Prof. Marek Weretka's

Econ 301 Intermediate Microeconomics

Problem Set 2

Problem 1 (Marginal Rate of Substitution)

In this problem we review your knowledge of calculus.

a) Fill out the following table

$U(x_1, x_2)$	$\frac{\delta U}{\delta x_1}$	$\frac{\delta U}{\delta x_2}$	$MRS(x_1, x_2)$	$MRS(2, 3)$
$U() = x_1 x_2$				
$U() = (x_1)^3 (x_2)^5$				
$U() = 3ln x_1 + 5ln x_2$				

The first column specifies the utility function, the next two columns represent partial derivatives with respect to x_1(column 2) and x_2 (column 3). In column 4, we have MRS for bundle (x_1, x_2) and in the last column, MRS is calculated for a particular bundle $(2, 3)$. In other words, in the third column, we have MRS as a function of bundles, and in the fourth column, the function is evaluated for a given bundle values. (Hint: the derivative of $ln(x)$ is equal to $1/x$)

b) Consider utility function $U() = (x_1)^3 (x_2)^5$. Interpret the value of the MRS at $(2, 3)$ (see the last column of the table in a). Is good 1 (locally) more or less "valuable" than good 2? How much (approximately) of good 2 would you have to give to a consumer, after taking away 0.00001 of good 1, to keep him indifferent?

c) (Hard, extra points) Knowing that $ln(x)$ is a strictly increasing function and that $ln(x^\alpha y^\beta) = \alpha ln x + \beta ln y$, explain why the MRS of functions in the last two rows in the table coincide.

Problem 2 (Well behaved preferences)

Alicia likes watching DVDs (x_1) and listen to her favorite CDs (x_2) . Her utility function is given by

$$U(x_1, x_2) = (x_1)^3 (x_2)^1$$

a) Find Alicia's MRS, for an arbitrary bundle (x_1, x_2). (You are encouraged to use the "trick" with taking a logarithm of the utility function first.)

b) Plot Alicia's indifference curve that passes through the bundle $(1, 1)$. Find the MRS for this bundle, and depict it on the graph. Which good is (locally more valued) at $(1, 1)$ DVD or CD?

c) Let the price of a DVD be \$40 and the price of a CD be \$20, and Alicia's income be \$800. Write down the (two) conditions that guarantee the optimality of her choice. Illustrate them on the graph.

d) Find analytically Alicia's optimal bundle. Is it interior?

Problem 3 (Perfect Complements)

Trevor always begins his day with a strawberry milkshake. He makes it by mixing milk (x_1) with five strawberries (x_2). The secret of a really good milkshake lies in the optimal proportion of milk and fruit: one glass always comes with five strawberries!

a) Plot in a diagram Trevors' preferences (indifference map). Depict three indifference curves that pass through the following bundles $(5, 1)$, $(10, 10)$ and $(15, 4)$. What is the MRS at each of these points?

b) What utility function represents these preferences (make sure you use right coefficients defining proportion). On the graph from a), indicate the level of utility corresponding to each indifference curve.

c) Multiply your utility function by ten and add to it constant equal to two. How did the indifference map change (explain why)? How was the level of utility associated with the each indifference curve affected?

d) Trevor spends \$100 per month on his favorite milkshake, and pays \$1 for a glass of milk and \$1 for one strawberry (they are organic). Find analytically Trevor's demand for milk and strawberries, and depict it on the graph. Is it interior?

e) The organic strawberries are replaced by genetically modified ones. These are, on average, 2.5 times larger. Consequently, the optimal proportion of milk to strawberry becomes $1 : 2$. Plot the indifference curves for milk and genetically modified strawberries, and write down a utility function that represents them.

Problem 4 (Perfect Substitutes)

Kate has two favorite kinds of apples: Red Delicious (x_1) and Jonagold (x_2). Kate loves them both, and actually does not distinguish between the two kinds (that is a little bit strange we must admit, given that these two are quite different).

a) In a graph show Kate's indifference curves that pass through points (3,2), (3,3).

b) Suggest two distinct utility functions (analytically) that represents such preferences. (Hint: Would a monotone transformation do the job?)

c) Find MRS analytically. How does MRS depend on the values of (x_1, x_2). Intutitively explain why.

d) She spends her total income of \$100 paying p_1 =\$2 per each Red Delicious apple and p_2 =\$1 per a Jonagold. Find her optimal demand, and show it on the graph. Is it interior?

e) Suppose one day prices change so that now p_1 =\$1 per each Red Delicious and p_2 =\$2 per a Jonagold. Find her optimal demand, and show it on the graph.

f) Describe Kate's optimal choice(s) when p_1 =\$1 and p_2 =\$1.