Solutions to Spring 2014 ECON 301 Final Group A

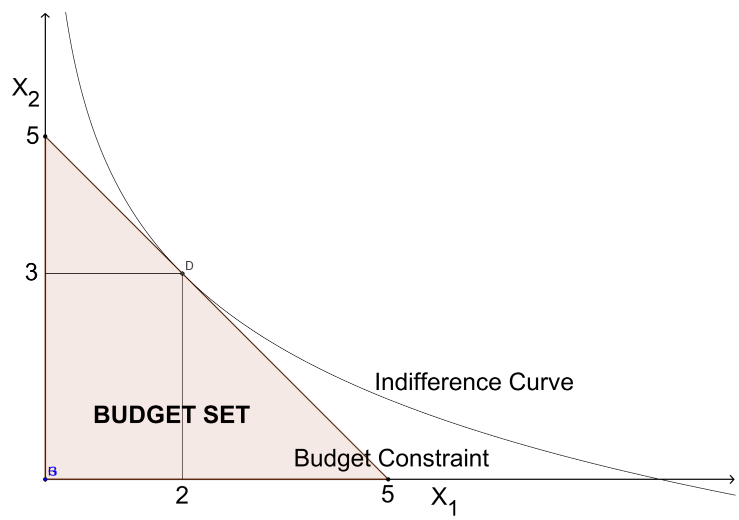
Problem 1. (Quasilinear income effect) (25 points)

Mirabella consumes chocolate candy bars x_1 and fruits x_2. The prices of the two goods are $p_1 = 4$ and $p_2 = 4$ respectively and Mirabella's income is $m = 20$. Her utility function is

$$U(x_1, x_2) = 2 \ln x_1 + x_2$$

a) (4 points) In the commodity space plot Mirabella's budget set. Find slope of budget line (one number). Provide the economic interpretation of the slope (one sentence).

Solution: See figure below for Mirabella's budget set - it's the entire shaded region. The slope of the budget line is equal to $-p_1/p_2 = -4/4 = -1$. The economic interpretation of the slope is that it represents the rate at which the market prices allow Mirabella to trade between the two goods.



b) (4 points) Find analytically the formula that gives Mirabella's MRS for any bundle (x_1, x_2) (a function). Give the economic and the geometric interpretation of the MRS (two sentences). Find the value of the MRS at bundle $(x_1, x_2) = (4, 4)$ (one number). Which of the two commodities is (locally) more valuable? (choose one)

Solution: $MRS = -\frac{MUx_1}{MUx_2} = -\frac{2/x_1}{1} = -\frac{2}{x_1}$. The economic interpretation of the MRS is that it represents the rate at which Mirabella would trade-off between goods while remaining indifferent. Geometrically, it is the slope of the indifference curve at that bundle. $MRS(4, 4) = -2/4 = -1/2$. Since at that bundle, $MUx_1 = 1/2 < 1 = MUx_2$, x_2 is locally more valuable.

c) (5 points) Write down two secrets to happiness that determine Mirabella's optimal choice (two equations). Provide the geometric interpretation of the conditions in the commodity space.

Solution: The two secrets to happiness are

$$\text{(1) Budget Constraint} : x_1p_1 + x_2p_2 = m, \text{ i.e. } 4x_1 + 4x_2 = 20$$

$$\text{(2) Equating Bang-Per-Buck} : \frac{MUx_1}{p_1} = \frac{MUx_2}{p_2}, \text{ i.e. } \frac{2/x_1}{4} = \frac{1}{4}$$

Note that (2) above is equivalent to $MRS = -\frac{p_1}{p_2}$. Using either is fine. The geometric interpretation of (1) is that the optimal bundle is on the budget constraint. The economic interpretation of (1) is that you spend all of your money. The geometric interpretation of (2) is that the optimal bundle is where the indifference curve and budget constraint have the same slope (i.e. the indifference curve is tangent to the budget constraint).

d) (4 points) Find Mirabella's optimal choice (two numbers). Is solution interior? (yes-no answer)

Solution: To solve, note that (2) above implied that $\frac{2/x_1}{4} = \frac{1}{4}$, which, when solved, yields $x_1^* = 2$. Plugging $x_1 = 2$ into (1) gives us that $4 \cdot 2 + 4x_2 = 20$, which implies that $x_2^* = 3$. Therefore the optimal bundle is $(2, 3)$. This is an interior solution as it includes a strictly positive quantity of each good. You can also see this in the diagram in the solution to a).

e) (4 points) Suppose the price of a chocolate candy bar goes down to $p_1 = 2$, while other price $p_2 = 4$ and income $m = 20$ are unchanged. Find the new optimal choice (two numbers). Is a chocolate candy bar an ordinary or Giffen good? (pick one)

Solution: Now we have the following:

$$\frac{MUx_1}{p_1} = \frac{MUx_2}{p_2} \Leftrightarrow \frac{2/x_1}{2} = \frac{1}{4} \Leftrightarrow x_1^* = 4$$
$$x_1 p_1 + x_2 p_2 = m \Leftrightarrow 4 \cdot 2 + x_2 \cdot 4 = 20 \Leftrightarrow x_2^* = 3$$

Therefore the optimal choice is $(4, 3)$. Since demand for the chocolate candy bar x_1^* went up (from 2 to 4) when the price was reduced, the chocolate candy bar is an ordinary good.

f) (4 points) Decompose the change in demand for x_1 in points d) and e) into a substitution and income effect.

Solution:

To decompose the change in demand, we have to consider an auxiliary/Slutsky step in which we give the agent exactly enough income to purchase the original bundle (2,3) at the new prices:

$$m' = 2 \cdot 2 + 4 \cdot 3 = 16$$

We then consider what the agent's optimal bundle is given that income level, m', and the new prices.

$$\frac{MUx_1}{p_1} = \frac{MUx_2}{p_2} \Leftrightarrow \frac{2/x_1}{2} = \frac{1}{4} \Leftrightarrow x_1^* = 4$$
$$x_1 p_1 + x_2 p_2 = m' \Leftrightarrow 4 \cdot 2 + x_2 \cdot 4 = 16 \Leftrightarrow x_2^* = 2$$

Then the substitution effect is the auxiliary/Slutsky x_1^* minus the original x_1^*, which equals $4 - 2 = 2$. The income effect is the new x_1^* minus the auxiliary/Slutsky x_1^*, which equals $4 - 4 = 0$. Therefore, $SE = 2$ and $IE = 0$.

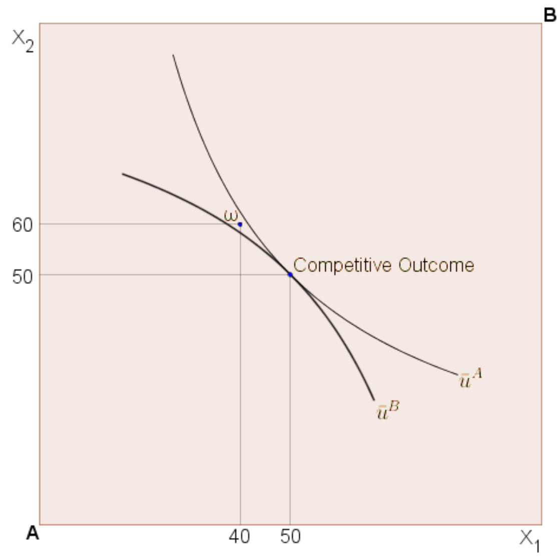
Problem 2. (Equilibrium) (20 points)

Consider an economy with two consumers, Adalia and Briana and two goods: bicycles, x_1, and flowers, x_2. Adalia's initial endowment of the commodities is $\omega^A = (40, 60)$ and Brianna's endowment is $\omega^B = (60, 40)$. Adalia and Briana's utility functions are given by, for $i = A, B$,

$$U^i(x_1, x_2) = 4 \ln x_1 + 4 \ln x_2$$

a) (3 points) Plot an Edgeworth box and mark the point that corresponds to initial endowments.

Solution: See diagram below (which also includes solutions for later parts of this question).



b) (3 points) Give a definition of a Pareto efficient allocation (one sentence).

Solution: An allocation is Pareto efficient if it is impossible to make one person better off without making another worse off.

c) (3 points) Give a (general) equivalent condition for Pareto efficiency in terms of MRS. Provide arguments that demonstrate the necessity and sufficiency of the MRS condition for Pareto efficiency.

Solution: The equivalent condition is that $MRS^A = MRS^B$, i.e. all agents have the same MRS at the Pareto efficient allocation. The arguments for necessity and suffiency of the MRS condition are:

- Necessity: If MRS are not equal at a given point then the indifference curves through that point have different slopes. This implies that there exists a lens-shape region between the two indifference curves (better for both people). Any point inside of that region is a Pareto improvement over the original given point. Therefore, if MRS are not equal at a point, that point isn't Pareto optimal. Hence the MRS condition is necessary for Pareto optimality.
- Sufficiency: If the MRS of the agents are the same, then everything down and to the left of A's indifference curve is strictly worse for A. Everything up and to the right of B's indifference curve is stricty worse for B. Every point in the edgeworth box is either down and to the left of A's indifference curve or up and to the right of B's indifference curve, or both. Hence there is no feasible alternative allocation that makes one agent better off without hurting the other, and $MRS^A = MRS^B$ is a sufficient condition for Pareto efficiency.

d) (8 points) Find competitive equilibrium (six numbers). Depict the obtained equilibrium in the Edgeworth box. Using MRS condition verify that the equilibrium is Pareto efficient.

Solution: To solve this, we normalize $p_2 = 1$. (You could also normalize $p_1 = 1$ and solve for p_2 if you prefer.) Then, to solve for p_1, we use Cobb-Douglas formulas and clear the market for x_1 (you could clear the market for x_2 if you prefer):

$$\begin{aligned} \text{Demand for Good 1} &= \text{Supply for Good 1} \\ \frac{a}{a+b}\frac{m^A}{p_1} + \frac{a}{a+b}\frac{m^B}{p_1} &= 40+60 \\ \frac{4}{4+4}\frac{40p_1+60p_2}{p_1} + \frac{4}{4+4}\frac{60p_1+40p_2}{p_1} &= 100 \\ \frac{1}{2}\frac{40p_1+60}{p_1} + \frac{1}{2}\frac{60p_1+40}{p_1} &= 100 \\ 40p_1+60+60p_1+40 &= 200p_1 \\ 100 &= 100p_1 \\ p_1 &= 1 \end{aligned}$$

Now that we know that $p_1 = p_2 = 1$, we simply plug into the Cobb-Douglas demand formulas to solve for the demands:

$$\begin{aligned} x_1^A &= \frac{a}{a+b}\frac{m^A}{p_1} = \frac{1}{2}\frac{40+60}{1} = 50 \\ x_2^A &= \frac{b}{a+b}\frac{m^A}{p_2} = \frac{1}{2}\frac{40+60}{1} = 50 \\ x_1^B &= \frac{a}{a+b}\frac{m^B}{p_1} = \frac{1}{2}\frac{60+40}{1} = 50 \\ x_2^B &= \frac{b}{a+b}\frac{m^B}{p_2} = \frac{1}{2}\frac{60+40}{1} = 50 \end{aligned}$$

Therefore the market outcome is $p_1 = p_2 = 1$ and $x_1^A = x_2^A = x_1^B = x_2^B = 50$. This is depicted in the diagram above in part a). At this outcome, $MRS^A = -\frac{4/50}{4/50} = -1 = MRS^B$. Therefore we have verified that the equilibrium is Pareto efficient.

e) (3 points) Harder: Using (one of) the secrets of happiness, prove that a competitive equilibrium is Pareto efficient in any economy.

Solution: The secret to happiness for A is that $MRS^A = -p_1/p_2$. The secret to happiness for B is that $MRS^B = -p_1/p_2$. Since the secrets to happiness dictate that each sets her MRS equal to $-p_1/p_2$, it follows that $MRS^A = MRS^B$.

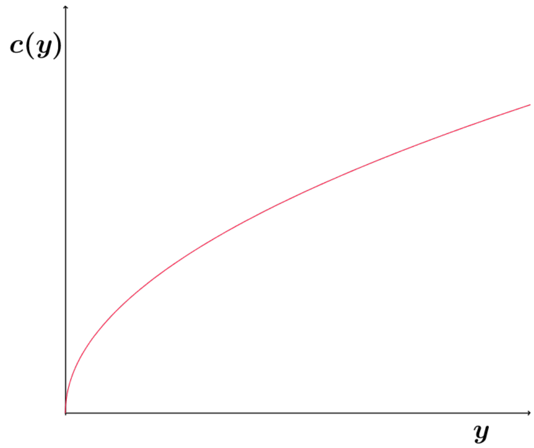
Problem 3. (Short Questions) (20 points)

a) (4 points) Using λ argument, prove that Cobb-Douglas production function $y = 2KL$ exhibits increasing returns to scale. Without any calculations, sketch total cost function $c(y)$ corresponding to the production function.

Solution: The λ argument is as follows:

$$F(\lambda K, \lambda L) = 2(\lambda K)(\lambda L) = \lambda^2 2KL = \lambda^2 F(K,L) > \lambda F(K,l) \text{ for all } \lambda > 1$$

This implies IRS. As for $c(y)$, note that you cannot actually do the calculations without the wage rates for capital and labor. However, we know that the cost function is increasing in output. We also know that, because of IRS, it should be concave. Therefore, any function you can draw that is increasing and concave is valid. One example:



b) (4 points) Now consider a firm (different from point a)) with variable cost $c(y) = 4y^2$ and fixed cost $F = 4$. Find ATC^{MES} and y^{MES} (two numbers). In a long-run equilibrium with free entry how many firms should be expected in the industry if demand is $D(p) = 16 - p$?

Solution: $TC = 4y^2 + 4$, $MC = 8y$, $ATC = 4y + 4/y$. To solve for y^{MES}, set $ATC = MC$ (note you can also set the derivative of ATC with respect to y equal to zero to find y^{MES}).

$$ATC = MC \quad \Leftrightarrow \quad 4y + 4/y = 8y \quad \Leftrightarrow \quad y^{MES} = 1$$

Plugging that into the ATC gives us that $ATC^{MES} = 4 + 4 = 8$. In the long-run, free-entry equilibrium, we know that each firm produces y^{MES} and equilibrium price is ATC^{MES}. Therefore

$$\begin{aligned} \text{Supply} &= \text{Demand} \\ N \cdot y^{MES} &= 16 - ATC^{MES} \\ N &= 8 \end{aligned}$$

So we would expect 8 firms in this equilibrium.

c) (4 points) Suppose a Bernoulli utility function is $u(x) = x^2$, and two states are equally likely (probability 1/2). Write down the corresponding von Neuman-Morgenstern utility function. Find the certainty equivalent and the expected value of lottery (0,2) (two numbers). Which of the two is bigger and why? (two numbers and one sentence.)

Solution: von Neuman-Morgenstern utility function: $U(x_g, x_b) = 1/2x_g^2 + 1/2x_b^2$.
Expected value of lottery: EV=$0 + 1/2 * 2 = 1$.
certainty equivalent, CE makes $U(CE, CE) = CE^2 =$ Expected Utility $= 0 + 1/2 * 2^2 = 2$, so CE=$\sqrt{2}$.
CE > EV, because this Bernoulli utility function represents preference of a risk lover, and risk lover enjoys the lottery more than its expected value (he loves uncertainty).

d) (4 points) Find Herfindahl-Hirschman Index (HHI) for industry with N = 50 identical firms (one number). Is the industry concentrated?

Solution: Each firm occupies 2% market share. So HHI= $50 * 2^2 = 200 < 1800$. This industry is not concentrated (or the industry is competitive).

e) (4 points) Derive formula for the present value of perpetuity.

Solution: Suppose the interest rate is r and the constant payment of the asset is x.

$$
\begin{aligned}
PV &= \frac{x}{1+r} + \frac{x}{(1+r)^2} + \frac{x}{(1+r)^3} + \dots \\
&= \frac{x}{1+r} + \frac{1}{1+r}[\frac{x}{1+r} + \frac{x}{(1+r)^2} + \dots] \\
&= \frac{x}{1+r} + \frac{1}{1+r}PV \\
\Rightarrow & PV[1 - \frac{1}{1+r}] = \frac{x}{1+r} \\
\Rightarrow & PV = \frac{x}{r}
\end{aligned}
$$

Problem 4. (Market Power) (15 points)
Consider an industry with inverse demand $p(y) = 8-y$, and a monopoly with cost function $TC(y) = 0$ who cannot discriminate.

a) **(2 points)** What are the total gains-to-trade (or potential total surplus) in this industry? (give one number)

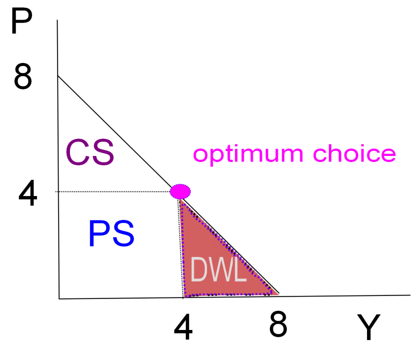
Solution: TS=$1/2 * 8 * 8 = 32$.

b) **(4 points)**Write down monopolys profit function. Derive the condition on MR and MC that gives profit maximizing level of production. Provide economic interpretation of this condition.

Solution: $\pi = p(y)y - TC((y) = (8-y)y - TC(y)$.
Derive: $\frac{\partial \pi}{\partial y} = MR - MC = 8 - 2y = 0$. (Write MR=MC explicitly.)
MR is decreasing in y and MC is increasing in y. So when MR>MC, the monopoly can increase its profits by producing more until MR=MC; when MR<MC, the last unit produced has negative profit, the monopoly can increase its profits by reducing production until MR=MC.

c) **(5 points)** Find the level of production, the price, the deadweight loss and the elasticity of the demand at optimum (four numbers). Illustrate the choice in a graph.

Solution: From (b),$8 - 2y = 0 \rightarrow y^M = 4, p^M = 4$, DWL=$1/2 * 4 * 4 = 8$, elasticity at the optimum choice: $\epsilon = \frac{1}{p'(y)}\frac{p}{y} = -1$.



d) **(4 points)** Assuming the same demand function find the individual and the aggregate level of production and the price in the Cournot-Nash equilibrium with N = 3 identical firms (give three numbers). Show the deadweight loss in the graph.

Solution: We need to derive the best response function first. Suppose there are three firms, 1,2,3.

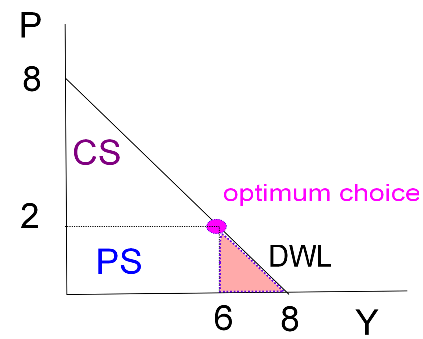
For firm 1,

$$\max_{y_1} \pi_1 = (8 - y_1 - y_2 - y_3)y_1$$
$$\Rightarrow \frac{\partial \pi_1}{\partial y_1} = 8 - 2y_1 - (y_2 + y_3) = 0$$
$$\Rightarrow y_1^* = \frac{8 - (y_2 + y_3)}{2}.$$

At the Nash equilibrium, since three firms are identical, $y_1^* = y_2^* = y_3^*$, plug this information into the best response function

$$\Rightarrow y_1^* = \frac{8 - 2y_1^*}{2} = 4 - y_1^*, \Rightarrow y_1^* = 2$$

So the aggreagate output is $3y_1^* = 6, p = 2$, DWL=2.



Problem 5.(Externality) (10 points)
Lucy is addicted to nicotine. Her utility from smoking c cigarettes (net of their cost) is given by

$$U^L(c) = 2 \ln c - c$$

Her sister Taja prefers healthy lifestyle, her favorite commodity is orange juice, j. The two sisters live together and Taja is exposed to second-hand smoke and hence her utility is adversely affected by Lucy consumption of cigarettes c. In particular, her utility function (net of cost of orange juice) is given by

$$U^T(j, c) = \ln(j - c) - j$$

.

a) **(4 points)** Market outcome: Find consumption of cigarettes c that maximizes the utility of Lucy and the amount of orange juice chosen by Taja (assuming c is optimal for Lucy) (two numbers).

Solution: For Lucy,

$$\frac{\partial U^L(c)}{\partial c} = \frac{2}{c} - 1 = 0 \Rightarrow c^* = 2$$

For Taja,

$$\frac{\partial U^T(j, c)}{\partial j} = \frac{1}{j - c} - 1 = 0 \Rightarrow j^* = c + 1 = 3.$$

b) **(6 points)** Find the Pareto efficient level of c and j: Is the value of c higher or smaller than in a)? Why? (two numbers + one sentence)

Solution: Joint utility is $U = 2\ln c + \ln(j-c) - c - j$.

$$\frac{\partial U}{\partial c} = \frac{2}{c} - \frac{1}{j-c} - 1 = 0$$
$$\frac{\partial U}{\partial j} = \frac{1}{j-c} - 1 = 0.$$

So $j - c = 1, \rightarrow \tilde{c} = 1, \tilde{j} = 2$. The social optimal $\tilde{c} = 1 < 2$. Because Lucy does not internalize the externality of her cigarettes to Taja, she smokes too much comparing to the social optimal level of c.

Problem 6. (Asymmetric information) (10 points) In Shorewood Hills area there are two types of homes: lemons (bad quality homes) and plums (good quality ones). The fraction of lemons is equal to 1/2 : The value of a home for the two parties depends on its type and is given by

	Lemon	Plum
Seller	0	12
Buyer	10	18

Both parties agree on the price that is in between the value of a buyer and a seller.

a) **(3 points)** Buyers and sellers can perfectly determine the quality of a house before transaction takes place. What is expected total, buyers and sellers surplus (three numbers)

Solution: The *expected* gains to trade (ETS) =1/2*10+1/2*6=8.
Expected Buyer surplus (EBS)=1/2*5+1/2*3=4,
and Expected Seller surplus (ESS) =1/2*5+1/2*3=4.

b) **(7 points)** Now assume that the buyers are not able to determine quality of a house. Find the price of a house, and the expected buyers and sellers surplus (three numbers). Is a pooling equilibrium sustainable, or will this market result in a separating equilibrium? Is outcome Pareto efficient (why or why not)?

Solution: With probability 1/2, the expected value of a house to a buyer is EV=$1/2*10+1/2*18 = 14$ **(1 point)**, which is larger than 12 (seller's value for plums), so both houses can be sold, and we end up with a pooling equilibrium **(1 point)**.
This pooling equilibrium is sustainable. **(1 point)**
The price is $p = \frac{12+14}{2} = 13$. **(1 point)**
Expected buyer surplus= 1/2*(10-13) + 1/2 (18-13)=1,**(1 point)**
expected seller surplus = 1/2(13-0) + 1/2(13-12)=7.**(1 point)**
So total expected gain to trade is EBS+ESS=8, which is the same as in (a) (full information), this outcome is Pareto efficient.**(1 point)**

Solutions to Spring 2014 ECON 301 Final Group B

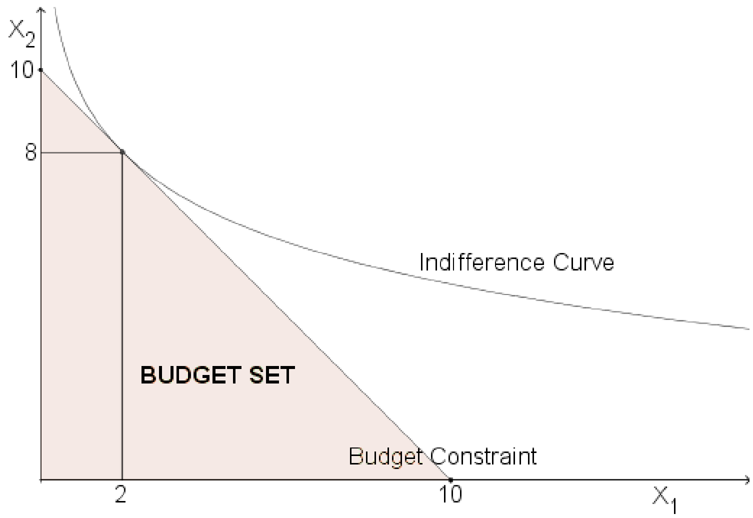
Problem 1. (Quasilinear income effect) (25 points)

Mirabella consumes chocolate candy bars x_1 and fruits x_2. The prices of the two goods are $p_1 = 2$ and $p_2 = 2$ respectively and Mirabella's income is $m = 20$. Her utility function is

$$U(x_1, x_2) = 2\ln x_1 + x_2$$

a) (4 points) In the commodity space plot Mirabella's budget set. Find slope of budget line (one number). Provide the economic interpretation of the slope (one sentence).

Solution: See figure below for Mirabella's budget set - it's the entire shaded region. The slope of the budget line is equal to $-p_1/p_2 = -2/2 = -1$. The economic interpretation of the slope is that it represents the rate at which the market prices allow Mirabella to trade between the two goods.



b) (4 points) Find analytically the formula that gives Mirabella's MRS for any bundle (x_1, x_2) (a function). Give the economic and the geometric interpretation of the MRS (two sentences). Find the value of the MRS at bundle $(x_1, x_2) = (8, 8)$ (one number). Which of the two commodities is (locally) more valuable? (choose one)

Solution: $MRS = -\frac{MUx_1}{MUx_2} = -\frac{2/x_1}{1} = -\frac{2}{x_1}$. The economic interpretation of the MRS is that it represents the rate at which Mirabella would trade-off between goods while remaining indifferent. Geometrically, it is the slope of the indifference curve at that bundle. $MRS(8, 8) = -2/8 = -1/4$. Since at that bundle, $MUx_1 = 1/4 < 1 = MUx_2$, x_2 is locally more valuable.

c) (5 points) Write down two secrets to happiness that determine Mirabella's optimal choice (two equations). Provide the geometric interpretation of the conditions in the commodity space.

Solution: The two secrets to happiness are

$$\text{(1) Budget Constraint}: x_1p_1 + x_2p_2 = m, \text{ i.e. } 2x_1 + 2x_2 = 20$$

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d) (4 points) Find Mirabella's optimal choice (two numbers). Is solution interior? (yes-no answer)

Solution: To solve, note that (2) above implied that $\frac{2/x_1}{2} = \frac{1}{2}$, which, when solved, yields $x_1^* = 2$. Plugging $x_1 = 2$ into (1) gives us that $2 \cdot 2 + 2x_2 = 20$, which implies that $x_2^* = 8$. Therefore the optimal bundle is $(2, 8)$. This is an interior solution as it includes a strictly positive quantity of each good. You can also see this in the diagram in the solution to a).

e) (4 points) Suppose the price of a chocolate candy bar goes down to $p_1 = 1$, while other price $p_2 = 2$ and income $m = 20$ are unchanged. Find the new optimal choice (two numbers). Is a chocolate candy bar an ordinary or Giffen good? (pick one)

Solution: Now we have the following:

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$$x_1p_1 + x_2p_2 = m \Leftrightarrow 4 \cdot 1 + x_2 \cdot 2 = 20 \Leftrightarrow x_2^* = 8$$

Therefore the optimal choice is $(4, 8)$. Since demand for the chocolate candy bar x_1^* went up (from 2 to 4) when the price was reduced, the chocolate candy bar is an ordinary good.

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Solution:

To decompose the change in demand, we have to consider an auxiliary/Slutsky step in which we give the agent exactly enough income to purchase the original bundle (2,8) at the new prices:

$$m' = 1 \cdot 2 + 2 \cdot 8 = 18$$

We then consider what the agent's optimal bundle is given that income level, m', and the new prices.

$$\frac{MUx_1}{p_1} = \frac{MUx_2}{p_2} \Leftrightarrow \frac{2/x_1}{1} = \frac{1}{2} \Leftrightarrow x_1^* = 4$$
$$x_1p_1 + x_2p_2 = m' \Leftrightarrow 4 \cdot 1 + x_2 \cdot 2 = 18 \Leftrightarrow x_2^* = 7$$

Then the substitution effect is the auxiliary/Slutsky x_1^* minus the original x_1^*, which equals $4-2 = 2$. The income effect is the new x_1^* minus the auxiliary/Slutsky x_1^*, which equals $4 - 4 = 0$. Therefore, $SE = 2$ and $IE = 0$.

Problem 2. (Equilibrium) (20 points)

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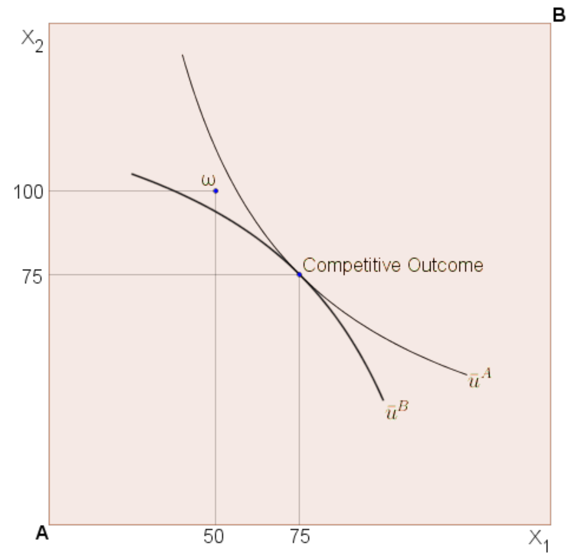
$$U^i(x_1, x_2) = 2 \ln x_1 + 2 \ln x_2$$

a) (3 points) Plot an Edgeworth box and mark the point that corresponds to initial endowments.

Solution: See diagram below (which also includes solutions for later parts of this question).



b) (3 points) Give a definition of a Pareto efficient allocation (one sentence).

Solution: An allocation is Pareto efficient if it is impossible to make one person better off without making another worse off.

c) (3 points) Give a (general) equivalent condition for Pareto efficiency in terms of MRS. Provide arguments that demonstrate the necessity and sufficiency of the MRS condition for Pareto efficiency.

Solution: The equivalent condition is that $MRS^A = MRS^B$, i.e. all agents have the same MRS at the Pareto efficient allocation. The arguments for necessity and suffiency of the MRS condition are:

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- Sufficiency: If the MRS of the agents are the same, then everything down and to the left of A's indifference curve is strictly worse for A. Everything up and to the right of B's indifference curve is stricty worse for B. Every point in the edgeworth box is either down and to the left of A's indifference curve or up and to the right of B's indifference curve, or both. Hence there is no feasible alternative allocation that makes one agent better off without hurting the other, and $MRS^A = MRS^B$ is a sufficient condition for Pareto efficiency.

d) (8 points) Find competitive equilibrium (six numbers). Depict the obtained equilibrium in the Edgeworth box. Using MRS condition verify that the equilibrium is Pareto efficient.

Solution: To solve this, we normalize $p_2 = 1$. (You could also normalize $p_1 = 1$ and solve for p_2 if you prefer.) Then, to solve for p_1, we use Cobb-Douglas formulas and clear the market for x_1 (you could clear the market for x_2 if you prefer):

$$\begin{aligned}
\text{Demand for Good 1} &= \text{Supply for Good 1} \\
\frac{a}{a+b}\frac{m^A}{p_1} + \frac{a}{a+b}\frac{m^B}{p_1} &= 50 + 100 \\
\frac{2}{2+2}\frac{50p_1 + 100p_2}{p_1} + \frac{2}{2+2}\frac{100p_1 + 50p_2}{p_1} &= 150 \\
\frac{1}{2}\frac{50p_1 + 100}{p_1} + \frac{1}{2}\frac{100p_1 + 50}{p_1} &= 150 \\
50p_1 + 100 + 100p_1 + 50 &= 300p_1 \\
150 &= 150p_1 \\
p_1 &= 1
\end{aligned}$$

Now that we know that $p_1 = p_2 = 1$, we simply plug into the Cobb-Douglas demand formulas to solve for the demands:

$$\begin{aligned}
x_1^A &= \frac{a}{a+b}\frac{m^A}{p_1} = \frac{1}{2}\frac{50+100}{1} = 75 \\
x_2^A &= \frac{b}{a+b}\frac{m^A}{p_2} = \frac{1}{2}\frac{50+100}{1} = 75 \\
x_1^B &= \frac{a}{a+b}\frac{m^B}{p_1} = \frac{1}{2}\frac{100+50}{1} = 75 \\
x_2^B &= \frac{b}{a+b}\frac{m^B}{p_2} = \frac{1}{2}\frac{100+50}{1} = 75
\end{aligned}$$

Therefore the market outcome is $p_1 = p_2 = 1$ and $x_1^A = x_2^A = x_1^B = x_2^B = 75$. This is depicted in the diagram above in part a). At this outcome, $MRS^A = -\frac{2/75}{2/75} = -1 = MRS^B$. Therefore we have verified that the equilibrium is Pareto efficient.

e) (3 points) Harder: Using (one of) the secrets of happiness, prove that a competitive equilibrium is Pareto efficient in any economy.

Solution: The secret to happiness for A is that $MRS^A = -p_1/p_2$. The secret to happiness for B is that $MRS^B = -p_1/p_2$. Since the secrets to happiness dictate that each sets her MRS equal to $-p_1/p_2$, it follows that $MRS^A = MRS^B$.

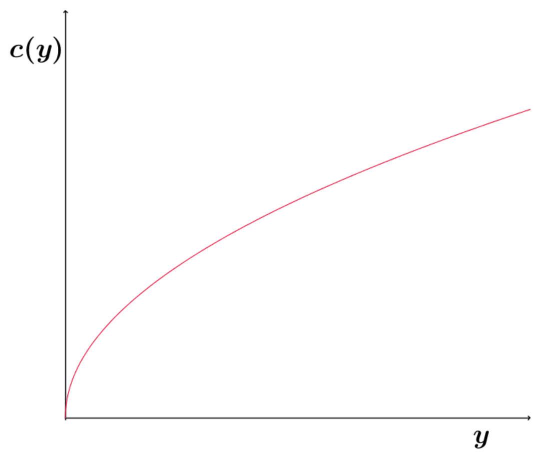
Problem 3. (Short Questions) (20 points)

a) (4 points) Using λ argument, prove that Cobb-Douglas production function $y = 2KL$ exhibits increasing returns to scale. Without any calculations, sketch total cost function $c(y)$ corresponding to the production function.

Solution: The λ argument is as follows:

$$F(\lambda K, \lambda L) = 2(\lambda K)(\lambda L) = \lambda^2 2KL = \lambda^2 F(K,L) > \lambda F(K,l) \text{ for all } \lambda > 1$$

This implies IRS. As for $c(y)$, note that you cannot actually do the calculations without the wage rates for capital and labor. However, we know that the cost function is increasing in output. We also know that, because of IRS, it should be concave. Therefore, any function you can draw that is increasing and concave is valid. One example:



b) (5 points) Now consider a firm (different from point a)) with variable cost $c(y) = 4y^2$ and fixed cost $F = 4$. Find ATC^{MES} and y^{MES} (two numbers). In a long-run equilibrium with free entry how many firms should be expected in the industry if demand is $D(p) = 16 - p$?

Solution: $TC = 4y^2 + 4$, $MC = 8y$, $ATC = 4y + 4/y$. To solve for y^{MES}, set $ATC = MC$ (note you can also set the derivative of ATC with respect to y equal to zero to find y^{MES}).

$$ATC = MC \;\Leftrightarrow\; 4y + 4/y = 8y \;\Leftrightarrow\; y^{MES} = 1$$

Plugging that into the ATC gives us that $ATC^{MES} = 4 + 4 = 8$. In the long-run, free-entry equilibrium, we know that each firm produces y^{MES} and equilibrium price is ATC^{MES}. Therefore

$$\begin{aligned} \text{Supply} &= \text{Demand} \\ N \cdot y^{MES} &= 16 - ATC^{MES} \\ N &= 8 \end{aligned}$$

So we would expect 8 firms in this equilibrium.

c) (4 points) Suppose a Bernoulli utility function is $u(x) = x^2$, and two states are equally likely (probability 1/2). Write down the corresponding von Neuman-Morgenstern utility function. Find the certainty equivalent and the expected value of lottery (0,2) (two numbers). Which of the two is bigger and why? (two numbers and one sentence.)

Solution: von Neuman-Morgenstern utility function: $U(x_g, x_b) = 1/2x_g^2 + 1/2x_b^2$.
Expected value of lottery: EV=$0 + 1/2 * 2 = 1$.
certainty equivalent, CE makes $U(CE, CE) = CE^2$ = Expected Utility $= 0 + 1/2 * 2^2 = 2$, so CE=$\sqrt{2}$.
CE > EV, because this Bernoulli utility function represents preference of a risk lover, and risk lover enjoys the lottery more than its expected value (he loves uncertainty).

d) (4 points) Find Herfindahl-Hirschman Index (HHI) for industry with N = 100 identical firms (one number). Is the industry concentrated?

Solution: Each firm occupies 1% market share. So HHI= $100 * 1^2 = 100 < 1800$. This industry is not concentrated.

e) (4 points) Derive formula for the present value of perpetuity. *Solution:* Suppose the interest rate is r and the constant payment of the asset is x.

$$\begin{aligned} PV &= \frac{x}{1+r} + \frac{x}{(1+r)^2} + \frac{x}{(1+r)^3} + \dots \\ &= \frac{x}{1+r} + \frac{1}{1+r}[\frac{x}{1+r} + \frac{x}{(1+r)^2} + \dots] \\ &= \frac{x}{1+r} + \frac{1}{1+r}PV \\ \Rightarrow & PV[1 - \frac{1}{1+r}] = \frac{x}{1+r} \\ \Rightarrow & PV = \frac{x}{r} \end{aligned}$$

Problem 4. (Market Power) (15 points)
Consider an industry with inverse demand $p(y) = 12 - y$, and a monopoly with cost function $TC(y) = 0$ who cannot discriminate.

a) **(2 points)** What are the total gains-to-trade (or potential total surplus) in this industry? (give one number)

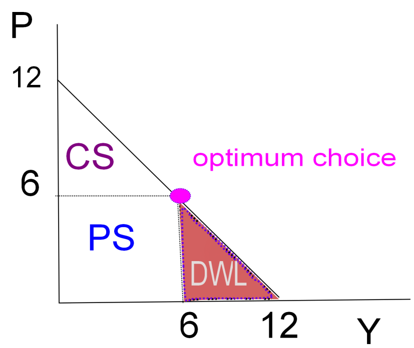
Solution: TS=$1/2 * 12 * 12 = 72$.

b) **(4 points)**Write down monopolys profit function. Derive the condition on MR and MC that gives profit maximizing level of production. Provide economic interpretation of this condition.

Solution: $\pi = p(y)y - TC((y) = (12 - y)y - TC(y)$.
Derive: $\frac{\partial \pi}{\partial y} = MR - MC = 12 - 2y = 0$. (Write MR=MC explicitly.)
MR is decreasing in y and MC is increasing in y. So when MR>MC, the monopoly can increase its profits by producing more until MR=MC; when MR<MC, the last unit produced has negative profit, the monopoly can increase its profits by reducing production until MR=MC.

c) **(5 points)** Find the level of production, the price, the deadweight loss and the elasticity of the demand at optimum (four numbers). Illustrate the choice in a graph.

Solution: From (b),$12 - 2y = 0 \to y^M = 6, p^M = 6$, DWL=$1/2 * 6 * 6 = 18$, elasticity at the optimum choice: $\epsilon = \frac{1}{p'(y)} \frac{p}{y} = -1$.



d) **(4 points)** Assuming the same demand function find the individual and the aggregate level of production and the price in the Cournot-Nash equilibrium with N = 3 identical firms (give three numbers). Show the deadweight loss in the graph.

Solution: We need to derive the best response function first. Suppose there are three firms, 1,2,3.

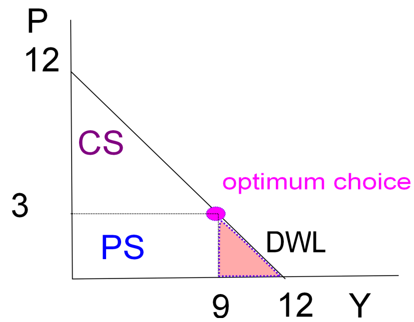
For firm 1,

$$\max_{y_1} \pi_1 = (12 - y_1 - y_2 - y_3)y_1$$
$$\Rightarrow \frac{\partial \pi_1}{\partial y_1} = 12 - 2y_1 - (y_2 + y_3) = 0$$
$$\Rightarrow y_1^* = \frac{12 - (y_2 + y_3)}{2}.$$

At the Nash equilibrium, since three firms are identical, $y_1^* = y_2^* = y_3^*$, plug this information into the best response function

$$\Rightarrow y_1^* = \frac{12 - 2y_1^*}{2} = 6 - y_1^*, \Rightarrow y_1^* = 3$$

So the aggregate output is $3y_1^* = 9, p = 3$, DWL=$1/2 * 3 * 3 = 4.5$.



Problem 5.(Externality) (10 points)
Lucy is addicted to nicotine. Her utility from smoking c cigarettes (net of their cost) is given by

$$U^L(c) = 2 \ln c - c$$

Her sister Taja prefers healthy lifestyle, her favorite commodity is orange juice, j. The two sisters live together and Taja is exposed to second-hand smoke and hence her utility is adversely affected by Lucy consumption of cigarettes c. In particular, her utility function (net of cost of orange juice) is given by

$$U^T(j, c) = \ln(j - c) - j$$

.

a) **(4 points)** Market outcome: Find consumption of cigarettes c that maximizes the utility of Lucy and the amount of orange juice chosen by Taja (assuming c is optimal for Lucy) (two numbers).

Solution: For Lucy,

$$\frac{\partial U^L(c)}{\partial c} = \frac{2}{c} - 1 = 0 \Rightarrow c^* = 2$$

For Taja,

$$\frac{\partial U^T(j,c)}{\partial j} = \frac{1}{j-c} - 1 = 0 \Rightarrow j^* = c + 1 = 3.$$

b) **(6 points)** Find the Pareto efficient level of c and j: Is the value of c higher or smaller than in a)? Why? (two numbers + one sentence)

Solution: Joint utility is $U = 2\ln c + \ln(j - c) - c - j$.

$$\frac{\partial U}{\partial c} = \frac{2}{c} - \frac{1}{j-c} - 1 = 0$$
$$\frac{\partial U}{\partial j} = \frac{1}{j-c} - 1 = 0.$$

So $j - c = 1, \rightarrow \tilde{c} = 1, \tilde{j} = 2$. The social optimal $\tilde{c} = 1 < 2$. Because Lucy does not internalize the externality of her cigarettes to Taja, she smokes too much comparing to the social optimal level of c.

Problem 6. (Asymmetric information) (10 points) In Shorewood Hills area there are two types of homes: lemons (bad quality homes) and plums (good quality ones). The fraction of lemons is equal to 1/2 : The value of a home for the two parties depends on its type and is given by

	Lemon	Plum
Seller	6	14
Buyer	10	22

Both parties agree on the price that is in between the value of a buyer and a seller.

a) **(3 points)** Buyers and sellers can perfectly determine the quality of a house before transaction takes place. What is expected total, buyers and sellers surplus (three numbers)

Solution: Two prices are $P_L = 8, P_P = 18$. The *expected* gains to trade (ETS) =1/2*4+1/2*8=6.
Expected Buyer surplus (EBS)=1/2*2+1/2*4=3,
and Expected Seller surplus (ESS) =1/2*2+1/2*4=3.

b) **(7 points)** Now assume that the buyers are not able to determine quality of a house. Find the price of a house, and the expected buyers and sellers surplus (three numbers). Is a pooling equilibrium sustainable, or will this market result in a separating equilibrium? Is outcome Pareto efficient (why or why not)?

Solution: With probability 1/2, the expected value of a house to a buyer is EV=$1/2*10+1/2*22 = 16$**(1 point)**, which is larger than 14 (seller's value for plums), so both houses can be sold, and we end up with a pooling equilibrium.**(1 point)**
This pooling equilibrium is sustainable.**(1 point)**
The price is $p = \frac{14+16}{2} = 15$.**(1 point)**
Expected buyer surplus= 1/2*(10-15) + 1/2 (22-15)=1,**(1 point)**
expected seller surplus = 1/2(15-6) + 1/2(15-14)=5.**(1 point)**
So total expected gain to trade is EBS+ESS=6, which is the same as in (a) (full information), this outcome is Pareto efficient.**(1 point)**