Problem Set 11: Solutions

ECON 301: Intermediate Microeconomics
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Problem 1 (Monopoly and the Labor Market)

(a) We find the optimal demand for labor for a monopoly firm (in the goods market as poposed to the labor market) through the profit maximization condition. Profit in terms of the labor choice is

$$\pi = TR - TC = TR(y(L)) - w_L L\,.$$

First-order condition $\frac{\partial \pi}{\partial L} = 0$ is equivalent to

$$\frac{\partial TR(y(L))}{\partial L} - w_L = 0$$

$$\implies \quad MR(y) \cdot \frac{\partial y(L)}{\partial L} - w_L = 0 \;\text{ chain rule on term } TR(y(L))\,.$$

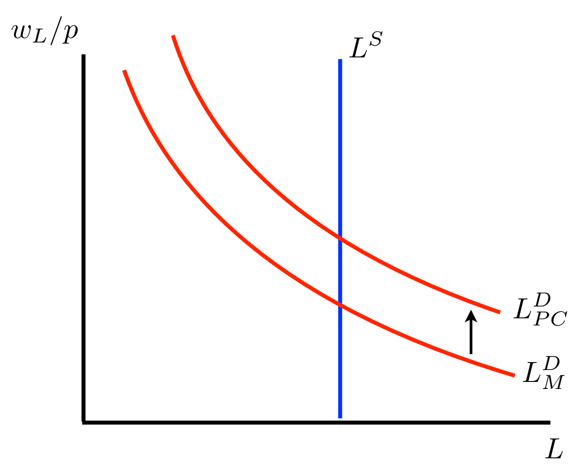
We know from Problem Set 10 that $MR = p[1 + 1/\varepsilon]$ and that $\frac{\partial y(L)}{\partial L} = MP_L$ so we can write this as

$$p\left[1 + \frac{1}{\varepsilon}\right] MP_L - w_L = 0 \quad \implies \quad \left[1 + \frac{1}{\varepsilon}\right] MP_L = \frac{w_L}{p}\,.$$

(b) Recall that with the perfectly competitive firm, demand for labor was such that $MP_L = \frac{w_L}{p}$. Now since with the monopoly firm $MR < p$, increasing labor by one unit gives less than MP_L in revenue, which is what gives rise to the term $[1 + 1/\varepsilon]$ which is less than 1.

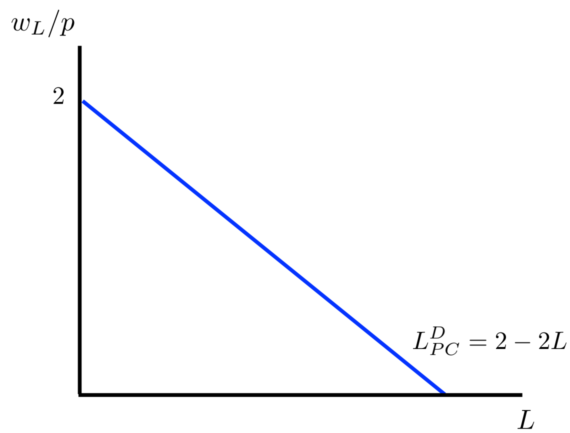
(c) Since labor demand is $L_M^D = MP_L = \frac{w_L}{p\left[1+\frac{1}{\varepsilon}\right]}$, as $|\varepsilon| \downarrow$, $L^D \downarrow$.

(d) If the firm is in a perfectly competitive market, demand is perfectly elastic and $\left[1 + \frac{1}{\varepsilon}\right] \to 1$ as $\varepsilon \to \infty$. So as a goods market becomes more competitive, $L_M^D \to L_{PC}^D$, and real wage increases as seen below:

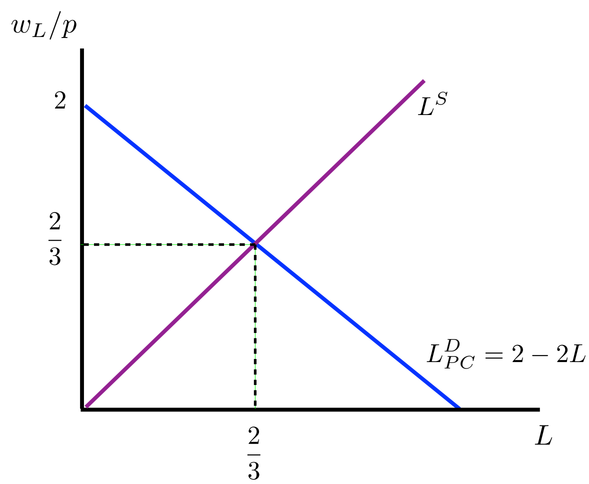


Problem 2 (Monosony and the Labor Market)

(a) We can the condition that, for a perfectly competitive firm, labor demand is such that $MP_L = w_L/p$ or, with the production function given and $p = 1$, $2 - 2L = w_L$, which is on the graph below:



(b) When $L^S = L$, equilibrium where $L^D = L^S \implies 2 - 2L = L \implies L^* = 2/3$ and $w_L^* = 2/3$.



(c) With $p = 1$, the monopsonist firm's profit can be written as

$$\pi = p \cdot y(L) - C_L(L) = 1 \cdot (2L - L^2) - w(L) \cdot L$$

Then since $w(L) = L$ we have

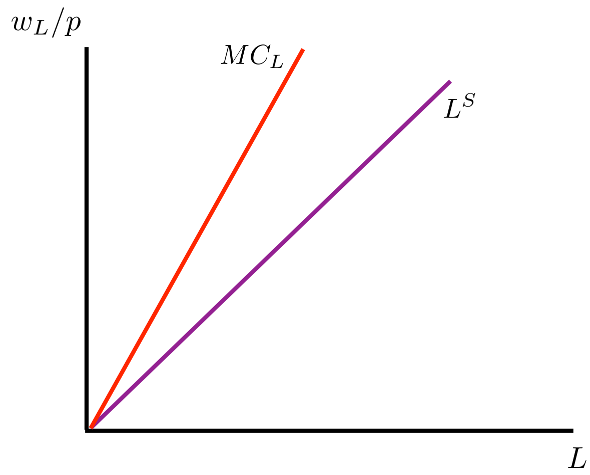
$$\pi = 1 \cdot (2L - L^2) - L \cdot L$$

(It is assumed that there is no capital K input here.) Our first-order condition for profit maximization is

$$2 - 2L - 2L = 0 \implies L = 1/2\,.$$

(d) This is similar to the reason why the MR curve in a monopoly lies below the demand curve: Here, the reason MC_L lies *above* the supply curve since to buy an additional unit of labor, the monopsonist must pay a higher wage on that additional unit of labor *as well as* all the preceding units.

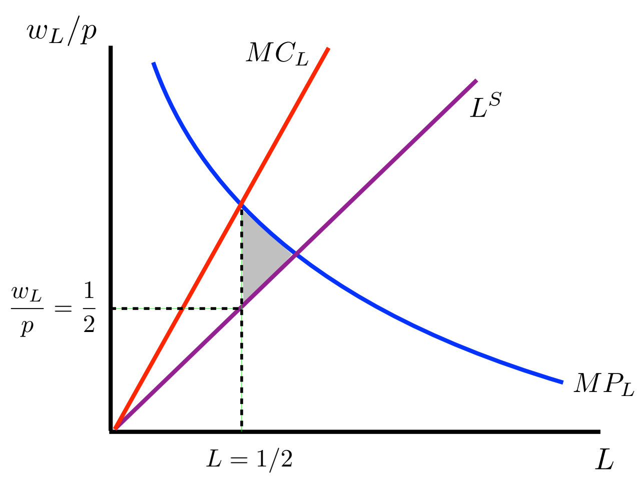
The two curves are shown below:



(e) The monopsonist's first-order condition can be written (more generally than we did in part **(c)**) as

$$\frac{\partial \pi}{\partial L} = 0 \implies p \cdot MP_L - MC_L(L) = 0 \implies MP_L = MC_L$$

(f) This is shown on the graph below. We can see the deadweight loss that results from fewer hires made in the gray shaded area:



Problem 3 (Oligopolistic Industry)

(a) The "big four" concentration ratio of the light beer industry in the U.S. is

$$CR = 36.8\% + 19.1\% + 18.5\% + 9.2\% = 83.6\% \,.$$

(b) The HHI is given by

$$HHI = 36.8^2 + 19.1^2 + 18.5^2 + 9.2^2 + 6.1^2 + 3.3^2 + 2.6^2 + 2.3^2 + 0.8^2 + 0.7^2 \approx 2{,}117 \,.$$

(c) This industry can be considered highly concentrated since $HHI > 1{,}800$.

(d) The FTC would likely attempt to block the merger as a monopoly would result from it.

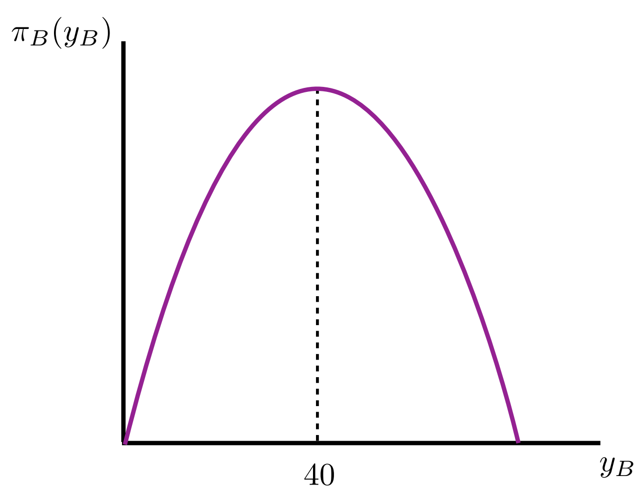
Problem 4 (Aircraft Industry)

(a) Boeing's profit function is

$$\pi_B(y_B) = TR - TC = (200 - y_A - y_B)y_B - 20y_B$$

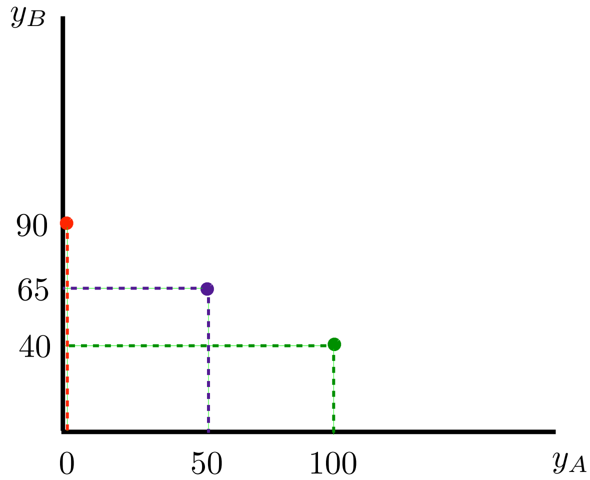
and given that Airbus is production at $y_A = 100$ this reduces to

$$\pi_B(y_B) = 80y_B - (y_B)^2$$



This attains an optimum at $y_B = 40$ (using first-order condition $\pi'_B(y_B) = 0$).

(b) No, we found in part **(a)** that Boeing's best response to Airbus producing $y_A = 100$ is $y_B = 40$ (not $y_B = 100$). For $y_A = 50$, Boing's profit-maximizing output is $y_B = 65$; when $y_A = 0$, we have $y_B = 90$. (You can verify this by going through the steps in part **(a)**.)



(c) The best response function for Boeing, which tells us their optimal output for any level of production Airbus might choose, we take the first-order condition of Boeing's profit function

$$\pi_B(y_B) = (200 - y_A - y_B)y_B - 20y_B\,,$$

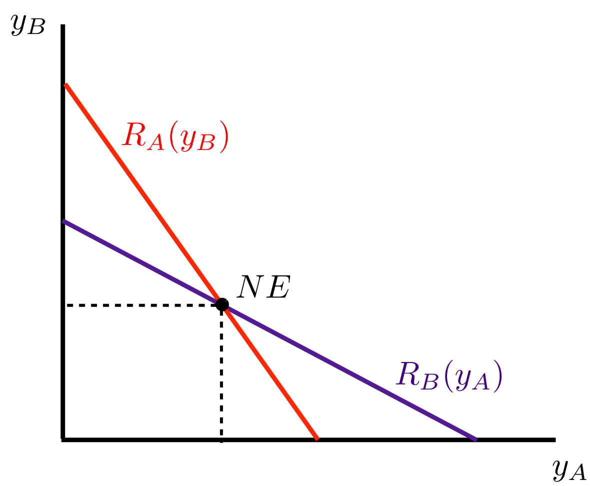
which is

$$\frac{\partial \pi_B}{\partial y_B} = 0 \implies 200 - y_A - 2y_B - 20 = 0\,.$$

Solving this for y_B, we find $y_B = \frac{180-y_A}{2}$, which is what gives us the best response function, so

$$R_B(y_A) = \frac{180 - y_A}{2}\,.$$

This is shown in the graph below:



(d) The best response function for Airbus is symmetric (they have the same cost functions). To find Airbus' best response function in general though we take the same steps as we did to find Boeing's. We take the profit function

$$\pi_A(y_A) = (200 - y_A - y_B)y_A - 20y_A$$

and the first-order condition

$$\frac{\partial \pi_A}{\partial y_A} = 0 \implies 200 - 2y_A - y_B - 20 = 0\,.$$

Solving this for y_A, we find $y_A = \frac{180-y_B}{2}$, which is what gives us the best response function

$$R_A(y_B) = \frac{180 - y_B}{2}\,.$$

This is shown on the graph above along with Boeing's best response function $R_B(y_A)$.

(e) The Cournot-Nash equilibrium level of output produced by each firm solves the best response functions simultaneously. You can set $y_A = R_B(y_A)$ and $y_B = R_A(y_B)$ to get

$$y_A = y_B = 60$$

so total output in the market is $y = y_A + y_B = 120$.

Turning to the inverse demand curve, this output is associated with a market price of

$$p(120) = 200 - 120 = 80$$

and the profits for each firm are

$$\pi_A = \pi_B = 80 \times 60 - 20 \times 60 = 3{,}600\,.$$

(f) To find the DWL, we first need to find the Pareto efficient output (where $p = MC$): In this problem $MC = 20$ and $p = MC \implies 200 - y = 20 \implies y = 180$.

The DWL then is the triangular area

$$DWL^{Duopoly} = \frac{1}{2}(80 - 20)(180 - 120) = 1{,}800\,.$$

(g) If the two firms were to form a cartel, they would determine output as a monopolist, maximizing cartel profit

$$\pi_{Cartel} = (200 - y)y - 20y\,.$$

Using the first order condition we get a profit-maximizing output of $y = 90$ and a cartel price of $p(90) = 200 - 90 = 110$. Each of them produce half of the optimal output, so $y_A = y_B = 45$, and they split the profits:

$$\pi_A = \pi_B = \frac{\pi_{Cartel}}{2} = \frac{110 \times 90 - 20 \times 90}{2} = 4{,}050\,.$$

This gives a higher level of profit than what we found with Cournot competition, so the cartel would be beneficial to both firms.

(h) The deadweight loss here is now the triangular area

$$DWL^{Cartel} = \frac{1}{2}(110 - 20)(180 - 90) = 4{,}050 > DWL^{Duopoly}.$$

(i) If the interactions are only in a short run, cartel is not sustainable as each firm has incentives to "cheat" the other firm by unilaterally increasing a production above 45 jets; you can see this by checking what their best response is to the other firm producing 45. This is, because given the other firm produces 45 jets, producing more leads to the firm that is even higher than in the case of the cartel.

When interactions are repeated however, the loss of reputation associated with cheating "today" may make the cooperation of the two firms tomorrow possible. If the firm cheats today, it gains some profit today but loses out on all of the cartel profit it could make in the future if ti cooperates (which is always higher than the duopoly profit it gets if it cheats). So such costs of cheating (i.e., the loss of future cartel profits), might make the cooperation sustainable.

Problem 5 (Accounting and Audit Services in the U.S.)

(a) *Monolopy:* Monopoly profit is

$$\pi_{Monopoly} = (1{,}000 - y)y - 10y$$

and taking the first-order condition we get optimal output $y = 495$ and price

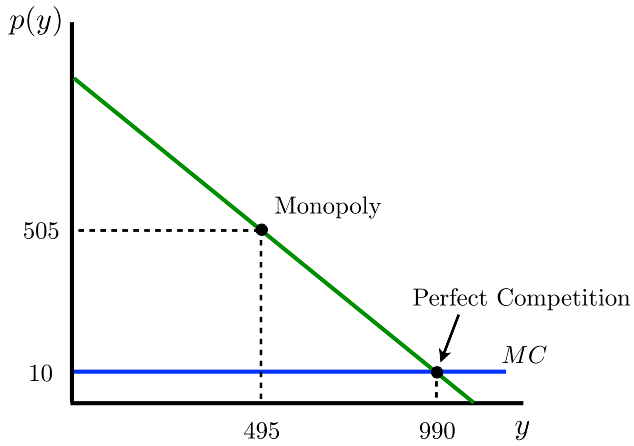
$$p(495) = 1{,}000 - 495 = 505\,.$$

Perfect Competition: With perfect competition, we will have that $p = MC = 10$ and so output is (turning to demand function $y(p) = 1000 - p$) then $y(10) = 990$

(b) The two points from part **(a)** are shown below along the inverse demand curve:

(c) Let $\sum_{j\neq i} y_j$ represent the output of all of the $N-1$ firms other than firm i. (So that $\sum_{j\neq i} y_j = y_1 + y_2 + \cdots + y_{i-1} + y_{i+1} + \cdots + y_N$.) In Cournot-Nash competition, profit for firm i is

$$\pi^i = (1{,}000 - y_i - \sum_{j\neq i} y_j)y_i - 10y_i\,.$$



The first-order optimality condition gives

$$1{,}000 - 2y_i - \sum_{j \neq i} y_j - 10 = 0$$

or

$$y_i = \frac{990 - \sum_{j \neq i} y_j}{2}$$

Since the firms are symmetric, $y_i = y_j$ and so we can write this as

$$y_i = \frac{990 - (N-1)y_i}{2}$$

and solving for y_i we get

$$y_i = \frac{990}{N+1}$$

Aggregate production with N firms is

$$y = Ny_i = \frac{N}{N+1} 990 \,.$$

This gives price

$$p(y) = 1{,}000 - \frac{N}{N+1} 990 \,.$$

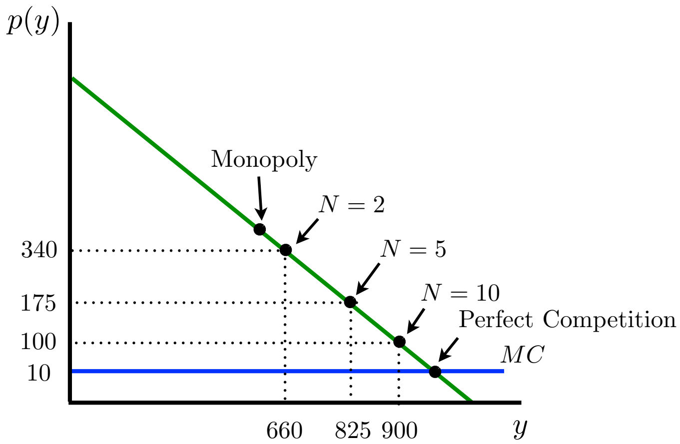
The DWL will depend on the number of firms in the market and is given by

$$DWL = \frac{1}{2} \times \left[(1{,}000 - \frac{N}{N+1} 990) - 10 \right] \times \left[990 - (\frac{N}{N+1} 990) \right] = \frac{990^2}{2} \times \left(\frac{1}{N+1} \right)^2$$

(d) We have the following aggregate output y and prices p for $N = 2, 5$, and 10:

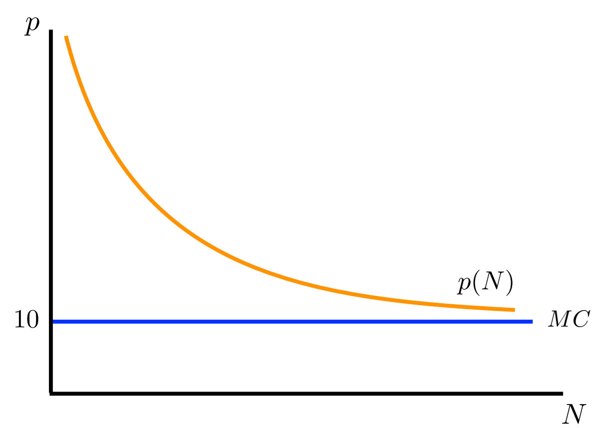
N	y	p
2	660	340
5	825	175
10	900	100

These are shown on the graph below:



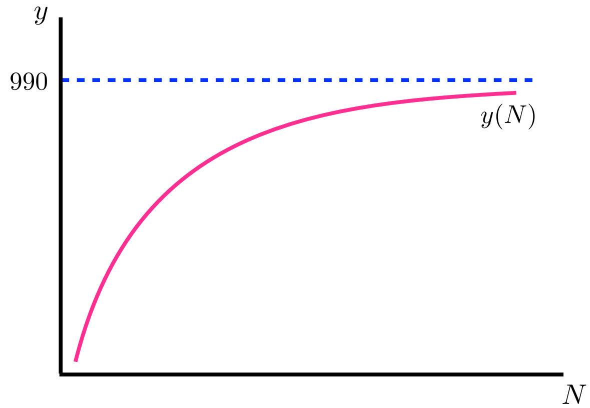
(e) As the number of firms N becomes large, the market looks more like a perfectly competitive market. Since $\lim_{N\to\infty} \frac{N}{N+1} = 1$, we can see that

$$\lim_{N\to\infty} p = \lim_{N\to\infty} \left[1{,}000 - \frac{N}{N+1}990\right] = 1{,}000 - 990\left(\lim_{N\to\infty} \frac{N}{N+1}\right) = 1{,}000 - 990(1) = 10\,.$$



(f) As N increases, we will get that output also converges to its competitive limit:

$$\lim_{N\to\infty} y = \lim_{N\to\infty} \frac{N}{N+1}990 = 990\,.$$



(g) Again, here we have that as the number of firms becomes very large, the DWL converges to its competitive limit. Since $\lim_{N\to\infty}\frac{1}{N+1}=0$, we have that

$$\lim_{N\to\infty} DWL = \frac{990^2}{2}\times\left(\lim_{N\to\infty}\frac{1}{N+1}\right)^2 = \frac{990^2}{2}\times(0)^2 = 0\,.$$