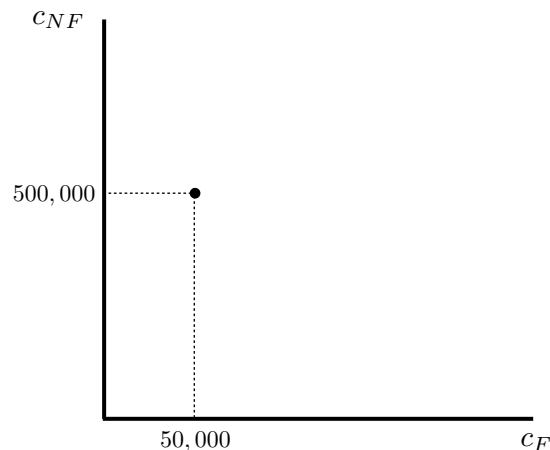


Problem Set 6: Solutions

ECON 301: Intermediate Microeconomics
Prof. Marek Weretka

Problem 1 (Insurance)

- (a) Ben's affordable bundle if there is no insurance market is his endowment:
 $(c_F, c_{NF}) = (50,000, 500,000)$.



- (b) Letting x be the amount of insurance coverage Ben purchases at a cost/premium of $0.1x$, note that in the state of the world in which there is a flood, his consumption is

$$c_F \leq 50,000 - 0.1x + x \quad (1)$$

and in the state of the world in which the house does not flood, his consumption is

$$c_{NF} \leq 500,000 - 0.1x. \quad (2)$$

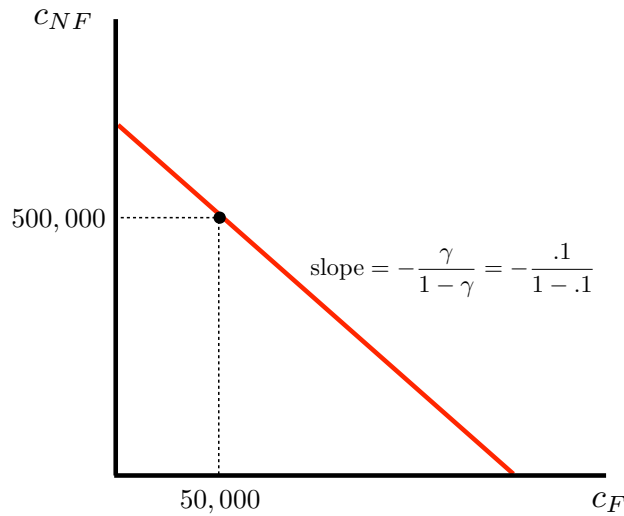
(We use \leq for the budget *constraint* and $=$ for the budget *line*. Of course, we know that consumption will end up being on the budget line, so using $=$ for these equations will for the most part not hurt even though when we're talking about the budget constraint we should technically be using \leq .)

The budget line will be in terms of c_F and c_{NF} (we will be determining what x is later), so to eliminate x , solve at equality one of the equations for x and plug it for x in the other equation.

From (1), at equality we get $x = \frac{10}{9}(c_F - 50,000)$. Plugging this into equation (2) we get

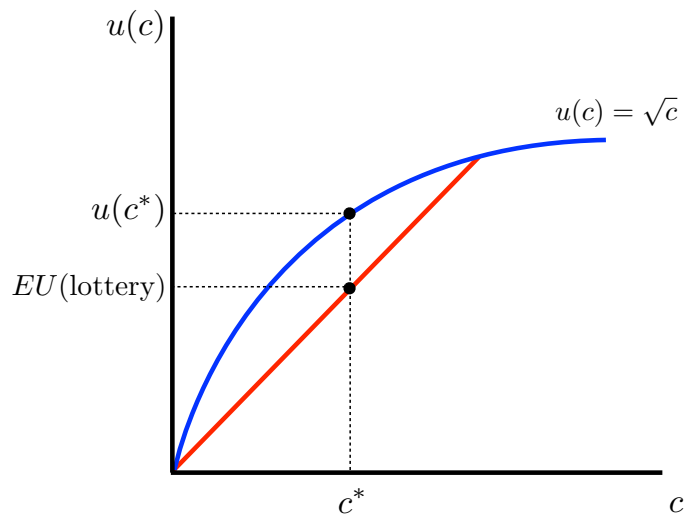
$$c_{NF} \leq 500,000 - 0.1 \left(\frac{10}{9}(c_F - 50,000) \right) \implies c_{NF} \leq 505,556 - \frac{1}{9}c_F \text{ (approximately).}$$

His budget line is shown below:



(c) Ben is risk averse. We can think about this from three different but consistent perspectives:

- (Analytically) Here, his Bernoulli utility function, $u(c) = \sqrt{c}$ is concave over c (i.e., its second derivative is negative).
- (Economically) The utility he gets from a riskless, definite amount of c^* , which is $u(c^*) = \sqrt{c^*}$, is greater than the expected utility of some lottery that gives an expected value of c^* , for instance a 50-50 lottery where he wins either \$0 or $2c^*$ where $EU(\text{lottery}) = \frac{1}{2}\sqrt{0} + \frac{1}{2}\sqrt{2c^*}$, as shown below.
- (Graphically) The points along a straight line connecting two points on $u(c)$ lie under $u(c)$.



(d) Ben's MRS is

$$MRS(c_F, c_{NF}) = -\frac{0.1(\frac{1}{2})c_F^{-\frac{1}{2}}}{0.9(\frac{1}{2})c_{NF}^{-\frac{1}{2}}} = -\frac{1}{9} \left(\frac{c_{NF}}{c_F} \right)^{\frac{1}{2}}$$

and at the endowment point $(50,000, 500,000)$, $MRS(50,000, 500,000) = -\frac{1}{9} \cdot 10^{\frac{1}{2}}$. Notice that this is not optimal since $MRS \neq -\frac{1}{9}$.

(e) These are well-behaved preferences, so we can use the “secrets of happiness” to find the optimal (c_F, c_{NF}) point (which will allow us to find his optimal insurance amount x):

- Secret 1: $MRS = -\frac{\gamma}{1-\gamma}$ (Marginal utility per dollar equalized across “goods” c_F and c_{NF} , and the price ratio $\frac{p_1}{p_2}$ is $\frac{\gamma}{1-\gamma}$ in this context, where $\gamma = 0.1$.)
- Secret 2: $c_{NF} = 505,556 - \frac{1}{9}c_F$ (Ben is consuming along his budget line)

From Secret 1 we get

$$-\frac{1}{9} \left(\frac{c_{NF}}{c_F} \right)^{\frac{1}{2}} = -\frac{1}{9} \implies c_F = c_{NF}.$$

Plugging $c_F = c_{NF}$ into the equation for Secret 2 for c_F , we get

$$c_{NF} = 505,556 - \frac{1}{9}c_{NF} \implies c_{NF} = 455,000 \text{ and so } c_F = 455,000 \text{ also.}$$

To find the insurance amount x , we can plug this value into equation (1), $c_{NF} \leq 500,000 - 0.1x$ (or equation (2)) to get

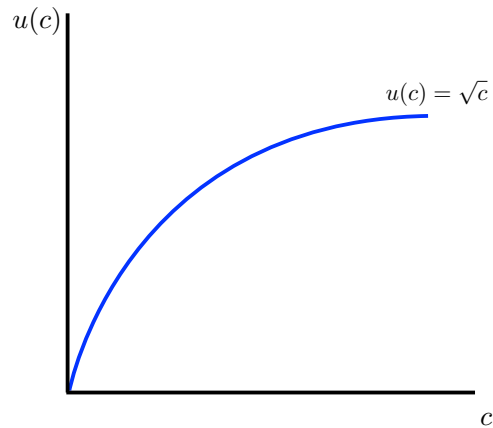
$$c_{NF} = 500,000 - 0.1x \implies 455,000 = 500,000 - 0.1x \implies x = 450,000.$$

The fact that $c_F = c_{NF}$ tells us that Ben fully insures (in either state of the world, he consumes the same amount).

(f) When the insurance premium is instead $0.2x$, the new budget line has a slope of $-\frac{\gamma}{1-\gamma} = -\frac{0.2}{1-0.2} = -\frac{1}{4}$, rotating about the endowment point $(50,000, 500,000)$. Optimal consumption will no longer be on the 45° line (along which $c_{NF} = c_F$, i.e., full insurance).

Problem 2 (Risk Aversion and Certainty Equivalence)

(a) Frank McGambler's Bernoulli utility over c , $u(c) = \sqrt{c}$, is shown below:



Frank is risk averse. His utility over c is concave, and he would prefer to have the expected value of the lottery with certainty than to take the gamble.

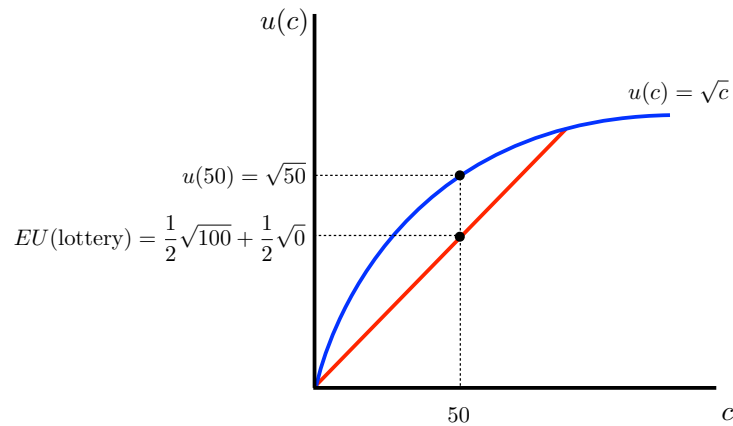
(b) The expected value of the lottery is

$$EV(\text{lottery}) = \frac{1}{2}100 + \frac{1}{2}0 = 50.$$

(c) The expected utility is

$$EU(\text{lottery}) = \frac{1}{2}\sqrt{100} + \frac{1}{2}\sqrt{0} = 5.$$

This is shown below:



Notice that since Frank is risk averse, the lottery gives him less utility than does a “sure” amount equal to the expected value of the lottery, which is \$50: $EU(\text{lottery}) < u(50)$.

(d) The certainty equivalent (CE) is the amount of “sure” cash that makes Frank indifferent between the \$CE amount and the lottery, so $u(CE) = EU(\text{lottery})$.¹ Since $EU(\text{lottery}) = 5$, setting the two equal, we get

$$u(CE) = EU(\text{lottery}) \implies \sqrt{CE} = 5 \implies CE = 25.$$

This means Frank is indifferent between getting $CE = 25$ with no risk and the (risky) lottery which gives him $EV(\text{lottery}) = 50$ in expectation. The fact that Frank’s $CE = 25 < 50 = EV(\text{lottery})$ is another way of seeing that Frank is risk averse: He would be willing to give up \$15 on average to avoid the risk.

(e) Frank would be better off choosing the \$40 (or any sure amount that is greater than his certainty equivalent).

(f) The expected value of the lottery is unchanged (it does not depend on his utility). With $u(c) = c$, his expected utility is

$$EU(\text{lottery}) = \frac{1}{2}100 + \frac{1}{2}0 = 50.$$

Setting the $u(CE) = EU(\text{lottery})$ equal and solving for CE, we get

$$u(CE) = EU(\text{lottery}) \implies CE = 50,$$

So he is indifferent between a sure \$50 and this lottery from which he would get an expected amount of \$50; he is risk neutral. With Bernoulli utility $u(c) = c$, he would prefer the lottery to the sure \$40.

(g) Again, the expected value of the lottery is unchanged (it does not depend on his utility). With $u(c) = c^2$, his expected utility is

$$EU(\text{lottery}) = \frac{1}{2}100^2 + \frac{1}{2}0^2 = 5,000.$$

Setting the $u(CE) = EU(\text{lottery})$ equal and solving for CE, we get

$$u(CE) = EU(\text{lottery}) \implies CE^2 = 5,000 \implies CE \approx 70.7,$$

Now Frank is risk loving/seeking, and he prefers this lottery from which he would get an expected amount of \$50 to a sure \$70.70! With Bernoulli utility $u(c) = c^2$, he would *definitely* prefer the lottery to the sure \$40 (he prefers the lottery over any amount up to \$70.7).

¹Alternatively, we can think of the utility from the “sure” CE amount as being a lottery in which Frank receives \$CE in both states of the world, then $EU(CE) = \frac{1}{2}U(CE) + \frac{1}{2}u(CE) = u(CE)$ and we get the exact same answer.

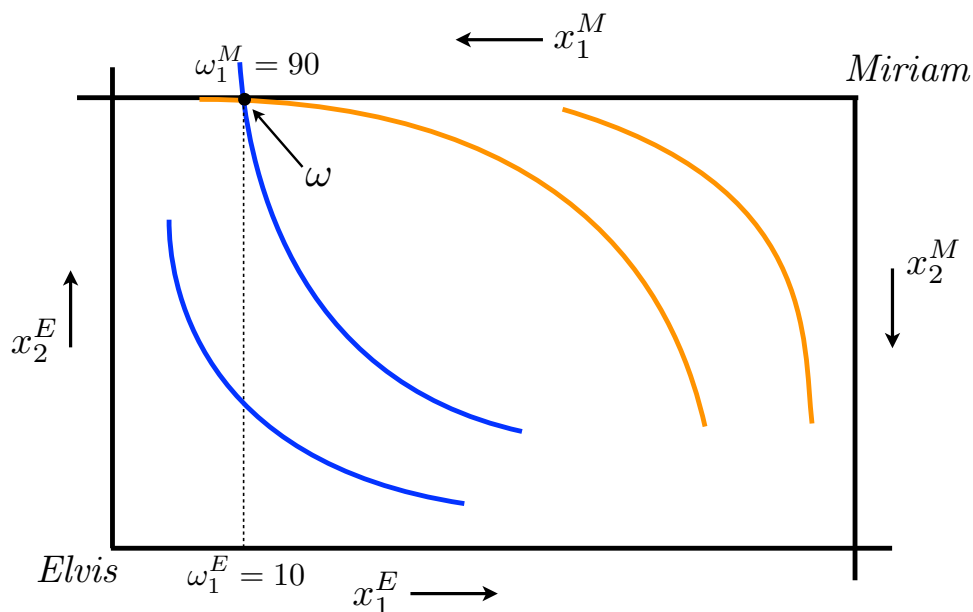
Problem 3 (Standard Edgeworth Box)

(a) The total resources in this economy are

$$\text{MP3s: } \omega_1 = \omega_1^E + \omega_1^M = 10 + 90 = 100$$

$$\text{DVDs: } \omega_2 = \omega_2^E + \omega_2^M = 10 + 0 = 10$$

(b) Allocation ω is shown in the Edgeworth box below:



To determine whether the initial allocation is efficient, we need to check whether $MRS^E(\omega^E) = MRS^M(\omega^M)$. Here $MRS^i(x_1^i, x_2^i) = -\frac{x_2^i}{5x_1^i}$, so

$$MRS^E(\omega^E) = -\frac{\omega_2^E}{5\omega_1^E} = -\frac{10}{5 \cdot 10} = -\frac{1}{5} \quad \text{and}$$

$$MRS^M(\omega^M) = -\frac{\omega_2^M}{5\omega_1^M} = -\frac{0}{5 \cdot 90} = 0.$$

Since $MRS^E(\omega^E) \neq MRS^M(\omega^M)$, the endowment is not Pareto efficient (notice how their indifference curves intersect). As we'll see next, it is off the contract curve.

(c) The contract curve is characterized by all the Pareto efficient points, i.e. points for which

$$MRS^M(x_1^M, x_2^M) = MRS^E(x_1^E, x_2^E) \quad (3)$$

and

$$x_1^M + x_1^E = \omega_1 \quad \text{and} \quad x_2^M + x_2^E = \omega_2 \quad (4)$$

(the allocation is feasible and all resources are used—another way of saying that we’re at some point in the Edgeworth box).

From equation (3),

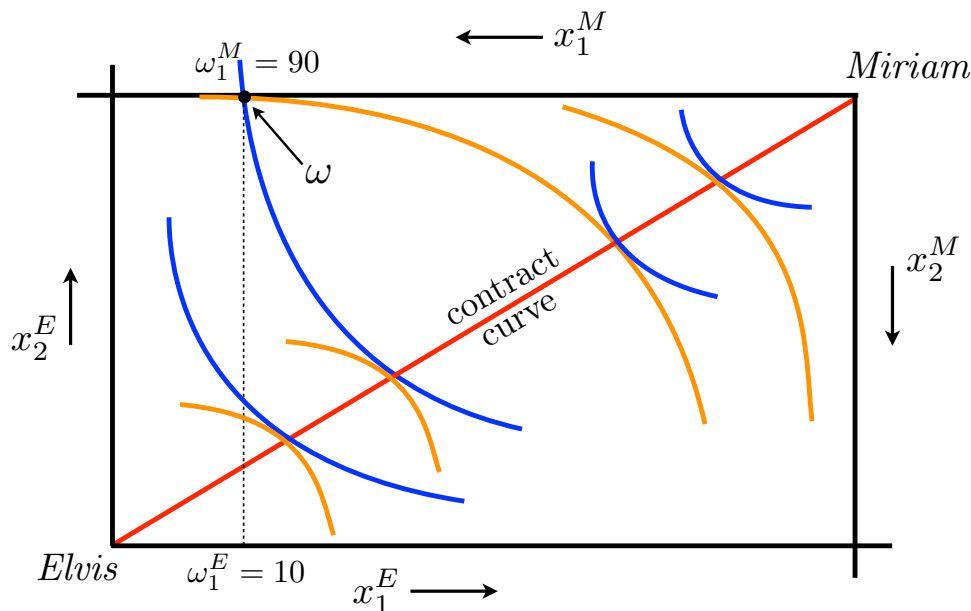
$$-\frac{x_2^M}{5x_1^M} = -\frac{x_2^E}{5x_1^E}. \quad (5)$$

Since $x_1^E = 100 - x_1^M$ and $x_2^E = 10 - x_2^M$ (from (4)), we can rewrite (5) as follows and solve for x_2^M :

$$-\frac{x_2^M}{5x_1^M} = -\frac{10 - x_2^M}{5(100 - x_1^M)} \implies x_2^M = \frac{1}{10}x_1^M$$

after some algebra. So the contract curve is characterized by $x_2^M = \frac{1}{10}x_1^M$, which represents all the Pareto efficient points in the Edgeworth box. Along this line, the indifference curves of Elvis and Miriam are tangent to each other.

(Note: You also could have solved for x_2^E instead and get an equation representing the same line in the Edgeworth box.)



(d) When we find the equilibrium consumption with Elvis and Miriam trading, graphically we’re determining the point in the Edgeworth box for which $MRS^E = MRS^M$ (indifference curves are tangent to each other, so we’ll be on the contract curve) as well as relative prices p_1 and p_2 such that the budget line passes through the endowment point and is tangent to MRS^i at the equilibrium point.

The equilibrium in this market is the allocation (x_1^E, x_2^E) and (x_1^M, x_2^M) with prices (p_1, p_2) that satisfy

- *Condition 1:* For both consumers $i = E, M$, (x_1^i, x_2^i) is optimal given prices (p_1, p_2)
- *Condition 2:* At prices (p_1, p_2) market clear. This means

$$\begin{aligned}x_1^E + x_1^M &= \omega_1 \quad \text{and} \\x_2^E + x_2^M &= \omega_2.\end{aligned}$$

We'll take the following steps to find the equilibrium allocation:

Step 1: Normalize p_2 . Equilibrium determines relative prices, so we can always normalize at least one of the prices to be some constant. The easiest is to just let $p_2 = 1$. (Now we already have one of the six components we need for the equilibrium!) Thinking graphically, the slope of the budget line is $-\frac{p_1}{p_2}$, so any multiple a of p_1 and p_2 work since $-\frac{p_1}{p_2} = -\frac{a \cdot p_1}{a \cdot p_2}$.

Step 2: Find incomes m^i . In the next step, we'll find Elvis's and Miriam's demand functions, but first we need to know what their incomes are given their endowments (in terms of p_1 since we haven't found that yet.) We have that (recalling that $p_2 = 1$):

$$\begin{aligned}m^E &= p_1 \omega_1^E + p_2 \omega_2^E = 10p_1 + 10 \\m^M &= p_1 \omega_1^M + p_2 \omega_2^M = 90p_1.\end{aligned}$$

Step 3: Find demand functions x_1^i . For this we use the "magic formula" for demand for Cobb-Douglas type utility, $x_1^i = \frac{a}{a+b} \frac{m^i}{p_1}$. (Of course, if we were dealing with any other type of utility we could derive demand just as we did earlier in the course.) This gives us:

$$\begin{aligned}x_1^E &= \frac{1}{6} \left(\frac{10p_1 + 10}{p_1} \right) \\x_1^M &= \frac{1}{6} \left(\frac{90p_1}{p_1} \right)\end{aligned}$$

We only really need to get the demand functions for good 1 since we can figure out what x_2^E and x_2^M are from the market clearing conditions, which we'll use next.

Step 4: Solve for p_1 using demand functions and market clearing conditions. Plugging the demand functions we just found for good 1 into the market clearing condition for good 1 we get:

$$x_1^E + x_1^M = \omega_1 \implies \frac{1}{6} \left(\frac{10p_1 + 10}{p_1} \right) + \frac{1}{6} \left(\frac{90p_1}{p_1} \right) = 100$$

and solving this for p_1 we get

$$p_1 = \frac{1}{50}.$$

Step 5: Use p_1 and to get optimal consumption. Now, we can plug $p_1 = \frac{1}{50}$ into the demand function we found in Step 3 to get optimal consumption levels

$$x_1^E = \frac{1}{6} \left(\frac{10p_1 + 10}{p_1} \right) = 85$$

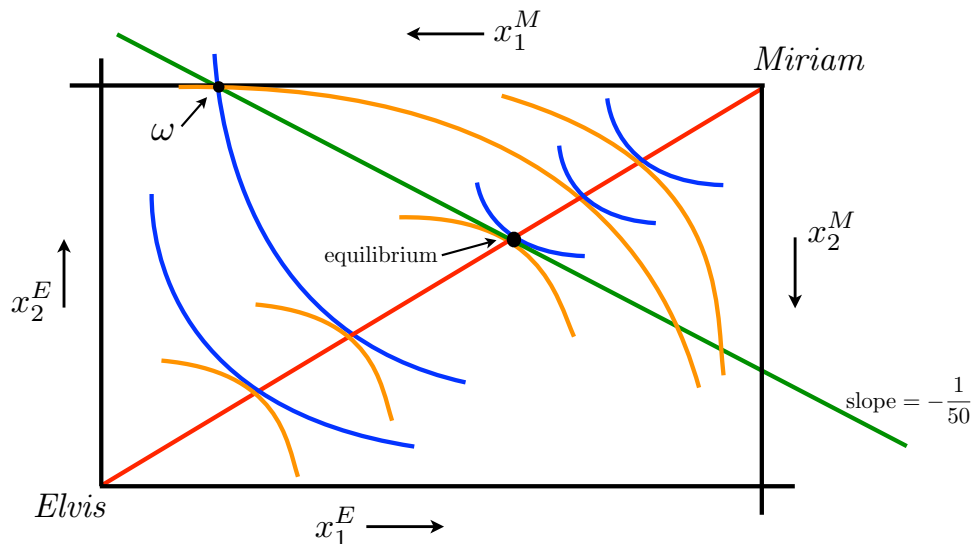
$$x_1^M = \frac{1}{6} \left(\frac{90p_1}{p_1} \right) = 15$$

We're almost there. We just need to plug in $p_1 = \frac{1}{50}$ and $p_2 = 1$ into the demand functions for Good 2 now (again, these demand functions come from the “magic formula” for demand with Cobb-Douglas utility):

$$x_2^E = \frac{5}{6} \left(\frac{10p_1 + 10}{p_2} \right) = 8.5$$

$$x_2^M = \frac{5}{6} \left(\frac{90p_1}{p_2} \right) = 1.5$$

So (finally!) we have found equilibrium allocation $x^E = (85, 8.5)$, $x^M = (15, 1.5)$ and prices $(p_1, p_2) = (\frac{1}{50}, 1)$.



(e) Any pair of prices that give us the same ratio $\frac{p_1}{p_2} = \frac{1/50}{1} = \frac{1}{50}$ would work. So, for instance, prices $(p_1, p_2) = (1, 50)$ support this equilibrium. Prices $(p_1, p_2) = (2, 100)$, so do $(p_1, p_2) = (\frac{1}{2}, 25)$, etc.

(f) Yes, the market efficiently allocates resources. To see this, observe that the MRS for Elvis and Miriam are

$$MRS^E = -\frac{x_2^E}{5x_1^E} = -\frac{8.5}{5 \cdot 85} = -\frac{1}{50} \quad \text{and}$$

$$MRS^M = -\frac{x_2^M}{5x_1^M} = -\frac{1.5}{5 \cdot 15} = -\frac{1}{50}.$$

Their indifference curves are tangent, the allocation is Pareto optimal and we are on the contract curve. Notice also that, even though the total resources are fixed, through trade both of them are better off than they were before! (Utility is higher, they are both on higher indifference curves.)

(g) In this case, all the allocations in the Edgeworth box are Pareto efficient (moving away from any point will not be a Pareto improvement; one cannot be made better off without the other being worse off). Now, let's think about what prices would work in this market. In equilibrium, with these goods being perfect substitutes for both Elvis and Miriam, for any relative price

$$\frac{p_1}{p_2} < MRS^i = \frac{1}{5}$$

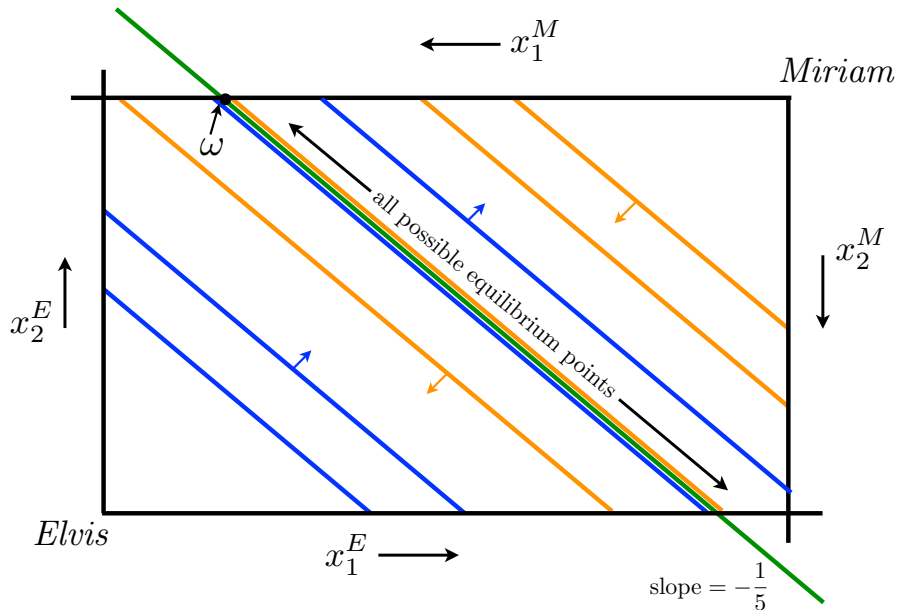
both Elvis and Miriam would want to spend all of their income on x_1 (and none on x_2). This would result in excess demand for good 1 and markets don't clear. For any relative price

$$\frac{p_1}{p_2} > MRS^i = \frac{1}{5}$$

they would want to buy only x_2 (and no x_1), which would result in excess demand for good 2. Therefore, it must be that

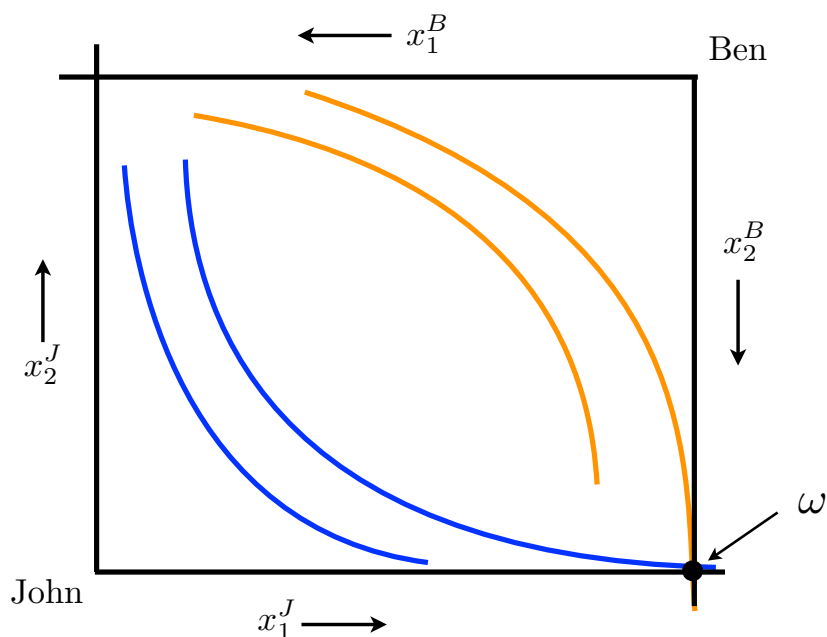
$$\frac{p_1}{p_2} = MRS^i = \frac{1}{5}$$

for markets to clear. Now, all of the allocation along the budget line going through the endowment with prices such that $\frac{p_1}{p_2} = \frac{1}{5}$ is an equilibrium allocation.



Problem 4 (Uncertainty and Asset Pricing)

(a) The Edgeworth box and the point corresponding to their initial endowments (shares held) is shown below. The initial endowment cannot be Pareto efficient since $MRS^J(\omega^J) \neq MRS^B(\omega^B)$: From this utility function we have that $MRS^i(x_1^i, x_2^i) = -\frac{x_2^i}{x_1^i}$ and so at the endowment point $MRS^J(100, 0) = -\frac{0}{100} = 0$ and $MRS^B(0, 100) = -\frac{100}{0} = -\infty$.



The endowment (where they are not trading shares) is in fact risky because both of them have different levels of consumption in different states of the world (rainy or no rain).

(b) To find this equilibrium, we go through the same steps as above (so for a more thorough explanation see Problem 3). We normalize $p_2 = 1$ first. Then we get both John and Benjamin's demand for x_1 , shares of Rainalot Inc.:

$$x_1^J = \frac{a}{a+b} \frac{m^J}{p_1} = \frac{1}{2} \frac{p_1 \times 100 + p_2 \times 0}{p_1} = 50$$

$$x_1^B = \frac{a}{a+b} \frac{m^B}{p_1} = \frac{1}{2} \frac{p_1 \times 0 + p_2 \times 100}{p_1} = \frac{50}{p_1}$$

Next we use the market clearing condition $x_1^J + x_1^B = 100$ to find p_1 :

$$x_1^J + x_1^B = 100 \implies 50 + \frac{50}{p_1} = 100 \implies p_1 = 1$$

Now given prices $p_1 = p_2 = 1$, we have demand

$$x_1^J = 50$$

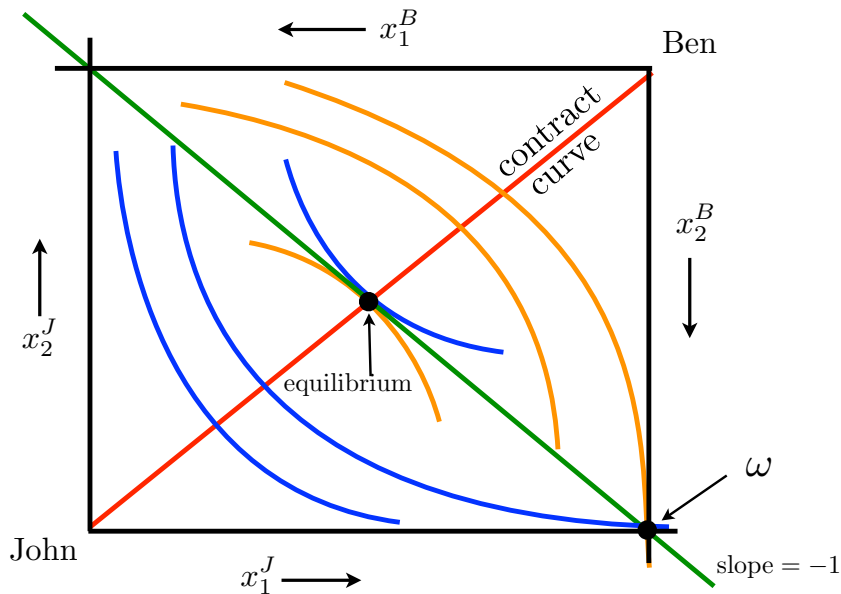
$$x_1^B = \frac{50}{p_1} = 50$$

So all we have left to do now is to find x_2^J and x_2^B . We can plug $p_1 = p_2 = 1$ into their demand for x_2 , shares of HateRain Inc.:

$$x_2^J = \frac{b}{a+b} \frac{m^J}{p_2} = \frac{1}{2} \frac{p_1 \times 100 + p_2 \times 0}{p_2} = 50$$

$$x_2^B = \frac{b}{a+b} \frac{m^B}{p_2} = \frac{1}{2} \frac{p_1 \times 0 + p_2 \times 100}{p_2} = 50$$

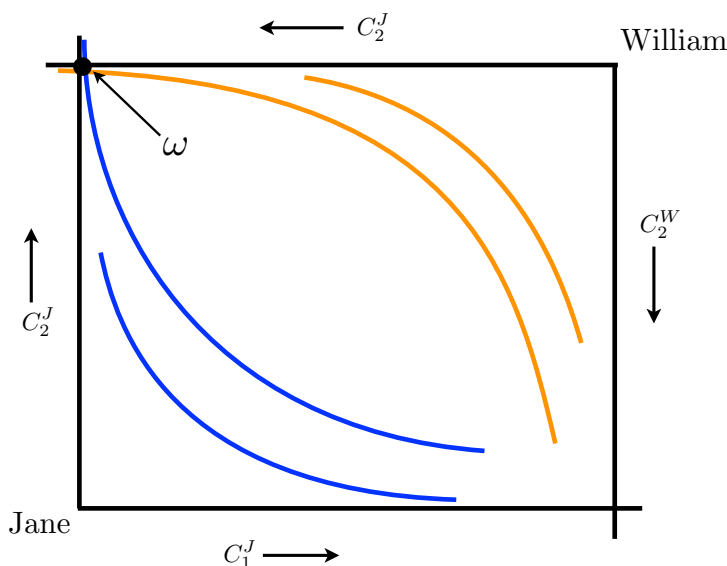
So our equilibrium allocation is described by prices $(p_1, p_2) = (1, 1)$, shares of x_1 held $(x_1^J, x_1^B) = (50, 50)$, and shares of x_2 held $(x_2^J, x_2^B) = (50, 50)$.



(c) The allocation is efficient since $MRS^J(50, 50) = -1 = MRS^B(50, 50)$. Also, it is not risky since each of them consumes that same amount regardless of the state of the world (rainy or no rain).

Problem 5 (Irving Fisher Determination of Interest Rate)

(a) The allocation corresponding to the initial endowment is shown below. It is not Pareto efficient since $MRS^J(\omega^J) \neq MRS^W(\omega^W)$: From this utility function we have that $MRS^i(x_1^i, x_2^i) = -\frac{x_2^i}{\beta x_1^i}$ and so at the endowment point $MRS^J(0, 1,000) = -\frac{1,000}{\beta \cdot 0} = -\infty$ and $MRS^W(1,000, 0) = -\frac{0}{\beta \cdot 1,000} = 0$.



(b) To find the equilibrium interest rate r , we first need to find p_1 and p_2 , since we'll use that $\frac{p_1}{p_2} = 1 + r$. Our first step is to normalize $p_2 = 1$. Again, the steps toward finding the market equilibrium are the same as in Problems 3 and 4 (see Problem 4 for more detail.)

We can use the “magic formulas” for demand for consumption today for Jane and William:

$$C_1^J = \frac{a}{a+b} \frac{m^J}{p_1} = \frac{1}{1+\beta} \cdot \frac{p_1 \times 0 + p_2 \times 1,000}{p_1} = \frac{2}{3} \cdot \frac{1,000 p_2}{p_1} = \frac{2}{3} \cdot \frac{1,000}{p_1}$$

$$C_1^W = \frac{a}{a+b} \frac{m^W}{p_1} = \frac{1}{1+\beta} \cdot \frac{p_1 \times 1,000 + p_2 \times 0}{p_1} = \frac{2}{3} \cdot 1,000$$

Next we use market clearing condition $C_1^J + C_1^W = 1,000$ to get p_1 :

$$C_1^J + C_1^W = 1,000 \implies \frac{2}{3} \cdot \frac{1,000}{p_1} + \frac{2}{3} \cdot 1,000 = 1,000 \implies p_1 = 2$$

Now plugging this into demand for C_1 and C_2 for both Jane and William, we get:

$$C_1^J = \frac{2}{3} \cdot \frac{1,000}{p_1} = 333\frac{1}{3}$$

$$C_1^W = \frac{2}{3} \cdot 1,000 = 666\frac{2}{3}$$

$$C_2^J = \frac{b}{a+b} \frac{m^J}{p_2} = 333\frac{1}{3}$$

$$C_2^W = \frac{b}{a+b} \frac{m^W}{p_2} = 666\frac{2}{3}$$

The interest rate r is such that $\frac{p_1}{p_1} = 1 + r \implies r = 1 = 100\%$

(c) Yes, the equilibrium is Pareto efficient since $MRS^J(333\frac{1}{3}, 333\frac{1}{3}) = -2 = MRS^W(666\frac{2}{3}, 666\frac{2}{3})$.

(d) When $\beta = 1$, going through the same steps as in part (b) we get that $p_1 = 1$ and hence $r = 1$. First, getting demand for consumption in period 1:

$$C_1^J = \frac{a}{a+b} \frac{m^J}{p_1} = \frac{1}{1+\beta} \cdot \frac{p_1 \times 0 + p_2 \times 1,000}{p_1} = \frac{1}{2} \cdot \frac{1,000p_2}{p_1} = \frac{1}{2} \cdot \frac{1,000}{p_1}$$

since $p_2 = 1$ and

$$C_1^W = \frac{a}{a+b} \frac{m^W}{p_1} = \frac{1}{1+\beta} \cdot \frac{p_1 \times 1,000 + p_2 \times 0}{p_1} = \frac{1}{2} \cdot 1,000$$

Next we use market clearing condition $C_1^J + C_1^W = 1,000$ to get p_1 :

$$C_1^J + C_1^W = 1,000 \implies \frac{1}{2} \cdot \frac{1,000}{p_1} + \frac{1}{2} \cdot 1,000 = 1,000 \implies p_1 = 1$$

which gives us $r = 0 = 0\%$. Remember that β , here the discount factor, measures patience or how today's consumption is weighted relative to tomorrow's consumption. With $\beta = 1$, they are now indifferent between the two and therefore do not find saving as costly. In order to equilibrate the market interest rate must go down. Interest rate r is a reflection of β .

(e) Now with $\beta = \frac{1}{2}$ again with Jane's income tomorrow being 2,000 we'll get

$$C_1^J = \frac{a}{a+b} \frac{m^J}{p_1} = \frac{1}{1+\beta} \cdot \frac{p_1 \times 0 + p_2 \times 2,000}{p_1} = \frac{2}{3} \cdot \frac{2,000p_2}{p_1} = \frac{2}{3} \cdot \frac{2,000}{p_1}$$

$$C_1^W = \frac{a}{a+b} \frac{m^W}{p_1} = \frac{1}{1+\beta} \cdot \frac{p_1 \times 1,000 + p_2 \times 0}{p_1} = \frac{2}{3} \cdot 1,000$$

Using market clearing condition $C_1^J + C_1^W = 1,000$ to get p_1 :

$$C_1^J + C_1^W = 1,000 \implies \frac{2}{3} \cdot \frac{2,000}{p_1} + \frac{2}{3} \cdot 1,000 = 1,000 \implies p_1 = 4$$

Now since $p_1 = 4$ and we normalize $p_2 = 1$, we'll get $\frac{p_1}{p_2} = 1 + r \implies r = 3 = 300\%$. The intuition is that Jane has a larger endowment tomorrow and she is now able and wants to borrow more. In order to equilibrate the savings market interest rate must go up. This partially reduces her willingness to borrow (relative to a lower interest rate) and also encourages William to lend more.