Prof. Marek Weretka's

Econ 301 Intermediate Microeconomics

Problem Set 8

Problem 1 (Cost Functions)

Consider the following production functions:

$$F(K, L) = K^2 L^2$$

$$F(K, L) = K^{(1/3)} L^{(2/3)}$$

$$F(K, L) = K^{(1/4)} L^{(1/4)}$$

a) What is the returns to scale for each function (use formal argument with λ)?

Let $w_L = w_K = 1$

b) Find the cost functions for each of the production functions.

c) Plot the cost function on the same graph with y on the horizontal axis and cost on the vertical one.

d) Find and plot the average and marginal cost functions with y on the horizontal axis and average cost on the vertical one.

Problem 2 (Perfect Complements)

Consider the following production functions:

$$F(K, L) = min(K, L))$$

$$F(K, L) = [min(K, L)]^2$$

$$F(K, L) = \sqrt{}(min(K, L)$$

a) what are the returns to scale for each function (use formal argument with λ)?

Let $w_L = w_K = 1$

b) Find the cost functions for each of the production functions.

c) Plot the cost function on the same graph with y on the horizontal axis and cost on the vertical one.

d) Find and plot the average and marginal cost functions with y on the horizontal axis and average cost on the vertical one.

Problem 3 (Perfect Substitutes)

Consider the following production functions:

$$F(K,L) = K + 0.5L$$

$$F(K,L) = [K + 0.5L]^2$$

$$F(K,L) = \surd(K + 0.5L)$$

a) what are the returns to scale for each function (use formal argument with λ)

$$\text{Let} w_L = w_K = 1$$

b) Find the cost functions for each of the production functions.

c) Plot the cost function on the same graph with y on the horizontal axis and cost on the vertical one.

d) Find and plot the average and marginal cost functions with y on the horizontal axis and average cost on the vertical one.

Problem 4 (Cost Curves)

The GMC company is considering building a new car factory in China. The total (fixed) cost of the investment is $F = 4$. When built, the factory will allow to produce y cars at the (variable) cost given by

$$c(y) = 4y^2$$

a) Does the technology used in the new factory exhibit increasing, decreasing or constant returns to scale (ignore the fixed costs in this point)?

b) Find a total costs (TC) of producing 1, 2 and 4 cars. In the graph $(y, COST)$ plot a TC curve, and decompose it into a fixed cost curve and a variable cost curve by adding the two curves to your graph.

c) Find the values of the average fixed cost (AFC) for three levels of production $y = 1, 2$ and 4. Plot an AFC curve in a separate graph. What happens to the AFC when production becomes very large (close to infinity) and when it is very small (close to zero). Explain.

d) Find the values of the average variable cost AVC for $y = 1, 2$ and 4, and mark them in the graph from question c). Connect the three points to obtain the AVC curve.

e) Find the values of the average total cost ATC for $y = 1, 2$ and 4 and mark them in your graph from c). Connect the three points to obtain the ATC curve. What are the values of ATC when the production is very small and very large? Explain which of the two components of ATC - AFC or AVC- dominates in each of the two extremes. Why?

f) Find analytically the minimal efficient scale (MES), y^{MES}, ATC^{MES} for the considered car technology.

g) Find analytically marginal cost MC curve. In a new graph plot the MC curve, together with the ATC, marking the MES.

h) Explain intuitively why or why not the MCcurve cuts or does not cut the ATC curve at the MES.

i) Harder: find analytically a minimal efficient scale y^{MES}, and ATC^{MES} as a function of F (parameter). How do the two values depend on the level of F?

Problem 5 (Supply Curve of GMC)

Suppose GMC from Problem 1 is maximizing its profit given by:

$$\pi = py - TC(y)$$

a) Find analytically the optimal level of production for each of the three price levels $p = 4, p = 8, p = 16$? (Hint: first derive the secret of happiness $MC = p$, then find the level of production (y) and finally check whether the maximal profit is non-negative. If it is non-negative, then y you have found is optimal, theorizes the optimal production is zero.

b) Find analytically a car supply function of GMC,$y(p)$. Hint: it should have the following form:

$$y(p) = \{0 \text{ if } p < sth$$

and

$$y(p) = \{sth \text{ if } p \geq sth$$

where sth should be replaced with proper numbers or functions.

c) Plot your supply function on the graph, adding the ATC function.

d) Find supply as in b), c) for $F = 1$ (instead of $F = 4$). How is your supply function affected by the change of F? Is it steeper? Hint: use the value you have calculated in f), Problem 1.