Problem Set 2: Solutions

ECON 301: Intermediate Microeconomics
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Problem 1 (Marginal Rate of Substitution)

(a) For the third column, recall that by definition $MRS(x_1, x_2) = -\frac{\left(\frac{\partial U}{\partial x_1}\right)}{\left(\frac{\partial U}{\partial x_2}\right)}$.

	Utility Function	$\frac{\partial U}{\partial x_1}$	$\frac{\partial U}{\partial x_2}$	$MRS(x_1, x_2)$	$MRS(2,3)$
(i)	$U(x_1, x_2) = x_1 x_2$	x_2	x_1	$-\frac{x_2}{x_1}$	$-\frac{3}{2}$
(ii)	$U(x_1, x_2) = x_1^3 x_2^5$	$3x_1^2 x_2^5$	$5x_1^3 x_2^4$	$-\frac{3x_2}{5x_1}$	$-\frac{9}{10}$
(iv)	$U(x_1, x_2) = 3\ln x_1 + 5\ln x_2$	$\frac{3}{x_1}$	$\frac{5}{x_2}$	$-\frac{3x_2}{5x_1}$	$-\frac{9}{10}$

(b) $MRS(2,3) = -9/10$ for utility function $U(x_1, x_2) = x_1^3 x_2^5$ has the following interpretation: At bundle $(2,3)$, to remain indifferent about the change (i.e., remain at the same utility level), a consumer is willing to give up $9/10$ of x_2 for one additional unit of x_1. (Or, after losing one unit of x_1, he must receive $9/10$ of a unit of x_2 to be as well off as he was at bundle $(2,3)$.) So *at the point* $(2,3)$, good two is more valuable since he needs to get less of it than he lost of the other good to remain as satisfied. If 0.00001 of good one is taken away, he would have to receive approximately $0.00001 \times (\frac{9}{10}) = 0.000009$ units of good two to remain indifferent to the change.

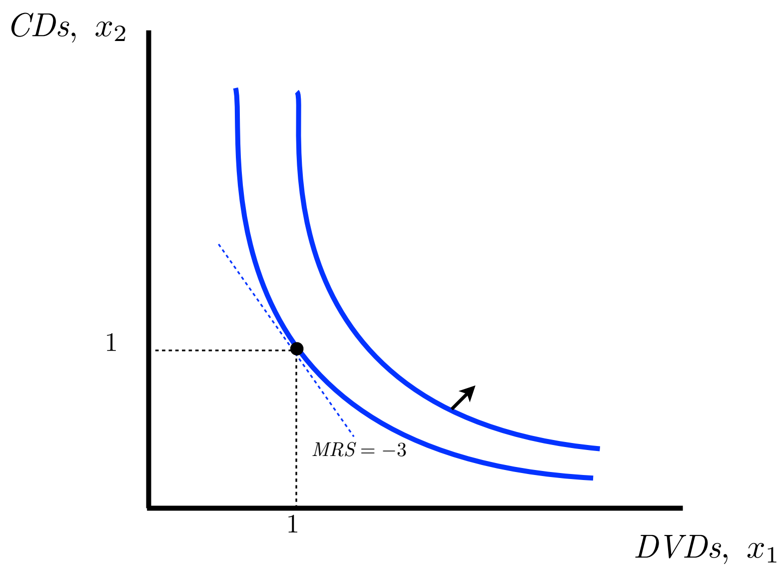
(c) The two utility functions share the same MRS functions because $U(x_1, x_2) = 3\ln x_1 + 5\ln x_2$ is a monotonic transformation of $U(x_1, x_2) = x_1^3 x_2^5$. To see this, let $f(u) = \ln(u)$ ($f(u)$ is a monotonic function). Then letting $u = x_1^3 x_2^5$, we have that $f(u) = \ln(x_1^3 x_2^5) = 3\ln x_1 + 5\ln x_2$. If one function is a monotonic transformation of another, the two describe the same preferences since they will they rank bundles in the same way. (They assign different values to the bundle, but we do not use these *cardinal* numbers in determining the utility-maximizing choices—we only care about *ordinal* comparisons.)

Problem 2 (Well-Behaved Preferences)

(a) Instead of using utility function $U(x_1, x_2) = x_1^3 x_2^1$, we can use a monotonic transformation instead: $U(x_1, x_2) = 3\ln x_1 + \ln x_2$. (To get this, let $f(u) = \ln(u)$. Then $f(u) = \ln(x_1^3 x_2) = 3\ln x_1 + \ln x_2$. Again, even though these are not the same utility functions, they'll give the same MRS and thus the same results.)

Using $U(x_1, x_2) = 3\ln x_1 + \ln x_2$, we get than $MU_1 = \frac{\partial U}{\partial x_1} = \frac{3}{x_1}$ and $MU_2 = \frac{\partial U}{\partial x_2} = \frac{1}{x_2}$. Since $MRS(x_1, x_2) = -\frac{MU_1}{MU_2} = -\frac{\left(\frac{\partial U}{\partial x_1}\right)}{\left(\frac{\partial U}{\partial x_2}\right)}$, we have here that $MRS(x_1, x_2) = -\frac{3x_2}{x_1}$. This is the MRS for any bundle (x_1, x_2), which is also the slope of the indifference curve passing through that point.

(b) Using our answer in **(a)**, we get that $MRS(1,1) = -\frac{3 \cdot 1}{1} = -3$. This tells us that the slope of the indifference curve passing through the point $(1,1)$ is -3:

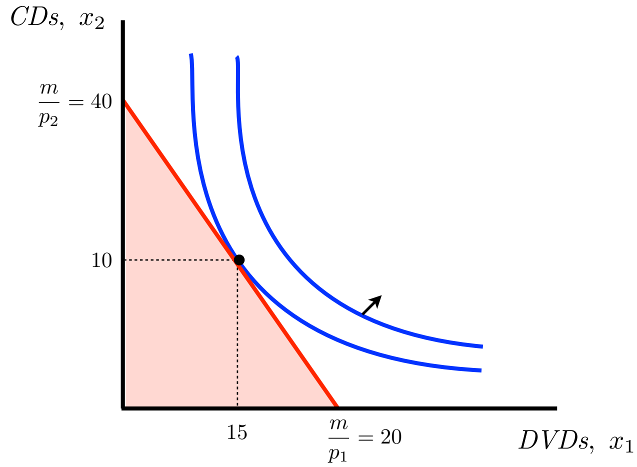


At $(1,1)$, good one is locally more valued since, to compensate for a loss of 3 units of good two (the CDs), Alicia only needs 1 unit of good one (the DVDs) to maintain the initial level of happiness.

(c) The two secrets of happiness for well-behaved preferences are:

- **(1)** $p_1x_1 + p_2x_2 = m$ (Since more is preferred to less, spend all of your income.)
- **(2)** $MRS = -\frac{p_1}{p_2}$ (Marginal utility per dollar spent is equalized.)
  - Note: An equivalent way of writing this is $-\frac{MU_1}{MU_2} = -\frac{p_1}{p_2}$ (using the definition of MRS) or $\frac{MU_1}{p_1} = \frac{MU_2}{p_2}$. All three ways are exactly the same.

Graphically, we're finding the bundle for which the budget line is tangent to an indifference curve:



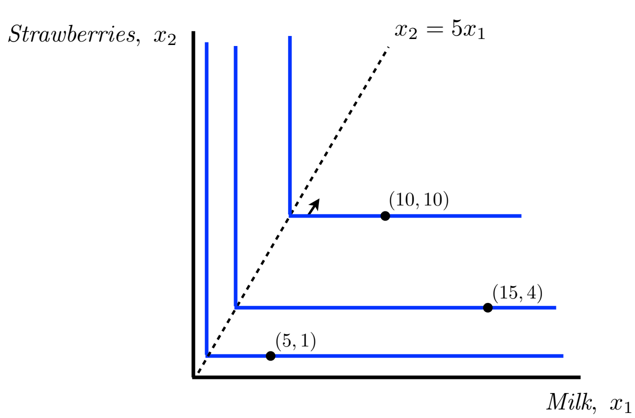
Given that $p_1 = 40$, $p_2 = 20$, and $m = 800$, we can rewrite these two equations as

- **(1)** $40x_1 + 20x_2 = 800$
- **(2)** $-\frac{3x_2}{x_1} = -\frac{40}{20} \implies x_2 = \frac{2}{3}x_1$

(d) To find Alicia's optimal bundle, we just use the two equations above to solve for our two unknowns, which are x_1 and x_2. (So there's no economics here, only Algebra.) You can just take $x_2 = \frac{2}{3}x_1$ from equation **(2)** and plug it into **(1)** to get $40x_1 + 20(\frac{2}{3}x_1) = 800 \implies x_1 = 15$. Plug $x_1 = 15$ into either equation to find that $x_2 = 10$. Alicia's optimal bundle, given these prices and her income, is $(15, 10)$, which is interior (she's consuming non-zero amounts of both). This is shown in the figure above.

Problem 3 (Perfect Complements)

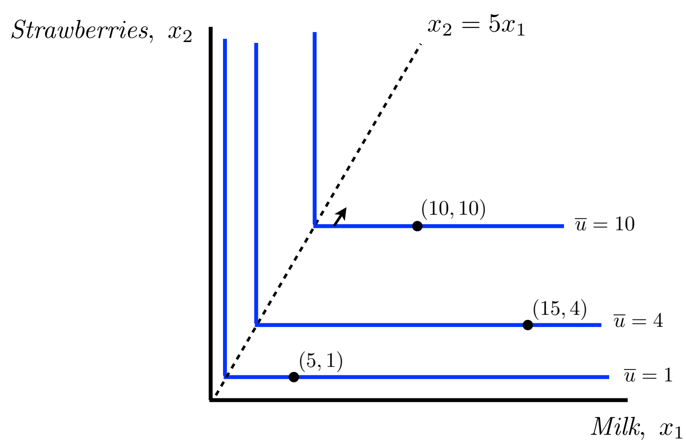
(a) The indifference curves passing through $(5, 1)$, $(10, 10)$, and $(15, 4)$ are shown below. The vertices all fall along the dotted line along which $x_2 = 5x_1$. (It shows the combinations for which Trevor consumes five times as many strawberries (x_2) as he does units of milk (x_1)). The MRS at each of these points (without using any formulas and only looking at the graph) is zero: $MRS(5, 1) = MRS(10, 10) = MRS(15, 4) = 0$.



(b) His preferences can be represented by the utility function $U(x_1, x_2) = \min\{5x_1, x_2\}$. In general, if preferences are perfect complements where a of x_1 must be consumed for every b of x_2, the utility function can be expressed as $U(x_1, x_2) = \min\{\frac{1}{a}x_1, \frac{1}{b}x_2\}$, and the line along which all of the vertices of those L-shaped indifference curves lie is $\frac{1}{a}x_1 = \frac{1}{b}x_2$. So using this formula directly $U(x_1, x_2) = \min\{x_1, \frac{1}{5}x_2\}$ but, multiplying everything through by 5 (which would be a monotonic transformation!) we get $U(x_1, x_2) = \min\{5x_1, x_2\}$.

To find the level of utility associated with the indifference curves passing through $(5, 1)$, $(10, 10)$, and $(15, 4)$, we use this utility function to find that:

- $U(5, 1) = \min\{5 \cdot 5,\ 1\} = \min\{25, 1\} = 1$
- $U(10, 10) = \min\{5 \cdot 10,\ 10\} = \min\{50, 10\} = 10$
- $U(15, 4) = \min\{5 \cdot 15,\ 4\} = \min\{75, 4\} = 4$



Notice that if you would have used utility function $U(x_1, x_2) = \min\{x_1, \frac{1}{5}x_2\}$, you would get:

- $U(5, 1) = \min\{5, \frac{1}{5} \cdot 1\} = \min\{5, \frac{1}{5}\} = \frac{1}{5}$
- $U(10, 10) = \min\{10, \frac{1}{5} \cdot 10\} = \min\{10, 2\} = 2$
- $U(15, 4) = \min\{15, \frac{1}{5} \cdot 4\} = \min\{15, \frac{4}{5}\} = \frac{4}{5}$

Either way, we see that $(10, 10)$ is the most preferred (i.e., gives the highest utility among the three), followed by $(15, 4)$ and then $(5, 1)$.

(c) Multiplying our utility function by ten and adding two is equivalent to taking a monotonic transformation $f(u) = 10u + 2$. If we take our utility $U(x_1, x_2) = \min\{5x_1, x_2\}$, we get $U^{\text{trans}}(x_1, x_2) = 10 \cdot \min\{5x_1, x_2\} + 2$. Then

- $U(5, 1) = 10 \cdot \min\{25, 1\} + 2 = 10 \cdot 1 + 2 = 12$
- $U(10, 10) = 10 \cdot \min\{50, 10\} + 2 = 10 \cdot 10 + 2 = 102$
- $U(15, 4) = 10 \cdot \min\{75, 4\} + 2 = 10 \cdot 4 + 2 = 42$

Again, the indifference curves do not move and the preference ranking among the bundles is preserved, we just have the above levels of utility attached to each of the indifference curves.

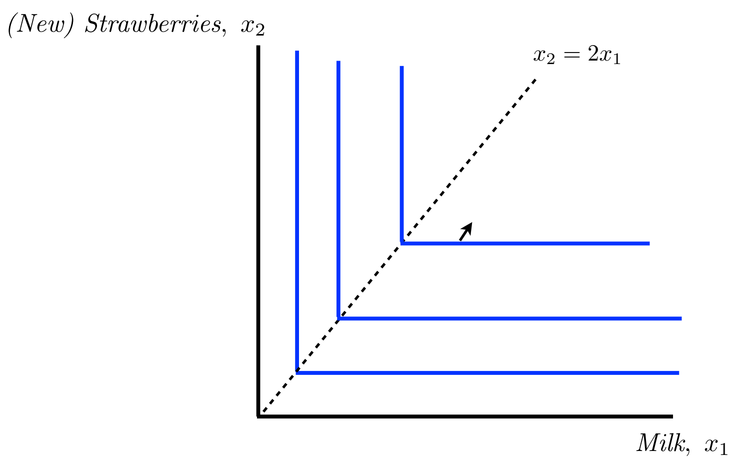
(d) Letting $p_1 = 1$, $p_2 = 1$, and $m = 100$, the two secrets of happiness for perfect complements are

- **(1)** $p_1x_1 + p_2x_2 = m \implies x_1 + x_2 = 100$ (Trevor spends all of his income.)
- **(2)** $x_2 = 5x_1$ (He consumes only to optimal proportions along the dotted line along which $x_2 = 5x_1$.)
  - Note: These types of preferences are not "well behaved" like Cobb Douglas preferences are, so we use a different second secret of happiness for these preferences. We can no longer use $MRS = -\frac{p_1}{p_2}$ since the MRS of the indifference curve is not defined at the kink.

To find the demand for both milk (x_1) and strawberries (x_2) we solve the equations in **(1)** and **(2)**: Plug $x_2 = 5x_1$ into equation **(1)** for x_2, so $x_1 + (5x_1) = 100 \implies x_1 = 100/6$. Plug this into either equation to solve for x_2 and get $x_2 = 500/6$.

This is interior since Trevor is consuming non-zero amounts of both goods (i.e., $x_1 > 0$ and $x_2 > 0$).

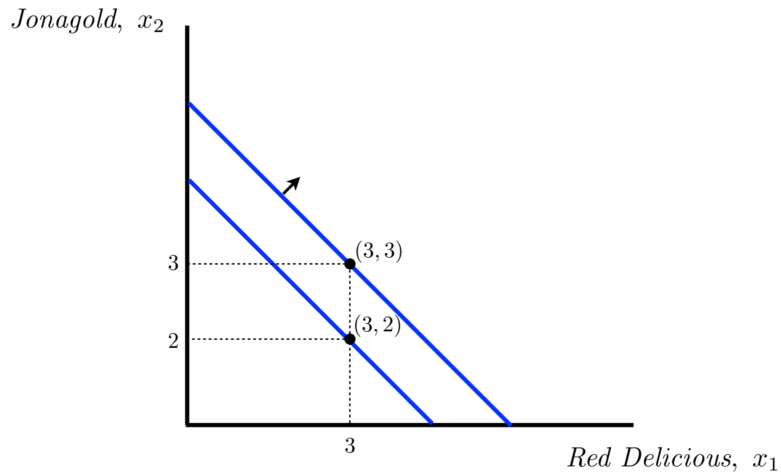
(e) With larger strawberries, the new optimal proportion of milk (x_1) and strawberries (x_2) is two strawberries for every unit of milk, or $x_2 = 2x_1$. Our indifference curves are the same shape as they were before, but now the vertices of these L-shaped indifference curves (which will be the optimal bundles) lie along $x_2 = 2x_1$, as seen below.



These new preferences can be represented by utility function $U(x_1, x_2) = \min\{2x_1, x_2\}$.

Problem 4 (Perfect Substitutes)

(a) When two goods are perfect substitutes, we know the indifference curves are linear and downward-sloping, in this case having a constant slope of -1. The indifference curves passing through points $(3, 2)$ and $(3, 3)$ are shown below:



(b) Some utility functions that could represent these functions:

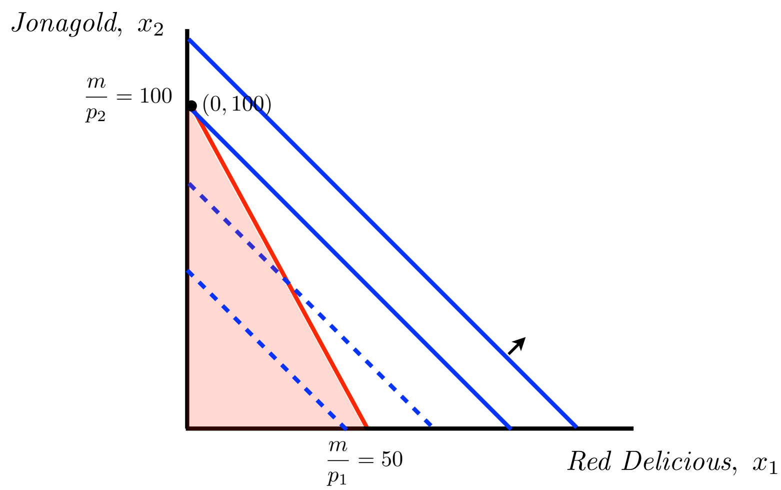
- **(i)** $U(x_1, x_2) = x_1 + x_2$
- **(ii)** $U(x_1, x_2) = \ln(x_1 + x_2)$
- **(iii)** $U(x_1, x_2) = (x_1 + x_2)^2$
- **(iv)** $U(x_1, x_2) = \frac{1}{8}x_1 + \frac{1}{8}x_2$
- **(v)** $U(x_1, x_2) = 6x_1 + 6x_2$

Each of these five utility functions represents a monotonic transformation of any of the others; they all represent the same underlying perfect-substitute preferences over Red Delicious (x_1) and Jonagold (x_2) apples and give indifference curves having the same $MRS = -1$ (you can verify this).

(c) $MRS(x_1, x_2) = -1$ for any (x_1, x_2). This means that starting from any (x_1, x_2) bundle, Kate is always willing to give up one Red Delicious (x_1) apple to get one additional Jonagold (x_2) (or vice versa). This means, as noted above, that the indifferences curves will be linear everywhere.

(d) Letting $p_1 = 2$, $p_2 = 1$, and $m = 100$ we can first turn to the commodity space to see how to go about funding the optimal bundle. We know that the first secret of happiness (spending all of one's income) will always hold, so the optimal choice is along the budget line. But the budget line is not tangent to any indifference curve here: The budget line has a slope of $-\frac{p_1}{p_2} = -\frac{2}{1}$ and we just determined that $MRS(x_1, x_2) = -1$ everywhere, so

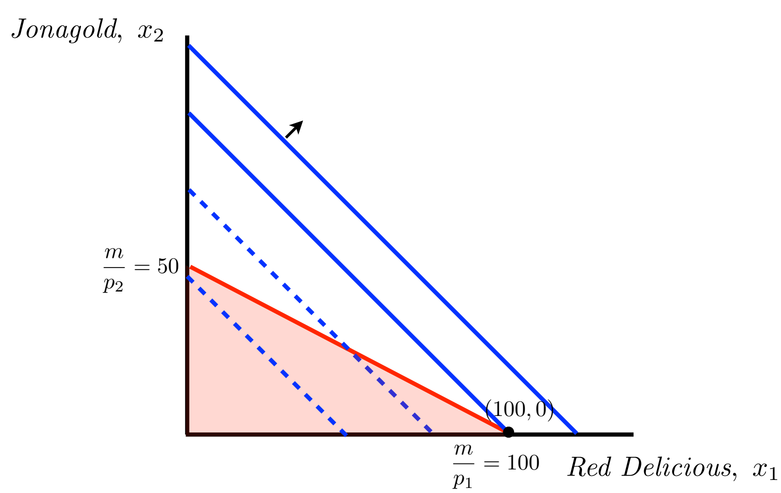
$MRS \neq -\frac{p_1}{p_2}$ anywhere.



We can see that at the indifference curve furthest out from the origin (the one with highest utility) but still touching the budget line, Kate is consuming only Jonagolds (x_2) and no Red Delicious (x_1): $x_1 = 0$, $x_2 = \frac{m}{p_2} = \frac{100}{1} = 100$. This is not an interior solution (since $x_1 = 0$) but rather is a corner solution.

As a general rule, when $|MRS| < \frac{p_1}{p_2}$ (the indifference curves are more flat than the budget line), the consumer chooses to consume only x_2. Here, $|MRS| = 1 < \frac{p_1}{p_2} = 2$. This makes sense intuitively here: If the two types of apples are 1:1 perfect substitutes for Kate, she should just consume the one that is cheaper.

(e) Now we have that $|MRS| = 1 > \frac{p_1}{p_2} = \frac{1}{2}$ and Kate consumes only the cheaper apple, the Red Delicious (x_1), with $x_1 = \frac{m}{p_1} = \frac{100}{1} = 100$, $x_2 = 0$.



(f) With this price change, $|MRS| = \frac{p_1}{p_2}$ everywhere, so all of the combinations along the budget line lie along the same indifference curve, and thus any bundle is equally as good as any other bundle on the budget line. Kate can choose any (x_1, x_2) combination such that $p_1x_1 + p_2x_2 = m$ or $x_1 + x_2 = 100$: