Econ 301

Intermediate Microeconomics

Prof. Marek Weretka

Midterm 2 (Group A)

You have 70 minutes to complete the exam. The midterm consists of 4 questions (25,30,25 and 20 points)+Just For Fun question.

Problem 1. (25p) (Uncertainty)

Oscar is an owner of Lamborghini Veneno, one of the most expensive cars ever made. Its market value is estimated at 8 million dollars. In case of a car collision the value of the car drops to 4 million dollars. The probability of a collision is π_c= 0.5. In short, Lamborghini is a lottery $(8, 4)$.

a) Oscar's Bernoulli utility function is given by $u(c) = 10 \ln c$. Write down his Von Neumann-Morgenstern (expected) utility function over lotteries $U(C_c, C_{nc})$ (give a formula). Is Oscar risk averse, risk neutral or risk loving (choose one)? Plot Oscar's indifference curves in the commodity space (C_c, C_{nc}).

b) In Madison area insurance for ridiculously expensive cars is provided by State Farm. Derive Oscar's budget constraint if State Farm insurance premium is $\gamma = 0.5$ (give equation for C_c and C_{nc} in terms of coverage x, and then reduce the two equations to one budget constraint.) Plot the corresponding budget set in the commodity space.

c) Find the optimal level of wealth (C_c, C_{nc}) and the coverage x. (three numbers) Is Oscar fully insured (yes-no answer).

d) Demonstrate that if the premium is greater than the probability of flood, $\gamma > \pi_c$ Oscar will not purchase full insurance. (Use "MRS" secret of happiness to show that in optimum C_c< C_{nc}).

Problem 2. (30p) (Edgeworth box and equilibrium)

Consider an economy with two goods (apples and oranges) and two agents, Elisa and Bob. Elisa is initially endowed with $\omega^E = (5, 30)$ of apples and oranges respectively and Ben's endowment is $\omega^B = (20, 20)$. Elisa and Ben have the same utility given by

$$U(x_1, x_2) = 2 \ln x_1 + 2 \ln x_2$$

a) Plot an Edgeworth box and mark the initial endowments.

b) Give a definition of Pareto efficient allocation (one sentence). Using graph argue that the necessary and sufficient condition for Pareto efficiency of a (interior) feasible allocation is MRS^E= MRS^B.

c) Derive the contract curve (give formula) and depict it in the Edgeworth box.

d) Find the competitive equilibrium (give six numbers) and show it in the Edgeworth box.

e) Verify that the allocation in the competitive equilibrium is Pareto efficient.

f) Give two other prices that are consistent with a competitive equilibrium? (give two numbers without any calculations)

Problem 3. (25p) (Producers)

A producer has the following technology

$$y = \sqrt{K + L}$$

a) Show that the production function exhibits decreasing returns to scale (provide formal argument).

b) Find analytically (the variable) cost function given prices of inputs $w_K = 1$ and $w_L = 2$ (formula). Plot the cost function in the graph.

c) Assuming fixed cost $F = 1$ find analytically y^{MES} and ATC^{MES} (give two numbers) and plot a supply of the firm, marking a threshold for non-zero production.[1]

d) Determine the number of firms operating in the industry if demand is $D(p) = 40 - p$, firms are competitive and there is free entry in the market. (one number)

Problem 4 (20p). (Short questions)

a) Robert's Bernoulli utility function is given by $u(c) = 10c$. A lottery ticket pays 10 with probability $\frac{1}{2}$ and zero otherwise and thus the lottery is $C = (10, 0)$. Find certainty equivalent CE of lottery C (one number) and the expected value $E(C)$ of the lottery (one number). Is the certainty equivalent bigger or smaller than the expected value? Why? (one sentence)

b) Consider economy with two agents, Andy and Bob. They have identical utility functions $U(x_1, x_2) = x_1 + x_2$ but different endowments $\omega^A = (10, 30)$ and $\omega^B = (10, 20)$, for Andy and Bob, respectively. Without any calculations, plot the contract curve in an Edgeworth box and suggest (one) competitive equilibrium (six numbers).

c) Given production function $y = 4\bar{K}^7 L^{\frac{1}{2}}$ and short-run level of capital $\bar{K} = 1$ derive labor demand (formula) of a competitive firm. Find equilibrium real wage rate if labor supply is given by $L^s = 16$ (one number). Find the unemployment rate if the minimal real wage is $w^{min}/p = 2$ (one number+graph).

Just For Fun

Give a formal argument that from the perspective of the whole society free (competitive) markets efficiently allocate resources. (i.e., show that an allocation in competitive equilibrium is Pareto efficient).

[1] If you do not know how to answer b) to get partial credit you can assume $c(y) = y^2$

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Midterm 2 (Group B)

You have 70 minutes to complete the exam. The midterm consists of 4 questions (25,30,25 and 20 points)+Just For Fun question.

Problem 1. (25p) (Uncertainty)

Oscar is an owner of Lamborghini Veneno, one of the most expensive cars ever made. Its market value is estimated at 16 million dollars. In case of a car collision the value of the car drops to 8 million dollars. The probability of a collision is π_c= 0.5. In short, Lamborghini is a lottery $(16, 8)$.

a) Oscar's Bernoulli utility function is given by $u(c) = 10 \ln c$. Write down his Von Neumann-Morgenstern (expected) utility function over lotteries $U(C_c, C_{nc})$ (give a formula). Is Oscar risk averse, risk neutral or risk loving (choose one)? Plot Oscar's indifference curves in the commodity space (C_c, C_{nc}).

b) In Madison area insurance for ridiculously expensive cars is provided by State Farm. Derive Oscar's budget constraint if State Farm insurance premium is $\gamma = 0.5$ (give equation for C_c and C_{nc} in terms of coverage x, and then reduce the two equations to one budget constraint.) Plot the corresponding budget set in the commodity space.

c) Find the optimal level of wealth (C_c, C_{nc}) and the coverage x. (three numbers) Is Oscar fully insured (yes-no answer).

d) Demonstrate that if the premium is greater than the probability of flood, $\gamma > \pi_c$ Oscar will not purchase full insurance. (Use "MRS" secret of happiness to show that in optimum C_c< C_{nc}).

Problem 2. (30p) (Edgeworth box and equilibrium)

Consider an economy with two goods (apples and oranges) and two agents, Elisa and Bob. Elisa is initially endowed with $\omega^E = (1, 6)$ of apples and oranges respectively and Ben's endowment is $\omega^B = (4, 4)$. Elisa and Ben have the same utility given by

$$U(x_1, x_2) = 3 \ln x_1 + 3 \ln x_2$$

a) Plot an Edgeworth box and mark the initial endowments.

b) Give a definition of Pareto efficient allocation (one sentence). Using graph argue that the necessary and sufficient condition for Pareto efficiency of a (interior) feasible allocation is MRS^E= MRS^B.

c) Derive the contract curve (give formula) and depict it in the Edgeworth box.

d) Find the competitive equilibrium (give six numbers) and show it in the Edgeworth box.

e) Verify that the allocation in the competitive equilibrium is Pareto efficient.

f) Give two other prices that are consistent with a competitive equilibrium? (give two numbers without any calculations)

Problem 3. (25p) (Producers)

A producer has the following technology

$$y = \sqrt{K + L}$$

a) Show that the production function exhibits decreasing returns to scale (provide formal argument).

b) Find analytically (the variable) cost function given prices of inputs $w_K = 2$ and $w_L = 1$ (formula). Plot the cost function in the graph.

c) Assuming fixed cost $F = 1$ find analytically y^{MES} and ATC^{MES} (give two numbers) and plot a supply of the firm, marking a threshold for non-zero production.[2]

d) Determine the number of firms operating in the industry if demand is $D(p) = 20 - p$, firms are competitive and there is free entry in the market. (one number)

Problem 4 (20p). (Short questions)

a) Robert's Bernoulli utility function is given by $u(c) = 2c$. A lottery ticket pays 12 with probability $\frac{1}{2}$ and zero otherwise and thus the lottery is $C = (12, 0)$. Find certainty equivalent CE of lottery C (one number) and the expected value $E(C)$ of the lottery (one number). Is the certainty equivalent bigger or smaller than the expected value? Why? (one sentence)

b) Consider economy with two agents, Andy and Bob. They have identical utility functions $U(x_1, x_2) = x_1 + x_2$ but different endowments $\omega^A = (10, 40)$ and $\omega^B = (20, 20)$, for Andy and Bob, respectively. Without any calculations, plot the contract curve in an Edgeworth box and suggest (one) competitive equilibrium (six numbers).

c) Given production function $y = 12\bar{K}^{7}L^{\frac{1}{2}}$ and short-run level of capital $\bar{K} = 1$ derive labor demand (formula) of a competitive firm. Find equilibrium real wage rate if labor supply is given by $L^s = 9$ (one number). Find the unemployment rate if the minimal real wage is $w^{min}/p = 6$ (one number+graph).

Just For Fun

Give a formal argument that from the perspective of the whole society free (competitive) markets efficiently allocate resources. (i.e., show that an allocation in competitive equilibrium is Pareto efficient).

[2] If you do not know how to answer b) to get partial credit you can assume $c(y) = y^2$

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Midterm 2 (Group C)

You have 70 minutes to complete the exam. The midterm consists of 4 questions (25,30,25 and 20 points)+Just For Fun question.

Problem 1. (25p) (Uncertainty)

Oscar is an owner of Lamborghini Veneno, one of the most expensive cars ever made. Its market value is estimated at 2 million dollars. In case of a car collision the value of the car drops to 1 million dollars. The probability of a collision is $\pi_c = 0.5$. In short, Lamborghini is a lottery $(2, 1)$.

a) Oscar's Bernoulli utility function is given by $u(c) = 10 \ln c$. Write down his Von Neumann-Morgenstern (expected) utility function over lotteries $U(C_c, C_{nc})$ (give a formula). Is Oscar risk averse, risk neutral or risk loving (choose one)? Plot Oscar's indifference curves in the commodity space (C_c, C_{nc}).

b) In Madison area insurance for ridiculously expensive cars is provided by State Farm. Derive Oscar's budget constraint if State Farm insurance premium is $\gamma = 0.5$ (give equation for C_c and C_{nc} in terms of coverage x, and then reduce the two equations to one budget constraint.) Plot the corresponding budget set in the commodity space.

c) Find the optimal level of wealth (C_c, C_{nc}) and the coverage x. (three numbers) Is Oscar fully insured (yes-no answer).

d) Demonstrate that if the premium is greater than the probability of flood, $\gamma > \pi_c$ Oscar will not purchase full insurance. (Use "MRS" secret of happiness to show that in optimum $C_c < C_{nc}$).

Problem 2. (30p) (Edgeworth box and equilibrium)

Consider an economy with two goods (apples and oranges) and two agents, Elisa and Bob. Elisa is initially endowed with $\omega^E = (10, 60)$ of apples and oranges respectively and Ben's endowment is $\omega^B = (40, 40)$. Elisa and Ben have the same utility given by

$$U(x_1, x_2) = 8 \ln x_1 + 8 \ln x_2$$

a) Plot an Edgeworth box and mark the initial endowments.

b) Give a definition of Pareto efficient allocation (one sentence). Using graph argue that the necessary and sufficient condition for Pareto efficiency of a (interior) feasible allocation is $MRS^E = MRS^B$.

c) Derive the contract curve (give formula) and depict it in the Edgeworth box.

d) Find the competitive equilibrium (give six numbers) and show it in the Edgeworth box.

e) Verify that the allocation in the competitive equilibrium is Pareto efficient.

f) Give two other prices that are consistent with a competitive equilibrium? (give two numbers without any calculations)

Problem 3. (25p) (Producers)

A producer has the following technology

$$y = \sqrt{2K + 2L}$$

a) Show that the production function exhibits decreasing returns to scale (provide formal argument).

b) Find analytically (the variable) cost function given prices of inputs $w_K = 2$ and $w_L = 4$ (formula). Plot the cost function in the graph.

c) Assuming fixed cost $F = 1$ find analytically y^{MES} and ATC^{MES} (give two numbers) and plot a supply of the firm, marking a threshold for non-zero production.[3]

d) Determine the number of firms operating in the industry if demand is $D(p) = 40 - p$, firms are competitive and there is free entry in the market. (one number)

Problem 4 (20p). (Short questions)

a) Robert's Bernoulli utility function is given by $u(c) = 10c$. A lottery ticket pays 16 with probability $\frac{1}{2}$ and zero otherwise and thus the lottery is $C = (16, 0)$. Find certainty equivalent CE of lottery C (one number) and the expected value $E(C)$ of the lottery (one number). Is the certainty equivalent bigger or smaller than the expected value? Why? (one sentence)

b) Consider economy with two agents, Andy and Bob. They have identical utility functions $U(x_1, x_2) = x_1 + x_2$ but different endowments $\omega^A = (10, 40)$ and $\omega^B = (20, 20)$, for Andy and Bob, respectively. Without any calculations, plot the contract curve in an Edgeworth box and suggest (one) competitive equilibrium (six numbers).

c) Given production function $y = 16\bar{K}^{7}L^{\frac{1}{2}}$ and short-run level of capital $\bar{K} = 1$ derive labor demand (formula) of a competitive firm. Find equilibrium real wage rate if labor supply is given by $L^s = 16$ (one number). Find the unemployment rate if the minimal real wage is $w^{min}/p = 8$ (one number+graph).

Just For Fun

Give a formal argument that from the perspective of the whole society free (competitive) markets efficiently allocate resources. (i.e., show that an allocation in competitive equilibrium is Pareto efficient).

[3] If you do not know how to answer b) to get partial credit you can assume $c(y) = y^2$

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Midterm 2 (Group D)

You have 70 minutes to complete the exam. The midterm consists of 4 questions (25,30,25 and 20 points)+Just For Fun question.

Problem 1. (25p) (Uncertainty)

Oscar is an owner of Lamborghini Veneno, one of the most expensive cars ever made. Its market value is estimated at 8 million dollars. In case of a car collision the value of the car drops to 4 million dollars. The probability of a collision is $\pi_c = 0.5$. In short, Lamborghini is a lottery $(8, 4)$.

a) Oscar's Bernoulli utility function is given by $u(c) = 10 \ln c$. Write down his Von Neumann-Morgenstern (expected) utility function over lotteries $U(C_c, C_{nc})$ (give a formula). Is Oscar risk averse, risk neutral or risk loving (choose one)? Plot Oscar's indifference curves in the commodity space (C_c, C_{nc}).

b) In Madison area insurance for ridiculously expensive cars is provided by State Farm. Derive Oscar's budget constraint if State Farm insurance premium is $\gamma = 0.5$ (give equation for C_c and C_{nc} in terms of coverage x, and then reduce the two equations to one budget constraint.) Plot the corresponding budget set in the commodity space.

c) Find the optimal level of wealth (C_c, C_{nc}) and the coverage x. (three numbers) Is Oscar fully insured (yes-no answer).

d) Demonstrate that if the premium is greater than the probability of flood, $\gamma > \pi_c$ Oscar will not purchase full insurance. (Use "MRS" secret of happiness to show that in optimum $C_c < C_{nc}$).

Problem 2. (30p) (Edgeworth box and equilibrium)

Consider an economy with two goods (apples and oranges) and two agents, Elisa and Bob. Elisa is initially endowed with $\omega^E = (5, 30)$ of apples and oranges respectively and Ben's endowment is $\omega^B = (20, 20)$. Elisa and Ben have the same utility given by

$$U(x_1, x_2) = 2 \ln x_1 + 2 \ln x_2$$

a) Plot an Edgeworth box and mark the initial endowments.

b) Give a definition of Pareto efficient allocation (one sentence). Using graph argue that the necessary and sufficient condition for Pareto efficiency of a (interior) feasible allocation is $MRS^E = MRS^B$.

c) Derive the contract curve (give formula) and depict it in the Edgeworth box.

d) Find the competitive equilibrium (give six numbers) and show it in the Edgeworth box.

e) Verify that the allocation in the competitive equilibrium is Pareto efficient.

f) Give two other prices that are consistent with a competitive equilibrium? (give two numbers without any calculations)

Problem 3. (25p) (Producers)

A producer has the following technology

$$y = \sqrt{2K + 2L}$$

a) Show that the production function exhibits decreasing returns to scale (provide formal argument).

b) Find analytically (the variable) cost function given prices of inputs $w_K = 4$ and $w_L = 2$ (formula). Plot the cost function in the graph.

c) Assuming fixed cost $F = 1$ find analytically y^{MES} and ATC^{MES} (give two numbers) and plot a supply of the firm, marking a threshold for non-zero production.[4]

d) Determine the number of firms operating in the industry if demand is $D(p) = 20 - p$, firms are competitive and there is free entry in the market. (one number)

Problem 4 (20p). (Short questions)

a) Robert's Bernoulli utility function is given by $u(c) = 10c$. A lottery ticket pays 10 with probability $\frac{1}{2}$ and zero otherwise and thus the lottery is $C = (10, 0)$. Find certainty equivalent CE of lottery C (one number) and the expected value $E(C)$ of the lottery (one number). Is the certainty equivalent bigger or smaller than the expected value? Why? (one sentence)

b) Consider economy with two agents, Andy and Bob. They have identical utility functions $U(x_1, x_2) = 3x_1 + 3x_2$ but different endowments $\omega^A = (10, 30)$ and $\omega^B = (10, 20)$, for Andy and Bob, respectively. Without any calculations, plot the contract curve in an Edgeworth box and suggest (one) competitive equilibrium (six numbers).

c) Given production function $y = 18\bar{K}^{7}L^{\frac{1}{2}}$ and short-run level of capital $\bar{K} = 1$ derive labor demand (formula) of a competitive firm. Find equilibrium real wage rate if labor supply is given by $L^s = 9$ (one number). Find the unemployment rate if the minimal real wage is $w^{min}/p = 9$ (one number+graph).

Just For Fun

Give a formal argument that from the perspective of the whole society free (competitive) markets efficiently allocate resources. (i.e., show that an allocation in competitive equilibrium is Pareto efficient).

[4] If you do not know how to answer b) to get partial credit you can assume $c(y) = y^2$

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Prof.Marek Weretka
Solution to Midterm 2 (Group A)

Problem 1. (a) expected utility function over lotteries

$$U(C_c, C_{nc}) = \frac{1}{2} \times 10 \ln C_c + \frac{1}{2} \times 10 \ln C_{nc}$$
$$= 5 \ln C_c + 5 \ln C_{nc} \qquad (3')$$

Oscar is risk averse since $U(c) = 10 \ln C$ is concave. (1')
Let $U(C_c, C_{nc}) = \bar{u}$ Then $C_{nc} = \frac{e^{\bar{u}/5}}{C_c}$. So we can obtain the indifference curve as below. (1')

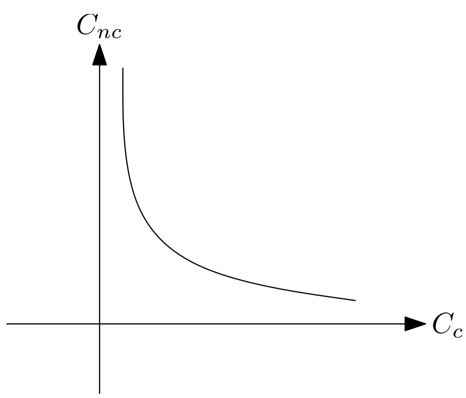

Figure 1: Problem 1(a)

(b) The C_c and C_{nc} can be represented as

$$C_c = 4 - \gamma x + x = 4 + 0.5x$$
$$C_{nc} = 8 - \gamma x = 8 - 0.5x \qquad (2')$$

Then,

$$C_{nc} + \frac{\gamma}{1-\gamma} C_c = 8 + 4\frac{\gamma}{1-\gamma} \Longrightarrow C_{nc} + C_c = 12 \qquad (3')$$

The budget line is shown as below: (2')

(c) By the utility function in part (a) and budget constraint in part (b), optimal choice of wealth levels are given by the short-cut formula:

$$C_c = \frac{5}{5+5} \times \frac{12}{1} = 6 \qquad (2')$$

$$C_{nc} = \frac{5}{5+5} \times \frac{12}{1} = 6 \qquad (2')$$

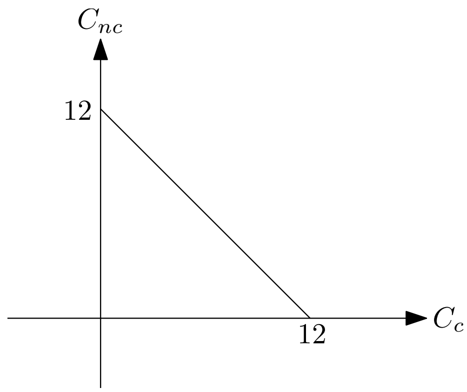

Figure 2: Problem 1(b)

Then $C_c = C_{nc}$ So Oscar is fully insured. (1')
By the formula in part(b), $x = 4$. (2')

(d) First calculate

$$MRS = \frac{5/C_{nc}}{5/C_c} = \frac{C_c}{C_{nc}} \qquad (2')$$

then by the 'secret of happiness',

$$MRS = \frac{p_{nc}}{p_c} \Longrightarrow \frac{C_c}{C_{nc}} = \frac{1-\gamma}{\gamma} \qquad (2')$$

If $\gamma > \pi_c = 0.5$, then $\frac{1-\gamma}{\gamma} < 1$. Therefor, $C_nc > C_c$. That is, Oscar is not fully insured. (2')

Problem 2. (a) Total endowment $\omega = \omega^E + \omega^B = (5, 30) + (20, 20) = (25, 50)$. (1')
The Edgeworth box and initial endowment are in the following figure.

(b) An allocation is Pareto Efficient if there is no other allocation which would make one of them strictly better off without hurting any of the others. (2')
$\Longrightarrow$) Suppose that $MRS^E \neq MRS^B$ when an allocation is Pareto Efficient. Then the two indifference curves passing through this point will intersect as in figure 4. It's obvious that the points in the shadowed area will make both of them not worse off and at least one of them strictly better off. That's a contradiction with the assumption. Then the necessity follows immediately. (3')
$\Longleftarrow$) Now assume the condition $MRS^E = MRS^B$ is satisfied at an allocation a. Then the two indifference curves passing through this point will be tangent with each other as in figure 5. Then check all

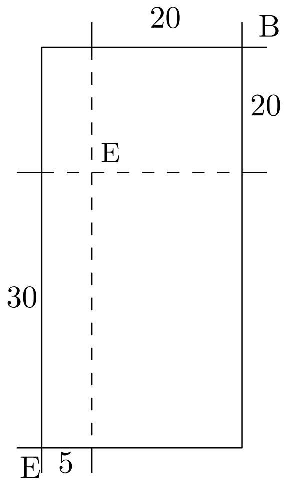

Figure 3: Problem 2(a)

the points in the areas A, B, C, D except point a in the Edgeworth box and all of them will make at least of Elisa and Ben strictly worse off. So allocation a is Pareto efficient by definition. (3')

(c)

$$MRS^E = MRS^B \Longrightarrow \frac{x_2^E}{x_1^E} = \frac{x_2^E}{x_1^E} \qquad (2')$$

Also

$$x_1^E + x_1^B = 25, x_2^E + x_2^B = 50 \qquad (2')$$

Substitute the last two equation to the first one.

$$x_2^E = 2x_1^E, x_2^B = 2x_1^E \qquad (2')$$

the contract curve is plotted in figure 6. (2')

(d) Normalize $p_2 = 1$. the budget constraints are:

$$\begin{aligned} 2x_1^E + x_2^E &= 5p_1 + 30 \\ 2x_1^B + x_2^B &= 20P_1 + 20 \qquad (1') \end{aligned}$$

Then

$$\begin{aligned} x_1^E &= \frac{2}{2+2}\frac{5p_1 + 30}{p_1} \\ x_1^B &= \frac{2}{2+2}\frac{20P_1 + 20}{p_1} \qquad (1') \end{aligned}$$

Thus

$$\frac{2}{2+2}\frac{5p_1 + 30}{p_1} + \frac{2}{2+2}\frac{20P_1 + 20}{p_1} = 25 \Longrightarrow p_1 = 2 \qquad (2')$$

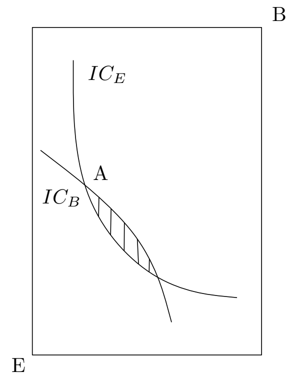

Figure 4: Problem 2(b)

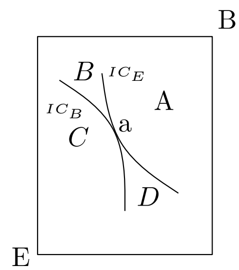

Figure 5: Problem 2(b)

Solve the equations and the result follows:

$$x_1^E = 10, x_2^E = 20 \qquad (2')$$

$$x_1^B = 15, x_2^B = 30 \qquad (2')$$

The competitive equilibrium is indicated in the Edgeworth box in Figure 7.

(e) Calculate and compare the MRS's.

$$MRS^E = \frac{x_2^E}{x_1^E} = \frac{20}{10} = 2 \qquad (1')$$

$$MRS^B = \frac{x_2^B}{x_1^B} = \frac{30}{15} = 2 \qquad (1')$$

So obviously $MRS^E = MRS^B$.

(f) any numbers satisfying $\frac{p_1}{p_2} = 2$, for example, $p_1 = 4, p_2 = 2$. (2')

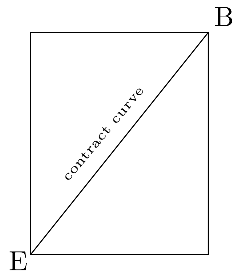

Figure 6: Problem 2(c)

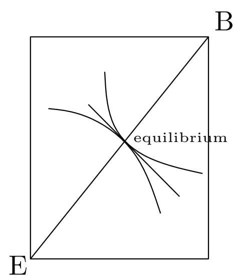

Figure 7: Problem 2(d)

Problem 3. (a) The production function $y = f(K, L) = \sqrt{K + L}$

$f(rK, rL) = \sqrt{rK + rL} = \sqrt{r(K + L)} = \sqrt{r}\sqrt{K + L} = r^{\frac{1}{2}} f(K, L) < rf(K, L)$ (3′)

So production function is decreasing returns to scale(DRS).

(b)

$$TRS = \frac{\partial y/\partial L}{\partial y/\partial K} = \frac{\frac{1}{2\sqrt{K+L}}}{\frac{1}{2\sqrt{K+L}}} = 1 \qquad (2')$$

Notice $\frac{\omega_L}{\omega_K} = 2$ so $TRS < \frac{\omega_L}{\omega_K}$. (2')
Hence $L = 0$ and $y = \sqrt{K} \Longrightarrow K = y^2$ (2')
Then $c(y) = 2 \times 0 + 1 \times y^2 = y^2$ (1')
The cost function is plotted by Figure 8. (1')

(c) $TC(y) = FC + VC(y) = 1 + y^2$ (1')

$$ATC(y) = \frac{TC(y)}{y} = \frac{1 + y^2}{y} \qquad (1')$$

$$MC(y) = 2y \qquad (1')$$

To find minimum efficient scale(MES), let $MC(y) = ATC(y) \Longrightarrow 2y = \frac{1+y^2}{y} \Longrightarrow y^{MES} = 1$ (2')

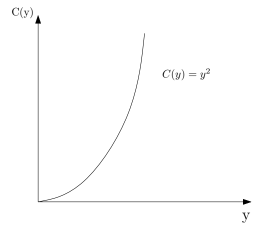

Figure 8: Problem 3(b)

Then $p^{MES} = MC(y^{MES}) = 2 \times 1 = 2$ (1')
When the firm supply positive quantity, $p = MC(y) \Longrightarrow y = \frac{p}{2}$. (1')
Supply curve is given by (2')

$$y = \begin{cases} p/2 & \text{if } p \geq 2, \\ 0 & \text{if } p < 2. \end{cases}$$

The supply curve is provided by Figure 9. (1')

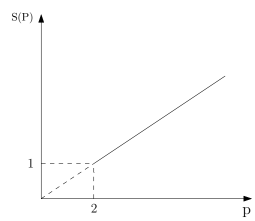

Figure 9: Problem 3(c)

(d) When there is free entry, the firm will production quantity $y^{MSE} = 1$ and the market price is $p^{MSE} = 2$. (2')
Total demand $D(p^{MSE}) = 40 - p^{MSE} = 38$ (1')
Total number of firms $N = \frac{D(p^{MSE})}{y^{MSE}} = 38$ (1')

Problem 4. (a) $E(C) = \frac{1}{2} \times 10 + \frac{1}{2} \times 0 = 5$ (2')
$E(u(C)) = \frac{1}{2} \times 10 \times 10 + \frac{1}{2} \times 10 \times 0 = 50$ Then $u(CE) = Eu(C) \Longrightarrow CE = 5$ (2')
Thus $E(C) = CE$ (1')
That's because Robert's utility function is linear thus he is risk neutral. (1')

(b) Notice both Andy and Bob's utility function is $U(x_1, x_2) = x_1 + x_2$ and thus $MRS^E = MRS^B \equiv 1$ for all allocations. Therefore, any points in the Edgeworth box is Pareto efficient. (2')
Then $\frac{p_1}{p_2} = MRS^E = 1$. Say $p_1 = 1 = p_2$. (2')
The competitive equilibrium is any points on the budget line within the Edgeworth box: (4')

$$x_1^E + x_2^E = 10 + 30 = 40$$
$$x_1^B + x_2^B = 10 + 20 = 30$$
$$x_1^E + x_1^B = 10 + 10 = 20$$
$$x_2^E + x_2^B = 30 + 20 = 50$$

for example $x_1^E = 5, x_2^E = 35; x_1^B = 15, x_2^B = 25$.

(c) When $\bar{K} = 1$, the production function is $y = 4L^{\frac{1}{2}}$. So $MPL = \frac{\partial y}{\partial L} = \frac{2}{\sqrt{L}} = \frac{\omega}{p}$ Thus $L^d = (\frac{p}{2\omega})^2$ (2')
If $L^s = 16$, real wage $\frac{\omega}{p} = \frac{2}{\sqrt{16}} = \frac{1}{2}$ (2')
Similarly, if $\frac{\omega^{min}}{p} = 2$, $L^d = (\frac{2}{2})^2 = 1$. So unemployment rate = (16-1)/16 = 15/16. (1')
The graph is given by Figure 10. (1')

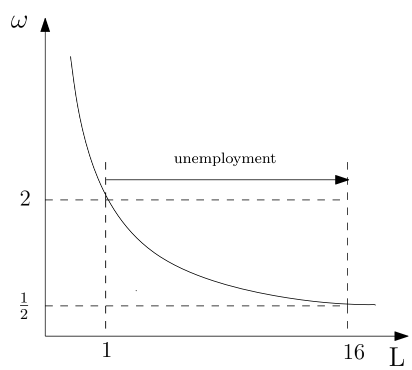

Figure 10: Problem 4(c)

Econ 301 Intermediate Microeconomics Spring 2013
Prof.Marek Weretka
Solution to Midterm 2 (Group B)

Problem 1. (a) expected utility function over lotteries

$$U(C_c, C_{nc}) = \frac{1}{2} \times 10 \ln C_c + \frac{1}{2} \times 10 \ln C_{nc}$$
$$= 5 \ln C_c + 5 \ln C_{nc} \qquad (3')$$

Oscar is risk averse since $U(c) = 10 \ln C$ is concave. (1')
Let $U(C_c, C_{nc}) = \bar{u}$ Then $C_{nc} = \frac{e^{\bar{u}/5}}{C_c}$. So we can obtain the indifference curve as below. (1')

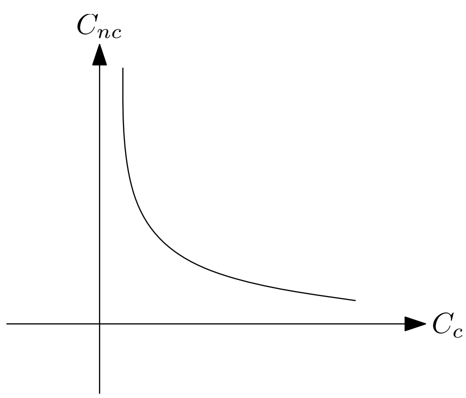

Figure 1: Problem 1(a)

(b) The C_c and C_{nc} can be represented as

$$C_c = 8 - \gamma x + x = 8 + 0.5x$$
$$C_{nc} = 16 - \gamma x = 16 - 0.5x \qquad (2')$$

Then,

$$C_{nc} + \frac{\gamma}{1-\gamma} C_c = 16 + 8\frac{\gamma}{1-\gamma} \Longrightarrow C_{nc} + C_c = 24 \qquad (3')$$

The budget line is shown as below: (2')

(c) By the utility function in part (a) and budget constraint in part (b), optimal choice of wealth levels are given by the short-cut formula:

$$C_c = \frac{5}{5+5} \times \frac{24}{1} = 12 \qquad (2')$$

$$C_{nc} = \frac{5}{5+5} \times \frac{24}{1} = 12 \qquad (2')$$

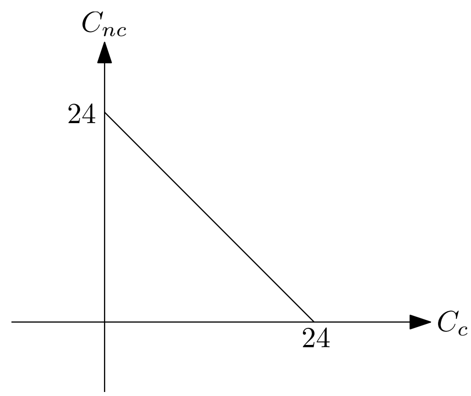

Figure 2: Problem 1(b)

Then $C_c = C_{nc}$ So Oscar is fully insured. (1')
By the formula in part(b), $x = 8$. (2')

(d) First calculate

$$MRS = \frac{5/C_{nc}}{5/C_c} = \frac{C_c}{C_{nc}} \qquad (2')$$

then by the 'secret of happiness',

$$MRS = \frac{p_{nc}}{p_c} \Longrightarrow \frac{C_c}{C_{nc}} = \frac{1-\gamma}{\gamma} \qquad (2')$$

If $\gamma > \pi_c = 0.5$, then $\frac{1-\gamma}{\gamma} < 1$. Therefor, $C_nc > C_c$. That is, Oscar is not fully insured. (2')

Problem 2. (a) Total endowment $\omega = \omega^E + \omega^B = (1,6) + (4,4) = (5,10)$. (1')
The Edgeworth box and initial endowment are in the following Figure 1. (1')

(b) An allocation is Pareto Efficient if there is no other allocation which would make one of them strictly better off without hurting any of the others. (2')
$\Longrightarrow$) Suppose that $MRS^E \neq MRS^B$ when an allocation is Pareto Efficient. Then the two indifference curves passing through this point will intersect as in figure 4. It's obvious that the points in the shadowed area will make both of them not worse off and at least one of them strictly better off. That's a contradiction with the assumption. Then the necessity follows immediately. (3')
$\Longleftarrow$) Now assume the condition $MRS^E = MRS^B$ is satisfied at an allocation a. Then the two indifference curves passing through this point will be tangent with each other as in figure 5. Then check all

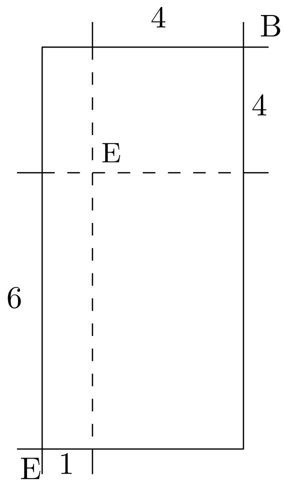

Figure 3: Problem 2(a)

the points in the areas A, B, C, D except point a in the Edgeworth box and all of them will make at least of Elisa and Ben strictly worse off. So allocation a is Pareto efficient by definition. (3')

(c)

$$MRS^E = MRS^B \Longrightarrow \frac{x_2^E}{x_1^E} = \frac{x_2^E}{x_1^E} \qquad (2')$$

Also

$$x_1^E + x_1^B = 10, x_2^E + x_2^B = 5 \qquad (2')$$

Substitute the last two equation to the first one.

$$x_2^E = 2x_1^E, x_2^B = 2x_1^E \qquad (2')$$

the contract curve is plotted in figure 6. (2')

(d) Normalize $p_2 = 1$. the budget constraints are:

$$2x_1^E + x_2^E = p_1 + 6$$
$$2x_1^B + x_2^B = 4P_1 + 4 \qquad (1')$$

Then

$$x_1^E = \frac{3}{3+3}\frac{p_1+6}{p_1}$$
$$x_1^B = \frac{3}{3+3}\frac{4P_1+4}{p_1} \qquad (1')$$

Thus

$$\frac{3}{3+3}\frac{p_1+6}{p_1} + \frac{3}{3+3}\frac{4P_1+4}{p_1} = 25 \Longrightarrow p_1 = 2 \qquad (2')$$

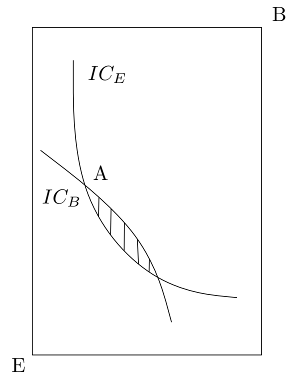

Figure 4: Problem 2(b)

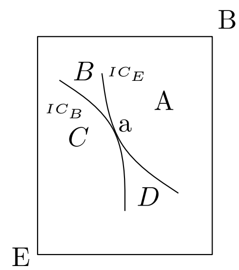

Figure 5: Problem 2(b)

Solve the equations and the result follows:

$$x_1^E = 2, x_2^E = 4 \qquad (2')$$

$$x_1^B = 3, x_2^B = 6 \qquad (2')$$

The competitive equilibrium is indicated in the Edgeworth box in Figure 7.

(e) Calculate and compare the MRS's.

$$MRS^E = \frac{x_2^E}{x_1^E} = \frac{4}{2} = 2 \qquad (1')$$

$$MRS^B = \frac{x_2^B}{x_1^B} = \frac{6}{3} = 2 \qquad (1')$$

So obviously $MRS^E = MRS^B$.

(f) any numbers satisfying $\frac{p_1}{p_2} = 2$, for example, $p_1 = 4, p_2 = 2$. (2')

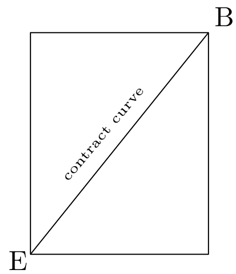

Figure 6: Problem 2(c)

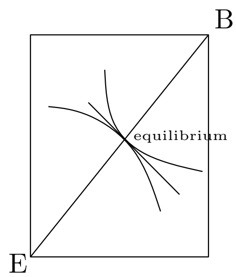

Figure 7: Problem 2(d)

Problem 3. (a) The production function $y = f(K, L) = \sqrt{K + L}$

$$f(rK, rL) = \sqrt{rK + rL} = \sqrt{r(K + L)} = \sqrt{r}\sqrt{K + L} = r^{\frac{1}{2}} f(K, L) < rf(K, L) \qquad (3')$$

So production function is decreasing returns to scale(DRS).

(b)

$$TRS = \frac{\partial y/\partial L}{\partial y/\partial K} = \frac{\frac{1}{2\sqrt{K+L}}}{\frac{1}{2\sqrt{K+L}}} = 1 \qquad (2')$$

Notice $\frac{\omega_L}{\omega_K} = \frac{1}{2}$ so $TRS > \frac{\omega_L}{\omega_K}$. (2')
Hence $K = 0$ and $y = \sqrt{L} \Longrightarrow L = y^2$ (2')
Then $c(y) = 2 \times 0 + 1 \times y^2 = y^2$ (1')
The cost function is plotted by Figure 8. (1')

(c) $TC(y) = FC + VC(y) = 1 + y^2$ (1')

$$ATC(y) = \frac{TC(y)}{y} = \frac{1 + y^2}{y} \qquad (1')$$

$$MC(y) = 2y \qquad (1')$$

To find minimum efficient scale(MES), let $MC(y) = ATC(y) \Longrightarrow 2y = \frac{1+y^2}{y} \Longrightarrow y^{MES} = 1$ (2')

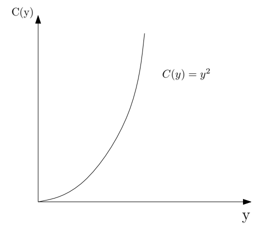

Figure 8: Problem 3(b)

Then $p^{MES} = MC(y^{MES}) = 2 \times 1 = 2$ (1')
When the firm supply positive quantity, $p = MC(y) \Longrightarrow y = \frac{p}{2}$. (1')
Supply curve is given by (2')

$$y = \begin{cases} p/2 & \text{if } p \geq 2, \\ 0 & \text{if } p < 2. \end{cases}$$

The supply curve is provided by Figure 9. (1')

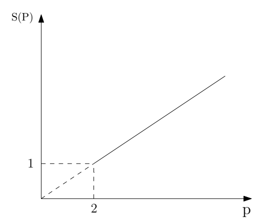

Figure 9: Problem 3(c)

(d) When there is free entry, the firm will production quantity $y^{MSE} = 1$ and the market price is $p^{MSE} = 2$. (2')
Total demand $D(p^{MSE}) = 20 - p^{MSE} = 18$ (1')
Total number of firms $N = \frac{D(p^{MSE})}{y^{MSE}} = 18$ (1')

Problem 4. (a) $E(C) = \frac{1}{2} \times 12 + \frac{1}{2} \times 0 = 6$ (2')
$E(u(C)) = \frac{1}{2} \times 2 \times 12 + \frac{1}{2} \times 2 \times 0 = 12$ Then $u(CE) = Eu(C) \Longrightarrow CE = 6$ (2')
Thus $E(C) = CE$ (1')
That's because Robert's utility function is linear thus he is risk neutral. (1')

(b) Notice both Andy and Bob's utility function is $U(x_1, x_2) = x_1 + x_2$ and thus $MRS^E = MRS^B \equiv 1$ for all allocations. Therefore, any points in the Edgeworth box is Pareto efficient. (2')
Then $\frac{p_1}{p_2} = MRS^E = 1$. Say $p_1 = 1 = p_2$. (2')
The competitive equilibrium is any points on the budget line within the Edgeworth box: (4')

$$\begin{aligned} x_1^E + x_2^E &= 10 + 40 = 50 \\ x_1^B + x_2^B &= 20 + 20 = 40 \\ x_1^E + x_1^B &= 10 + 20 = 30 \\ x_2^E + x_2^B &= 40 + 20 = 60 \end{aligned}$$

for example $x_1^E = 5, x_2^E = 45; x_1^B = 25, x_2^B = 15$.

(c) When $\bar{K} = 1$, the production function is $y = 12L^{\frac{1}{2}}$. So $MPL = \frac{\partial y}{\partial L} = \frac{6}{\sqrt{L}} = \frac{\omega}{p}$ Thus $L^d = (\frac{p}{6\omega})^2$ (2')
If $L^s = 9$, real wage $\frac{\omega}{p} = \frac{6}{\sqrt{9}} = 2$ (2')
Similarly, if $\frac{\omega^{min}}{p} = 6$, $L^d = (\frac{6}{6})^2 = 1$. So unemployment rate = (9-1)/9 = 8/9. (1')
The graph is given by Figure 10. (1')

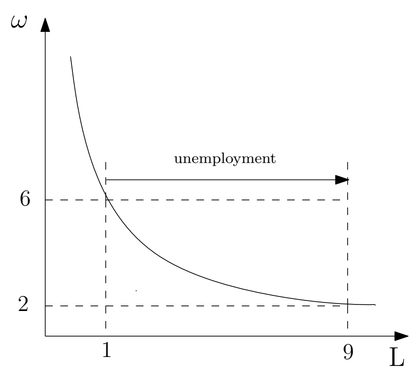

Figure 10: Problem 4(c)

Econ 301 Intermediate Microeconomics Spring 2013
Prof.Marek Weretka
Solution to Midterm 2 (Group C)

Problem 1. (a) expected utility function over lotteries

$$U(C_c, C_{nc}) = \frac{1}{2} \times 10 \ln C_c + \frac{1}{2} \times 10 \ln C_{nc}$$
$$= 5 \ln C_c + 5 \ln C_{nc} \qquad (3')$$

Oscar is risk averse since $U(c) = 10 \ln C$ is concave. (1')
Let $U(C_c, C_{nc}) = \bar{u}$ Then $C_{nc} = \frac{e^{\bar{u}/5}}{C_c}$. So we can obtain the indifference curve as below. (1')

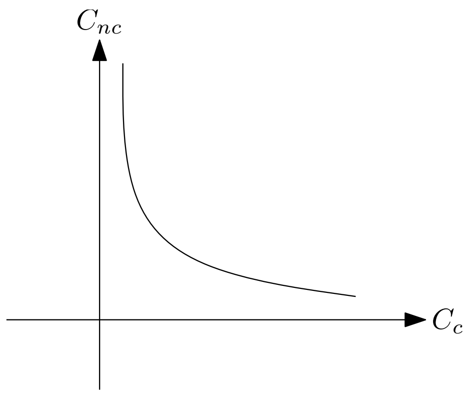

Figure 1: Problem 1(a)

(b) The C_c and C_{nc} can be represented as

$$C_c = 1 - \gamma x + x = 1 + 0.5x$$
$$C_{nc} = 2 - \gamma x = 2 - 0.5x \qquad (2')$$

Then,

$$C_{nc} + \frac{\gamma}{1-\gamma} C_c = 2 + 1 \frac{\gamma}{1-\gamma} \Longrightarrow C_{nc} + C_c = 3 \qquad (3')$$

The budget line is shown as below: (2')

(c) By the utility function in part (a) and budget constraint in part (b), optimal choice of wealth levels are given by the short-cut formula:

$$C_c = \frac{5}{5+5} \times \frac{3}{1} = 1.5 \qquad (2')$$

$$C_{nc} = \frac{5}{5+5} \times \frac{3}{1} = 1.5 \qquad (2')$$

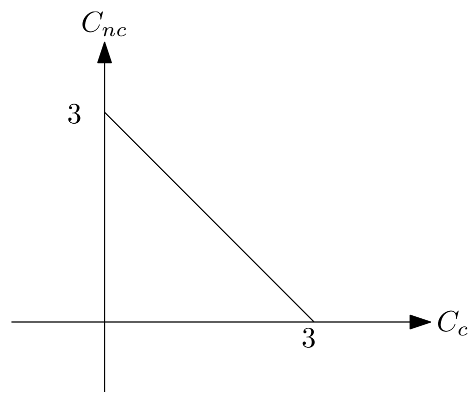

Figure 2: Problem 1(b)

Then $C_c = C_{nc}$ So Oscar is fully insured. (1')
By the formula in part(b), $x = 1$. (2')

(d) First calculate

$$MRS = \frac{5/C_{nc}}{5/C_c} = \frac{C_c}{C_{nc}} \qquad (2')$$

then by the 'secret of happiness',

$$MRS = \frac{p_{nc}}{p_c} \Longrightarrow \frac{C_c}{C_{nc}} = \frac{1-\gamma}{\gamma} \qquad (2')$$

If $\gamma > \pi_c = 0.5$, then $\frac{1-\gamma}{\gamma} < 1$. Therefor, $C_nc > C_c$. That is, Oscar is not fully insured. (2')

Problem 2. (a) Total endowment $\omega = \omega^E + \omega^B = (10, 60) + (40, 40) = (50, 100)$. (1')
The Edgeworth box and initial endowment are in the following figure.

(b) An allocation is Pareto Efficient if there is no other allocation which would make one of them strictly better off without hurting any of the others. (2')
$\Longrightarrow$) Suppose that $MRS^E \neq MRS^B$ when an allocation is Pareto Efficient. Then the two indifference curves passing through this point will intersect as in figure 4. It's obvious that the points in the shadowed area will make both of them not worse off and at least one of them strictly better off. That's a contradiction with the assumption. Then the necessity follows immediately. (3')
$\Longleftarrow$) Now assume the condition $MRS^E = MRS^B$ is satisfied at an allocation a. Then the two indifference curves passing through this point will be tangent with each other as in figure 5. Then check all

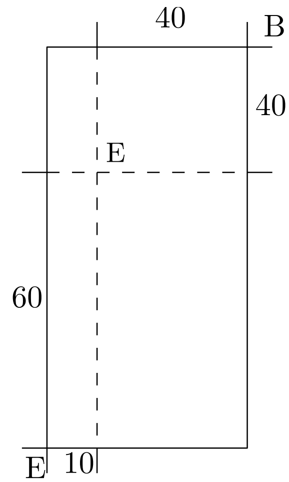

Figure 3: Problem 2(a)

the points in the areas A, B, C, D except point a in the Edgeworth box and all of them will make at least of Elisa and Ben strictly worse off. So allocation a is Pareto efficient by definition. (3')

(c)

$$MRS^E = MRS^B \Longrightarrow \frac{x_2^E}{x_1^E} = \frac{x_2^E}{x_1^E} \qquad (2')$$

Also

$$x_1^E + x_1^B = 50, x_2^E + x_2^B = 100 \qquad (2')$$

Substitute the last two equation to the first one.

$$x_2^E = 2x_1^E, x_2^B = 2x_1^E \qquad (2')$$

the contract curve is plotted in figure 6. (2')

(d) Normalize $p_2 = 1$. the budget constraints are:

$$2x_1^E + x_2^E = 10p_1 + 60$$
$$2x_1^B + x_2^B = 40P_1 + 40 \qquad (1')$$

Then

$$x_1^E = \frac{8}{8+8}\frac{10p_1 + 60}{p_1}$$
$$x_1^B = \frac{8}{8+8}\frac{40P_1 + 40}{p_1} \qquad (1')$$

Thus

$$\frac{8}{8+8}\frac{10p_1 + 60}{p_1} + \frac{8}{8+8}\frac{40P_1 + 40}{p_1} = 50 \Longrightarrow p_1 = 2 \qquad (2')$$

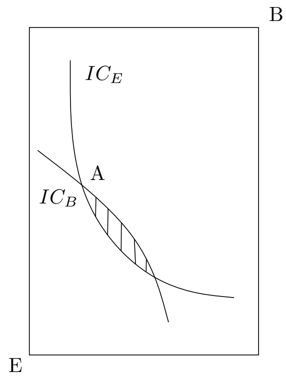

Figure 4: Problem 2(b)

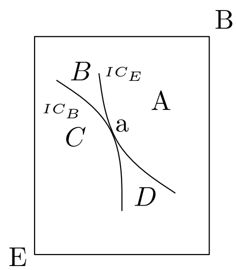

Figure 5: Problem 2(b)

Solve the equations and the result follows:

$$x_1^E = 20, x_2^E = 40 \qquad (2')$$

$$x_1^B = 30, x_2^B = 60 \qquad (2')$$

The competitive equilibrium is indicated in the Edgeworth box in Figure 7.

(e) Calculate and compare the MRS's.

$$MRS^E = \frac{x_2^E}{x_1^E} = \frac{40}{20} = 2 \qquad (1')$$

$$MRS^B = \frac{x_2^B}{x_1^B} = \frac{60}{30} = 2 \qquad (1')$$

So obviously $MRS^E = MRS^B$.

(f) any numbers satisfying $\frac{p_1}{p_2} = 2$, for example, $p_1 = 4, p_2 = 2$. (2')

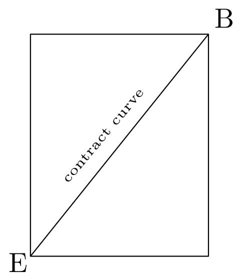

Figure 6: Problem 2(c)

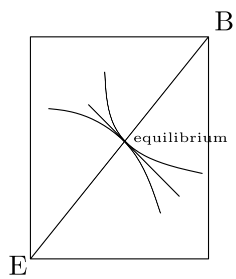

Figure 7: Problem 2(d)

Problem 3. (a) The production function $y = f(K, L) = \sqrt{2K + 2L}$

$$f(rK, rL) = \sqrt{2rK + 2rL} = \sqrt{r(2K + 2L)} = \sqrt{r}\sqrt{2K + 2L} = r^{\frac{1}{2}} f(K, L) < rf(K, L) \qquad (3')$$

So production function is decreasing returns to scale(DRS).

(b)

$$TRS = \frac{\partial y/\partial L}{\partial y/\partial K} = \frac{\frac{1}{\sqrt{K+L}}}{\frac{1}{\sqrt{K+L}}} = 1 \qquad (2')$$

Notice $\frac{\omega_L}{\omega_K} = 2$ so $TRS < \frac{\omega_L}{\omega_K}$. (2')
Hence $L = 0$ and $y = \sqrt{2K} \Longrightarrow K = \frac{1}{2}y^2$ (2')
Then $c(y) = 4 \times 0 + 2 \times \frac{1}{2}y^2 = y^2$ (1')
The cost function is plotted by Figure 8. (1')

(c) $TC(y) = FC + VC(y) = 1 + y^2$ (1')

$$ATC(y) = \frac{TC(y)}{y} = \frac{1 + y^2}{y} \qquad (1')$$

$$MC(y) = 2y \qquad (1')$$

To find minimum efficient scale(MES), let $MC(y) = ATC(y) \Longrightarrow 2y = \frac{1+y^2}{y} \Longrightarrow y^{MES} = 1$ (2')

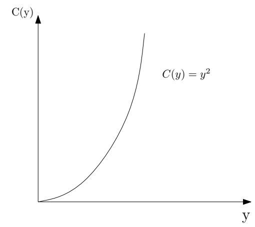

Figure 8: Problem 3(b)

Then $p^{MES} = MC(y^{MES}) = 2 \times 1 = 2$ (1')
When the firm supply positive quantity, $p = MC(y) \implies y = \frac{p}{2}$. (1')
Supply curve is given by (2')

$$y = \begin{cases} p/2 & \text{if } p \geq 2, \\ 0 & \text{if } p < 2. \end{cases}$$

The supply curve is provided by Figure 9. (1')

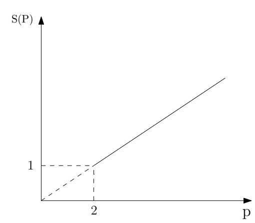

Figure 9: Problem 3(c)

(d) When there is free entry, the firm will production quantity $y^{MSE} = 1$ and the market price is $p^{MSE} = 2$. (2')
Total demand $D(p^{MSE}) = 40 - p^{MSE} = 38$ (1')
Total number of firms $N = \frac{D(p^{MSE})}{y^{MSE}} = 38$ (1')

Problem 4. (a) $E(C) = \frac{1}{2} \times 16 + \frac{1}{2} \times 0 = 8$ (2')
$E(u(C)) = \frac{1}{2} \times 10 \times 16 + \frac{1}{2} \times 10 \times 0 = 80$ Then $u(CE) = Eu(C) \implies CE = 8$ (2')
Thus $E(C) = CE$ (1')
That's because Robert's utility function is linear thus he is risk neutral. (1')

(b) Notice both Andy and Bob's utility function is $U(x_1, x_2) = x_1 + x_2$ and thus $MRS^E = MRS^B \equiv 1$ for all allocations. Therefore, any points in the Edgeworth box is Pareto efficient. (2')
Then $\frac{p_1}{p_2} = MRS^E = 1$. Say $p_1 = 1 = p_2$. (2')
The competitive equilibrium is any points on the budget line within the Edgeworth box: (4')

$$x_1^E + x_2^E = 10 + 40 = 50$$
$$x_1^B + x_2^B = 20 + 20 = 40$$
$$x_1^E + x_1^B = 10 + 20 = 30$$
$$x_2^E + x_2^B = 40 + 20 = 60$$

for example $x_1^E = 5, x_2^E = 45; x_1^B = 25, x_2^B = 15$.

(c) When $\bar{K} = 1$, the production function is $y = 16L^{\frac{1}{2}}$. So $MPL = \frac{\partial y}{\partial L} = \frac{8}{\sqrt{L}} = \frac{\omega}{p}$ Thus $L^d = (\frac{p}{8\omega})^2$ (2')
If $L^s = 16$, real wage $\frac{\omega}{p} = \frac{8}{\sqrt{16}} = 2$ (2')
Similarly, if $\frac{\omega^{min}}{p} = 8$, $L^d = (\frac{8}{8})^2 = 1$. So unemployment rate = (16-1)/16 = 15/16. (1')
The graph is given by Figure 10. (1')

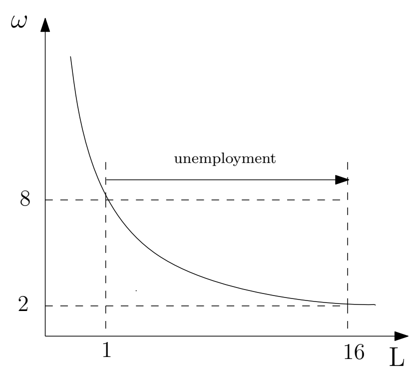

Figure 10: Problem 4(c)

Econ 301 Intermediate Microeconomics Spring 2013
Prof.Marek Weretka
Solution to Midterm 2 (Group D)

Problem 1. (a) expected utility function over lotteries

$$U(C_c, C_{nc}) = \frac{1}{2} \times 10 \ln C_c + \frac{1}{2} \times 10 \ln C_{nc}$$
$$= 5 \ln C_c + 5 \ln C_{nc} \qquad (3')$$

Oscar is risk averse since $U(c) = 10 \ln C$ is concave. (1')
Let $U(C_c, C_{nc}) = \bar{u}$ Then $C_{nc} = \frac{e^{\bar{u}/5}}{C_c}$. So we can obtain the indifference curve as below. (1')

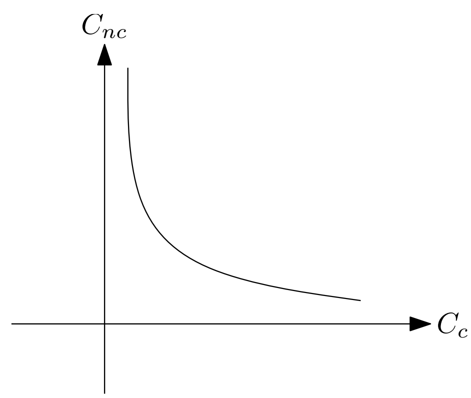

Figure 1: Problem 1(a)

(b) The C_c and C_{nc} can be represented as

$$C_c = 4 - \gamma x + x = 4 + 0.5x$$
$$C_{nc} = 8 - \gamma x = 8 - 0.5x \qquad (2')$$

Then,

$$C_{nc} + \frac{\gamma}{1-\gamma} C_c = 8 + 4\frac{\gamma}{1-\gamma} \Longrightarrow C_{nc} + C_c = 12 \qquad (3')$$

The budget line is shown as below: (2')

(c) By the utility function in part (a) and budget constraint in part (b), optimal choice of wealth levels are given by the short-cut formula:

$$C_c = \frac{5}{5+5} \times \frac{12}{1} = 6 \qquad (2')$$

$$C_{nc} = \frac{5}{5+5} \times \frac{12}{1} = 6 \qquad (2')$$

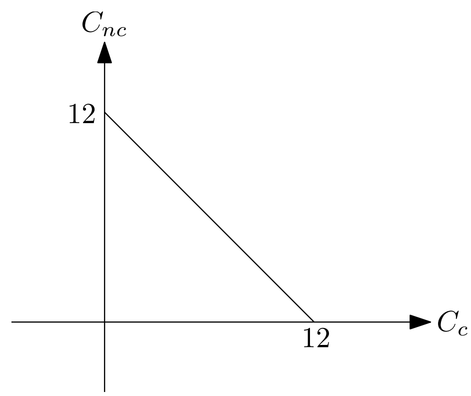

Figure 2: Problem 1(b)

Then $C_c = C_{nc}$ So Oscar is fully insured. (1')
By the formula in part(b), $x = 4$. (2')

(d) First calculate

$$MRS = \frac{5/C_{nc}}{5/C_c} = \frac{C_c}{C_{nc}} \qquad (2')$$

then by the 'secret of happiness',

$$MRS = \frac{p_{nc}}{p_c} \Longrightarrow \frac{C_c}{C_{nc}} = \frac{1-\gamma}{\gamma} \qquad (2')$$

If $\gamma > \pi_c = 0.5$, then $\frac{1-\gamma}{\gamma} < 1$. Therefor, $C_nc > C_c$. That is, Oscar is not fully insured. (2')

Problem 2. (a) Total endowment $\omega = \omega^E + \omega^B = (5, 30) + (20, 20) = (25, 50)$. (1')
The Edgeworth box and initial endowment are in the following figure.

(b) An allocation is Pareto Efficient if there is no other allocation which would make one of them strictly better off without hurting any of the others. (2')
$\Longrightarrow$) Suppose that $MRS^E \neq MRS^B$ when an allocation is Pareto Efficient. Then the two indifference curves passing through this point will intersect as in figure 4. It's obvious that the points in the shadowed area will make both of them not worse off and at least one of them strictly better off. That's a contradiction with the assumption. Then the necessity follows immediately. (3')
$\Longleftarrow$) Now assume the condition $MRS^E = MRS^B$ is satisfied at an allocation a. Then the two indifference curves passing through this point will be tangent with each other as in figure 5. Then check all

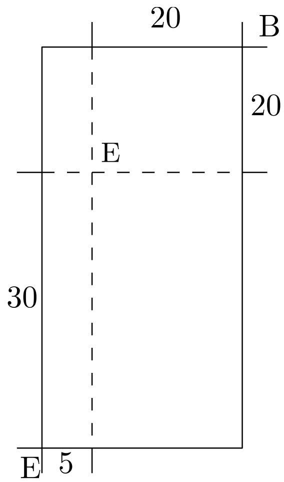

Figure 3: Problem 2(a)

the points in the areas A, B, C, D except point a in the Edgeworth box and all of them will make at least of Elisa and Ben strictly worse off. So allocation a is Pareto efficient by definition. (3')

(c)

$$MRS^E = MRS^B \Longrightarrow \frac{x_2^E}{x_1^E} = \frac{x_2^E}{x_1^E} \qquad (2')$$

Also

$$x_1^E + x_1^B = 25, x_2^E + x_2^B = 50 \qquad (2')$$

Substitute the last two equation to the first one.

$$x_2^E = 2x_1^E, x_2^B = 2x_1^E \qquad (2')$$

the contract curve is plotted in figure 6. (2')

(d) Normalize $p_2 = 1$. the budget constraints are:

$$\begin{aligned} 2x_1^E + x_2^E &= 5p_1 + 30 \\ 2x_1^B + x_2^B &= 20P_1 + 20 \qquad (1') \end{aligned}$$

Then

$$\begin{aligned} x_1^E &= \frac{2}{2+2}\frac{5p_1 + 30}{p_1} \\ x_1^B &= \frac{2}{2+2}\frac{20P_1 + 20}{p_1} \qquad (1') \end{aligned}$$

Thus

$$\frac{2}{2+2}\frac{5p_1 + 30}{p_1} + \frac{2}{2+2}\frac{20P_1 + 20}{p_1} = 25 \Longrightarrow p_1 = 2 \qquad (2')$$

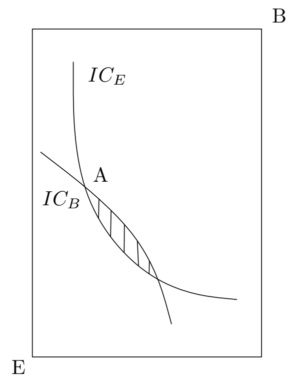

Figure 4: Problem 2(b)

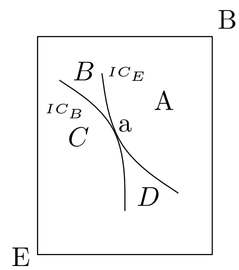

Figure 5: Problem 2(b)

Solve the equations and the result follows:

$$x_1^E = 10, x_2^E = 20 \qquad (2')$$

$$x_1^B = 15, x_2^B = 30 \qquad (2')$$

The competitive equilibrium is indicated in the Edgeworth box in Figure 7.

(e) Calculate and compare the MRS's.

$$MRS^E = \frac{x_2^E}{x_1^E} = \frac{20}{10} = 2 \qquad (1')$$

$$MRS^B = \frac{x_2^B}{x_1^B} = \frac{30}{15} = 2 \qquad (1')$$

So obviously $MRS^E = MRS^B$.

(f) any numbers satisfying $\frac{p_1}{p_2} = 2$, for example, $p_1 = 4, p_2 = 2$. (2')

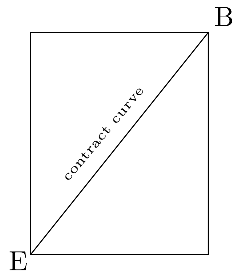

Figure 6: Problem 2(c)

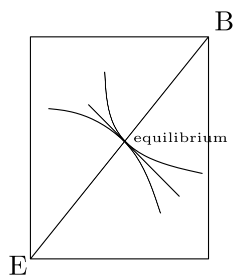

Figure 7: Problem 2(d)

Problem 3. (a) The production function $y = f(K, L) = \sqrt{2K + 2L}$

$$f(rK, rL) = \sqrt{2rK + 2rL} = \sqrt{r(2K + 2L)} = \sqrt{r}\sqrt{2K + 2L} = r^{\frac{1}{2}} f(K, L) < rf(K, L) \qquad (3')$$

So production function is decreasing returns to scale(DRS).

(b)

$$TRS = \frac{\partial y/\partial L}{\partial y/\partial K} = \frac{\frac{1}{2\sqrt{K+L}}}{\frac{1}{2\sqrt{K+L}}} = 1 \qquad (2')$$

Notice $\frac{\omega_L}{\omega_K} = \frac{1}{2}$ so $TRS > \frac{\omega_L}{\omega_K}$. (2')
Hence $K = 0$ and $y = \sqrt{2L} \Longrightarrow L = \frac{1}{2}y^2$ (2')
Then $c(y) = 4 \times 0 + 2 \times \frac{1}{2}y^2 = y^2$ (1')
The cost function is plotted by Figure 8. (1')

(c) $TC(y) = FC + VC(y) = 1 + y^2$ (1')

$$ATC(y) = \frac{TC(y)}{y} = \frac{1 + y^2}{y} \qquad (1')$$

$$MC(y) = 2y \qquad (1')$$

To find minimum efficient scale(MES), let $MC(y) = ATC(y) \Longrightarrow 2y = \frac{1+y^2}{y} \Longrightarrow y^{MES} = 1$ (2')

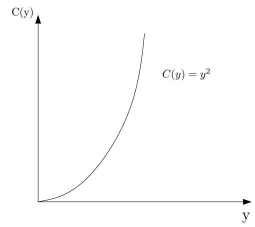

Figure 8: Problem 3(b)

Then $p^{MES} = MC(y^{MES}) = 2 \times 1 = 2$ (1')
When the firm supply positive quantity, $p = MC(y) \Longrightarrow y = \frac{p}{2}$. (1')
Supply curve is given by (2')

$$y = \begin{cases} p/2 & \text{if } p \geq 2, \\ 0 & \text{if } p < 2. \end{cases}$$

The supply curve is provided by Figure 9. (1')

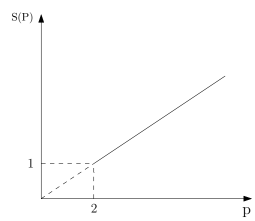

Figure 9: Problem 3(c)

(d) When there is free entry, the firm will production quantity $y^{MSE} = 1$ and the market price is $p^{MSE} = 2$. (2')
Total demand $D(p^{MSE}) = 20 - p^{MSE} = 18$ (1')
Total number of firms $N = \frac{D(p^{MSE})}{y^{MSE}} = 18$ (1')

Problem 4. (a) $E(C) = \frac{1}{2} \times 10 + \frac{1}{2} \times 0 = 5$ (2')
$E(u(C)) = \frac{1}{2} \times 10 \times 10 + \frac{1}{2} \times 10 \times 0 = 50$ Then $u(CE) = Eu(C) \Longrightarrow CE = 5$ (2')
Thus $E(C) = CE$ (1')
That's because Robert's utility function is linear thus he is risk neutral. (1')

(b) Notice both Andy and Bob's utility function is $U(x_1, x_2) = x_1 + x_2$ and thus $MRS^E = MRS^B \equiv 1$ for all allocations. Therefore, any points in the Edgeworth box is Pareto efficient. (2')
Then $\frac{p_1}{p_2} = MRS^E = 1$. Say $p_1 = 1 = p_2$. (2')
The competitive equilibrium is any points on the budget line within the Edgeworth box: (4')

$$x_1^E + x_2^E = 10 + 30 = 40$$
$$x_1^B + x_2^B = 10 + 20 = 30$$
$$x_1^E + x_1^B = 10 + 10 = 20$$
$$x_2^E + x_2^B = 30 + 20 = 50$$

for example $x_1^E = 5, x_2^E = 35; x_1^B = 15, x_2^B = 15$.

(c) When $\bar{K} = 1$, the production function is $y = 18L^{\frac{1}{2}}$. So $MPL = \frac{\partial y}{\partial L} = \frac{9}{\sqrt{L}} = \frac{\omega}{p}$ Thus $L^d = (\frac{p}{9\omega})^2$ (2')
If $L^s = 9$, real wage $\frac{\omega}{p} = \frac{9}{\sqrt{9}} = 3$ (2')
Similarly, if $\frac{\omega^{min}}{p} = 9$, $L^d = (\frac{9}{9})^2 = 1$. So unemployment rate = (9-1)/9 = 8/9. (1')
The graph is given by Figure 10. (1')

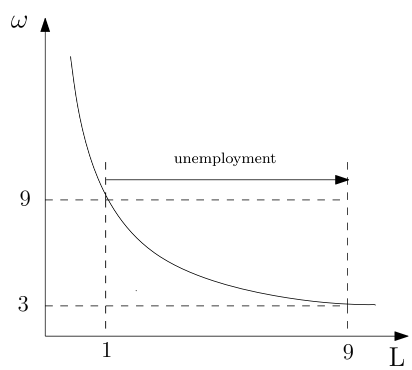

Figure 10: Problem 4(c)