.e. a la Cournot) with competition in demand functions.

Since most of the literature on information revelation by prices has focused on understanding when prices are fully revealing, it is worth providing a brief discussion on this prior to our analysis. More specifically, we want to clarify in what sense the same outcome under different information structures can lead to different qualifications on the information revelation properties of prices. Since this is not central to our discussion, we relegate this to the end of the section.

5.1 Cournot Competition

We first explain briefly how to think of Cournot competition in our model, and why it is the natural benchmark model we compare with. We consider N agents competing in quantities, keeping the payoff structure the same as before. In this case, agents get a information structure $\{\mathcal{I}_i\}_{i \in N}$, then they submit quantities $\{q_i\}_{i \in N}$ ³ and the market clearing prices are calculated by,

$$p = P_0 + r \sum_{i \in N} q_i.$$

Studying competition in quantities is equivalent to studying a model of agents taking actions simultaneously, with best response given by,

$$q_i = \mathbb{E}[\theta_i - rN\bar{q}|\mathcal{I}_i] - P_0.$$

It is worth formally defining a Bayes Nash equilibrium in quantity competition.

Definition 10 (Equilibrium in Competition in Quantities).

The random variables $\{q_i\}_{i \in N}$ form a normal Bayes Nash equilibrium under competition in quantities and information structure $\{\mathcal{I}_i\}_{i \in N}$ if they are normally distributed and,

$$q_i = \mathbb{E}[\theta_i - (P_0 + rN\bar{q})|\mathcal{I}_i].$$

Note that the best response of agents changes in two aspects with respect to a correlated REE. First, agents don't have the endogenous information of prices when choosing a quantity. Second, agents have no market power. In a model with a continuum of agents there is no market power in a correlated REE either, and thus in this case we have that the only difference between Cournot competition and demand function competition is the fact that in the latter case agents know the equilibrium price when they choose the quantity they submit. Thus, this can be seen as a restriction on the set of possible information structures considered under Cournot competition. To see this, there is an easy characterization that relates the set of outcomes that can be achieved in competition in supply functions with competition in quantities.

Lemma 11 (Relation between Supply Functions and Quantity Competition).

Let $(a_i, \bar{a}, \theta_i, \bar{\theta})$

be the outcome of a REE in demand functions when agents get signals $\{s_i\}_{i \in N}$ (with s_i described as in (8)), then $(a_i, \bar{a}, \theta_i, \bar{\theta})$ is the outcome of the Bayes Nash Equilibrium when players receive information structure $\mathcal{I}_i = (\Delta s_i, \bar{s})$ and the best response of agents is given by,

$$q_i = \frac{1}{1 + \tau} (\mathbb{E}[\theta_i - rN\bar{a}|\mathcal{I}_i] - P_0),$$

where τ is given by (10).

Proof. First, note that $\text{cov}(\bar{s}, \Delta\theta_i) = \text{cov}(\bar{\theta}, \Delta s_i) = 0$. Thus,

$$\mathbb{E}[\theta_i|\bar{s}, \Delta s_i] = \mathbb{E}[\bar{\theta}|\bar{s}] + \mathbb{E}[\Delta\theta_i|\Delta s_i].$$

³We change the variable from a_i to q_i only to emphasize that the competition structure we are considering has changed.

By definition $\sum_{i \in N} \Delta s_i = 0$, thus in equilibrium,

$$\mathbb{E}[\bar{a} | \bar{s}, \Delta s_i] = \mathbb{E}[\bar{a} | \bar{s}].$$

Thus, it is easy to see that the equilibrium actions will be given by,

$$q_i = \frac{\mathbb{E}[\Delta \theta_i | \Delta s_i]}{1 + \tau} + \frac{\mathbb{E}[\bar{\theta} | \bar{s}]}{1 + r + \tau}.$$

Yet, this is exactly the characterization provided in Proposition 6. ■

Thus, the outcome of any REE can be described as a competition in quantities, but where agents get two signals, equal to the common and idiosyncratic part of the signals agents get in the REE. Nevertheless, to make the problem completely equivalent it is necessary to normalize the best response of agents to allow for the fact in a REE the best response is rescaled by the market power agents have. Once again, if we think of a model with a continuum of agents, this rescaling would not be necessary. It is perhaps worth noting that the set of signals of $\mathcal{I}_i = (\Delta s_i, \bar{s})$ are equivalent to the case in which each agent receives a private signal s_i , and then agents pooled their signals together before they. Thus, in this case it is easy to see that the REE is privately revealing, as defined by Vives (2011). That is, in the REE when agents receive one dimensional signals and compete in demand functions, for an agent forecasting his type prices provide a sufficient statistic of all the information other agents have. Yet, this is only because we have assumed that agents received originally a one dimensional signal. As we have already explained, for more general signal structures the equilibrium need not be privately revealing.

The outcomes of competition in quantities can also be characterized in statistical terms. To keep the analysis a bit more compact we will just say the random variables $(\theta_i, \bar{\theta}, q_i, \bar{q})$ form a Bayes correlated equilibrium if they are normally distributed and they can be rationalized as the Bayes Nash equilibrium outcome under competition in quantities for some information structure. The complete formalization of a Bayes correlated equilibrium in this environment and the respective characterization can be found in Bergemann, Heumann, and Morris (2013). We just provide the characterization, keeping some of the formalities aside as we only intend to compare both set of outcomes. For competition in quantities we have the following characterization of the feasible distribution of quantities and types.

Proposition 12 (Statistical Characterization of Competition in Quantities).

The normal random variables $(\theta_i, \bar{\theta}, q_i, \bar{q})$ form a Bayes correlated equilibrium if and only if,

1. *The joint distribution of variables is given by (14)*

2. *The mean individual action is given by,*

$$\mu_q = \frac{\mu_\theta - P_0}{1 + Nr} \tag{22}$$

3. *The second moments satisfy the following equalities,*

$$\sigma_q = \frac{\rho_{q\theta} \sigma_\theta}{1 - \rho_{qq} r}$$

4. The correlations satisfy the following inequalities,

$$\frac{(N-1)^2}{N^2} (1 - \rho_{aa})(1 - \rho_{\theta\theta}) \geq (\rho_{a\theta} - \rho_{a\phi})^2 ; \frac{((N-1)\rho_{aa} + 1)((N-1)\rho_{\theta\theta} + 1)}{N^2} \geq \rho_{a\phi}^2 ;$$

$$\rho_{aa} \in \left[\frac{-1}{N-1}, 1 \right].$$

Proof. See Bergemann, Heumann, and Morris (2013) ■

5.2 Comparing Feasible Outcomes

Comparing both set of outcomes there are two things that come to mind. Competition in quantities allows for a more general structure of correlation matrices. Moreover, the set of feasible correlations is independent of the payoff structure of the game, and can be chosen freely. That is, for each possible (valid) correlations matrix, there exists a information structure that decentralizes that given set of correlations as a BCE. Yet, when competing in quantities the average quantity is completely pin down by the average type and r (see (22)), thus it is independent of the information structure assumed. On the other hand, for competition in supply functions the average realized action is no longer constant, but depends on the information structure as it depends on the market power. Moreover, the set of feasible correlation matrix is more restrictive than under competition in quantities. In the case of correlated REE not all feasible correlation matrices can be decentralized as a correlated REE, moreover the set of feasible correlation matrices is no longer independent of r .

To be more specific. Just using the assumption of symmetry in the joint distribution of variables, we know from Lemma 7 that the correlation matrix is determined by three parameters. Namely $(\rho_{aa}, \rho_{a\theta}, \rho_{a\phi})$. The joint distribution of variables of a Bayes correlated equilibrium is completely determined by these three parameters, which are the relevant correlations of the joint distribution of variables. Thus, an equilibrium is completely determined by the parameters $(\rho_{aa}, \rho_{a\theta}, \rho_{a\phi})$, moreover the set of feasible correlations live in a three dimensional space and are only bounded by the statistical constraints imposed by the restriction that a valid correlation matrix must be positive semi-definite. On the other hand, on a correlated REE the set of feasible correlations $(\rho_{aa}, \rho_{a\theta}, \rho_{a\phi})$ live in a two dimensional space. This is because constraint (18) imposes a restriction on the correlations, and thus they can no longer be chosen freely. Nevertheless, in a correlated REE there is an extra free parameter which is the market power.

The same analysis can be seen from the perspective of the information structures needed to decentralize all feasible outcomes. As Bergemann, Heumann, and Morris (2013) show, the set of all feasible outcomes of the competition in quantities can also be decentralized by signals of the form given by (8). In the case of competition in quantities, the variance and correlation of the error term, plus the parameter λ , allow to span all feasible correlations $(\rho_{aa}, \rho_{a\theta}, \rho_{a\phi})$. As it is easy to realize, since the set of signals needed to span all possible signals is three dimensional, the set of feasible correlations must also three dimensional. On the other hand, as we have show for demand function competition, now one of the parameters is used to span the set of feasible market powers. Yet, the other two parameters are used to span the two dimensional set of feasible correlations $(\rho_{aa}, \rho_{a\theta}, \rho_{a\phi})$.

We will now explore how the set of feasible correlations are restricted in a correlated REE. As we already briefly discussed, the definition of correlated REE presents two variations with respect to a Bayes correlated equilibrium.

The first one is informational, as agents condition on prices, and thus condition on the average action taken by other player. This is equivalent to imposing an additional restriction on the possible information structures that need to be considered on the set of Bayes correlated equilibrium. The second variation comes from the fact that market power changes the levels of the responses of agents to the different components of the information agents have. To illustrate how both of these changes affect the set of feasible correlations we separate the discussion in two parts.

First, we will show how the set of feasible correlations $(\rho_{a\theta}, \rho_{aa})$ compare in a correlated REE with Bayes correlated equilibrium in a model with a continuum of players. Therefore, we assume $\tau = 0$ and take the limit $N \rightarrow \infty$ and $N \cdot r \rightarrow \tilde{r}$ of (18) and (19). This will allow us to illustrate how the informational constraint that agents know the average action of other players when they take actions, constraint the set of feasible correlations. In this case, the set of feasible correlations $(\rho_{a\theta}, \rho_{aa})$ in a correlated REE with a continuum of players is given by⁴,

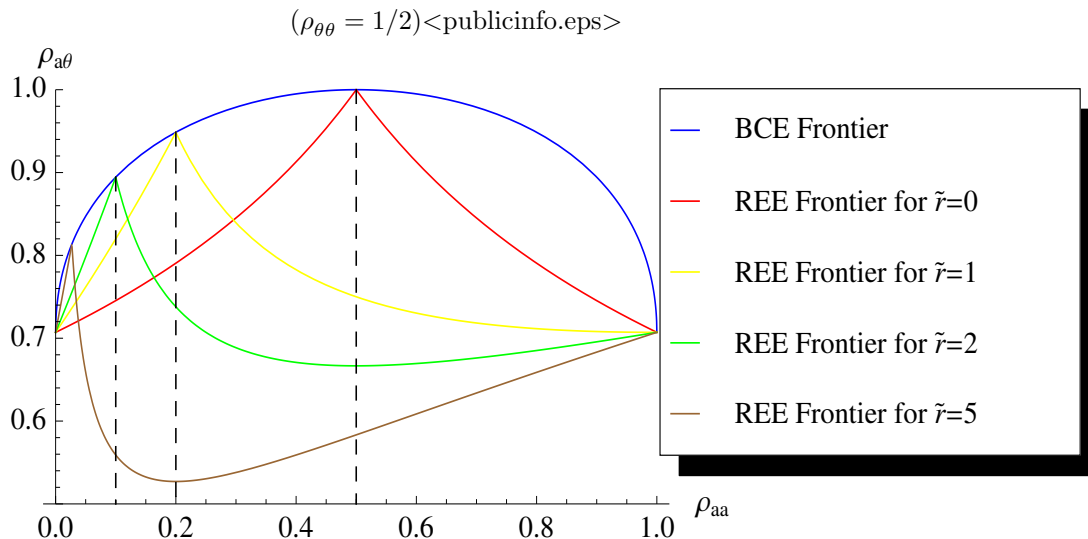
$$\rho_{a\theta} \leq \begin{cases} \frac{\sqrt{\rho_{\theta\theta}(1+\tilde{r}\rho_{aa})}}{(1+\tilde{r})\sqrt{\rho_{aa}}} & \frac{(1-\rho_{aa})\rho_{\theta\theta}}{1+\tilde{r}} \leq (1+\tilde{r})(1-\rho_{\theta\theta})\rho_{aa} \\ \frac{2(1+\tilde{r})(1-\rho_{\theta\theta})}{\sqrt{\rho_{aa}\rho_{\theta\theta}} + \sqrt{4(1+\tilde{r})^2 + \rho_{\theta\theta}(\rho_{aa} - 4(1+\tilde{r})^2)}} & \frac{(1-\rho_{aa})\rho_{\theta\theta}}{1+\tilde{r}} \geq (1+\tilde{r})(1-\rho_{\theta\theta})\rho_{aa} \end{cases}. \quad (23)$$

The set of feasible correlations $(\rho_{a\theta}, \rho_{aa})$ in a Bayes correlated equilibrium is characterized by the following inequality,

$$\rho_{a\theta} \leq \sqrt{\rho_{aa}\rho_{\theta\theta}} + \sqrt{(1+\rho_{aa})(1+\rho_{\theta\theta})}.$$

It is perhaps worth highlighting that the set of feasible correlations for a Bayes correlated equilibrium is independent of \tilde{r} as these are restricted only by the statistical constraint that the correlation matrix is positive semi-definite. In Plot 1 we compare the feasible correlations $(\rho_{a\theta}, \rho_{aa})$ for both models for different values of \tilde{r} . The curves plotted are the upper bound on the set of correlations $(\rho_{a\theta}, \rho_{aa})$ that are feasible⁵.

Figure 1: Comparing Feasible Correlations with a Continuum of Agents



⁴The calculation is not trivial, this can be found in the appendix.

⁵BCE in the plots stands for Bayes Correlated equilibrium.

Without going through a rigorous analysis and just by looking at Figure 1, we can interpret the difference in the feasible correlations under both forms of competition and compare how does the equilibrium behave for different r . We can see that the frontier of feasible correlations under correlated REE is strictly lower than under Cournot competition, except for three points. These points correspond, from left to right, to the equilibria in which agents know perfectly $\Delta\theta_i$ and nothing else, the complete information equilibria and the equilibria in which agents know perfectly $\bar{\theta}$ and nothing else. This implies that there are only 3 values of ρ_{aa} for which the maximum value of $\rho_{a\theta}$ is the same for both forms of competition. The intuition behind this is simple. Under Cournot competition, one can change the weights between $\bar{\theta}$ and $\Delta\theta_i$ in the signal of agents and change the value of ρ_{aa} , without the need to add noise to the signals. In particular, the signal structure in which agents know θ_i and nothing else always yields a correlation $\rho_{a\theta} = 1$.

Under demand function competition things change as now agents know the average action of other players, thus the information structure of agents cannot be chosen freely. For example, it is no longer possible to achieve a information in which agents know θ_i and nothing else. In particular, if the signal agents get is noiseless, then in all equilibria agents know perfectly the realization of $\bar{\theta}$ and $\Delta\theta_i$. With a continuum of players there is no market power, and thus the exact composition in the weights of the signals of agents is irrelevant. Thus, this always yields the complete information equilibria with 0 market power. Under demand function competition the only way to change the correlation in the action of agents is by adding noise. In particular, by adding idiosyncratic noise, the response to the idiosyncratic part of signals is diminished, and thus the action correlation increases with respect to the complete information equilibrium. By adding common value noise the response to the common part of signals is diminished, and thus the action correlation decreases with respect to the complete information equilibrium. Thus, the frontier is spanned by adding either common value noise or idiosyncratic noise to the signals of agents. Yet, as the signals of agents are noisy, necessarily the correlation of actions with types decreases, thus $\rho_{a\theta}$ is lower than in a noiseless world.

We can also interpret the effects of increasing \tilde{r} . The straightforward effect is that with a higher \tilde{r} the correlations of the complete information equilibrium change. By increasing \tilde{r} agents respond less strongly to $\bar{\theta}$, which implies that the action correlation decreases. This is intuitive, as a high \tilde{r} means that agents do not want to co-move, and thus in relative terms respond stronger to the idiosyncratic shock. This also explains the shape of the frontier for very large \tilde{r} . Remember that to get a ρ_{aa} higher than under complete information it is necessary to add idiosyncratic noise to the signal of agents. If \tilde{r} is large, at the beginning agents are still responding very strongly in relative terms to the idiosyncratic part of their signals, which now is very noisy. Thus, $\rho_{a\theta}$ decreases without having a big increase in ρ_{aa} . At some point the effect of the noise in the idiosyncratic part of signals of agents is strong enough that agents no longer respond strongly to the idiosyncratic part of their signal. This has two consequences. In relative terms the response to the common part of the signal gets stronger, which increases ρ_{aa} , but also this part of the signal is noiseless so $\rho_{a\theta}$ also increases.

We can now analyze the more general case. If we take the complete model, then the set of feasible correlations

$(\rho_{aa}, \rho_{a\theta})$ in a correlated REE is given by the following inequality⁶:

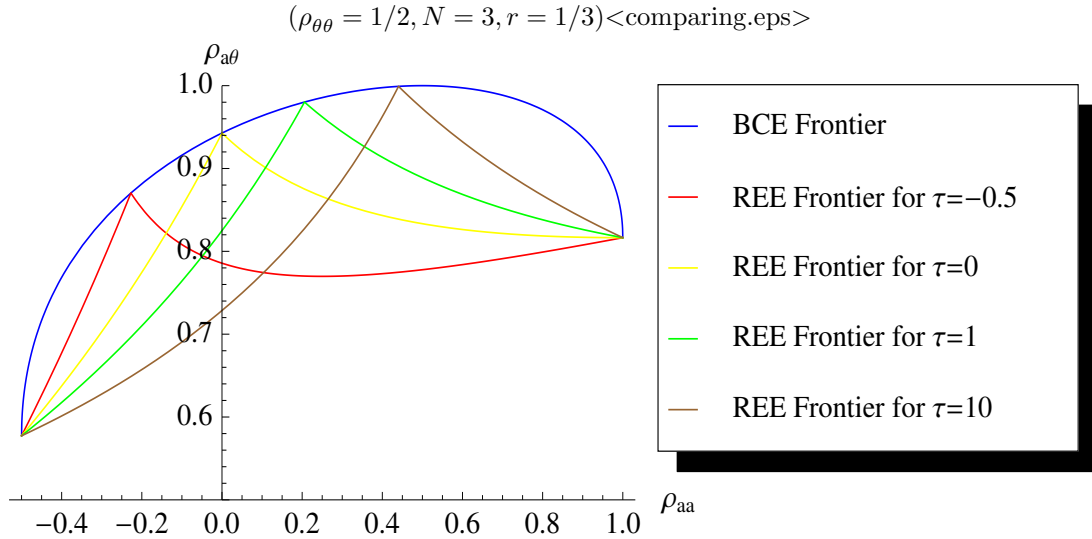
$$\rho_{a\theta}^f(\rho_{aa}) = \begin{cases} \frac{\sqrt{(n-1)\rho_{\theta\theta}+1}(r((n-1)\rho_{aa}+1)+\tau+1)}{(nr+\tau+1)\sqrt{((n-1)\rho_{aa}+1)}} & \rho_{aa} > \frac{r^2(n-n\rho_{\theta\theta})-\rho_{\theta\theta}(\tau+1)^2-2(\rho_{\theta\theta}-1)r(\tau+1)}{(n-1)n(\rho_{\theta\theta}-1)r^2+2(n-1)(\rho_{\theta\theta}-1)r(\tau+1)-(\tau+1)^2} \\ \frac{r((n-1)\rho_{aa}+1)+\tau+1}{\sqrt{\frac{(\tau+1)^2(1-\rho_{aa})}{1-\rho_{\theta\theta}}}} & \rho_{aa} \leq \frac{r^2(n-n\rho_{\theta\theta})-\rho_{\theta\theta}(\tau+1)^2-2(\rho_{\theta\theta}-1)r(\tau+1)}{(n-1)n(\rho_{\theta\theta}-1)r^2+2(n-1)(\rho_{\theta\theta}-1)r(\tau+1)-(\tau+1)^2} \end{cases} \quad (24)$$

Similarly to the case with a continuum of players in a Bayes correlated equilibrium is characterized by the following inequality,

$$\rho_{a\theta} \leq \frac{1}{N} \left(\sqrt{((N-1)\rho_{aa}+1)((N-1)\rho_{\theta\theta}+1)} + (N-1)\sqrt{(1+\rho_{aa})(1+\rho_{\theta\theta})} \right).$$

In Figure 2 we present a plot to show how does the set of feasible correlations change with the market power with a finite number of agents⁷.

Figure 2: Comparing Feasible Correlations with Finite Number of Agents



Just as before, we can now interpret the effects of market power on the set of feasible correlations without going through a rigorous analysis. Just by looking at Figures 1 and 2 we can see that increasing τ has an effect similar to decreasing \tilde{r} . At first this seems counter intuitive, as τ is increasing in r . Yet, the intuition is simple. By increasing τ agents dampen their response to the idiosyncratic and common part of their signals equally. One can see this from part i of Proposition 6. Thus, if the response of agents to both kinds of shocks is equally dampen, then the relative effects of r are smaller. Thus, when looking at feasible correlations, increasing τ is effectively as decreasing r . Other than that, the intuitions remain essentially the same as with a continuum of players, which we previously explained.

⁶The calculation is not trivial, this can be found in the appendix.

⁷Note that the frontier of the Bayes Correlated equilibrium looks different between Figures 1 and 2. This is because in one we are considering a finite number of agents while in the other one we are considering a continuum of agents. This implies we also need to consider a different range for ρ_{aa} , which for a finite number of agents is in $[-1/(n-1), 1]$

5.3 Price Volatility

We will now discuss how the endogenous information in prices allows to bound the equilibrium price volatility. Throughout the rest of the subsection we will think of the model with a continuum of agents. One of the remarkable comparative statics found in Bergemann, Heumann, and Morris (2013) is that the volatility of the aggregate action need not be bounded by the volatility. More specifically, they show that,

$$\lim_{\sigma_{\Delta\theta}^2 \rightarrow \infty} \left(\max\{\sigma_p^2 : \sigma_p^2 \text{ is the price volatility in a Bayes correlated equilibrium}\} \right) = \infty.$$

That is, as the idiosyncratic uncertainty goes to infinity, we can find an information structure that decentralizes a Bayes Nash equilibrium with arbitrary large price volatility. This kind of result is also closely related to results that related idiosyncratic uncertainty with aggregate volatility. For example, Angeletos and La'O (2013) provide a model of an economy in which there is no aggregate uncertainty, but there may be aggregate fluctuations. One of the key aspects is that in the economy the production decisions are done prior to the exchange phase, and thus there are no endogenous information through prices.

We can now understand how this changes in a model with endogenous information.

Lemma 13 (Price Volatility).

In a correlated REE with a continuum of agents, the aggregate price volatility is given by:

$$\sigma_p = \frac{r\rho_{\bar{\theta}p}\sigma_{\bar{\theta}}}{1+r}.$$

Thus, if $r \geq 0$, we have that $\sigma_p^2 \leq \sigma_{\bar{\theta}}^2$.

Proof. In a correlated REE with a continuum of agents, we must have that,

$$a_i = \mathbb{E}[\theta_i | p, a_i] - p. \tag{25}$$

Taking expectations of the previous equation:

$$\mu_{a_i} = \mu_{\theta_i} - \mu_p. \tag{26}$$

We now subtract (26) from (25), multiply by p , and taking expectations. We get:

$$\text{cov}(a_i, p) = \text{cov}(\theta_i, p) - \sigma_p^2.$$

Taking expectations with respect to the agents, we get,

$$\text{cov}(\bar{a}, p) = \text{cov}(\mathbb{E}_i[a_i], p) = \mathbb{E}_i[\text{cov}(a_i, p)] = \mathbb{E}_i[\text{cov}(\theta_i, p)] - \sigma_p^2 = \text{cov}(\bar{\theta}, p) - \sigma_p^2.$$

Thus, we get,

$$\begin{aligned} \frac{1}{r}\sigma_p^2 &= \text{cov}(\bar{\theta}, p) - \sigma_p^2. \\ \Rightarrow \sigma_p &= \frac{r \cdot \text{cov}(\bar{\theta}, p)}{1+r}. \end{aligned}$$

■

It is clear that in a correlated REE price volatility is bounded by the volatility of aggregate shocks. It is worth noting that the proof does not require any symmetry assumption, and thus the same would hold if we allowed for asymmetric REE. The only thing we used for the result was the fact that the first order condition of agents is satisfied, and agents know the equilibrium price when they submit their demands. Even if we allowed for exogenous noise traders, this would just act as adding a population of agents in a asymmetric environment. We would have to add the volatility added by the noise traders, but the main idea would remain unchanged.

Although we present the bounds for a model with a continuum of agents, the same kind of bounds would hold with a finite number of agents. In case of a finite number of agents, we need to consider the effects of market power, which might be negative. Yet, although in an asymmetric environment this makes the bound algebraically more cumbersome, we can always use the bounds found for a continuum and multiply it by a factor of 2. This just comes from the fact that with a finite number of agents the market power amplifies the decisions of agents by at-most a factor of 2.

We can see that in an economy with endogenous information, the volatility of prices is bounded by the volatility of aggregate shocks. This is unrelated to the discussion of whether prices fully aggregate information or not. Yet, this can be seen as a argument that, even when prices are not fully revealing, they are always “revealing enough”, to keep aggregate fluctuations bounded in terms of aggregate shocks. A key aspect of the previous argument is that all agents observe the same contemporaneous prices.

5.4 Information Revelation by Prices

In the previous analysis we have provided a sharp characterization on how the set of feasible outcomes compare when endogenous prices aggregate information. Yet, this provides no additional insights, as to when are prices completely aggregating the information of all agents. That is, when can we say that prices provide a sufficient statistic for the pooled information all agents have in the economy.

If we were certain about the information structure agents receive in an economy we would be able to provide such a qualification. For example, under the information structure assumed in Vives (2011), the author shows that prices do provide a sufficient statistic for all the information in the economy. This argument would clearly extend to the more general class of one dimensional signals here proposed. On the other hand, Rostek and Weretka (2012) show that by keeping the information structure unchanged, but assuming a asymmetric type space (that is, $(\theta_1, \dots, \theta_N)$ asymmetrically distributed), this result is reversed. Keeping the information structure the same, but changing the distribution of the type space leads to the conclusion that prices are not fully revealing.

It is not difficult to show that by extending our argument to a asymmetric environment, one can always decentral-ized a REE with information structures in which prices are privately revealing. Of course, this would require picking a specific information structure for each player, nevertheless a priori one could think such information structures are feasible. Moreover, note that such information structures would be efficient, in the sense that all information is aggregate. Thus, one might be tempted to think that they are plausible, even when they seem unusual when one considers information structures as exogenous objects.

On the other hand, one can also show that in a symmetric payoff environment, it is also possible to find information

structures under which prices are not fully revealing. Nevertheless, for an arbitrary information structure, one can always decentralize an equilibrium that yields the same outcome, but in which prices are privately revealing. We will now provide such an example. This will allow us to show how the outcomes of multidimensional signals can be mapped in outcome equivalent one dimensional signals. It will illustrate the importance of considering λ in one dimensional signals of the form 8. Finally, it will allow us to connect our results to the ones in REE with noise traders, as in Manzano and Vives (2011)

A Simple Example

We provide a simple example to show how a two dimensional signal is mapped into a one dimensional signal. As mentioned, this will help illustrate how different information structures with different properties concerning information revelation of prices lead to the same outcomes. We first consider a model with a continuum of agents as we want to illustrate how different information structures allow to decentralize different equilibria, and how this leads to different properties on the information revelation properties of prices. Therefore, we want to avoid the complications associated with market power. It will be clear that all intuitions remain the same for a model with finite number of players, only with finite number of players one needs to add the difficulty of finding the right market power of agents. We later address an example in which the number of players is finite. This also will help understand why studying the model with a continuum of agents would inevitably lead to missing the importance of the information structure in market power.

We consider a payoff structure as the one previously described, where the type of agents can be written as follows,

$$\theta_i = \bar{\theta} + \Delta\theta_i = \theta_1 + \theta_2 + \Delta\theta_i,$$

where θ_1, θ_2 are common valued variables that by definition add up to $\bar{\theta}$. All variables are independent between each other, with 0 mean and a variance of 1. We also assume $r = 1$ and $P_0 = 0$. Consider the information structure \mathcal{I}_i in which agents get two signals of the form,

$$s_1 = \theta_1 + \varepsilon_i ; s_2 = \theta_2 + \Delta\theta_i. \tag{27}$$

Manzano and Vives (2011) provide a justification for such an information structure in a similar environment. We can think of θ_i being the value of an asset for an agent, where the value is the sum of a liquidity shock and a common valued payoff shock. The liquidity shock is equal to s_2 . Each agent knows its own liquidity shock, but the liquidity shock has some correlation across agents and thus agents don't know the average liquidity shock. On the other hand, the asset has a common valued payoff given by θ_1 . Each agent gets a private signal on the payoff of the asset, which is given by s_1 .

Since we are considering a continuum of agents, no agent will have market power. Thus, we can solve for a equilibria in supply functions by solving the equilibrium when the first order condition of agents is given by

$$a_i = \mathbb{E}[\theta_i | s_1, s_2, p] - p,$$

where the equilibrium price satisfies,

$$p = r\mathbb{E}_i[a_i] = r\bar{a}.$$

Given the information structure we are considering, we could also make the following change of variables and solve for an equivalent equilibrium. We consider

$$\hat{a}_i = a_i - s_2$$

and solve for an equilibrium in which the first order condition of agents is given by

$$\hat{a}_i = \mathbb{E}[\theta_1 | s_1, s_2, p] - p,$$

where the equilibrium price satisfies,

$$p = r\bar{a} = r(\mathbb{E}_i[\hat{a}_i] + \theta_2).$$

Note that the second formulation can be interpreted as a equilibrium in which agents have common values over the good, given by θ_1 , but there are “noise traders”, given by θ_2 .

After some algebraic manipulations one can check that in equilibrium the action of players are given by,

$$a_i = \frac{2}{5}s_1 + \frac{4}{5}s_2 - \frac{1}{3}p,$$

or in terms of fundamentals:

$$a_i = \frac{3}{10}\theta_1 + \frac{3}{5}\theta_2 + \frac{2}{5}\varepsilon_i + \frac{4}{5}\Delta\theta_i.$$

Note that this is evidently not a fully revealing equilibrium. In any fully revealing equilibrium all variables would be revealed by looking at the signals of all players pooled together. In the complete information case it is not difficult to see that the equilibrium actions would be given by,

$$a_i^* = \frac{\theta_1 + \theta_2}{2} + \Delta\theta_i.$$

Finally, note that we can calculate the volatility of a_i and decompose a_i in terms of the payoff relevant fundamentals and noise. We get the following,

$$\begin{aligned} \text{var}(a_i) &= \left(\frac{3}{10}\right)^2 + \left(\frac{3}{5}\right)^2 + \left(\frac{2}{5}\right)^2 + \left(\frac{4}{5}\right)^2 = \frac{5}{4}; \quad \text{var}(\bar{a}) = \left(\frac{3}{10}\right)^2 + \left(\frac{3}{5}\right)^2 = \frac{9}{20} \\ \text{cov}(a_i, \Delta\theta_i) &= \frac{4}{5}; \quad \text{cov}(a_i, \bar{\theta}) = \text{cov}(a_i, \theta_1 + \theta_2) = \frac{9}{10}. \end{aligned}$$

Now consider a different information structure, in which players receive the following signal,

$$\tilde{s}_i = \Delta\theta_i + \varepsilon_{i1} + \lambda(\bar{\theta} + \varepsilon_2)$$

where ε_{i1} is a idiosyncratic noise term with variance of $1/4$, ε_2 is a common value noise term with variance of $2/9$ and λ can be any real number (the analysis does not change with the value of λ when there are a continuum of players). To illustrate the solution it is convenient to describe the solution in terms of the following signals,

$$\tilde{s}_1 = \Delta\theta_i + \varepsilon_{i1}; \quad \tilde{s}_2 = \bar{\theta} + \varepsilon_2.$$

If we calculate the equilibrium under this alternative information structure we get the following equilibrium actions,

$$a'_i = \mathbb{E}[\Delta\theta_i | \tilde{s}_{1i}] + \frac{\mathbb{E}[\bar{\theta} | \tilde{s}_2]}{1+r} = \frac{4}{5}(\Delta\theta_i + \varepsilon_{i1}) + \frac{9}{20}(\bar{\theta} + \varepsilon_2).$$

If we calculate the variance of a'_i , \bar{a}' and decompose a'_i in terms of the payoff relevant fundamentals and noise, we get the following,

$$\begin{aligned} \text{var}(a'_i) &= \left(\frac{4}{5}\right)^2 \left(1 + \frac{1}{4}\right) = \frac{5}{4} ; \text{var}(\bar{a}') = \left(\frac{9}{20}\right) \left(2 + \frac{2}{9}\right) = \frac{9}{20} \\ \text{cov}(a'_i, \Delta\theta_i) &= \frac{4}{5} ; \text{cov}(a'_i, \bar{\theta}) = \text{cov}(a_i, \bar{\theta}) = \frac{9}{10}. \end{aligned}$$

By looking at the distribution of moments we can note that the equilibrium under both information structures previously explained are equivalent in terms of outcomes. To be more precise, for an analyst observing the joint distribution of traded quantities, prices and types it would be impossible to distinguish between both information structures. This doesn't mean that both equilibria are equivalent in terms of higher order beliefs. For the second information structure described the average expectation of $\bar{\theta}$ is common knowledge. Moreover, all higher order beliefs on $\bar{\theta}$ are common knowledge. On the other hand, for the first information structure the average expectation of $\bar{\theta}$ is not common knowledge. The set of outcomes will always be possible to describe in terms of a canonical information structure in which the average expectation of $\bar{\theta}$ is common knowledge, analogous to the second information structure described. Note that with a continuum of agents the equilibrium with the second information structure is independent of λ , and thus this can be considered a free parameter.

If we solve for a finite number of agents most of the analysis remains unchanged, but we need to consider the additional variable of the market power of each agent. For example, consider the case in which we keep everything as previously described but we consider that the number of agents is $N = 9$. In this case, under the first information structure, the market power of agents will be given by $\tau = 1/3$. On the other hand, the equilibrium action will be given by:

$$a_i = \frac{1}{1 + \tau} \left(\frac{2}{5} s_1 + \frac{4}{5} s_2 - \frac{1}{3} p \right) = \frac{3}{4} \left(\frac{2}{5} s_1 + \frac{4}{5} s_2 - \frac{1}{3} p \right)$$

If we go back to the second information structure and consider $N = 9$, we will have that the outcome is no longer independent of λ . To be more specific, the equilibrium actions will be given by,

$$a'_i = \frac{4}{5(1 + \tau)} (\Delta\theta_i + \varepsilon_{i1}) + \frac{1}{1 + r + \tau} \frac{9}{10} (\bar{\theta} + \varepsilon_2),$$

where τ is the market power. In this numerical example, τ will depend on λ and will be given by,

$$\tau = \frac{-320\lambda + \sqrt{64\lambda(1600\lambda - 279) + 2025} + 36}{64\lambda + 9}.$$

It is easy to check that only if we impose $\lambda = 117/248$ we will get $\tau = 1/3$. Thus, in this the case information structure,

$$s_i = \Delta\theta_i + \varepsilon_{i1} + \lambda(\bar{\theta} + \varepsilon_2)$$

with $\lambda = 117/248$ and the variances for the noise terms previously specified is outcome equivalent to information structure (27) when $N = 9$. Thus, most of the analysis is the same as with a continuum of agents, only in this case we also need to adjust λ to match for the equilibrium market power.

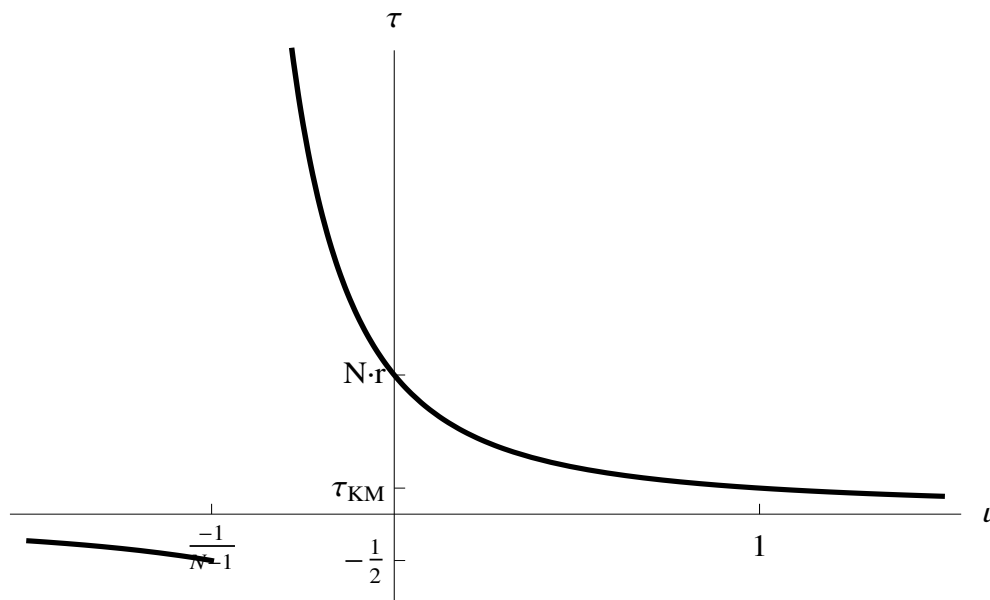
6 Market Power

We will now study how market power behaves in a demand function equilibrium. We first provide a qualitative description on how market power changes with the information structure and provide the intuitions behind the drivers of market power. We then provide an analysis on the profit maximizing level of market power. Finally, we analyze how market power changes with an exogenous introduction of a public signal.

6.1 Qualitative Description and Intuitions

From Proposition 6, we can see that all the effect of the information structure in the market power is summarized in the parameter ι . In Plot 3 we show a schematic figure to illustrate how the market power changes as a function of ι . We can identify two important cases, $\iota \rightarrow \infty$ and $\iota \rightarrow 0$. When $\iota \rightarrow \infty$ an agent has no market power, which implies that any additional increase in the demand function he submits does not change the equilibrium prices. This implies that a change in the submitted quantity must be offset by the demand functions of the other agents. On the other hand, as $\iota \rightarrow 0$ we have that the market power of an agent converges to Nr , which implements the collusive price level (we are more specific on how to calculate the collusive price when we discuss the profit maximizing market power). This is as if any additional increase in the demand function he submits is perfectly replicated by all other agents. Thus, this yields the collusive level of output. Moreover, we can see the market power is decreasing in ι almost everywhere, except for a discontinuity at $\lambda = -1/(N-1)$. The market power converges to infinity if ι approaches $-1/(N-1)$ from the right, while it converges to $-1/2$ if it approaches $-1/(N-1)$ from the left.

Figure 3: Market Power $v/s \iota$ <marketpower.eps>



If we impose $B = b = \lambda = 1$, then we recover the (robust) equilibrium with complete information, which is studied in Klemperer and Meyer (1989). In this case we have that,

$$\iota_{KM} \triangleq \frac{n-2}{n}.$$

We can also recover the model studied by Vives (2011)⁸. This model corresponds to imposing $\lambda = 1$ and $\rho_{\varepsilon\varepsilon} = 0$, in which case we get,

$$\iota_V \triangleq \frac{(1 - \rho_{\theta\theta})((n-1)\rho_{\theta\theta} + \frac{\sigma_\varepsilon^2}{\sigma_\theta^2} + 1)}{((n-1)\rho_{\theta\theta} + 1)(1 + \frac{\sigma_\varepsilon^2}{\sigma_\theta^2} - \rho_{\theta\theta})} \in \left[\frac{1 - \rho_{\theta\theta}}{(1 - \rho_{\theta\theta}) + n\rho_{\theta\theta}}, 1 \right].$$

Note that in any equilibrium, the information agents have on $\Delta\theta_i$ and $\bar{\theta}$ is measured by b and B respectively, while λ only affects the market power. To understand why λ affects market power, we begin by studying the case in which $b = B = 1$. Thus, in equilibrium agents perfectly know the realization of $\Delta\theta_i$ and $\bar{\theta}$. We now proceed to provide an intuition behind the determination of the market power.

There are two forces which determines the market power under any information structure. To understand the drivers of market power we look at the response of other players if player i decides to submit a higher demand than the one dictated by the equilibrium. First, agent observe higher prices, and thus they interpret this as a higher realization of $\bar{\theta}$ than they would have originally estimated. This results in all agents increasing their demand as well through the supply function they submitted. On the other hand, a unexpected increase in price also results in agents interpreting this as a lower realization of $\Delta\theta_i$ than they would have originally estimated. Thus, they reduce their demand through the supply function they submitted. We now explain which of these forces dominates depending on the information structure. For this we look at the two limit case of $\lambda = 0$ and $\lambda = \infty$, and analyze the equilibrium in this case (with the obvious interpretation when $\lambda = \infty$).

We begin by looking at the case $\lambda = 0$. In this case the signals agents have are purely idiosyncratic and by definition sum up to 0. Therefore, agents have no information on what the equilibrium price should be based on the signal they have. The equilibrium in this case consists in agents submitting a demand function that is perfectly collinear with the supply function, and with prices adjusting to the average signal (which for $\lambda = 0$ is actually equal to 0, but we take the obvious limit). This equilibrium suffers from the classic Grossman-Stiglitz paradox that prices are not measurable with respect to the information agent have. Yet, in the limit we get the same intuitions without suffering from the paradox. As $\lambda \rightarrow 0$, all information agents have is concerning their idiosyncratic shocks. Thus, they forecast the average type from the equilibrium price they observe. Thus, if an agent deviates and decreases the quantity he submits this is responded by agents forecasting a lower average type and thus also decreasing the quantity they submit. Thus, the deviations of an agent are reinforced by the best response of other agents, thus as $\lambda \rightarrow 0$ if an agent increases the price by increasing the quantity he submits, he expects that all other agents should do the same. Therefore, in the limit case we get the collusive price level (we are more specific on the precise meaning of collusive price level later).

On the other hand, the case $\lambda = \infty$ suffers from a similar paradox. In this case agents know exactly what is the equilibrium price based only on their signals, and thus they submit a perfectly elastic supply. The traditional approach would be to split the equilibrium demand equally among all players, yet this is not necessary. In this

⁸This is for the baseline model. He later studies the effect of introducing a public signal, which we also discuss later.

case the Walrasian auctioneer could split the demand between agents according to the information contained in Δs_i (which is equal to 0 when $\lambda = \infty$, but one can take the limit). In this case we would have a paradox as the Walrasian auctioneer would be splitting the demand between agents according to information no agent has. Note that a priori we could allow for $\tau = 0$ in the definition of correlated REE, but this would not be implementable in demand functions. This is just because a perfectly elastic demand function cannot be submitted as a limit order function. Nevertheless, as we take the limit $\lambda \rightarrow \infty$ we get the same intuitions without the paradox. As $\lambda \rightarrow \infty$ the private signal of agents is a very good predictor of the average signal. Thus, they can correctly anticipate what will be the equilibrium prices. Thus, if agents see a higher equilibrium price than the one they anticipated, they attribute this to having a negative shock to the value of $\Delta\theta_i$. Therefore, if they see a high price they reduce their equilibrium demand which reduces prices, which implies a very elastic demand. Hence, if an agent deviated and attempts to increase the prices, this is responded by the best response of other players which offsets the deviation.

For negative values of λ we have that both forces reinforce each other. Depending on the value of λ , agents interpret a high price as a lower shock to $\bar{\theta}$ and $\Delta\theta$, in which case the market power gets to be above Nr . The other situation is that agents interpret a high price as a higher shock to $\bar{\theta}$ and $\Delta\theta$, in which case the market power gets to be below 0, as agents decrease the demand of other agents by increasing the price level.

We can finally understand what are the effects in the market power of having $b, B \neq 1$. Having noise in signals has the only effect that it adds residual uncertainty to the information of agents and thus dampens the response of the agents to the signals. For each agent i we can define a modified type as follows,

$$\varphi_i \triangleq \mathbb{E}[\Delta\theta_i | \Delta s_i] + \mathbb{E}[\bar{\theta} | \bar{s}] = b\Delta s_i + B\bar{s} + (1 - B)\mu_\theta.$$

We can rewrite the signal an agent receives as follows,

$$s_i = \frac{1}{b}(b\Delta s_i + \frac{b}{B}B\bar{s}) = \frac{1}{b}(\Delta\varphi_i + \iota\bar{\varphi} - (1 - B)\mu_\theta).$$

Yet, this signal is informationally equivalent to the following signal,

$$s'_i = \Delta\varphi_i + \frac{\lambda b}{B}\bar{\varphi}.$$

Thus, it is easy to see that we can repeat the previous analysis but using the definition already made $\iota = \frac{\lambda b}{B}$. Therefore, the errors in the signals affect the market power by dampening the response to the average and idiosyncratic part of the signal, which serves as a reweighing of the informational content of the signal.

Finally, it is worth highlighting from the previous analysis that the actual correlation in agents type ρ is irrelevant for market power, beyond the impact it has on the information structure. Keeping ι fixed, varying the correlation in agents type does not change market power. This implies that a priori, the lemons problem can be exacerbated or dampen without the need for a change in the fundamentals. A change in the information structure agents have may have arbitrary large effects on the amount of trade in equilibrium, going from a fully competitive efficient allocation, to a situation of market shutdown.

Profit Maximizing Market Power

Before we provide a characterization of the profit maximizing market power it is useful to provide a characterization of what would agents pick as traded quantities if they could collude. That is, what is the maximum added profits that agents could get under any realization of types. For this we solve the following maximization problem,

$$\begin{aligned} \{a_1^*, \dots, a_N^*\} &= \arg \max_{\{a_1, \dots, a_N\}} \sum_{i \in N} \theta_i a_i - \frac{1}{2} a_i^2 - a_i p; \\ &\text{subject to } p = P_0 + r \sum_{i \in N} a_i. \end{aligned}$$

Calculating the optimal traded quantities we get:

$$\bar{a}^* = \frac{\bar{\theta}}{1 + 2rN}; \Delta a_i^* = \Delta \theta_i. \quad (28)$$

If we compare the optimal traded quantities with the ones provided in Proposition 6, we can see that for any information structure the profits in equilibrium under demand function competition will always be below the maximum possible that can be achieved. Going back to Proposition 5 and assuming agents get noiseless signals ($\sigma_\varepsilon = 0$). If $\tau = Nr$ then we would have that the price is always equal to the collusive price, and thus agents get the maximum profits from the variations in $\bar{\theta}$. Yet, in this case the trade between agents is too low, and thus the profits are lower than the maximum achievable. On the other hand, as $\tau \rightarrow 0$ the trade between agents approaches the optimal one, but in this case the average price is too responsive to the average type. Thus, the competition lowers the profits agents get from the exogenous supply. It will be clear that there is an optimal market power which will be between $\tau = Nr$ and $\tau = 0$.

Using Lemma 9 we can easily find the optimal market power for the case in which agents receive noiseless signals ($\sigma_\varepsilon = 0$). If agents receive noiseless signals, then the profits of agents can be written in terms of the market power as follows⁹:

$$\mathbb{E}[\pi] = (1/2 + \tau) \left(\frac{(\mu_\theta - P_0)^2 + \sigma_\theta^2}{(1 + \tau + Nr)^2} + \frac{\sigma_{\Delta\theta}^2}{(1 + \tau)^2} \right) \quad (29)$$

If we maximize (29) with respect to τ , we get the optimal market power, which we denote τ^* . This can be easily decentralized by allowing agents to play the REE in demand functions with a noiseless signal given by (8), where λ is found using the value of τ^* and (10). Before we go on, we provide a simple result on the existence of τ^*

Lemma 14 (Optimal Market Power).

The function $\mathbb{E}[\pi]$ has a unique maximum τ^ , moreover $\tau^* \in [0, N \cdot r]$.*

Proof. We first prove that $\mathbb{E}[\pi]$ has a unique maximum in τ , and the maximum is in $(0, Nr)$. For this, first note that the function $(1/2 + x)/(1 + c + x)^2$ is quasi-concave in x , with a unique maximum at $x = c$. Second, note that the

⁹As we have explained, the case in which agents receive noisy signals just implies one needs to renormalize certain terms using Proposition 6. Moreover, it is easy to see from Lemma 9 that the optimal information structure is always noiseless, as one wants to maximize the variance of the individual action.

function $(1/2 + x)/(1 + c + x)^2$ is strictly concave in x for $x < 1/2 + c$. Since the sum of concave functions is concave, it is easy to see that $\mathbb{E}[\pi]$ is strictly concave for $\tau \leq 1/2 + N \cdot r$. Moreover, we have that,

$$0 = \arg \max_{\tau} (1/2 + \tau) \frac{\sigma_{\Delta\theta}^2}{(1 + \tau)^2} ; Nr = \arg \max_{\tau} (1/2 + \tau) \frac{(\mu_{\theta} - P_0)^2 + \sigma_{\theta}^2}{(1 + \tau + Nr)^2}$$

Thus, it is easy to see that $\mathbb{E}[\pi]$ is decreasing for $\tau \geq 1/2 + N \cdot r$ (which is the part we cannot check it is concave) and has a unique maximum in $[0, Nr]$.

Using the previous result we can also check the τ^* is monotonic increasing in $\rho_{\theta\theta}$. We note that,

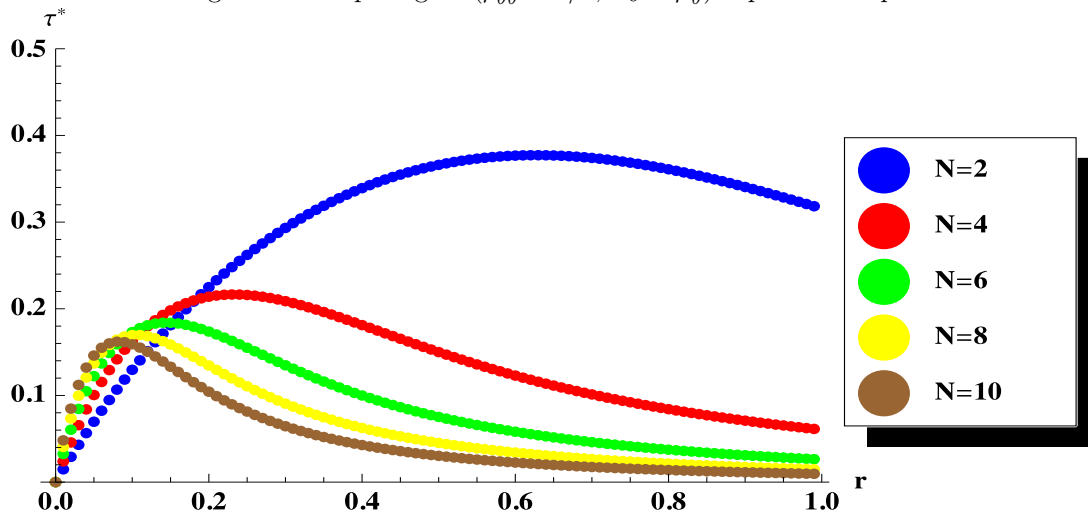
$$\frac{\partial}{\partial \tau} \left((1/2 + \tau) \frac{\sigma_{\Delta\theta}^2}{(1 + \tau)^2} \right) \Big|_{\tau=\tau^*} < 0 ; \frac{\partial}{\partial \tau} \left((1/2 + \tau) \frac{(\mu_{\theta} - P_0)^2 + \sigma_{\theta}^2}{(1 + \tau + Nr)^2} \right) \Big|_{\tau=\tau^*} > 0.$$

Using the fact that $\sigma_{\Delta\theta}^2 = \frac{N-1}{N}(1 - \rho)\sigma_{\theta}$ and $\sigma_{\theta}^2 = \frac{(N-1)\rho+1}{N}\sigma_{\theta}^2$ we have that τ^* is increasing in $\rho_{\theta\theta}$. ■

To understand the bounds provided in Lemma 14 we find the optimal market power in some extreme cases. If $\sigma_{\Delta\theta} = 0$, then it is easy to see that it is optimal to impose $\tau^* = Nr$. Intuitively, when there are no gains from trade between agents, it is best to impose the optimal price level. If $(\mu_{\theta} - P_0)^2 + \sigma_{\theta}^2 = 0$, then it is optimal to maximize trade between agents and impose $\tau^* = 0$.

Although the solution is algebraically cumbersome, and thus it is not worth providing it, some comparative statics are easy to find and the problem is also numerically easy to solve. In the proof of Lemma 14 we checked that τ^* is increasing in $\rho_{\theta\theta}$. On the other hand, the comparative statics with respect to r is non-monotonic. To illustrate this, in Figure 4 we plot τ^* as a function of r .

Figure 4: Comparing τ^* ($\rho_{\theta\theta} = 1/2, P_0 = \mu_{\theta}$)<optimaltau.eps>

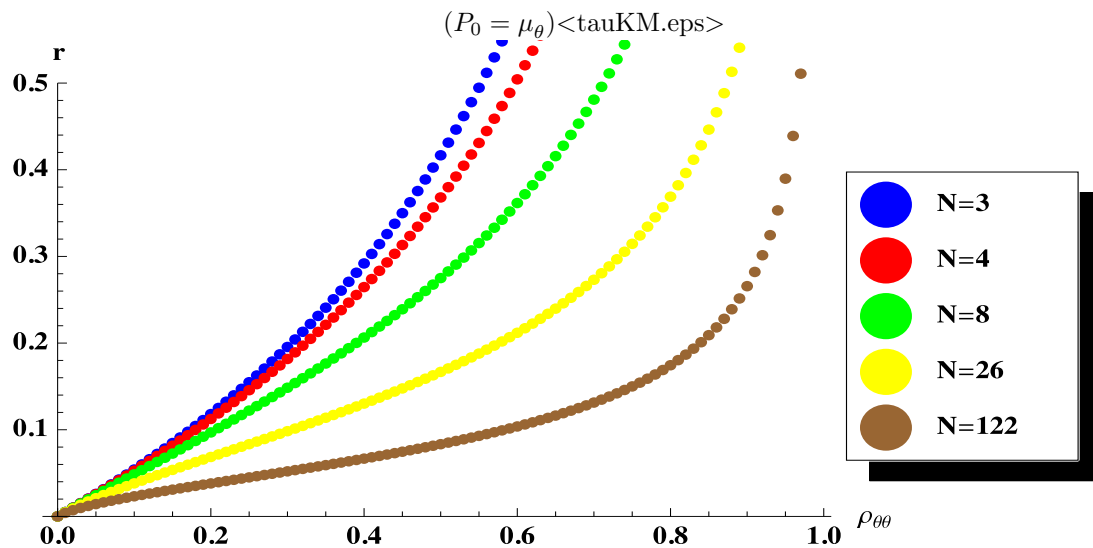


The intuition on why τ^* is non-monotonic in r is simple. For a very small r the exogenous supply is very elastic, and thus the optimal price level is almost constant. This implies that agents would like to maximize the gains from trade between each other, as the aggregate price level is already constant. Thus, agents would like a small market power. On the other hand, for a very large r , once again the maximum profits from the exogenous supply are unresponsive

to shocks in $\bar{\theta}$. Thus, agents rather maximize the profits from trade between each other. For intermediate levels of r it is the case it is most important for agents to pin-down the optimal price level to maximize profits from the exogenous supply, and thus it is optimal to have a higher market power. Of course, as N becomes large, the gains from trade between agents becomes larger. This can be seen from the fact that $\sigma_{\Delta\theta}^2$ increases. Thus, agents have bigger gains from trading within each other, which decreases the optimal level of market power.

Since the complete information market power (τ_{KM}) is also non-monotonic in r , it is not clear from Figure 4 whether the optimal market power is above or below the complete information one. Thus, we compare the optimal equilibrium market power compare to the complete information market power, given by τ_{KM} . In Figure 5 we show for what values of $(r, \rho_{\theta\theta})$, the optimal market power is equal to τ_{KM} . Since τ^* is monotonic increasing with respect to $\rho_{\theta\theta}$ and τ_{KM} is constant with respect to $\rho_{\theta\theta}$, we know that the points above the line corresponds to values of $(r, \rho_{\theta\theta})$ for which τ^{KM} is too low, while below the line corresponds to values $(r, \rho_{\theta\theta})$ for which τ^{KM} is too high.

Figure 5: Parameters for which Optimal Market Power is Equal to Complete Information Market Power ($\tau^* = \tau_{KM}$).



We can see that as $\rho_{\theta\theta}$ increases, it is more likely that agents will rather have the optimal price level, and thus the complete information market power will be too low. On the other hand, as $\rho_{\theta\theta}$ decreases, agents would like to increase trade between each other, which implies that agents would like a lower market power than the complete information one.

Public Signals

In one of the extensions, Vives (2011) studies the impact of introducing an exogenous public signal. In what may be a bit surprising result, he finds that a precise enough signal may reduce the market power level below the complete information equilibria, even when this is not a possibility under the benchmark information structure (at least for positive values of ρ). We now will provide a simple discussion to understand how a public signal can increase or decrease the equilibrium market power.

Suppose agents get a signal of the following form,

$$s_i = \Delta\theta_i + \bar{\theta} + \bar{\varepsilon} + \Delta\theta_i = \Delta\theta_i + \theta_1 + \theta_2 + \varepsilon_1 + \varepsilon_1 + \Delta\theta_i,$$

where by definition θ_1 and θ_2 are common valued random variables that add up to $\bar{\theta}$ and independent between each other. The same hold for ε_1 and ε_2 . We know that in equilibrium, the market power will be characterized by the parameters b and B (for the example we impose $\lambda = 1$). In particular we will have that the value of B , which we denote B_0 for this example, is given by,

$$B_0 = \frac{\sigma_{\theta_1}^2 + \sigma_{\theta_2}^2}{\sigma_{\theta_1}^2 + \sigma_{\theta_2}^2 + \sigma_{\varepsilon_1}^2 + \sigma_{\varepsilon_2}^2}.$$

We now study two counterfactual situations. In the first, there is a public signal that discloses the value θ_1 . In this case, evidently the equilibrium will correspond to agents using the public signal to shift the expected value of θ in an amount θ_1 , and then filter θ_1 from their private signals. The new equilibrium value for B , denoted by B' will be:

$$B' = \frac{\sigma_{\theta_2}^2}{\sigma_{\theta_2}^2 + \sigma_{\varepsilon_1}^2 + \sigma_{\varepsilon_2}^2} > B_0.$$

Thus, under this first counterfactual situation, the value of B decreases. This implies that the value of ι increases, which in turn implies that the market power will decrease. Thus, we note (we follow the obvious notation),

$$\tau' < \tau_0.$$

This effect of a public signal is noted in Vives (2011).

We now consider a second counterfactual situation in which there is a public signal that discloses the value of ε_1 . As before, all the effect of the public signal will be that agents use the public signal to filter out ε_1 from their private signals. The new equilibrium value for B , denoted by B'' will be:

$$B'' = \frac{\sigma_{\theta_1}^2 + \sigma_{\theta_2}^2}{\sigma_{\theta_1}^2 + \sigma_{\theta_2}^2 + \sigma_{\varepsilon_2}^2}.$$

Thus, under this first counterfactual situation, the value of B increases. This implies that the value of ι decreases, which in turn implies that the market power will increase. Thus, we note (we follow the obvious notation),

$$\tau'' > \tau_0.$$

Thus, under these second counterfactual situation the value of τ increases.

The previous examples illustrate two things. First, in general it is necessary to consider a rich set of one dimensional signals to encompass all possible outcomes that can be achieved under some information structure. Considering a particular set of information structures may leave possible outcomes out of the analysis. Moreover, since the information structures agents have is generally complex and not observable, any robust prediction of the possible outcomes must encompass a rich set of information structures.

Second, since the market power is rooted in the information structure agents have, it is important to understand precisely how the information structures changes to run comparative statics. In particular, two counterfactual situations in which public signals are disclosed may have very different effects. Moreover, under the two counterfactual situations the correlation between fundamentals and actions may stay the same, but the effect of the public signal on market power may have opposite effects.

7 Conclusions

We studied how the information structure of agents affects the Bayes Nash equilibrium of a game in which agents compete in demand functions. We have shown that market power strongly depends on the private information agents. The analysis also provides a very clear understanding on how the information in prices affects the set of feasible outcomes. This allow us to provide a sharp distinction between the set of feasible outcomes that can be achieved under demand function competition and quantity competition.

We have also provided a new solution concept that extends the methodologies developed by Bergemann and Morris (2013), allowing for endogenous information and market power. This provides a unified framework to study linear best response games as Bayes Nash equilibrium and Rational Expectations equilibrium. We have provided an example to show that the impact of public information can be ambiguous, yet the exact connection to the results found Morris and Shin (2002) is still left as an open question. More generally, we leave for further research understanding the optimal use and value of information in demand function competition, which would allow us to understand the connection with Angeletos and Pavan (2007). We also leave for future work the study of asymmetric environments, noisy prices and non-centralized markets.

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