Problem Set 9: Solutions

ECON 301: Intermediate Microeconomics
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Problem 1 (Equilibrium with N Firms)

(a) First note that marginal cost is $MC(y) = c'(y) = 8y$. Using the condition that $p = MC(y)$, we have $p = 8y \implies y = \frac{1}{8}p$. For prices equal to above ATC^{MES}, the supply function is $y = \frac{1}{8}p$, for prices below ATC^{MES}, the firm would be operating at a loss if producing and so supply is $y = 0$.

We can find the minimum of ATC, which is what ATC^{MES} is, either by (1) setting ATC' equal to 0 and solving for y, or (2) equating $ATC = MC$ and solving for y (since the minimum of ATC corresponds to the point at which $ATC = MC$). Both give us $y^{MES} = 1$ and $ATC^{MES} = ATC(y^{MES}) = 8$.

The individual supply is then:

$$y(p) = \begin{cases} 0 & \text{for } p < 8 \\ \frac{1}{8}p & \text{for } p \geq 8 \end{cases}$$

(b) The aggregate supply with the three identical cost structures is $y^{AGG}(p) = 3y(p)$ (the supply curves are added horizontally over the y-axis). Letting $S(p) = y^{AGG}(p)$:

$$S(p) = \begin{cases} 0 & \text{for } p < 8 \\ \frac{3}{8}p & \text{for } p \geq 8 \end{cases}$$

(c) We'll first find the equilibrium price and aggregate level of production by equating $S(p) = D(p)$:

$$S(p) = D(p) \implies \frac{3}{8}p = 8 - \frac{1}{8}p \implies p = 16 \text{ (which is } \geq 8)$$

and the aggregate level of production is $S(16) = D(16) = 6$.

The production of each firm is $y(p) = 2$ (since $S(p) = 3y(p)$, $6 = 3y(p)$), which gives profit $\pi = TR - TC = 16 \cdot 2 - (4 \cdot 2^2 + 4) = 12$ for each factory.

(d) The highest amount a firm would pay for the license is 12 (its profit when producing in this market, leaving it with 0 economic profit after paying the license fee).

Problem 2 (Free Entry and Market Structure)

(a) We'll first solve this for a general fixed cost of F, as we saw in Problem Set 8. The price in equilibrium with free entry must be $p = ATC^{MES} = 4\sqrt{F}$ where F is fixed cost with $y^{MES} = \frac{1}{2}\sqrt{F}$. At $p = ATC^{MES} = 4\sqrt{F}$, quantity demanded is

$$D(4\sqrt{F}) = 8 - \frac{1}{8}(4\sqrt{F})$$

and aggregate supply with N firms is

$$S(4\sqrt{F}) = N\frac{1}{2}\sqrt{F}\,.$$

Equating the two, we get

$$S(4\sqrt{F}) = D(4\sqrt{F}) \implies N\frac{1}{2}\sqrt{F} = 8 - \frac{1}{8}(4\sqrt{F}) \implies N = \frac{16}{\sqrt{F}} - 1\,.$$

So for fixed cost $F = 4$, the number of firms in the market will be $N = 7$.

(b) The number of firms in the market for various fixed costs are shown below, using the formula we found above, $N = \frac{16}{\sqrt{F}} - 1$:

Fixed Cost F:	64	16	4	$\frac{1}{4}$	$\frac{1}{16}$
Firms N:	1	3	7	31	63

(c) The market structures are:

- Monopoly at $F = 64$
- Oligopoly at $F = 16,\ F = 4$
- Nearly perfect competition at $F = \frac{1}{4},\ F = \frac{1}{16}$