Problem Set 10: Solutions

ECON 301: Intermediate Microeconomics
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Problem 1 (Reasons why Monopolies Exist)

(1) Nuclear power plant: Large fixed costs.

(2) Microsoft's Vista operating system: Patent.

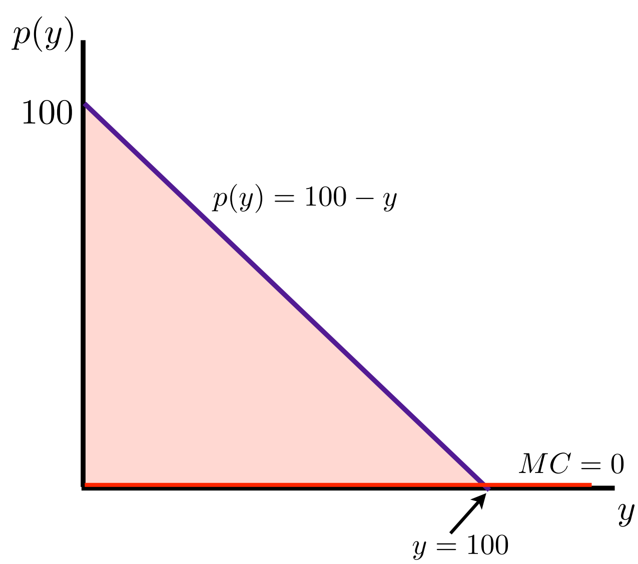
(3) Casinos, U.S. Post Office: Legal fiat.

(4) Niagara Falls State Park: Sole owner of waterfalls.

Problem 2 (Monopoly, Uniform Price)

(a) The total gains to trade is the pink area in the figure below *minus* the fixed cost of $F = \$1{,}000$ and is calculated as

$$TS = \frac{1}{2} \times 100 \times 100 - F = 5{,}000 - 1{,}000 = 4{,}000\,.$$



If this were a competitive market and Microsoft were a price taker, output would occur where $p = MC$. Since $MC = 0$, we get output $y = 100$. Consumer surplus is the area under the inverse demand curve and above price ($p = 0$ here), which corresponds to the shaded pink area above and is:

$$CS = \frac{1}{2} \times 100 \times 100 = 5{,}000\,.$$

At output $y = 100$ and price $p = 0$, producer surplus is

$$PS = -1{,}000\,.$$

Note that with price being equal to zero the firm has a negative profit and should exit the industry.

An answer where fixed cost is not subtracted (so $TS = 5{,}000$ and $PS = 1{,}000$) would also be correct; in such a case F would be considered a sunk cost.

(b) If Microsoft cannot discriminate among customers' willingness to pay, its profit-maximizing output is where $MR = MC$.

Marginal revenue is the derivative of total revenue, which is

$$TR(y) = p(y) \cdot y = (100 - y)y = 100y - y^2\,,$$

and so total revenue is

$$MR(y) = TR'(y) = 100 - 2y\,.$$

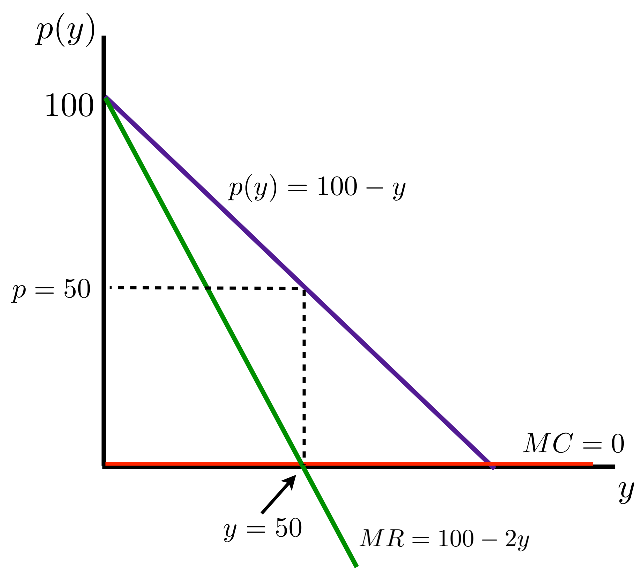
Since $MC(y) = 0$,

$$MR(y) = MC(y) \quad \Longrightarrow \quad 100 - 2y = 0 \quad \Longrightarrow \quad y = 50$$

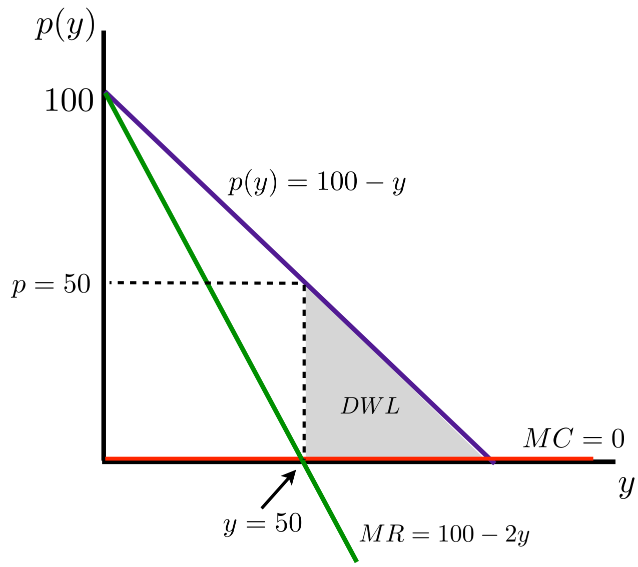
giving price

$$p(50) = 100 - (50) = 50\,.$$

Profit is $\pi = TR(50) - TC(50) = 50 \cdot 50 - 1{,}000 = 1{,}500$.



(c) The outcome in part **(b)** is not Pareto efficient; there are only 50 trades made while in the competitive market (which is Pareto efficient) there are 100 trades. The deadweight loss is the gray shaded area seen in the figure below:



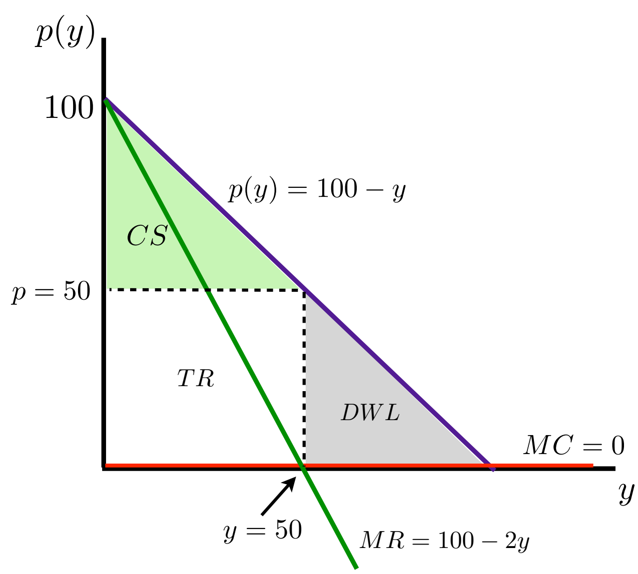
Analytically, the deadweight loss is

$$DWL = \frac{1}{2} \times 50 \times (100 - 50) = 1{,}250\,.$$

(d) The consumer surplus is the area under the demand curve and above the price:

$$CS = \frac{1}{2} \times (100 - 50) \times 50$$

This is the green shaded area in the figure below:



Producer surplus is the total revenue (TR) area *minus* the fixed cost of $F = 1{,}000$:

$$PS = 50 \times 50 - 1{,}000 = 1{,}500$$

Notice that consumer and producer surplus add up to less than the total surplus we found in part **(a)**; the difference is the deadweight loss of $DWL = 1{,}250$.

(e) Elasticity is defined as

$$\varepsilon = \frac{\Delta y / y}{\Delta p / p} = \frac{\Delta y}{\Delta p} \times \frac{p}{y}$$

where $\frac{\Delta y}{\Delta p}$ is the slope of the demand function $y(p)$ (not inverse demand $p(y)$, though they happen to have the same slope in this problem). Here, $y(p) = 100 - p$, and since $p = 50$ and $y = 50$,

$$\varepsilon = y'(p) \times \frac{p}{y} = (-1) \times \frac{50}{50} = -1\,.$$

Since $|\varepsilon| = 1$, the firm is operating at a point along the demand curve that is neither elastic ($|\varepsilon| > 1$) nor inelastic ($|\varepsilon| < 1$); it's at the threshold between the two regions along the demand curve.

(f) Markup is p/MC. Since we are operating where $MR = MC$, we can write markup as p/MR. Since $MR = p[1 + 1/\varepsilon]$, we have

$$\frac{p}{MC} = \frac{p}{MR} = \frac{p}{p[1 + 1/\varepsilon]} = \frac{1}{1 + 1/\varepsilon}$$

since $\varepsilon = -1$, we have $\frac{p}{MC} = \infty$. Markup is infinite with marginal cost of zero.

Problem 3 (Price Discrimination)

(a) If Microsoft can perfectly price discriminate, its profit (and the producer surplus PS) is the total surplus area (accounting for the fixed costs of $F = 1{,}000$) that we found in **Problem 2 (a)**:

$$PS = \pi = TS = \frac{1}{2} \times 100 \times 100 - F = 5{,}000 - 1{,}000 = 4{,}000\,.$$

Consumer surplus is zero and the outcome is Pareto efficient since there is no deadweight loss.

(b) To find the aggregate demand with these two segments, we sum over quantity y. (Note that if the firm does not price discriminate, it must charge the same price so $p = p^I = p^F$.)

$$y^{AGG}(p) = y^I(p) + y^F(p) = (50 - \frac{4}{5}p) + (50 - \frac{1}{5}p) = 100 - p$$

which gives us the inverse aggregate demand we had in **Problem 2**:

$$p^{AGG}(y) = 100 - y\,.$$

(c) With third-degree price discrimination, Microsoft charges price p^I to the individual buyers market segment and price p^F to the firm buyers. We can simply solve the profit maximizing problem for each segment separately.

Individual Buyers: The inverse demand curve in this market is

$$p^I(y) = \frac{125}{2} - \frac{5}{4}y^I$$

and so the marginal revenue in the individual buyers market is

$$MR^I(y) = \frac{125}{2} - \frac{10}{4}y^I\,.$$

Profit maximizing condition $MR(y^I) = MC(y^I) \implies \frac{125}{2} - \frac{10}{4}y^I = 0 \implies y^I = 25$.

Price is then $p^I(25) = 31\frac{1}{4}$. Total revenue in the individual segment is then $TR^I = 25 \times \frac{1}{4} =$ 1,781$\frac{1}{4}$, and consumer surplus is $CS^I = \frac{1}{2} \times (\frac{125}{2} - 31\frac{1}{4}) \times 25 = 390\frac{5}{8}$.

Firm Buyers: The inverse demand curve in this market is

$$p^F(y) = 250 - 5y^F$$

and so the marginal revenue in the firm buyers market is

$$MR^F(y) = 250 - 10y^F\,.$$

Profit maximizing condition $MR(y^F) = MC(y^F) \implies 250 - 10y^F = 0 \implies y^F = 25$.

Price is then $p^F(25) = 125$. Total revenue in the firm segment is then $TR^F = 25 \times 125 =$ 3,125, and consumer surplus is $CS^I = \frac{1}{2} \times (250 - 125) \times 25 = 1{,}562\frac{1}{2}$.

To find Microsoft's total profit (which corresponds to producer surplus here), we add the total revenue from each segment and subtract the total cost of $F = 1{,}000$:

$$\pi = TR^I - TR^F - TC = 1{,}781\frac{1}{4} + 4{,}125 - 1{,}000 = 3{,}906\frac{1}{4}$$

(It would also be fine to assume that the fixed cost of $F = 1{,}000$ is incurred in each market, giving instead profit $\pi = 2{,}906\frac{1}{4}$.)

(d) Here we compare consumer and producer surplus in the three scenarios.

Uniform pricing:

- CS=1,250
- PS=1,500

Perfect price discrimination (first-degree):

- CS=0
- PS=4,000

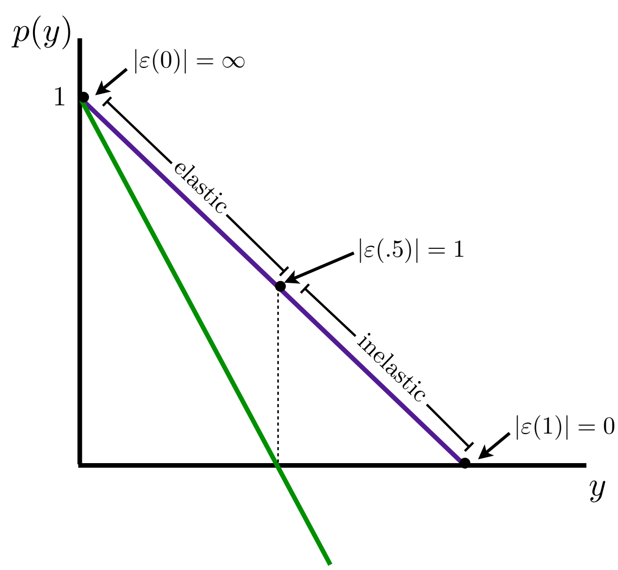
Third-degree price discrimination:

- CS= $390\frac{5}{8} + 1{,}562\frac{1}{2} = 1{,}953\frac{1}{8}$
- PS=$3{,}906\frac{1}{4}$ (or $2{,}906\frac{1}{4}$)

Notice that the producer surplus is the greatest with first-degree price discrimination and lowest with uniform pricing.

Problem 4 (Demand Elasticity)

(a) The inverse demand curve, $p(y) = 1 - y$, is shown below:



(b) For this demand function, we have $\varepsilon = y'(p) \times \frac{p(y)}{y} = (-1)\frac{1-y}{y}$, so

- $\varepsilon(0) = -\frac{1-0}{0} = -\infty$
- $\varepsilon(.5) = -\frac{1-.5}{.5} = -1$
- $\varepsilon(1) = -\frac{1-1}{1} = 0$

These points are marked on the graph in part **(a)**.

(c) When $TC(y) = cy$, we have constant marginal cost of $MC(y) = TC'(y) = c$. Total revenue is $TR(y) = p(y) \cdot y$ and so marginal revenue is $MR(y) = TR'(y) = p(y) + y \cdot p'(y)$ (product rule). Since the profit-maximizing firm chooses output such that $MR(y) = MC(y)$, we have

$$MR(y) = MC(y) \quad \Longrightarrow \quad p(y) + y \cdot p'(y) = c\,.$$

Changing notation so that $p(y) = p$ and $p'(y) = \frac{dp}{dy}$, we have

$$p + y\frac{dp}{dy} = c \quad \Longrightarrow \quad p\left[1 + \frac{y}{p}\frac{dp}{dy}\right] = c\,.$$

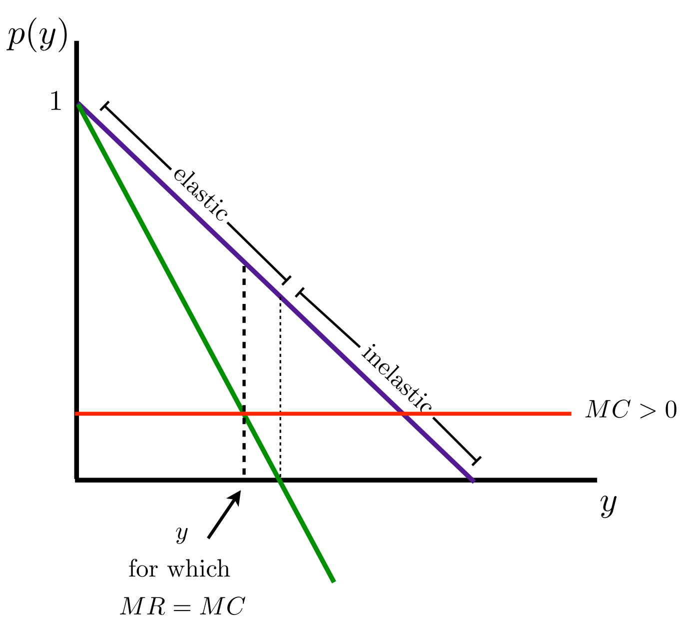
Recognizing that $1/\varepsilon = \frac{dp}{dy}\frac{y}{p}$, we have

$$c = p\left[1 + \frac{1}{\varepsilon}\right]\,.$$

Then since $c > 0$, we must have $p\left[1 + \frac{1}{\varepsilon}\right] > 0$. Since $p > 0$, we can solve for ε to get

$$p\left[1 + \frac{1}{\varepsilon}\right] \quad \Longrightarrow \quad 1 + \frac{1}{\varepsilon} > 0 \quad \Longrightarrow \quad \frac{1}{\varepsilon} > -1 \quad \Longrightarrow \quad \varepsilon < -1 \quad \Longrightarrow \quad \varepsilon > 1$$

and so the optimal output is on the elastic portion of the demand curve for any $MC = c > 0$.



As we can see in the figure, whenever $MC > 0$, output occurs where MR=MC which is necessarily on the elastic region of the demand curve.

(d) Equating $MR(y) = MC(y)$, we get

$$p\left[1 + \frac{1}{\varepsilon}\right] = MC \quad \Longrightarrow \quad p = \left[\frac{1}{1 + 1/\varepsilon}\right] MC$$

so the price markup over marginal cost is $\left[\frac{1}{1+1/\varepsilon}\right]$.