Econ 301
Intermediate Microeconomics
Prof. Marek Weretka

Midterm 2 (Group A)

You have 70 minutes to complete the exam. The midterm consists of 4 questions (25, 20, 30, 25 points)+Just For Fun question.

Problem 1. (25p) (Intertemporal Choice)

Joseph earns m_1= \$10 when young and m_2= \$90 when old.

a) Write down Joseph's budget constraint (one inequality) and plot his budget set given interest rate $r = 200\%$ in the graph. Find PV and FV of income and mark it in your graph (give two numbers).

b) Joseph's utility is given by $U(C_1, C_2) = \ln(c_1) + \frac{1}{1+\delta}\ln(c_2)$ where discount rate is $\delta = 2$. Using magic formulas, find the optimal consumption plan and the optimal saving strategy (give three numbers C_1, C_2, S).

c) (Annuity) Joseph is contemplating leasing a car. Leasing would require three annual payments, each \$1600, starting next year (after which you can keep your car). Alternatively, he can buy a car for \$1,500 (payment this year). Which of the two alternatives should he choose if annual interest is $r = 100\%$? Why? (calculate PV of the two alternatives)

d) (Perpetuity) Derive the (general) formula for PV of perpetuity.

Problem 2. (20p) (Uncertainty and Insurance)

Trevor's motorbike is worth 16 (thousand \$). In an event (state of the world) of a crash, its value drops to 0 (and, hence, the bike is a lottery with $w = (w_1, w_2) = (16, 0)$). Assume that the probability of a crash is equal to $\frac{1}{2}$.

a) Find the expected value of the "bike" lottery $(16, 0)$ (one number) and its certainty equivalent CE, assuming Bernoulli utility function $u(y) = \sqrt{y}$ (one number). Is the certainty equivalent bigger or smaller than the expected value? Why? (one sentence) .

b) Write down the Von Neumann-Morgenstern (expected) utility function over lotteries $U(C_1; C_2)$ for Bernoulli utility function $u(y) = \ln y$. Is Trevor risk averse, risk neutral or risk loving? (chose one+ one sentence)

c) Assume insurance premium $\gamma = \frac{1}{2}$ and the utility function as in point b), find the optimal consumption (C_1, C_2) and insurance coverage x (give three numbers, you can use Magic Formulas). Is Trevor fully insured? Why? (one sentence Hint: is insurance fair?)

Problem 3. (30p) (Edgeworth box and equilibrium)

Jayden and Olivia are consume two types of commodities, ice cream x_1 and burgers x_2. Jayden is initially endowed with $\omega^J = (9, 1)$ of the two goods and Olivia's endowment is $\omega^O = (1, 9)$. The utility function of Jayden and Olivia is the same and is given by

$$U(x_1, x_2) = \frac{1}{3}\ln x_1 + \frac{1}{3}\ln x_2.$$

a) Plot an Edgeworth box and mark the initial endowments.

b) Find the competitive equilibrium (give six numbers) and depict the equilibrium in the Edgeworth box.

c) Verify that the allocation in the competitive equilibrium is Pareto efficient (compare two numbers).

d) Find analytically the contract curve (write down the condition for Pareto efficiency and solve for a contract curve line). Are the initial endowments located on this curve? (a yes or no answer)

Problem 4. (25p) (Producers)

A producer has the following technology

$$y = K^{\frac{1}{2}} L^{\frac{1}{2}}$$

a) Is MPL increasing, decreasing or constant (choose one) What are the returns to scale? (choose: IRS, DRS or CRS)

b) Given short-run level of capital $\bar{K} = 1$ derive the labor demand (formula) of a competitive firm. Find the equilibrium real wage rate if labor supply is given by $L^s = 9$ (one number).

c) Find the unemployment rate if the minimal (real) wage is $w^{min}/p = \frac{1}{2}$ (one number+graph).

d) Find analytically a cost function if $w_K = w_L = 2$ (formula). Plot the cost function in the graph.

Just For Fun

Give a definition of the Pareto efficient allocation. Prove that allocation is Pareto efficient if and only if "MRS" of consumers are equal (show the necessary and sufficient condition).

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Midterm 2 (Group B)

You have 70 minutes to complete the exam. The midterm consists of 4 questions (25, 20, 30, 25 points)+Just For Fun question.

Problem 1. (25p) (Intertemporal Choice)

Joseph earns m_1= \$20 when young and m_2= \$180 when old.

a) Write down Joseph's budget constraint (one inequality) and plot his budget set given interest rate $r = 200\%$ in the graph. Find PV and FV of income and mark it in your graph (give two numbers).

b) Joseph's utility is given by $U(C_1, C_2) = \ln(c_1) + \frac{1}{1+\delta}\ln(c_2)$ where discount rate is $\delta = 2$. Using magic formulas, find the optimal consumption plan and the optimal saving strategy (give three numbers C_1, C_2, S).

c) (Annuity) Joseph is contemplating leasing a car. Leasing would require three annual payments, each \$8,000, starting next year (after which you can keep your car). Alternatively, he can buy a car for \$7,500 (payment this year). Which of the two alternatives should he choose if annual interest is $r = 100\%$? Why? (calculate PV of the two alternatives)

d) (Perpetuity) Derive the (general) formula for PV of perpetuity.

Problem 2. (20p) (Uncertainty and Insurance)

Trevor's motorbike is worth 36 (thousand \$). In an event (state of the world) of a crash, its value drops to 0 (and, hence, the bike is a lottery with $w = (w_1, w_2) = (36, 0)$). Assume that the probability of a crash is equal to $\frac{1}{2}$.

a) Find the expected value of the "bike" lottery $(36, 0)$ (one number) and its certainty equivalent CE, assuming Bernoulli utility function $u(y) = \sqrt{y}$ (one number). Is the certainty equivalent bigger or smaller than the expected value? Why? (one sentence).

b) Write down the Von Neumann-Morgenstern (expected) utility function over lotteries $U(C_1; C_2)$ for Bernoulli utility function $u(y) = \ln y$. Is Trevor risk averse, risk neutral or risk loving? (chose one+ one sentence)

c) Assume insurance premium $\gamma = \frac{1}{2}$ and the utility function as in point b), find the optimal consumption (C_1, C_2) and insurance coverage x (give three numbers, you can use Magic Formulas). Is Trevor fully insured? Why? (one sentence Hint: is insurance fair?)

Problem 3. (30p) (Edgeworth box and equilibrium)

Jayden and Olivia are consume two types of commodities, ice cream x_1 and burgers x_2. Jayden is initially endowed with $\omega^J = (4, 6)$ of the two goods and Olivia's endowment is $\omega^O = (6, 4)$. The utility function of Jayden and Olivia is the same and is given by

$$U(x_1, x_2) = \frac{1}{5}\ln x_1 + \frac{1}{5}\ln x_2.$$

a) Plot an Edgeworth box and mark the initial endowments.

b) Find the competitive equilibrium (give six numbers) and depict the equilibrium in the Edgeworth box.

c) Verify that the allocation in the competitive equilibrium is Pareto efficient (compare two numbers).

d) Find analytically the contract curve (write down the condition for Pareto efficiency and solve for a contract curve line). Are the initial endowments located on this curve? (a yes or no answer)

Problem 4. (25p) (Producers)

A producer has the following technology

$$y = K^{\frac{1}{2}} L^{\frac{1}{2}}$$

a) Is MPL increasing, decreasing or constant (choose one) What are the returns to scale? (choose: IRS, DRS or CRS)

b) Given short-run level of capital $\bar{K} = 1$ derive the labor demand (formula) of a competitive firm. Find the equilibrium real wage rate if labor supply is given by $L^s = 9$ (one number).

c) Find the unemployment rate if the minimal (real) wage is $w^{min}/p = \frac{1}{2}$ (one number+graph).

d) Find analytically a cost function if $w_K = w_L = 3$ (formula). Plot the cost function in the graph.

Just For Fun

Give a definition of the Pareto efficient allocation. Prove that allocation is Pareto efficient if and only if "MRS" of consumers are equal (show the necessary and sufficient condition).

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Midterm 2 (Group C)

You have 70 minutes to complete the exam. The midterm consists of 4 questions (25, 20, 30, 25 points)+Just For Fun question.

Problem 1. (25p) (Intertemporal Choice)

Joseph earns $m_1 = \$10$ when young and $m_2 = \$60$ when old.

a) Write down Joseph's budget constraint (one inequality) and plot his budget set given interest rate $r = 100\%$ in the graph. Find PV and FV of income and mark it in your graph (give two numbers).

b) Joseph's utility is given by $U(C_1, C_2) = \ln(c_1) + \frac{1}{1+\delta}\ln(c_2)$ where discount rate is $\delta = 1$. Using magic formulas, find the optimal consumption plan and the optimal saving strategy (give three numbers C_1, C_2, S).

c) (Annuity) Joseph is contemplating leasing a car. Leasing would require three annual payments, each $\$2400$, starting next year (after which you can keep your car). Alternatively, he can buy a car for $\$2,300$ (payment this year). Which of the two alternatives should he choose if annual interest is $r = 100\%$? Why? (calculate PV of the two alternatives)

d) (Perpetuity) Derive the (general) formula for PV of perpetuity.

Problem 2. (20p) (Uncertainty and Insurance)

Trevor's motorbike is worth 64 (thousand \$). In an event (state of the world) of a crash, its value drops to 0 (and, hence, the bike is a lottery with $w = (w_1, w_2) = (64, 0)$). Assume that the probability of a crash is equal to $\frac{1}{2}$.

a) Find the expected value of the "bike" lottery $(64, 0)$ (one number) and its certainty equivalent CE, assuming Bernoulli utility function $u(y) = \sqrt{y}$ (one number). Is the certainty equivalent bigger or smaller than the expected value? Why? (one sentence) .

b) Write down the Von Neumann-Morgenstern (expected) utility function over lotteries $U(C_1; C_2)$ for Bernoulli utility function $u(y) = \ln y$. Is Trevor risk averse, risk neutral or risk loving? (chose one+ one sentence)

c) Assume insurance premium $\gamma = \frac{1}{2}$ and the utility function as in point b), find the optimal consumption (C_1, C_2) and insurance coverage x (give three numbers, you can use Magic Formulas). Is Trevor fully insured? Why? (one sentence Hint: is insurance fair?)

Problem 3. (30p) (Edgeworth box and equilibrium)

Jayden and Olivia are consume two types of commodities, ice cream x_1 and burgers x_2. Jayden is initially endowed with $\omega^J = (6, 14)$ of the two goods and Olivia's endowment is $\omega^O = (14, 6)$. The utility function of Jayden and Olivia is the same and is given by

$$U(x_1, x_2) = 3\ln x_1 + 3\ln x_2.$$

a) Plot an Edgeworth box and mark the initial endowments.

b) Find the competitive equilibrium (give six numbers) and depict the equilibrium in the Edgeworth box.

c) Verify that the allocation in the competitive equilibrium is Pareto efficient (compare two numbers).

d) Find analytically the contract curve (write down the condition for Pareto efficiency and solve for a contract curve line). Are the initial endowments located on this curve? (a yes or no answer)

Problem 4. (25p) (Producers)

A producer has the following technology

$$y = K^{\frac{1}{2}} L^{\frac{1}{2}}$$

a) Is MPL increasing, decreasing or constant (choose one) What are the returns to scale? (choose: IRS, DRS or CRS)

b) Given short-run level of capital $\bar{K} = 1$ derive the labor demand (formula) of a competitive firm. Find the equilibrium real wage rate if labor supply is given by $L^s = 9$ (one number).

c) Find the unemployment rate if the minimal (real) wage is $w^{min}/p = \frac{1}{2}$ (one number+graph).

d) Find analytically a cost function if $w_K = w_L = 5$ (formula). Plot the cost function in the graph.

Just For Fun

Give a definition of the Pareto efficient allocation. Prove that allocation is Pareto efficient if and only if "MRS" of consumers are equal (show the necessary and sufficient condition).

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Midterm 2 (Group D)

You have 70 minutes to complete the exam. The midterm consists of 4 questions (25, 20, 30, 25 points)+Just For Fun question.

Problem 1. (25p) (Intertemporal Choice)

Joseph earns m_1= \$10 when young and m_2= \$100 when old.

a) Write down Joseph's budget constraint (one inequality) and plot his budget set given interest rate $r = 100\%$ in the graph. Find PV and FV of income and mark it in your graph (give two numbers).

b) Joseph's utility is given by $U(C_1, C_2) = \ln(c_1) + \frac{1}{1+\delta}\ln(c_2)$ where discount rate is $\delta = 1$. Using magic formulas, find the optimal consumption plan and the optimal saving strategy (give three numbers C_1, C_2, S).

c) (Annuity) Joseph is contemplating leasing a car. Leasing would require three annual payments, each \$2400, starting next year (after which you can keep your car). Alternatively, he can buy a car for \$2,300 (payment this year). Which of the two alternatives should he choose if annual interest is $r = 100\%$? Why? (calculate PV of the two alternatives)

d) (Perpetuity) Derive the (general) formula for PV of perpetuity.

Problem 2. (20p) (Uncertainty and Insurance)

Trevor's motorbike is worth 100 (thousand \$). In an event (state of the world) of a crash, its value drops to 0 (and, hence, the bike is a lottery with $w = (w_1, w_2) = (100, 0)$). Assume that the probability of a crash is equal to $\frac{1}{2}$.

a) Find the expected value of the "bike" lottery $(100, 0)$ (one number) and its certainty equivalent CE, assuming Bernoulli utility function $u(y) = \sqrt{y}$ (one number). Is the certainty equivalent bigger or smaller than the expected value? Why? (one sentence) .

b) Write down the Von Neumann-Morgenstern (expected) utility function over lotteries $U(C_1; C_2)$ for Bernoulli utility function $u(y) = \ln y$. Is Trevor risk averse, risk neutral or risk loving? (chose one+ one sentence)

c) Assume insurance premium $\gamma = \frac{1}{2}$ and the utility function as in point b), find the optimal consumption (C_1, C_2) and insurance coverage x (give three numbers, you can use Magic Formulas). Is Trevor fully insured? Why? (one sentence Hint: is insurance fair?)

Problem 3. (30p) (Edgeworth box and equilibrium)

Jayden and Olivia are consume two types of commodities, ice cream x_1 and burgers x_2. Jayden is initially endowed with $\omega^J = (3, 17)$ of the two goods and Olivia's endowment is $\omega^O = (17, 3)$. The utility function of Jayden and Olivia is the same and is given by

$$U(x_1, x_2) = 5\ln x_1 + 5\ln x_2.$$

a) Plot an Edgeworth box and mark the initial endowments.

b) Find the competitive equilibrium (give six numbers) and depict the equilibrium in the Edgeworth box.

c) Verify that the allocation in the competitive equilibrium is Pareto efficient (compare two numbers).

d) Find analytically the contract curve (write down the condition for Pareto efficiency and solve for a contract curve line). Are the initial endowments located on this curve? (a yes or no answer)

Problem 4. (25p) (Producers)

A producer has the following technology

$$y = K^{\frac{1}{2}} L^{\frac{1}{2}}$$

a) Is MPL increasing, decreasing or constant (choose one) What are the returns to scale? (choose: IRS, DRS or CRS)

b) Given short-run level of capital $\bar{K} = 1$ derive the labor demand (formula) of a competitive firm. Find the equilibrium real wage rate $\frac{w_L}{p}$ if labor supply is given by $L^s = 9$ (one number).

c) Find the unemployment rate if the minimal (real) wage is $w^{min}/p = \frac{1}{2}$ (one number+graph).

d) Find analytically a cost function if $w_K = w_L = 8$ (formula). Plot the cost function in the graph.

Just For Fun

Give a definition of the Pareto efficient allocation. Prove that allocation is Pareto efficient if and only if "MRS" of consumers are equal (show the necessary and sufficient condition).

Second Midterm Solutions

Econ 301 - Spring 2012

Maximum Points per Question

1a. = 3	2a. = 7	3a. = 3	4a. = 4
1b. = 10	2b. = 4	3b. = 15	4b. = 8
1c. = 5	2c. = 9	3c. = 6	4c. = 5
1d. = 7		3d. = 6	4d. = 8

Group A

Problem #1

A) The budget constraint maps consumption between the two time periods. With the "young" period on the x-axis and the "old" period on the y-axis, the budget constraint appears as

$$\begin{aligned} C_1P_1 + C_2P_2 &\leq m_1 + m_2P_2 \\ P_2 &= \frac{1}{1+r} \\ C_1P_1 + C_2\frac{1}{3} &\leq \$10 + \$90\frac{1}{3} \\ \implies C_1P_1 + C_2\frac{1}{3} &\leq \$40 \end{aligned}$$

Future Value is the \$90 that Joseph gets when he is "old" in addition to the value of the \$10 he gets when he is "young" when he is old:

$$\$10(1+r) = \$30$$

This means that FV = \$120.

Present value is the \$10 Joseph has when he is young plus the value when he is "young" of the \$90 he will get when he is old:

$$\$90(\frac{1}{1+r}) + \$10 = \$40$$

This means that PV = \$40

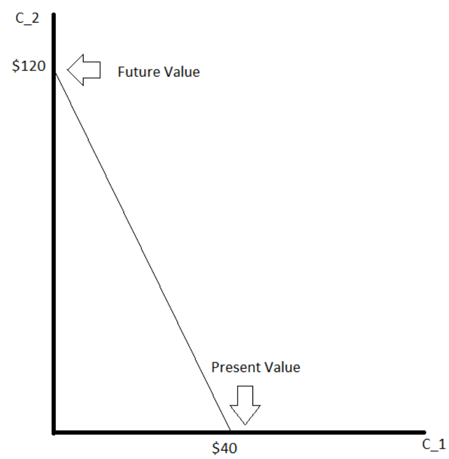


Figure 1: Budget Constraint

B) In terms of PV, the optimal demands for consumption are:

$$C_1 = \frac{1}{1+\frac{1}{1+\delta}} \times \frac{\$10P_1 + \$90P_2}{P_1}$$

$$\implies C_1 = \frac{3}{4} \times \frac{\$10 + \$90P_2}{1}$$

$$C_2 = \frac{\frac{1}{1+\delta}}{1+\frac{1}{1+\delta}} \times \frac{\$10P_1 + \$90P_2}{P_2}$$

$$\implies C_2 = \frac{1}{4} \times \frac{\$10 + \$90P_2}{P_2}$$

From the budget constraint we know that $C_1 + P_2C_2 \leq 40$, so combining this with the above equations we can confirm the value for P_2 we previously derived:

$$\begin{aligned}
\frac{3}{4} \times \frac{\$10 + \$90P_2}{1} + P_2(\frac{1}{4} \times \frac{\$10 + \$90P_2}{P_2}) &= \$40 \\
\$7.5 + \$67.5P_2 + \$2.5 + \$22.5P_2 &= \$40 \\
\$90P_2 &= \$30 \\
\Longrightarrow \; P_2 &= \frac{1}{3}
\end{aligned}$$

Plugging P_2 into the magic formulas for C_1 and C_2, we get:

$$C_1 = \frac{3}{4} \times \frac{\$10 + \$90\frac{1}{3}}{1} = \$30$$
$$C_2 = \frac{1}{4} \times \frac{\$10 + \$90\frac{1}{3}}{\frac{1}{3}} = \$30$$

Since Joseph is only endowed with \$10 in the first period, but consumes \$30, this means that he is borrowing (in present value) \$20 from the future (\$60 in FV). This makes his savings vector S = (-\$20,0).

C) The formula for an annuity of \$1,600 lasting for three periods, starting next period is:

$$\frac{\$1,600}{r}(1 - (\frac{1}{1+r}^{3})) = \$1,400 < \$1,500$$

This means that the lease is a better option than the purchase since it costs less in terms of present value and available information (assuming you ignore the value of the car remaining after three years).

D) The formula of a stream of payments that never ends is:

$$\begin{aligned}
PV &= \frac{x}{(1+r)} + \frac{x}{(1+r)^2} + \frac{x}{(1+r)^3} + ... \\
&= \frac{1}{(1+r)}[x + \frac{x}{(1+r)} + \frac{x}{(1+r)^2} + ...] \\
&= \frac{1}{(1+r)}[x + PV]
\end{aligned}$$

so we can solve for PV to get a more concise solution:

$$\begin{aligned}
(1 - \frac{1}{1+r})PV &= \frac{1}{1+r}x \\
(\frac{1+r}{1+r} - \frac{1}{1+r})PV &= \frac{1}{1+r}x \\
(\frac{r}{1+r})PV &= \frac{1}{1+r}x \\
PV &= \frac{x}{r}
\end{aligned}$$

Problem 2

A) To calculate the expected value it is necessary to find the value in each state and multiply it by the probability that each state will occur. The summation is then:

$$EV = 0 * \pi + 16 * (1 - \pi) = 8$$

The certainty equivalent is the amount of money that Trevor would take *for certain* in order to avoid the gamble that would leave him with the *expectation* of \$8. To calculate this you must first find the utility that the gamble provides, and then find the amount of money that would provide the same utility with a 100% probability:

$$\begin{aligned}
U(E(8)) &= \frac{1}{2}\sqrt{0} + \frac{1}{2}\sqrt{16} \\
&= 2 \\
U(CE) = 2 &\implies \sqrt{CE} = 2 \\
\implies CE &= 4
\end{aligned}$$

Here we clearly see that the certainty equivalent, 4, is smaller than the expected payoff from the gamble, 8. This makes sense because the concave shape of Trevor's utility function reflects his risk-averse preferences - meaning he is willing to take a smaller payout for certain than the one he would get in expectation to avoid the possibility of being left with no money at all.

B) The expected utility representation is:

$$U(C_1, C_2) = \frac{1}{2}\ln C_1 + \frac{1}{2}\ln C_2$$

The natural log is a concave function, meaning that Trevor is risk averse.
C) First, we calculate the formulas for consumption in each state of the world:

$$C_1 = 16 - x\gamma$$
$$C_2 = (1-\gamma)x$$

Knowing that x, the amount of insurance, is the same in both states of the world, we can solve both of the above equations for x and set them equal to one another:

$$\begin{aligned} x &= \frac{16 - C_1}{\gamma} \\ x &= \frac{C_2}{(1-\gamma)} \end{aligned}$$
$$\implies \frac{16 - C_1}{\gamma} = \frac{C_2}{(1-\gamma)}$$
$$\implies C_1 = 16 - \frac{\gamma}{1-\gamma} C_2$$

This final equation is our budget constraint. Normalizing P_1 to 1, this makes $P_2 = 1$ as well. Plugging these values into the magic formulas for demand we get:

$$\begin{aligned} C_1 &= \frac{1}{2} \times \frac{16}{P_1} \\ C_2 &= \frac{1}{2} \times \frac{16}{P_2} \\ C_1 = C_2 &= 8 \end{aligned}$$

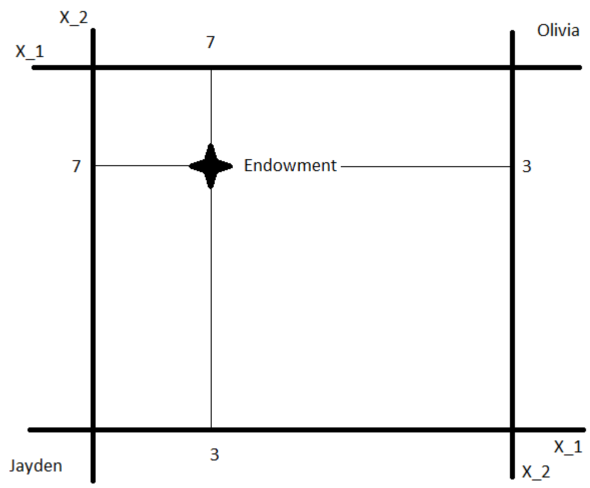
Going back to the first equation for C_1, we now have:

$$8 = 16 - \frac{1}{2}x$$
$$\implies x = 16$$

Consumption in both states of the world is 8, so that means that Trevor is fully insured. Full insurance means that your consumption does not depend on the state of the world. Trevor has chosen to fully insure in this case because the insurance premium, γ, is equal to the probability of loss, π.

Problem 3

A)



B) At an equilibrium, we need for MRS^J to equal MRS^O and also for MRS^J to be equal to $\frac{P_1}{P_2}$. We use the first identity to get the contract curve, the second to calculate the slope of the budget line. Given the endowment point, we can follow the budget line away from the endowment point to find its intersection with the contract curve, which is the equilibrium. Since the two utility functions are symmetrical, we can solve for Jayden and Olivia both by simply solving for Jayden. Analytically, this is done via the following equations: First, we normalize P_1 to equal 1 and use the Cobb-Douglas magic formulas to get the P_2:

$$x_1 = \frac{1}{2} \times \frac{3P_1 + 7P_2}{P_1}$$

$$x_2 = \frac{1}{2} \times \frac{3P_1 + 7P_2}{P_2}$$

$$x_1 + x_2 = 3P_1 + 7P_2$$

$$\implies \frac{1}{2} \times \frac{3P_1 + 7P_2}{P_1} + \frac{1}{2} \times \frac{3P_1 + 7P_2}{P_2} = 3P_1 + 7P_2$$

$$5P_2 + 5 = 3 + 7P_2$$

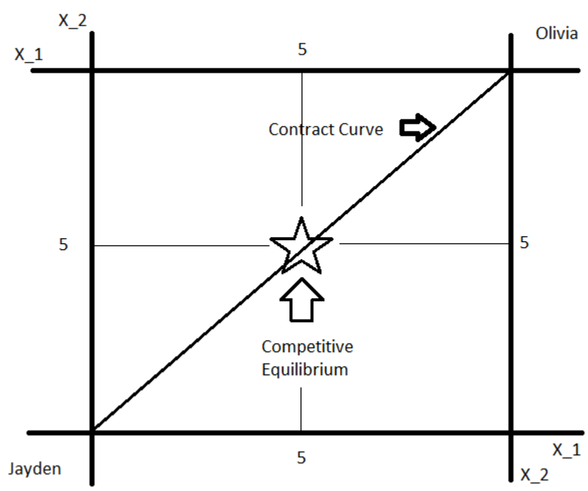
$$P_2 = 1$$

$$
\begin{aligned}
MRS^J = \frac{MU_1^J}{MU_2^J} &= \frac{\frac{1}{3x_1}}{\frac{1}{3x_2}} = \frac{P_1}{P_2} \\
\implies \frac{x_2}{x_1} &= \frac{P_1}{P_2} \\
\implies x_2 &= x_1
\end{aligned}
$$

This last step, however, was extraneous expect to show that the contract curve is the set of points where $x_1 = x_2$. Thus, our solution must satisfy this identity. Anyway, revisiting the Cobb-Douglas demands with P_2 in hand, we get:

$$
\begin{aligned}
x_1 &= \frac{1}{2} \times \frac{3P_1 + 7P_2}{P_1} \\
&= 5
\end{aligned}
$$

$$\implies x_1^J = x_2^J = x_1^O = x_2^O = 5 \ \& \ P_1 = P_2 = 1$$



C) As we previously stated, at the competitive equilibrium, it is required that $MRS^J = MRS^O$ which means that $\frac{x_2^J}{x_1^J} = \frac{x_2^O}{x_1^O}$ which is satisfied when $x_1^J = x_2^J = x_1^O = x_2^O = 5$ since $\frac{5}{5} = \frac{5}{5}$

D) Solving analytically for the contract curve requires knowledge of the total endowment of each good in the economy. Here, we can see from the individual endowments that there are 10 units of each good in the economy and that the edgeworth box depicting it is square. To solve for the slope of the contract curve use the equations:

$$MRS^J = \frac{MU_1^J}{MU_2^J} = \frac{\frac{1}{3x_1^J}}{\frac{1}{3x_2^J}} = \frac{x_2^J}{x_1^J} = MRS^O = \frac{MU_1^O}{MU_2^O} = \frac{10 - x_2^J}{10 - x_1^J}$$
$$\implies x_2^J = x_1^J$$

By symmetry this is true for both individuals. Clearly the initial endowments are not located on the contract curve (no).

Problem 4

A) The MPL is decreasing and there are constant returns to scale exhibited by this production function.

B) Labor demand can be calculated by setting MPL = $\frac{w}{p}$. Given that K is fixed, this makes the equation:

$$\begin{aligned}
\frac{1}{2}L^{-\frac{1}{2}} &= \frac{w}{p} \\
\frac{1}{2\sqrt{L}} &= \frac{w}{p} \\
\sqrt{L} &= \frac{p}{2w} \\
L^* &= (\frac{p}{2w})^2 \\
9 &= (\frac{p}{2w})^2 \\
\frac{1}{6} &= \frac{w}{p}
\end{aligned}$$

C)The unemployment rate is the ratio of the number of hours invidivduals are under-employed relative to the equilibrium level of employment in a free market. Here, when wages are free to adjust due to supply and demand the market clears at a wage real wage of $\frac{1}{6}$ and 9 hours of labor. When the wage is constrained to be $\frac{1}{2}$ the labor demand falls, while the supply remains constant at 9 hours. This yields the following unemployment rate:

$$\frac{1}{2\sqrt{L}} = \frac{1}{2}$$
$$2\sqrt{L} = 2$$
$$L = 1$$
$$\implies UnemploymentRate = \frac{(9-1)}{9} = \frac{8}{9}$$

D) First we must see that in order for the TRS = $\frac{W_K}{W_L}$ it must be the case that K = L:

$$TRS = \frac{\frac{1}{2}K^{-\frac{1}{2}}L^{\frac{1}{2}}}{\frac{1}{2}L^{-\frac{1}{2}}K^{\frac{1}{2}}} = \frac{W_K}{W_L} = 1$$
$$\implies K = L$$

plugging this result back into the production function to get costs as a function of output, we see that:

$$\begin{aligned} y &= K^{\frac{1}{2}}(K)^{\frac{1}{2}} \\ &= K \\ \implies K = y \end{aligned}$$

thus, the cost function is given by:

$$\begin{aligned} C(y) &= W_K K + W_L L \\ &= 8K + 8L \\ &= 16K \end{aligned} \quad (1)$$

Since above we learned that $K = y$, is must be that case that costs, as a function of output, is given by the equation $C(y) = 16y$.

Group B

Problem #1

A) The budget constraint maps consumption between the two time periods. With the "young" period on the x-axis and the "old" period on the y-axis, the budget constraint appears as

$$\begin{aligned} C_1P_1 + C_2P_2 &\leq m_1 + m_2P_2 \\ P_2 &= \frac{1}{1+r} \\ C_1P_1 + C_2\frac{1}{3} &\leq \$20 + \$180\frac{1}{3} \\ \implies C_1P_1 + C_2\frac{1}{3} &\leq \$80 \end{aligned}$$

Future Value is the $180 that Joseph gets when he is "old" in addition to the value of the $20 he gets when he is "young" when he is old:

$$\$20(1+r) = \$60$$

This means that FV = $240.

Present value is the $20 Joseph has when he is young plus the value when he is "young" of the $180 he will get when he is old:

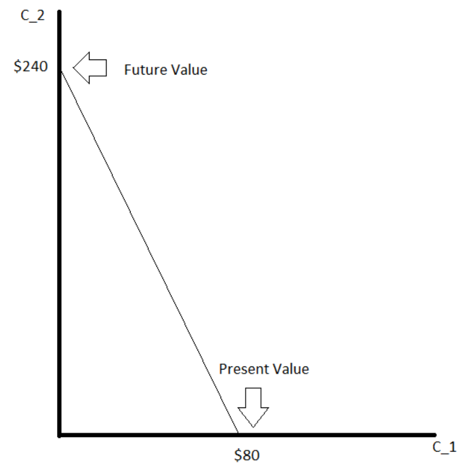
$$\$180(\frac{1}{1+r}) + \$20 = \$80$$

This means that PV = $80

B) In terms of PV, the optimal demands for consumption are:

$$\begin{aligned} C_1 &= \frac{1}{1+\frac{1}{1+\delta}} \times \frac{\$20P_1 + \$180P_2}{P_1} \\ \implies C_1 &= \frac{3}{4} \times \frac{\$20 + \$180P_2}{1} \\ C_2 &= \frac{\frac{1}{1+\delta}}{1+\frac{1}{1+\delta}} \times \frac{\$20P_1 + \$180P_2}{P_2} \\ \implies C_2 &= \frac{1}{4} \times \frac{\$20 + \$180P_2}{P_2} \end{aligned}$$

Figure 2: Budget Constraint



From the budget constraint we know that $C_1 + P_2 C_2 \leq 80$, so combining this with the above equations we can recover P_2:

$$\begin{aligned}
\frac{3}{4} \times \frac{\$20 + \$180P_2}{1} + P_2(\frac{1}{4} \times \frac{\$20 + \$180P_2}{P_2}) &= \$80 \\
\$15 + \$135P_2 + \$5 + \$45P_2 &= \$80 \\
\$180P_2 &= \$60 \\
\implies P_2 &= \frac{1}{3}
\end{aligned}$$

Plugging P_2 into the magic formulas for C_1 and C_2, we get:

$$\begin{aligned}
C_1 = \frac{3}{4} \times \frac{\$20 + \$180\frac{1}{3}}{1} = \$60 \\
C_2 = \frac{1}{4} \times \frac{\$20 + \$180\frac{1}{3}}{\frac{1}{3}} = \$60
\end{aligned}$$

Since Joseph is only endowed with \$20 in the first period, but consumes \$60, this means that he is borrowing (in present value) \$40 from the future (\$120 in FV). This makes his savings vector S = (-\$40,0).

C) The formula for an annuity of \$8,000 lasting for three periods, starting next period is:

$$\frac{\$8,000}{r}(1 - (\frac{1}{1+r}^{3})) = \$7,000 \leq \$7,500$$

This means that the lease is a better option than the purchase since it costs less in terms of present value and available information (assuming you ignore the value of the car remaining after three years).

D) The formula of a stream of payments that never ends is:

$$\begin{aligned} PV &= \frac{x}{(1+r)} + \frac{x}{(1+r)^2} + \frac{x}{(1+r)^3} + ... \\ &= \frac{1}{(1+r)}[x + \frac{x}{(1+r)} + \frac{x}{(1+r)^2} + ...] \\ &= \frac{1}{(1+r)}[x + PV] \end{aligned}$$

so we can solve for PV to get a more concise solution:

$$\begin{aligned} (1 - \frac{1}{1+r})PV &= \frac{1}{1+r}x \\ (\frac{1+r}{1+r} - \frac{1}{1+r})PV &= \frac{1}{1+r}x \\ (\frac{r}{1+r})PV &= \frac{1}{1+r}x \\ PV &= \frac{x}{r} \end{aligned}$$

Problem 2

A) To calculate the expected value it is necessary to find the value in each state and multiply it by the probability that each state will occur. The summation is then:

$$EV = 0 * \pi + 36 * (1 - \pi) = 18$$

The certainty equivalent is the amount of money that Trevor would take *for certain* in order to avoid the gamble that would leave him with the *expectation* of \$18. To calculate this you must first find the utility that the gamble provides, and then find the amount of money that would provide the same utility with a 100% probability:

$$\begin{aligned} U(E(18)) &= \frac{1}{2}\sqrt{0} + \frac{1}{2}\sqrt{36} \\ &= 3 \\ U(CE) = 3 &\implies \sqrt{CE} = 3 \\ \implies CE &= 9 \end{aligned}$$

Here we clearly see that the certainty equivalent, 9 is smaller than the expected payoff from the gamble, 18. This makes sense because the concave shape of Trevor's utility function reflects his risk-averse preferences - meaning he is willing to take a smaller payout for certain than the one he would get in expectation to avoid the possibility of being left with no money at all.

B) The expected utility representation is:

$$U(C_1, C_2) = \frac{1}{2}\ln C_1 + \frac{1}{2}\ln C_2$$

The natural log is a concave function, meaning that Trevor is risk averse.
C) First, we calculate the formulas for consumption in each state of the world:

$$C_1 = 36 - x\gamma$$
$$C_2 = (1-\gamma)x$$

Knowing that x, the amount of insurance, is the same in both states of the world, we can solve both of the above equations for x and set them equal to one another:

$$x = \frac{36 - C_1}{\gamma}$$
$$x = \frac{C_2}{(1-\gamma)}$$
$$\implies \frac{36 - C_1}{\gamma} = \frac{C_2}{(1-\gamma)}$$
$$\implies C_1 = 36 - \frac{\gamma}{1-\gamma}C_2$$

This final equation is our budget constraint. Normalizing P_1 to 1, this makes $P_2 = 1$ as well. Plugging these values into the magic formulas for demand we get:

$$C_1 = \frac{1}{2} \times \frac{36}{P_1}$$
$$C_2 = \frac{1}{2} \times \frac{36}{P_2}$$
$$C_1 = C_2 = 18$$

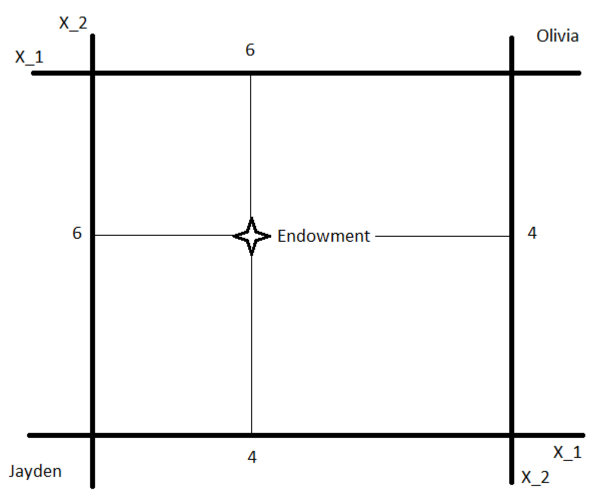
Going back to the first equation for C_1, we now have:

$$18 = 36 - \frac{1}{2}x$$
$$\implies x = 36$$

Consumption in both states of the world is 18, so that means that Trevor is fully insured. Full insurance means that your consumption does not depend on the state of the world. Trevor has chosen to fully insure in this case because the insurance premium, γ, is equal to the probability of loss, π.

Problem 3

A)



B) At an equilibrium, we need for MRS^J to equal MRS^O and also for MRS^J to be equal to $\frac{P_1}{P_2}$. We use the first identity to get the contract curve, the second to calculate the slope of the budget line. Given the endowment point, we can follow the budget line away from the endowment point to find its intersection with the contract curve, which is the equilibrium. Since the two unility functions are symetrical, we can solve for Jayden and Olivia both by simply solving for Jayden. Analytically, this is done via the following equations: First, we normalize P_1 to equal 1 and use the Cobb-Douglas magic forumlas to get the P_2:

$$\begin{aligned}
x_1 &= \frac{1}{2} \times \frac{4P_1 + 6P_2}{P_1} \\
x_2 &= \frac{1}{2} \times \frac{4P_1 + 6P_2}{P_2} \\
x_1P_1 + x_2P_2 &= 4P_1 + 6P_2 \\
\implies \frac{1}{2} \times \frac{4P_1 + 6P_2}{P_1} + P_2\frac{1}{2} \times \frac{4P_1 + 6P_2}{P_2} &= 4P_1 + 6P_2 \\
2 + 3P_2 + 2 + 3P_2 = 4 + 6P_2 &= 4 + 6P_2 \\
P_2 &= 1
\end{aligned}$$

$$\begin{aligned}
MRS^J = \frac{MU_1^J}{MU_2^J} &= \frac{\frac{1}{5x_1}}{\frac{1}{5x_2}} = \frac{P_1}{P_2} \\
\implies \frac{x_2}{x_1} &= \frac{P_1}{P_2} \\
\implies x_2 &= x_1
\end{aligned}$$

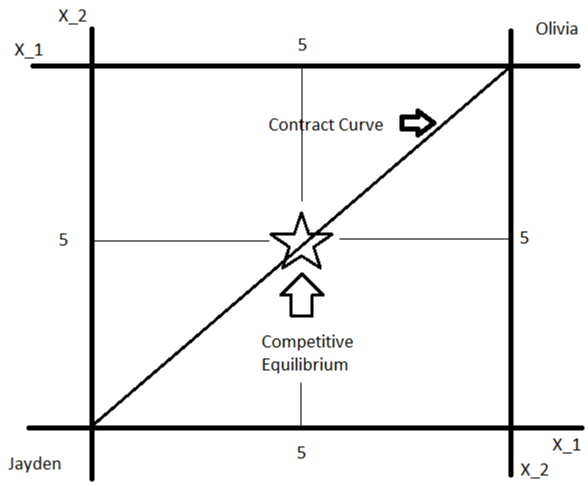
This last step, however, was extraneous expect to show that the contract curve is the set of points where $x_1 = x_2$. Thus, our solution must satisfy this identity. Anyway, revisiting the Cobb-Douglas demands with P_2 in hand, we get:

$$\begin{aligned}
x_1 &= \frac{1}{2} \times \frac{4P_1 + 6P_2}{P_1} \\
&= 5
\end{aligned}$$

$$\implies x_1^J = x_2^J = x_1^O = x_2^O = 5 \ \& \ P_1 = P_2 = 1$$

C) As we previously stated, at the competitive equilibrium, it is required that $MRS^J = MRS^O$ which means that $\frac{x_2^J}{x_1^J} = \frac{x_2^O}{x_1^O}$ which is satisfied when $x_1^J = x_2^J = x_1^O = x_2^O = 5$ since $\frac{5}{5} = \frac{5}{5}$

D) Solving analytically for the contract curve requires knowledge of the total endowment of each good in the economy. Here, we can see from the individual endowments that there are 10 units of each good in the economy and that the edgeworth box depicting it is square. To solve for the slope of the contract



curve use the equations:

$$MRS^J = \frac{MU_1^J}{MU_2^J} = \frac{\frac{1}{5x_1^J}}{\frac{1}{5x_2^J}} = \frac{x_2^J}{x_1^J} = MRS^O = \frac{MU_1^O}{MU_2^O} = \frac{10 - x_2^J}{10 - x_1^J}$$

$$\implies x_2^J = x_1^J$$

By symmetry this is true for both individuals. Clearly the initial endowments are not located on the contract curve (no).

Problem 4

A) The MPL is decreasing and there are constant returns to scale exhibited by this production function.

B) Labor demand can be calculated by setting MPL = $\frac{w}{p}$. Given that K is fixed, this makes the equation:

$$\frac{1}{2}L^{-\frac{1}{2}} = \frac{w}{p}$$
$$\frac{1}{2\sqrt{L}} = \frac{w}{p}$$
$$\sqrt{L} = \frac{p}{2w}$$
$$L^* = (\frac{p}{2w})^2$$
$$9 = (\frac{p}{2w})^2$$
$$\frac{1}{6} = \frac{w}{p}$$

C)The unemployment rate is the ratio of the number of hours invidivduals are under-employed relative to the equilibrium level of employment in a free market. Here, when wages are free to adjust due to supply and demand the market clears at a wage real wage of $\frac{1}{6}$ and 9 hours of labor. When the wage is constrained to be $\frac{1}{2}$ the labor demand falls, while the supply remains constant at 9 hours. This yields the following unemployment rate:

$$\frac{1}{2\sqrt{L}} = \frac{1}{2}$$
$$2\sqrt{L} = 2$$
$$L = 1$$
$$\implies UnemploymentRate = \frac{(9-1)}{9} = \frac{8}{9}$$

D) First we must see that in order for the TRS = $\frac{W_K}{W_L}$ it must be the case that K = L:

$$TRS = \frac{\frac{1}{2}K^{-\frac{1}{2}}L^{\frac{1}{2}}}{\frac{1}{2}L^{-\frac{1}{2}}K^{\frac{1}{2}}} = \frac{W_K}{W_L} = 1$$
$$\implies K = L$$

plugging this result back into the production function to get costs as a function of output, we see that:

$$
\begin{aligned}
y &= K^{\frac{1}{2}}(K)^{\frac{1}{2}} \\
&= K \\
\implies K = y &
\end{aligned}
$$

thus, the cost function is given by:

$$
\begin{aligned}
C(y) &= W_K K + W_L L \\
&= 5K + 5L \\
&= 10K
\end{aligned}
\tag{2}
$$

Since above we learned that $K = y$, is must be that case that costs, as a function of output, is given by the equation $C(y) = 10y$.

Group C

Problem #1

A) The budget constraint maps consumption between the two time periods. With the "young" period on the x-axis and the "old" period on the y-axis, the budget constraint appears as

$$
\begin{aligned}
C_1P_1 + C_2P_2 &\leq m_1 + m_2P_2 \\
P_2 &= \frac{1}{1+r} \\
C_1P_1 + C_2\frac{1}{2} &\leq \$10 + \$60\frac{1}{2} \\
\implies C_1P_1 + C_2\frac{1}{2} &\leq \$40
\end{aligned}
$$

Future Value is the \$60 that Joseph gets when he is "old" in addition to the value of the \$10 he gets when he is "young" when he is old:

$$\$10(1+r) + \$60 = \$80$$

This means that FV = \$80.

Present value is the \$20 Joseph has when he is young plus the value when he is "young" of the \$60 he will get when he is old:

$$\$60(\frac{1}{1+r}) + \$10 = \$40$$

This means that PV = \$40

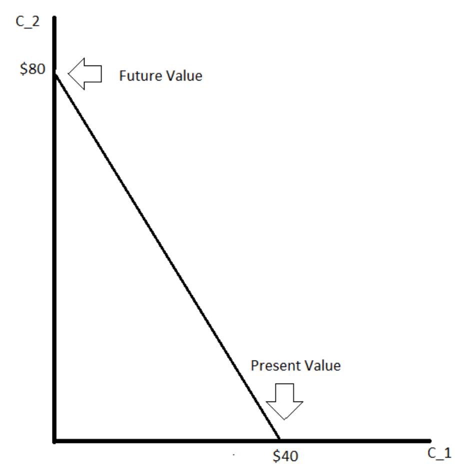


Figure 3: Budget Constraint

B) In terms of PV, the optimal demands for consumption are:

$$C_1 = \frac{1}{1+\frac{1}{1+\delta}} \times \frac{\$10P_1 + \$60P_2}{P_1}$$
$$\implies C_1 = \frac{2}{3} \times \frac{\$10 + \$60P_2}{1}$$
$$C_2 = \frac{\frac{1}{1+\delta}}{1+\frac{1}{1+\delta}} \times \frac{\$10P_1 + \$60P_2}{P_2}$$
$$\implies C_2 = \frac{1}{3} \times \frac{\$10 + \$60P_2}{P_2}$$

From the budget constraint we know that $C_1 + P_2C_2 \leq 40$, so combining this with the above equations we can recover P_2:

$$
\begin{aligned}
\frac{2}{3} \times \frac{\$10 + \$60P_2}{1} + P_2(\frac{1}{3} \times \frac{\$10 + \$60P_2}{P_2}) &= \$40 \\
\$6.66 + \$40P_2 + \$3.33 + \$20P_2 &= \$40 \\
\$60P_2 &= \$30 \\
\implies P_2 &= \frac{1}{2}
\end{aligned}
$$

Plugging P_2 into the magic formulas for C_1 and C_2, we get:

$$
\begin{aligned}
C_1 = \frac{2}{3} \times \frac{\$10 + \$60\frac{1}{2}}{1} = \$\frac{80}{3} \\
C_2 = \frac{1}{3} \times \frac{\$10 + \$60\frac{1}{2}}{\frac{1}{2}} = \$\frac{80}{3}
\end{aligned}
$$

Since Joseph is only endowed with \$10 in the first period, but consumes \$ $\frac{80}{3}$, this means that he is borrowing (in present value) $\$\frac{50}{3}$ from the future ($\$\frac{100}{3}$ in FV). This makes his savings vector S = (-$\$\frac{50}{3}$,0).

C) The formula for an annuity of \$2,400 lasting for three periods, starting next period is:

$$
\frac{\$2,400}{r}(1 - (\frac{1}{1+r}^{3})) = \$2,100 < \$2,300
$$

This means that the lease is a better option than the purchase since it costs less in terms of present value and available information (assuming you ignore the value of the car remaining after three years).

D) The formula of a stream of payments that never ends is:

$$
\begin{aligned}
PV &= \frac{x}{(1+r)} + \frac{x}{(1+r)^2} + \frac{x}{(1+r)^3} + ... \\
&= \frac{1}{(1+r)}[x + \frac{x}{(1+r)} + \frac{x}{(1+r)^2} + ...] \\
&= \frac{1}{(1+r)}[x + PV]
\end{aligned}
$$

so we can solve for PV to get a more concise solution:

$$\begin{aligned}(1-\frac{1}{1+r})PV &= \frac{1}{1+r}x\\(\frac{1+r}{1+r}-\frac{1}{1+r})PV &= \frac{1}{1+r}x\\(\frac{r}{1+r})PV &= \frac{1}{1+r}x\\PV &= \frac{x}{r}\end{aligned}$$

Problem 2

A) To calculate the expected value it is necessary to find the value in each state and multiply it by the probability that each state will occur. The summation is then:

$$EV = 0 * \pi + 64 * (1-\pi) = 32$$

The certainty equivalent is the amount of money that Trevor would take *for certain* in order to avoid the gamble that would leave him with the *expectation* of \$32. To calculate this you must first find the utility that the gamble provides, and then find the amount of money that would provide the same utility with a 100% probability:

$$\begin{aligned}U(E(18)) &= \frac{1}{2}\sqrt{0}+\frac{1}{2}\sqrt{64}\\&= 4\\U(CE)=4 &\implies \sqrt{CE}=4\\\implies CE &= 16\end{aligned}$$

Here we clearly see that the certainty equivalent, 16 is smaller than the expected payoff from the gamble, 32. This makes sense because the concave shape of Trevor's utility function reflects his risk-averse preferences - meaning he is willing to take a smaller payout for certain than the one he would get in expectation to avoid the possibility of being left with no money at all.

B) The expected utility representation is:

$$U(C_1, C_2) = \frac{1}{2}\ln C_1 + \frac{1}{2}\ln C_2$$

The natural log is a concave function, meaning that Trevor is risk averse.
C) First, we calculate the formulas for consumption in each state of the world:

$$C_1 = 64 - x\gamma$$
$$C_2 = (1-\gamma)x$$

Knowing that x, the amount of insurance, is the same in both states of the world, we can solve both of the above equations for x and set them equal to one another:

$$\begin{aligned} x &= \frac{64 - C_1}{\gamma} \\ x &= \frac{C_2}{(1-\gamma)} \end{aligned}$$
$$\implies \frac{64 - C_1}{\gamma} = \frac{C_2}{(1-\gamma)}$$
$$\implies C_1 = 64 - \frac{\gamma}{1-\gamma}C_2$$

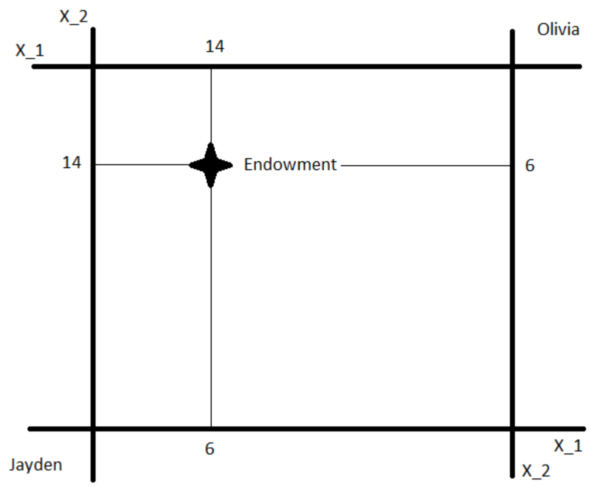
This final equation is our budget constraint. Normalizing P_1 to 1, this makes $P_2 = 1$ as well. Plugging these values into the magic formulas for demand we get:

$$\begin{aligned} C_1 &= \frac{1}{2} \times \frac{64}{P_1} \\ C_2 &= \frac{1}{2} \times \frac{64}{P_2} \\ C_1 = C_2 &= 32 \end{aligned}$$

Going back to the first equation for C_1, we now have:

$$32 = 64 - \frac{1}{2}x$$
$$\implies x = 64$$

Consumption in both states of the world is 32, so that means that Trevor is fully insured. Full insurance means that your consumption does not depend on the state of the world. Trevor has chosen to fully insure in this case



because the insurance premium, γ, is equal to the probability of loss, π.
Problem 3
A)
B) At an equilibrium, we need for MRS^J to equal MRS^O and also for MRS^J to be equal to $\frac{P_1}{P_2}$. We use the first identity to get the contract curve, the second to calculate the slope of the budget line. Given the endowment point, we can follow the budget line away from the endowment point to find its intersection with the contract curve, which is the equilibrium. Since the two unility functions are symetrical, we can solve for Jayden and Olivia both by simply solving for Jayden. Analytically, this is done via the following equations: First, we normalize P_1 to equal 1 and use the Cobb-Douglas magic forumlas to get the P_2:

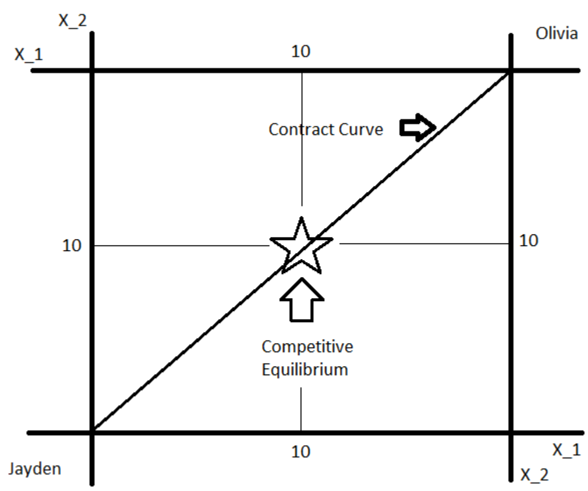
$$x_1 = \frac{1}{2} \times \frac{6P_1 + 14P_2}{P_1}$$
$$x_2 = \frac{1}{2} \times \frac{6P_1 + 14P_2}{P_2}$$
$$x_1 P_1 + x_2 P_2 = 6P_1 + 14P_2$$
$$\implies \frac{1}{2} \times \frac{6P_1 + 14P_2}{P_1} + P_2 \frac{1}{2} \times \frac{6P_1 + 14P_2}{P_2} = 6P_1 + 14P_2$$
$$3 + 7P_2 + 3 + 7P_2 = 6 + 14P_2 = 6 + 14P_2$$
$$P_2 = 1$$

$$
\begin{aligned}
MRS^J = \frac{MU_1^J}{MU_2^J} &= \frac{\frac{3}{x_1}}{\frac{3}{x_2}} = \frac{P_1}{P_2} \\
\implies \frac{x_2}{x_1} &= \frac{P_1}{P_2} \\
\implies x_2 &= x_1
\end{aligned}
$$

This last step, however, was extraneous expect to show that the contract curve is the set of points where $x_1 = x_2$. Thus, our solution must satisfy this identity. Anyway, revisiting the Cobb-Douglas demands with P_2 in hand, we get:

$$
\begin{aligned}
x_1 &= \frac{1}{2} \times \frac{6P_1 + 14P_2}{P_1} \\
&= 10
\end{aligned}
$$

$$
\implies x_1^J = x_2^J = x_1^O = x_2^O = 10 \ \& \ P_1 = P_2 = 1
$$



C) As we previously stated, at the competitive equilibrium, it is required that $MRS^J = MRS^O$ which means that $\frac{x_2^J}{x_1^J} = \frac{x_2^O}{x_1^O}$ which is satisfied when $x_1^J = x_2^J = x_1^O = x_2^O = 1$ since $\frac{10}{10} = \frac{10}{10}$

D) Solving analytically for the contract curve requires knowledge of the total endowment of each good in the economy. Here, we can see from the individual endowments that there are 10 units of each good in the economy and that the edgeworth box depicting it is square. To solve for the slope of the contract curve use the equations:

$$MRS^J = \frac{MU_1^J}{MU_2^J} = \frac{\frac{3}{x_1^J}}{\frac{3}{x_2^J}} = \frac{x_2^J}{x_1^J} = MRS^O = \frac{MU_1^O}{MU_2^O} = \frac{20 - x_2^J}{20 - x_1^J}$$
$$\implies x_2^J = x_1^J$$

By symmetry this is true for both individuals. Clearly the initial endowments are not located on the contract curve (no).

Problem 4

A) The MPL is decreasing and there are constant returns to scale exhibited by this production function.

B) Labor demand can be calculated by setting MPL = $\frac{w}{p}$. Given that K is fixed, this makes the equation:

$$\begin{aligned} \frac{1}{2}L^{-\frac{1}{2}} &= \frac{w}{p} \\ \frac{1}{2\sqrt{L}} &= \frac{w}{p} \\ \sqrt{L} &= \frac{p}{2w} \\ L^* &= (\frac{p}{2w})^2 \\ 9 &= (\frac{p}{2w})^2 \\ \frac{1}{6} &= \frac{w}{p} \end{aligned}$$

C)The unemployment rate is the ratio of the number of hours invidivduals are under-employed relative to the equilibrium level of employment in a free market. Here, when wages are free to adjust due to supply and demand the market clears at a wage real wage of $\frac{1}{6}$ and 9 hours of labor. When the wage is constrained to be $\frac{1}{2}$ the labor demand falls, while the supply remains constant at 9 hours. This yields the following unemployment rate:

$$\frac{1}{2\sqrt{L}} = \frac{1}{2}$$
$$2\sqrt{L} = 2$$
$$L = 1$$
$$\implies UnemploymentRate = \frac{(9-1)}{9} = \frac{8}{9}$$

D) First we must see that in order for the TRS = $\frac{W_K}{W_L}$ it must be the case that K = L:

$$TRS = \frac{\frac{1}{2}K^{-\frac{1}{2}}L^{\frac{1}{2}}}{\frac{1}{2}L^{-\frac{1}{2}}K^{\frac{1}{2}}} = \frac{W_K}{W_L} = 1$$
$$\implies K = L$$

plugging this result back into the production function to get costs as a function of output, we see that:

$$\begin{aligned} y &= K^{\frac{1}{2}}(K)^{\frac{1}{2}} \\ &= K \end{aligned}$$
$$\implies K = y$$

thus, the cost function is given by:

$$\begin{aligned} C(y) &= W_K K + W_L L \\ &= 3K + 3L \\ &= 6K \end{aligned} \tag{3}$$

Since above we learned that $K = y$, is must be that case that costs, as a function of output, is given by the equation $C(y) = 16y$.

Group D

Problem #1

A) The budget constraint maps consumption between the two time periods. With the "young" period on the x-axis and the "old" period on the y-axis, the budget constraint appears as

$$\begin{aligned} C_1P_1 + C_2P_2 &\leq m_1 + m_2P_2 \\ P_2 &= \frac{1}{1+r} \\ C_1P_1 + C_2\frac{1}{2} &\leq \$10 + \$100\frac{1}{2} \\ \implies C_1P_1 + C_2\frac{1}{2} &\leq \$60 \end{aligned}$$

Future Value is the $100 that Joseph gets when he is "old" in addition to the value of the $10 he gets when he is "young" when he is old:

$$\$10(1+r) + \$100 = \$120$$

This means that FV = $120.

Present value is the $10 Joseph has when he is young plus the value when he is "young" of the $100 he will get when he is old:

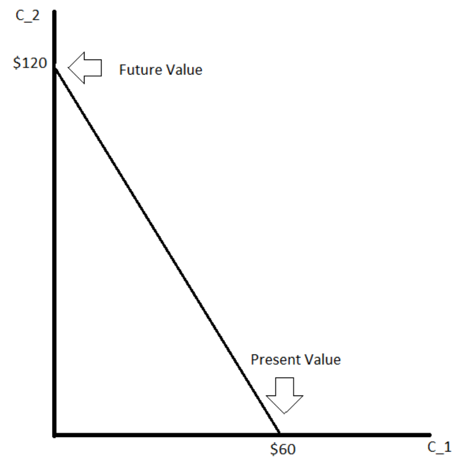
$$\$100(\frac{1}{1+r}) + \$10 = \$60$$

This means that PV = $60

B) In terms of PV, the optimal demands for consumption are:

$$\begin{aligned} C_1 &= \frac{1}{1+\frac{1}{1+\delta}} \times \frac{\$10P_1 + \$100P_2}{P_1} \\ \implies C_1 &= \frac{2}{3} \times \frac{\$10 + \$100P_2}{1} \\ C_2 &= \frac{\frac{1}{1+\delta}}{1+\frac{1}{1+\delta}} \times \frac{\$10P_1 + \$100P_2}{P_2} \\ \implies C_2 &= \frac{1}{3} \times \frac{\$10 + \$100P_2}{P_2} \end{aligned}$$

Figure 4: Budget Constraint



From the budget constraint we know that $C_1 + P_2C_2 \leq 60$, so combining this with the above equations we can recover P_2:

$$\begin{aligned}
\frac{2}{3} \times \frac{\$10 + \$100P_2}{1} + P_2(\frac{1}{3} \times \frac{\$10 + \$100P_2}{P_2}) &= \$60 \\
\$6.66 + \$66.6P_2 + \$3.33 + \$33.3P_2 &= \$60 \\
\$100P_2 &= \$50 \\
\implies P_2 &= \frac{1}{2}
\end{aligned}$$

Plugging P_2 into the magic formulas for C_1 and C_2, we get:

$$C_1 = \frac{2}{3} \times \frac{\$10 + \$100\frac{1}{2}}{1} = \$40$$
$$C_2 = \frac{1}{3} \times \frac{\$10 + \$100\frac{1}{2}}{\frac{1}{2}} = \$40$$

Since Joseph is only endowed with \$10 in the first period, but consumes \$ 40, this means that he is borrowing (in present value) \$30 from the future (\$60 in FV). This makes his savings vector S = (-\$30,0).

C) The formula for an annuity of \$2,400 lasting for three periods, starting next period is:

$$\frac{\$2,400}{r}(1 - (\frac{1}{1+r}^{3})) = \$2,100 < \$2,300$$

This means that the lease is a better option than the purchase since it costs less in terms of present value and available information (assuming you ignore the value of the car remaining after three years).

D) The formula of a stream of payments that never ends is:

$$\begin{aligned} PV &= \frac{x}{(1+r)} + \frac{x}{(1+r)^2} + \frac{x}{(1+r)^3} + ... \\ &= \frac{1}{(1+r)}[x + \frac{x}{(1+r)} + \frac{x}{(1+r)^2} + ...] \\ &= \frac{1}{(1+r)}[x + PV] \end{aligned}$$

so we can solve for PV to get a more concise solution:

$$\begin{aligned} (1 - \frac{1}{1+r})PV &= \frac{1}{1+r}x \\ (\frac{1+r}{1+r} - \frac{1}{1+r})PV &= \frac{1}{1+r}x \\ (\frac{r}{1+r})PV &= \frac{1}{1+r}x \\ PV &= \frac{x}{r} \end{aligned}$$

Problem 2

A) To calculate the expected value it is necessary to find the value in each state and multiply it by the probability that each state will occur. The summation is then:

$$EV = 0 * \pi + 100 * (1 - \pi) = 50$$

The certainty equivalent is the amount of money that Trevor would take *for certain* in order to avoid the gamble that would leave him with the *expectation* of \$50. To calculate this you must first find the utility that the gamble provides, and then find the amount of money that would provide the same utility with a 100% probability:

$$\begin{aligned} U(E(50)) &= \frac{1}{2}\sqrt{0} + \frac{1}{2}\sqrt{100} \\ &= 5 \\ U(CE) = 5 &\implies \sqrt{CE} = 5 \\ \implies CE &= 25 \end{aligned}$$

Here we clearly see that the certainty equivalent, 25 is smaller than the expected payoff from the gamble, 50. This makes sense because the concave shape of Trevor's utility function reflects his risk-averse preferences - meaning he is willing to take a smaller payout for certain than the one he would get in expectation to avoid the possibility of being left with no money at all.

B) The expected utility representation is:

$$U(C_1, C_2) = \frac{1}{2}\ln C_1 + \frac{1}{2}\ln C_2$$

The natural log is a concave function, meaning that Trevor is risk averse.
C) First, we calculate the formulas for consumption in each state of the world:

$$C_1 = 100 - x\gamma$$
$$C_2 = (1-\gamma)x$$

Knowing that x, the amount of insurance, is the same in both states of the world, we can solve both of the above equations for x and set them equal to one another:

$$x = \frac{100 - C_1}{\gamma}$$
$$x = \frac{C_2}{(1-\gamma)}$$
$$\implies \frac{100 - C_1}{\gamma} = \frac{C_2}{(1-\gamma)}$$
$$\implies C_1 = 100 - \frac{\gamma}{1-\gamma}C_2$$

This final equation is our budget constraint. Normalizing P_1 to 1, this makes $P_2 = 1$ as well. Plugging these values into the magic formulas for demand we get:

$$C_1 = \frac{1}{2} \times \frac{100}{P_1}$$
$$C_2 = \frac{1}{2} \times \frac{100}{P_2}$$
$$C_1 = C_2 = 50$$

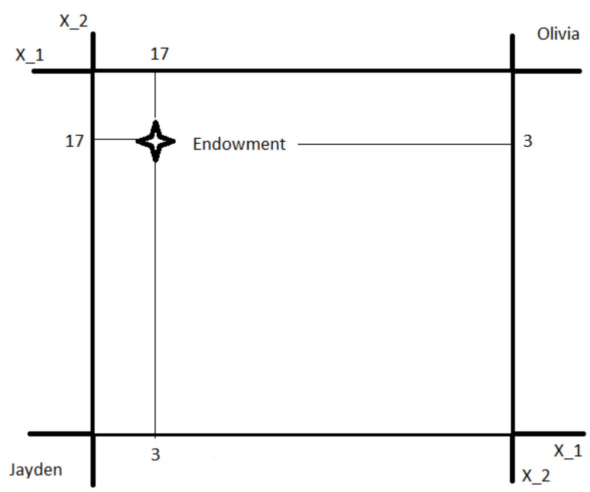
Going back to the first equation for C_1, we now have:

$$50 = 100 - \frac{1}{2}x$$
$$\implies x = 100$$

Consumption in both states of the world is 50, so that means that Trevor is fully insured. Full insurance means that your consumption does not depend on the state of the world. Trevor has chosen to fully insure in this case because the insurance premium, γ, is equal to the probability of loss, π.

Problem 3

A)



B) At an equilibrium, we need for MRS^J to equal MRS^O and also for MRS^J to be equal to $\frac{P_1}{P_2}$. We use the first identity to get the contract curve, the second to calculate the slope of the budget line. Given the endowment point, we can follow the budget line away from the endowment point to find its intersection with the contract curve, which is the equilibrium. Since the two unility functions are symetrical, we can solve for Jayden and Olivia both by simply solving for Jayden. Analytically, this is done via the following equations: First, we normalize P_1 to equal 1 and use the Cobb-Douglas magic forumlas to get the P_2:

$$x_1 = \frac{1}{2} \times \frac{3P_1 + 17P_2}{P_1}$$
$$x_2 = \frac{1}{2} \times \frac{3P_1 + 17P_2}{P_2}$$
$$x_1 P_1 + x_2 P_2 = 3P_1 + 17P_2$$
$$\implies \frac{1}{2} \times \frac{3P_1 + 17P_2}{P_1} + P_2 \frac{1}{2} \times \frac{3P_1 + 17P_2}{P_2} = 3P_1 + 17P_2$$
$$1.5 + 8.5P_2 + 1.5 + 8.5P_2 = 3 + 17P_2 = 3 + 17P_2$$
$$P_2 = 1$$

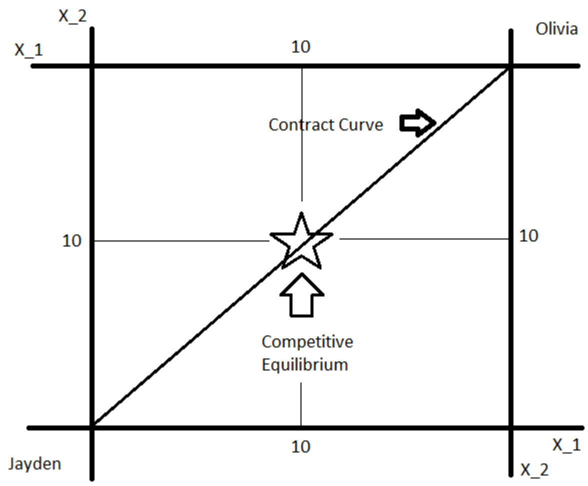
$$MRS^J = \frac{MU_1^J}{MU_2^J} = \frac{\frac{5}{x_1}}{\frac{5}{x_2}} = \frac{P_1}{P_2}$$
$$\implies \frac{x_2}{x_1} = \frac{P_1}{P_2}$$
$$\implies x_2 = x_1$$

This last step, however, was extraneous expect to show that the contract curve is the set of points where $x_1 = x_2$. Thus, our solution must satisfy this identity. Anyway, revisiting the Cobb-Douglas demands with P_2 in hand, we get:

$$x_1 = \frac{1}{2} \times \frac{3P_1 + 17P_2}{P_1}$$
$$= 10$$
$$\implies x_1^J = x_2^J = x_1^O = x_2^O = 10 \;\&\; P_1 = P_2 = 1$$

C) As we previously stated, at the competitive equilibrium, it is required that $MRS^J = MRS^O$ which means that $\frac{x_2^J}{x_1^J} = \frac{x_2^O}{x_1^O}$ which is satisfied when $x_1^J = x_2^J = x_1^O = x_2^O = 1$ since $\frac{10}{10} = \frac{10}{10}$

D) Solving analytically for the contract curve requires knowledge of the total endowment of each good in the economy. Here, we can see from the individual endowments that there are 10 units of each good in the economy and that the edgeworth box depicting it is square. To solve for the slope of the contract



curve use the equations:

$$MRS^J = \frac{MU_1^J}{MU_2^J} = \frac{\frac{5}{x_1^J}}{\frac{5}{x_2^J}} = \frac{x_2^J}{x_1^J} = MRS^O = \frac{MU_1^O}{MU_2^O} = \frac{20 - x_2^J}{20 - x_1^J}$$
$$\implies x_2^J = x_1^J$$

By symmetry this is true for both individuals. Clearly the initial endowments are not located on the contract curve (no).
Problem 4
A) The MPL is decreasing and there are constant returns to scale exhibited by this production function.
B) Labor demand can be calculated by setting MPL = $\frac{w}{p}$. Given that K is

fixed, this makes the equation:

$$\frac{1}{2}L^{-\frac{1}{2}} = \frac{w}{p}$$
$$\frac{1}{2\sqrt{L}} = \frac{w}{p}$$
$$\sqrt{L} = \frac{p}{2w}$$
$$L^* = (\frac{p}{2w})^2$$
$$9 = (\frac{p}{2w})^2$$
$$\frac{1}{6} = \frac{w}{p}$$

C)The unemployment rate is the ratio of the number of hours invidivduals are under-employed relative to the equilibrium level of employment in a free market. Here, when wages are free to adjust due to supply and demand the market clears at a wage real wage of $\frac{1}{6}$ and 9 hours of labor. When the wage is constrained to be $\frac{1}{2}$ the labor demand falls, while the supply remains constant at 9 hours. This yields the following unemployment rate:

$$\frac{1}{2\sqrt{L}} = \frac{1}{2}$$
$$2\sqrt{L} = 2$$
$$L = 1$$
$$\implies UnemploymentRate = \frac{(9-1)}{9} = \frac{8}{9}$$

D) First we must see that in order for the TRS = $\frac{W_K}{W_L}$ it must be the case that K = L:

$$TRS = \frac{\frac{1}{2}K^{-\frac{1}{2}}L^{\frac{1}{2}}}{\frac{1}{2}L^{-\frac{1}{2}}K^{\frac{1}{2}}} = \frac{W_K}{W_L} = 1$$
$$\implies K = L$$

plugging this result back into the production function to get costs as a function of output, we see that:

$$\begin{aligned} y &= K^{\frac{1}{2}}(K)^{\frac{1}{2}} \\ &= K \\ \Longrightarrow K = y \end{aligned}$$

thus, the cost function is given by:

$$\begin{aligned} C(y) &= W_K K + W_L L \\ &= 2K + 2L \\ &= 4K \end{aligned} \tag{4}$$

Since above we learned that $K = y$, is must be that case that costs, as a function of output, is given by the equation $C(y) = 4y$.

Econ 301
Intermediate Microeconomics
Prof. Marek Weretka

Midterm 2 (Group A)

You have 70 minutes to complete the exam. The midterm consists of 4 questions (25,30,25 and 20 points).

Problem 1 (25p). (Uncertainty and insurance)

You are an owner of a luxurious sailing boat, worth $10, that you use for recreation on Mendota lake. Unfortunately, there is a good (50%) chance of a tornado in Madison (probability is equal to $\frac{1}{2}$) that completely destroys it. Thus, your boat is in fact a lottery with payment $(0, 10)$.

a) What is the expected value of the "boat" lottery? (give one number)

b) Suppose your Bernoulli utility function is given by $u(c) = c^2$. Give von Neuman-Morgenstern utility function over lotteries $U(C_1; C_2)$. (formula) Are you risk averse, neutral or risk loving? (two words). Find the certainty equivalent (CE) of the "boat lottery" (one number). Which is bigger, CE or the expected value of a lottery from a)? Why? (one sentence)

c) Your Bernoulli utility function changes to $u(c) = \ln c$. Give von Neuman-Morgenstern utility function. (give a formula). Are you risk averse now?

d) You can insure your boat by buying insurance policy in which you specify coverage x. The insurance contract costs $\gamma \cdot x$ where the premium rate is equal to $\gamma = \frac{1}{2}$. Find analytically and depict in the graph your budget constraint. Mark the point that corresponds to no insurance.

e) Find optimal level of coverage x. Are you going to fully insure your boat? (one number and yes-no answer). Depict optimal consumption plan on the graph.

f) Propose a premium rate γ for which you will only partially insure your boat. (one number)

Problem 2 (30p). (Edgeworth box, and equilibrium)

Consider an economy with apples and oranges. Andy is initially endowed with $\omega^A = (0, 50)$ and Bob's endowment is $\omega^B = (50, 0)$.

The utility function of both Andy and Bob is the same and given by

$$U(x_1, x_2) = 3\ln x_1 + 3\ln x_2$$

a) Plot the Edgeworth box and mark the allocation representing the initial endowment.

b) Provide general definition of Pareto efficiency (one sentence starting with: Allocation is Pareto efficient if ...).

c) Prove, that an allocation is Pareto efficient if and only in such allocation satisfies $MRS^A = MRS^B$. Start with necessity by showing that if the MRS condition does not hold then allocation is not Pareto efficient. Then proceed to sufficiency by showing that if the condition MRS is satisfied then indeed allocation is efficient (use a graph and write two sentences for each of the two conditiions).

d) Find analytically a collection of all Pareto efficient allocations (contract curve) and depict it in the graph.

e) Find the competitive equilibrium (give six numbers).

g) Give some other prices that are consistent with competitive equilibrium (give two numbers).

f) Using MRS condition verify that equilibrium allocation is Pareto efficient and hence an invisible hand of a free (and competitive) market guides selfish Andy and Bob to a socially optimal outcome.

Problem 3 (25p). (Short questions)

a) Your sister has just promised to send you pocket money of $500 each month starting next month and she will keep doing it forever. What is the present value of "having such sister" if monthly interest rate is equal to 5% (one number).

b) Sam is a hockey player who earns \$100 when young and \$0 when old. Sam's intertemporal utility is given by $U(C_1, C_2) = \ln(c_1) + \frac{1}{1+\delta}\ln(c_1)$. Assuming $\delta = r = 0$ and using magic formulas find optimal consumption plan and optimal saving strategy (give three numbers C_1, C_2, S). Does Sam smooth his consumption? (yes/ no + one sentence) Is Sam tilting his consumption? (yes/ no + one sentence)

c) A production function is given by $y = 2\bar{K}^3 L^{\frac{1}{2}}$. Find analytically a short-run demand for labor (assume $\bar{K} = 1$). Find analytically equilibrium real wage rate if labor supply is given by $L^s = 16$. Depict it in a gaph.

d) You start you first job at the age of 21 and you work till 60, and then your retire. You live till 80. Your annual earnings between $21 - 60$ are \$100,000 and interest rate is $r = 5\%$. You want to maintain a constant level of consumption. Write down an equation that allows to determine C (write down the equation but you **do not need** to solve for C).

Problem 4 (20p). (Producers)

Consider a producer that has the following technology

$$y = K^{\frac{1}{4}} L^{\frac{1}{4}}.$$

a) What returns to scale are represented by this production function? (choose between CRS, IRS or DRS; prove your statement with λ argument).

b) Find analytically a (variable) cost function given $w_K = w_L = 2$. Plot it in the graph.

c) find y^{MES} and ATC^{MES} if a fixed cost is $F = 2$.

d) Find analitically a supply function of the firm and show it in the graph.

Just for fun

Using "secrets of happiness" show that if a firm is maximizing profit by producing y^*, it necessarily minimizes the cost of production of y^* (give two conditions for profit maximization and show that they imply condition for cost minimization).

Econ 301
Intermediate Microeconomics
Prof. Marek Weretka

Midterm 2 (Group B)

You have 70 minutes to complete the exam. The midterm consists of 4 questions (25,30,25 and 20 points).

Problem 1 (25p). (Uncertainty and insurance)

You are an owner of a luxurious sailing boat, worth \$4, that you use for recreation on Mendota lake. Unfortunately, there is a good (50%) chance of a tornado in Madison (probability is equal to $\frac{1}{2}$) that completely destroys it. Thus, your boat is in fact a lottery with payment $(0, 4)$.

a) What is the expected value of the "boat" lottery? (give one number)

b) Suppose your Bernoulli utility function is given by $u(c) = c^2$. Give von Neuman-Morgenstern utility function over lotteries $U(C_1; C_2)$. (formula) Are you risk averse, neutral or risk loving? (two words). Find the certainty equivalent (CE) of the "boat lottery" (one number). Which is bigger, CE or the expected value of a lottery from a)? Why? (one sentence)

c) Your Bernoulli utility function changes to $u(c) = \ln c$. Give von Neuman-Morgenstern utility function. (give a formula). Are you risk averse now?

d) You can insure your boat by buying insurance policy in which you specify coverage x. The insurance contract costs $\gamma \cdot x$ where the premium rate is equal to $\gamma = \frac{1}{2}$. Find analytically and depict in the graph your budget constraint. Mark the point that corresponds to no insurance.

e) Find optimal level of coverage x. Are you going to fully insure your boat? (one number and yes-no answer). Depict optimal consumption plan on the graph.

f) Propose a premium rate γ for which you will only partially insure your boat. (one number)

Problem 2 (30p). (Edgeworth box, and equilibrium)

Consider an economy with apples and oranges. Andy is initially endowed with $\omega^A = (20, 0)$ and Bob's endowment is $\omega^B = (0, 20)$.

The utility function of both Andy and Bob is the same and given by

$$U(x_1, x_2) = 5 \ln x_1 + 5 \ln x_2$$

a) Plot the Edgeworth box and mark the allocation representing the initial endowment.

b) Provide general definition of Pareto efficiency (one sentence starting with: Allocation is Pareto efficient if ...).

c) Prove, that an allocation is Pareto efficient if and only in such allocation satisfies $MRS^A = MRS^B$. Start with necessity by showing that if the MRS condition does not hold then allocation is not Pareto efficient. Then proceed to sufficiency by showing that if the condition MRS is satisfied then indeed allocation is efficient (use a graph and write two sentences for each of the two conditiions).

d) Find analytically a collection of all Pareto efficient allocations (contract curve) and depict it in the graph.

e) Find the competitive equilibrium (give six numbers).

g) Give some other prices that are consistent with competitive equilibrium (give two numbers).

f) Using MRS condition verify that equilibrium allocation is Pareto efficient and hence an invisible hand of a free (and competitive) market guides selfish Andy and Bob to a socially optimal outcome.

Problem 3 (25p). (Short questions)

a) Your sister has just promised to send you pocket money of \$100 each month starting next month and she will keep doing it forever. What is the present value of "having such sister" if monthly interest rate is equal to 5% (one number).

b) Sam is a hockey player who earns \$200 when young and \$0 when old. Sam's intertemporal utility is given by $U(C_1, C_2) = \ln(c_1) + \frac{1}{1+\delta}\ln(c_1)$. Assuming $\delta = r = 0$ and using magic formulas find optimal consumption plan and optimal saving strategy (give three numbers C_1, C_2, S). Does Sam smooth his consumption? (yes/ no + one sentence) Is Sam tilting his consumption? (yes/ no + one sentence)

c) A production function is given by $y = 2\bar{K}^3 L^{\frac{1}{2}}$. Find analytically a short-run demand for labor (assume $\bar{K} = 1$). Find analytically equilibrium real wage rate if labor supply is given by $L^s = 16$. Depict it in a gaph.

d) You start you first job at the age of 21 and you work till 60, and then your retire. You live till 80. Your annual earnings between $21 - 60$ are \$50,000 and interest rate is $r = 5\%$. You want to maintain a constant level of consumption. Write down an equation that allows to determine C (write down the equation but you **do not need** to solve for C).

Problem 4 (20p). (Producers)

Consider a producer that has the following technology

$$y = K^{\frac{1}{4}} L^{\frac{1}{4}}.$$

a) What returns to scale are represented by this production function? (choose between CRS, IRS or DRS; prove your statement with λ argument).

b) Find analytically a (variable) cost function given $w_K = w_L = 2$. Plot it in the graph.

c) find y^{MES} and ATC^{MES} if a fixed cost is $F = 2$.

d) Find analitically a supply function of the firm and show it in the graph.

Just for fun

Using "secrets of happiness" show that if a firm is maximizing profit by producing y^*, it necessarily minimizes the cost of production of y^* (give two conditions for profit maximization and show that they imply condition for cost minimization).

Econ 301
Intermediate Microeconomics
Prof. Marek Weretka

Midterm 2 (Group C)

You have 70 minutes to complete the exam. The midterm consists of 4 questions (25,30,25 and 20 points).

Problem 1 (25p). (Uncertainty and insurance)

You are an owner of a luxurious sailing boat, worth \$6, that you use for recreation on Mendota lake. Unfortunately, there is a good (50%) chance of a tornado in Madison (probability is equal to $\frac{1}{2}$) that completely destroys it. Thus, your boat is in fact a lottery with payment $(6, 0)$.

a) What is the expected value of the "boat" lottery? (give one number)

b) Suppose your Bernoulli utility function is given by $u(c) = c^2$. Give von Neuman-Morgenstern utility function over lotteries $U(C_1; C_2)$. (formula) Are you risk averse, neutral or risk loving? (two words). Find the certainty equivalent (CE) of the "boat lottery" (one number). Which is bigger, CE or the expected value of a lottery from a)? Why? (one sentence)

c) Your Bernoulli utility function changes to $u(c) = \ln c$. Give von Neuman-Morgenstern utility function. (give a formula). Are you risk averse now?

d) You can insure your boat by buying insurance policy in which you specify coverage x. The insurance contract costs $\gamma \cdot x$ where the premium rate is equal to $\gamma = \frac{1}{2}$. Find analytically and depict in the graph your budget constraint. Mark the point that corresponds to no insurance.

e) Find optimal level of coverage x. Are you going to fully insure your boat? (one number and yes-no answer). Depict optimal consumption plan on the graph.

f) Propose a premium rate γ for which you will only partially insure your boat. (one number)

Problem 2 (30p). (Edgeworth box, and equilibrium)

Consider an economy with apples and oranges. Andy is initially endowed with $\omega^A = (40, 0)$ and Bob's endowment is $\omega^B = (0, 40)$.

The utility function of both Andy and Bob is the same and given by

$$U(x_1, x_2) = 2 \ln x_1 + 2 \ln x_2$$

a) Plot the Edgeworth box and mark the allocation representing the initial endowment.

b) Provide general definition of Pareto efficiency (one sentence starting with: Allocation is Pareto efficient if ...).

c) Prove, that an allocation is Pareto efficient if and only in such allocation satisfies $MRS^A = MRS^B$. Start with necessity by showing that if the MRS condition does not hold then allocation is not Pareto efficient. Then proceed to sufficiency by showing that if the condition MRS is satisfied then indeed allocation is efficient (use a graph and write two sentences for each of the two conditiions).

d) Find analytically a collection of all Pareto efficient allocations (contract curve) and depict it in the graph.

e) Find the competitive equilibrium (give six numbers).

g) Give some other prices that are consistent with competitive equilibrium (give two numbers).

f) Using MRS condition verify that equilibrium allocation is Pareto efficient and hence an invisible hand of a free (and competitive) market guides selfish Andy and Bob to a socially optimal outcome.

Problem 3 (25p). (Short questions)

a) Your sister has just promised to send you pocket money of \$50 each month starting next month and she will keep doing it forever. What is the present value of "having such sister" if monthly interest rate is equal to 5% (one number).

b) Sam is a hockey player who earns $\$1000$ when young and $\$0$ when old. Sam's intertemporal utility is given by $U(C_1, C_2) = \ln(c_1) + \frac{1}{1+\delta}\ln(c_1)$. Assuming $\delta = r = 0$ and using magic formulas find optimal consumption plan and optimal saving strategy (give three numbers C_1, C_2, S). Does Sam smooth his consumption? (yes/ no + one sentence) Is Sam tilting his consumption? (yes/ no + one sentence)

c) A production function is given by $y = 2\bar{K}^3 L^{\frac{1}{2}}$. Find analytically a short-run demand for labor (assume $\bar{K} = 1$). Find analytically equilibrium real wage rate if labor supply is given by $L^s = 16$. Depict it in a gaph.

d) You start you first job at the age of 21 and you work till 60, and then your retire. You live till 80. Your annual earnings between $21 - 60$ are $\$40,000$ and interest rate is $r = 5\%$. You want to maintain a constant level of consumption. Write down an equation that allows to determine C (write down the equation but you **do not need** to solve for C).

Problem 4 (20p). (Producers)

Consider a producer that has the following technology

$$y = K^{\frac{1}{4}} L^{\frac{1}{4}}.$$

a) What returns to scale are represented by this production function? (choose between CRS, IRS or DRS; prove your statement with λ argument).

b) Find analytically a (variable) cost function given $w_K = w_L = 2$. Plot it in the graph.

c) find y^{MES} and ATC^{MES} if a fixed cost is $F = 2$.

d) Find analitically a supply function of the firm and show it in the graph.

Just for fun

Using "secrets of happiness" show that if a firm is maximizing profit by producing y^*, it necessarily minimizes the cost of production of y^* (give two conditions for profit maximization and show that they imply condition for cost minimization).

Econ 301
Intermediate Microeconomics
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Midterm 2 (Group D)

You have 70 minutes to complete the exam. The midterm consists of 4 questions (25,30,25 and 20 points).

Problem 1 (25p). (Uncertainty and insurance)

You are an owner of a luxurious sailing boat, worth \$2, that you use for recreation on Mendota lake. Unfortunately, there is a good (50%) chance of a tornado in Madison (probability is equal to $\frac{1}{2}$) that completely destroys it. Thus, your boat is in fact a lottery with payment $(2,0)$.

a) What is the expected value of the "boat" lottery? (give one number)

b) Suppose your Bernoulli utility function is given by $u(c)=c^2$. Give von Neuman-Morgenstern utility function over lotteries $U(C_1;C_2)$. (formula) Are you risk averse, neutral or risk loving? (two words). Find the certainty equivalent (CE) of the "boat lottery" (one number). Which is bigger, CE or the expected value of a lottery from a)? Why? (one sentence)

c) Your Bernoulli utility function changes to $u(c)=\ln c$. Give von Neuman-Morgenstern utility function. (give a formula). Are you risk averse now?

d) You can insure your boat by buying insurance policy in which you specify coverage x. The insurance contract costs $\gamma \cdot x$ where the premium rate is equal to $\gamma = \frac{1}{2}$. Find analytically and depict in the graph your budget constraint. Mark the point that corresponds to no insurance.

e) Find optimal level of coverage x. Are you going to fully insure your boat? (one number and yes-no answer). Depict optimal consumption plan on the graph.

f) Propose a premium rate γ for which you will only partially insure your boat. (one number)

Problem 2 (30p). (Edgeworth box, and equilibrium)

Consider an economy with apples and oranges. Andy is initially endowed with $\omega^A=(10,0)$ and Bob's endowment is $\omega^B=(0,10)$.

The utility function of both Andy and Bob is the same and given by

$$U(x_1,x_2)=8\ln x_1+8\ln x_2$$

a) Plot the Edgeworth box and mark the allocation representing the initial endowment.

b) Provide general definition of Pareto efficiency (one sentence starting with: Allocation is Pareto efficient if ...).

c) Prove, that an allocation is Pareto efficient if and only in such allocation satisfies $MRS^A=MRS^B$. Start with necessity by showing that if the MRS condition does not hold then allocation is not Pareto efficient. Then proceed to sufficiency by showing that if the condition MRS is satisfied then indeed allocation is efficient (use a graph and write two sentences for each of the two conditiions).

d) Find analytically a collection of all Pareto efficient allocations (contract curve) and depict it in the graph.

e) Find the competitive equilibrium (give six numbers).

g) Give some other prices that are consistent with competitive equilibrium (give two numbers).

f) Using MRS condition verify that equilibrium allocation is Pareto efficient and hence an invisible hand of a free (and competitive) market guides selfish Andy and Bob to a socially optimal outcome.

Problem 3 (25p). (Short questions)

a) Your sister has just promised to send you pocket money of \$200 each month starting next month and she will keep doing it forever. What is the present value of "having such sister" if monthly interest rate is equal to 5% (one number).

b) Sam is a hockey player who earns $1000 when young and $0 when old. Sam's intertemporal utility is given by $U(C_1, C_2) = \ln(c_1) + \frac{1}{1+\delta}\ln(c_1)$. Assuming $\delta = r = 0$ and using magic formulas find optimal consumption plan and optimal saving strategy (give three numbers C_1, C_2, S). Does Sam smooth his consumption? (yes/ no + one sentence) Is Sam tilting his consumption? (yes/ no + one sentence)

c) A production function is given by $y = 2\bar{K}^3 L^{\frac{1}{2}}$. Find analytically a short-run demand for labor (assume $\bar{K} = 1$). Find analytically equilibrium real wage rate if labor supply is given by $L^s = 16$. Depict it in a gaph.

d) You start you first job at the age of 21 and you work till 60, and then your retire. You live till 80. Your annual earnings between $21 - 60$ are $60,000 and interest rate is $r = 5\%$. You want to maintain a constant level of consumption. Write down an equation that allows to determine C (write down the equation but you **do not need** to solve for C).

Problem 4 (20p). (Producers)

Consider a producer that has the following technology

$$y = K^{\frac{1}{4}} L^{\frac{1}{4}}.$$

a) What returns to scale are represented by this production function? (choose between CRS, IRS or DRS; prove your statement with λ argument).

b) Find analytically a (variable) cost function given $w_K = w_L = 2$. Plot it in the graph.

c) find y^{MES} and ATC^{MES} if a fixed cost is $F = 2$.

d) Find analitically a supply function of the firm and show it in the graph.

Just for fun

Using "secrets of happiness" show that if a firm is maximizing profit by producing y^*, it necessarily minimizes the cost of production of y^* (give two conditions for profit maximization and show that they imply condition for cost minimization).

Econ 703
Intermediate Microeconomics
Prof. Marek Weretka

Answer Keys to midterm 2 (Group A)

"X and Y (2pt)." means that you get 2 pts if you answered both X and Y, and no pts if you missed either (or both).

Problem 1. [Here I denote by T the state with tornado and by N the state without tornado, instead of 1 and 2.]

a) $0.5 \cdot \$10 + 0.5 \cdot \$0 = \$5$(2pt).

b) With the Bernoulli utility function $u(c) = c^2$, the v.N.M. expected utility *function* is $U(C_T, C_N) = 0.5C_T^2 + 0.5C_N^2$ (1pt). Since $u(c) = c^2$ is a convex function, I am risk loving (2pt). The certainty equivalent CE is the amount of sure money s.t. $U(CE, CE) = CE^2 = U(0, 10) = 50$, i.e. $CE = 5\sqrt{2}$ (2pt). CE is larger than EV, because I am risk loving (2pt).

c) With the Bernoulli utility function $u(c) = c^2$, the v.N.M. expected utility *function* is $U(C_T, C_N) = 0.5 \ln C_T + 0.5 \ln C_N$ (1pt). Yes, I'm risk averse (2pt), since $u(c) = \ln c$ is a concave function.

d) As $C_T = (1-\gamma)x$ and $C_N = 4 - \gamma x$ with $\gamma = .5$, we obtain the budget constraint $C_T + C_N = 10$ (2pt). Its graph has the C_T intercept on $(C_T, C_N) = (10, 0)$, the C_N intercept on $(C_T, C_N) = (0, 10)$, and the slope -1 on the C_T-C_N plane (2pt). The endowment point should be plotted on $(C_T, C_N) = (0, 10)$ (1pt).

e) Now I should maximize the utility $U(C_T, C_N) = 0.5C_T^2 + 0.5C_N^2$ on the constraint $C_T + C_N = 10$. The magic formula yields $C_T = (1/2) \cdot (10/1) = 5$ (1pt) and $C_N = (1/2) \cdot (10/1) = 5$ (1pt). Plugging this into $C_N = 4 - \gamma x$, we obtain $x = 10$ (2pt). The optimal point should be plotted on $(5, 5)$ (1pt). Yes, I am fully insured (1pt) since $C_T = C_N$.

f) e.g. $\gamma = 1$ (2pt). Actually I would be partially insured, i.e. $C_T < C_N$ under any premium rate larger than 0.5.

Problem 2. [Here I denote apple by 1, orange by 2, Andy by A and Bob by B. You could use another notation, as long as you clarified it.]

a) The Edgeworth box should have length of 50 on each axis (1pt). The endowment is $(50, 0)$ looked from A's origin, i.e. $(0, 50)$ from B's origin (1pt). [This is a single point in an Edgeworth box. And, it cannot be an origin.]

b) ... if none could not be better off (by another feasible allocation) unless anyone is worth off (2pt). [$MRS^A = MRS^B$: no point since it is just a mathematical equivalent property and not the definition.[1]]

c) [Need to prove the claim directly from the definition of Pareto efficiency. Arguing only slopes or tangency is wrong. Besides, logical sequence (especially starting assumptions and ending conclusions) must be clarified. The curve's name, namely "an indifference curve", should be clarified.]

Necessity (4pt): If $MRS^A \neq MRS^B$ at an allocation x, both people's indifference curves should cross each other at x and thus we can find a point between them. Because this point is above each indifference curve looked from the people's origin, this allocation is better than x for both and thus the allocation x is not Pareto efficient. [The proof should start with $MRS^A \neq MRS^B$ and end with Pareto inefficiency of x. Graph is needed. On the graph, you need to *specify* another allocation that improves their utilities. If you wrote two separate points and two separate curves for a single allocation, you misunderstand an Edgeworth box.]

Sufficiency (4pt): If $MRS^A = MRS^B$ at an allocation x, both people's indifference curves should be tangent to each other at x and thus no point is below A's indifferent curve looked from A's origin, i.e. worse for A than x, or below B's indifferent curve looked from B's origin, i.e. worse for B, or below both. So any point (allocation) cannot be better than x for both people and x is Pareto efficient. [The proof should start with $MRS^A = MRS^B$ at x and end with Pareto efficiency of x. Graph is needed. On the graph, you need to clarify who is worse off than x in each region defined by the two indifference curves.]

d) As we proved above, the Pareto efficiency is equivalent to $MRS^A = MRS^B$, given the feasibility of the allocation $x_1^A + x_1^B = 50, x_2^A + x_2^B = 50$. So we solve

$$MRS^A(x_1^A, x_2^A) = \frac{3/x_1^A}{3/x_2^A} = \frac{3/(50 - x_1^A)}{3/(50 - x_2^A)} = MRS^B(50 - x_1^A, 50 - x_2^A).$$

[1] A Pareto efficient allocation may not hold this equation, unless the preference is convex, monotone, and smooth.

Then we obtain $x_1^A = x_2^A$ [or $x_1^B = x_2^B$] (3pt). This is the equation for the contract curve. [You need to clarify whose consumption it is.] Graphically it is the line starting from the origin of A with slope 1, i.e. the diagonal line connecting the two origins of the Edgeworth box (1pt).

e) Let the equilibrium price be (p_1, p_2). Then, Andy should maximize his utility $U^A(x_1^A, x_2^A) = 3\ln x_1^A + 3\ln x_2^A$ on the budget constraint $p_1 x_1^A + p_2 x_2^A = 50p_1$. The magic formula yields his optimal consumption bundle

$$x_1^A = \frac{1}{2}\frac{50p_1}{p_1} = 25, \quad x_2^A = \frac{1}{2}\frac{50p_1}{p_2} = 25\frac{p_1}{p_2}.$$

Bob should maximize his utility $U^B(x_1^B, x_2^B) = 3\ln x_1^B + 3\ln x_2^B$ on the budget constraint $p_1 x_1^B + p_2 x_2^B = 50p_2$. The magic formula yields his optimal consumption bundle

$$x_1^B = \frac{1}{2}\frac{50p_2}{p_1} = 25\frac{p_2}{p_1}, \quad x_2^B = \frac{1}{2}\frac{50p_2}{p_2} = 25.$$

The feasibility (a.k.a. market clearing) of the allocation requires[2]

$$x_1^A + x_1^B = 25 + 25\frac{p_2}{p_1} = 50, \quad \therefore\ p_2 = p_1 \neq 0.$$

Plugging this into the above optimal bundles, we obtain $x_1^A = 25$ (2pt), $x_2^A = 25$ (2pt), $x_1^B = 25$ (2pt) and $x_2^B = 25$ (2pt). The equilibrium price (p_1, p_2) can be any pair of two positive numbers as long as $p_1 = p_2$: for example, $p_1 = 1, p_2 = 1$ (2pt). [No partial credit for only p_1 or p_2.]

f) As we argued, p_1, p_2 can be any pair of two positive numbers as long as $p_1 = p_2$ and different from the answer in e): for example, $p_1 = 2, p_2 = 2$ (2pt).

g) At the equilibrium allocation $((x_1^A, x_2^A), (x_1^B, x_2^B)) = ((25, 25), (25, 25))$, the two's MRSs are

$$MRS^A(25, 25) = \frac{3/25}{3/25} = 1, \quad MRS^B(25, 25) = \frac{3/25}{3/25} = 1.$$

So we have $MRS^A = -1 = MRS^B$ and thus this equilibrium allocation is Pareto efficient (2pt). [MRS must be calculated.]

Problem 3. **a)** $PV = 100/(1.05) + 100/(1.05)^2 + \ldots = 10000$ (dollars, 4pt).

b) Sam should maximize his utility $U = \ln C_1 + \ln C_2$ on the budget constraint $C_1 + C_2 = 200$ (as $C_1 + S = 200, C_2 = S$.) The magic formula yields his optimal consumption bundle $C_1 = (1/2)\cdot(200/1) = 50$ (2pt), $C_2 = (1/2)\cdot(200/1) = 50$ (2pt). Plugging this into $C_2 = S$, we have $S = 50$ (2pt). Yes, he's smoothing (1pt) as $C_1 = C_2$. No, he's not tilting (1pt) as $C_1 = C_2$. [If you answered only either one question and did not clarify which question you answered, you get no point.]

c) The production function $y = 2K^3L^{1/2}$ implies the marginal productivity of labor $MP_L = (1/2)\cdot 2K^3L^{-1/2} = K^3L^{-1/2}$. In particular, $MP_L = L^{-1/2}$ at $K = \bar{K} = 1$. Solving the secret of happiness $MP_L = L^{-1/2} = w/p$, we find the short-run labor demand $L^D = (w/p)^{-2}$ where p is the product's price and w is wage (4pt). [Thus w/p is the real wage rate. It is not enough to state only the secret of happiness; the demand L^D should be explicitly determined.[3]] Solving the demand-supply equality $L^D = (w/p)^{-2} = 16 = L^S$, we obtain the equilibrium real wage $w/p = 1/4$ (2pt). The equilibrium point $(L, w/p) = (16, 1/4)$ must be plotted on a graph (1pt).

d) (6pt.) The annual consumption C (thousand dollars) is determined from

$$\frac{100}{1.05} + \cdots + \frac{100}{1.05^{40}} = \frac{C}{1.05} + \cdots + \frac{C}{1.05^{60}} \quad \therefore \left(1 - \frac{1}{1.05^{40}}\right)\frac{100}{1.05} = \left(1 - \frac{1}{1.05^{-60}}\right)\frac{C}{1.05}.$$

[Further simplification gets full points.]

[2] We do not have to consider the market clearing of the other good 2: Walras's theorem. Notice that if $p_1 = 0$ then $p_2/p_1 = \infty$ and the equation does not hold; so we need $p_1 \neq 0$ too.

[3] Also I saw so manyy answers "$L^D = L^{-1/2}$"; this does not make sense at all, as it is read as the short-run labor demand L^D is the inverse of the square root of L and we must ask what is L. $L = L^D$ is the solution of $MP_L = L^{-1/2} = w/p$, but not a number on either side of this equation.

Problem 4. **a)** DRS (1pt). This is because $F(\lambda K, \lambda L) = (\lambda^{1/4}K^{1/4})(\lambda^{1/4}L^{1/4}) = \lambda^{1/2}K^{1/4}L^{1/4} = \lambda^{1/2}F(K,L) < \lambda^{1/2}F(K,L)$ [if $\lambda > 1$] (4pt). [Here $F(k,l)$ is the output from $K = k$ and $L = l$.]

b) The secret of happiness is

$$\frac{MP_K}{MP_L} = \frac{0.25K^{-3/4}L^{1/4}}{0.25K^{1/4}L^{-3/4}} = \frac{2}{2} = \frac{w_K}{w_L}, \quad \therefore\ K = L$$

To achieve the production of $y = F(K,L)$, we need

$$y = F(K,K) = K^{1/2}, \quad \therefore\ K = L = y^2$$

So the cost function is $C = 2K + 2L = 2y^2 + 2y^2 = 4y^2$ (4pt).[4] Graph should be drawn on the y-C plane (1pt).

c) Solving $MC(y) = 8y = (4y^2+2)/y = ATC(y)$, we obtain $y^{MES} = 1/\sqrt{2}$ (2pt) and $ATC^{MES} = ATC(y^{MES}) = MC(y^{MES}) = 4\sqrt{2}$ (2pt).[5]

d) (6pt for giving both the function and the graph.) The optimal supply should satisfy $p = 8y^* = MC(y^*)$, i.e. $y^* = p/8$. But when $p < ATC^{MES} = 4\sqrt{2}$, the firm cannot get positive profit even from the optimal supply and thus should quit the production.

The supply function $S(p)$ is therefore

$$S(p) = \begin{cases} p/8 & \text{if } p \geq 4\sqrt{2} \\ 0 & \text{if } p \leq 4\sqrt{2}. \end{cases}$$

On the y-p plane, the graph is $y = p/8$ (i.e. $p = 8y$) for $p \geq 4\sqrt{2}$ and $y = 0$ (a part of the vertical axis) for $p \leq 4\sqrt{2}$.

Just for fun The secret of happiness for profit maximization is

$$MP_K = pw_K, \quad MP_L = pw_L.$$

Here p is the product price, MP_i is the marginal productivity of factor i, and w_i is the price of factor i. These two equations imply

$$\frac{MP_K}{MP_L} = \frac{w_K}{w_L};$$

i.e. the secret of happiness for cost minimization.[6]

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[4] Or, you can think of maximization of $Y = F(K,L) = K^{1/4}L^{1/4}$ on the constraint $2K + 2L = c$, thinking Y as a variable and c as a constant. Then the magic formula of Cobb-Douglas (utility) maximization implies $K = (1/2)(c/2) = c/4$ and $L = (1/2)(c/2) = c/4$. Then we obtain at the maximum $Y = (c/4)^{1/4}(c/4)^{1/4} = (c/4)^{1/2}$, i.e. $c = 4Y^2$. That is, when $Y = y$ is given, the budget/cost $C = 4y^2$ is needed to achieve this y at the optimum.

[5] Maybe ATC^{MES} is easier to calculate from $MC(y^{MES})$ than from $ATC(y^{MES})$, though they should yield the same number.

[6] So there's a close link between maximization and minimization. This link is called duality and was a driving force of mathematical economic theory during 1970s-80s: see Varian's textbook for graduate and advanced undergraduate, *Microeconomic Analysis*. And, you will use it in undergraduate linear programming, like Computer Science 525: see Ferris, Mangasarian, and Wright, *Linear Programming with MATLAB*, SIAM-MPS, 2007.

Econ 703
Intermediate Microeconomics
Prof. Marek Weretka

Solutions to midterm 2 (Group B)

"X and Y (2pt)." means that you get 2 pts if you answered both X and Y, and no pts if you missed either (or both).

Problem 1. [Here I denote by T the state with tornado and by N the state without tornado, instead of 1 and 2.] **a)** \$2 (2pt). **b)** $U(C_T, C_N) = 0.5C_T^2 + 0.5C_N^2$ (1pt). Risk loving (2pt). $CE = 2\sqrt{2}$ (2pt). Larger than EV, because I am risk loving (2pt). **c)** $U(C_T, C_N) = 0.5 \ln C_T + 0.5 \ln C_N$ (1pt). Yes, I'm risk averse (2pt). **d)** $C_T + C_N = 4$ (2pt). Graph is needed on the C_T-C_N plane and its position must be clarified with slope and intercepts (2pt). Plot a point on $(C_T, C_N) = (0, 4)$ for endowment (1pt). **e)** $C_T = 2$ (1pt). $C_N = 2$ (1pt). $x = 4$ (2pt). Plot a point on $(2, 2)$ (1pt). Yes, fully insured (1pt). **f)** e.g. $\gamma = 1$ (2pt). [Any number larger than 0.5 because we need $C_N > C_T$.]

Problem 2. [Here I denote apple by 1, orange by 2, Andy by A and Bob by B. You could use another notation, as long as you clarified it.] **a)** The Edgeworth box should have length of 20 on each axis (1pt). The endowment is $(20, 0)$ looked from A's origin, i.e. $(0, 20)$ from B's origin (1pt). [This is a single point in an Edgeworth box. And, it cannot be an origin.] **b)** ... if none could not be better off (by another feasible allocation) unless anyone is worth off (2pt). [$MRS^A = MRS^B$: no point since it is just a mathematical equivalent property and not the definition.[1]]

c) [Need to prove the claim directly from the definition of Pareto efficiency. Arguing only slopes or tangency is wrong. Besides, logical sequence (especially starting assumptions and ending conclusions) must be clarified. The curve's name, namely "an indifference curve", should be clarified.] **Necessity** (4pt): If $MRS^A \neq MRS^B$ at an allocation x, both people's indifference curves should cross each other at x and thus we can find a point between them. Because this point is above each indifference curve looked from the people's origin, this allocation is better than x for both and thus the allocation x is not Pareto efficient. [The proof should start with $MRS^A \neq MRS^B$ and end with Pareto inefficiency of x. Graph is needed. On the graph, you need to *specify* another allocation that improves their utilities. If you wrote two separate points and two separate curves for a single allocation, you misunderstand an Edgeworth box.]

Sufficiency (4pt): If $MRS^A = MRS^B$ at an allocation x, both people's indifference curves should be tangent to each other at x and thus no point is below A's indifferent curve looked from A's origin, i.e. worse for A than x, or below B's indifferent curve looked from B's origin, i.e. worse for B, or below both. So any point (allocation) cannot be better than x for both people and x is Pareto efficient. [The proof should start with $MRS^A = MRS^B$ at x and end with Pareto efficiency of x. Graph is needed. On the graph, you need to clarify who is worse off than x in each region defined by the two indifference curves.]

d) $x_1^A = x_2^A$ [or $x_1^B = x_2^B$] (3pt). [You need to clarify whose consumption it is.] The diagonal line connecting the two origins of the Edgeworth box (1pt). **e)** $x_1^A = 10$ (2pt). $x_2^A = 10$ (2pt). $x_1^B = 10$ (2pt). $x_2^B = 10$ (2pt) $p_1 = 1, p_2 = 1$ (2pt). [p_1, p_2 can be any pair of two positive numbers as long as $p_1 = p_2$. No partial credit for only p_1 or p_2.] **f)** $p_1 = 2, p_2 = 2$ (2pt). [p_1, p_2 can be any pair of two positive numbers as long as $p_1 = p_2$ and different from your answer in e).] **g)** $MRS^A = -1 = MRS^B$ and thus this equilibrium allocation is Pareto efficient (2pt). [MRS must be calculated.]

Problem 3. **a)** \$2000 (4pt). **b)** $C_1 = 100$ (2pt). $C_2 = 100$ (2pt). $S = 100$ (2pt). Yes, he's smoothing (1pt). No, he's not tilting (1pt). [If you answered only either one question and did not clarify which question you answered, you get no point.] **c)** Demand: $L^D = (w/p)^{-2}$ where p is the product's price and w is wage (4pt). [Thus w/p is the real wage rate.] Equilibrium real wage: $w/p = 1/4$ (2pt). The point $(L, w/p) = (16, 1/4)$ must be plotted on a graph (1pt). **d)** (6pt.) The annual consumption C (thousand dollars) is determined from $\left\{1 - (1.05)^{-40}\right\} \cdot 50/1.05 = \left\{1 - (1.05)^{-60}\right\} C/1.05$. [Further simplification gets full points.]

Problem 4. **a)** DRS (1pt). This is because $F(tK, tL) = t^{1/2} K^{1/4} L^{1/4} = t^{1/2} F(K, L) < tF(K, L)$ [if $t > 1$] (4pt). [Here $F(k, l)$ is the output from $K = k$ and $L = l$.] **b)** $C = 4y^2$ (4pt). Graph is needed on the y-C plane (1pt). **c)** $y^{MES} = 1/\sqrt{2}$ (2pt). $ATC^{MES} = 4\sqrt{2}$ (2pt). **d)** (6pt for giving both the function and the graph.) The supply function $S(p)$ is $p/8$ for $p \geq 4\sqrt{2}$, and 0 for $p \leq 4\sqrt{2}$. On the y-p plane, the graph is $y = p/8$ (i.e. $p = 8y$) for $p \geq 4\sqrt{2}$ and $y = 0$ (a part of the vertical axis) for $p \leq 4\sqrt{2}$.

[1] A Pareto efficient allocation may not hold this equation, unless the preference is convex, monotone, and smooth.

Econ 703
Intermediate Microeconomics
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Solutions to midterm 2 (Group C)

"X and Y (2pt)." means that you get 2 pts if you answered both X and Y, and no pts if you missed either (or both).

Problem 1. [Here I denote by T the state with tornado and by N the state without tornado, instead of 1 and 2.] **a)** \$3 (2pt). **b)** $U(C_T, C_N) = 0.5C_T^2 + 0.5C_N^2$ (1pt). Risk loving (2pt). $CE = 3\sqrt{2}$ (2pt). Larger than EV, because I am risk loving (2pt). **c)** $U(C_T, C_N) = 0.5 \ln C_T + 0.5 \ln C_N$ (1pt). Yes, I'm risk averse (2pt). **d)** $C_T + C_N = 6$ (2pt). Graph is needed on the C_T-C_N plane and its position must be clarified with slope and intercepts (2pt). Plot a point on $(C_T, C_N) = (0, 6)$ for endowment (1pt). **e)** $C_T = 3$ (1pt). $C_N = 3$ (1pt). $x = 6$ (2pt). Plot a point on $(3, 3)$ (1pt). Yes, fully insured (1pt). **f)** e.g. $\gamma = 1$ (2pt). [Any number larger than 0.5 because we need $C_N > C_T$.]

Problem 2. [Here I denote apple by 1, orange by 2, Andy by A and Bob by B. You could use another notation, as long as you clarified it.] **a)** The Edgeworth box should have length of 40 on each axis (1pt). The endowment is $(40, 0)$ looked from A's origin, i.e. $(0, 40)$ from B's origin (1pt). [This is a single point in an Edgeworth box. And, it cannot be an origin.] **b)** ... if none could not be better off (by another feasible allocation) unless anyone is worth off (2pt). [$MRS^A = MRS^B$: no point since it is just a mathematical equivalent property and not the definition.[1]]

c) [Need to prove the claim directly from the definition of Pareto efficiency. Arguing only slopes or tangency is wrong. Besides, logical sequence (especially starting assumptions and ending conclusions) must be clarified. The curve's name, namely "an indifference curve", should be clarified.] **Necessity** (4pt): If $MRS^A \neq MRS^B$ at an allocation x, both people's indifference curves should cross each other at x and thus we can find a point between them. Because this point is above each indifference curve looked from the people's origin, this allocation is better than x for both and thus the allocation x is not Pareto efficient. [The proof should start with $MRS^A \neq MRS^B$ and end with Pareto inefficiency of x. Graph is needed. On the graph, you need to *specify* another allocation that improves their utilities. If you wrote two separate points and two separate curves for a single allocation, you misunderstand an Edgeworth box.]

Sufficiency (4pt): If $MRS^A = MRS^B$ at an allocation x, both people's indifference curves should be tangent to each other at x and thus no point is below A's indifferent curve looked from A's origin, i.e. worse for A than x, or below B's indifferent curve looked from B's origin, i.e. worse for B, or below both. So any point (allocation) cannot be better than x for both people and x is Pareto efficient. [The proof should start with $MRS^A = MRS^B$ at x and end with Pareto efficiency of x. Graph is needed. On the graph, you need to clarify who is worse off than x in each region defined by the two indifference curves.]

d) $x_1^A = x_2^A$ [or $x_1^B = x_2^B$] (3pt). [You need to clarify whose consumption it is.] The diagonal line connecting the two origins of the Edgeworth box (1pt). **e)** $x_1^A = 20$ (2pt). $x_2^A = 20$ (2pt). $x_1^B = 20$ (2pt). $x_2^B = 20$ (2pt) $p_1 = 1, p_2 = 1$ (2pt). [p_1, p_2 can be any pair of two positive numbers as long as $p_1 = p_2$. No partial credit for only p_1 or p_2.] **f)** $p_1 = 2, p_2 = 2$ (2pt). [p_1, p_2 can be any pair of two positive numbers as long as $p_1 = p_2$ and different from your answer in e).] **g)** $MRS^A = -1 = MRS^B$ and thus this equilibrium allocation is Pareto efficient (2pt). [MRS must be calculated.]

Problem 3. **a)** \$1000 (4pt). **b)** $C_1 = 500$ (2pt). $C_2 = 500$ (2pt). $S = 500$ (2pt). Yes, he's smoothing (1pt). No, he's not tilting (1pt). [If you answered only either one question and did not clarify which question you answered, you get no point.] **c)** Demand: $L^D = (w/p)^{-2}$ where p is the product's price and w is wage (4pt). [Thus w/p is the real wage rate.] Equilibrium real wage: $w/p = 1/4$ (2pt). The point $(L, w/p) = (16, 1/4)$ must be plotted on a graph (1pt). **d)** (6pt.) The annual consumption C (thousand dollars) is determined from $\left\{1 - (1.05)^{-40}\right\} \cdot 40/1.05 = \left\{1 - (1.05)^{-60}\right\} C/1.05$. [Further simplification gets full points.]

Problem 4. **a)** DRS (1pt). This is because $F(tK, tL) = t^{1/2}K^{1/4}L^{1/4} = t^{1/2}F(K, L) < tF(K, L)$ [if $t > 1$] (4pt). [Here $F(k, l)$ is the output from $K = k$ and $L = l$.] **b)** $C = 4y^2$ (4pt). Graph is needed on the y-C plane (1pt). **c)** $y^{MES} = 1/\sqrt{2}$ (2pt). $ATC^{MES} = 4\sqrt{2}$ (2pt). **d)** (6pt for giving both the function and the graph.) The supply function $S(p)$ is $p/8$ for $p \geq 4\sqrt{2}$, and 0 for $p \leq 4\sqrt{2}$. On the y-p plane, the graph is $y = p/8$ (i.e. $p = 8y$) for $p \geq 4\sqrt{2}$ and $y = 0$ (a part of the vertical axis) for $p \leq 4\sqrt{2}$.

[1] A Pareto efficient allocation may not hold this equation, unless the preference is convex, monotone, and smooth.

Econ 703
Intermediate Microeconomics
Prof. Marek Weretka

Solutions to midterm 2 (Group D)

"X and Y (2pt)." means that you get 2 pts if you answered both X and Y, and no pts if you missed either (or both).

Problem 1. [Here I denote by T the state with tornado and by N the state without tornado, instead of 1 and 2.] **a)** \$1 (2pt). **b)** $U(C_T, C_N) = 0.5C_T^2 + 0.5C_N^2$ (1pt). Risk loving (2pt). $CE = \sqrt{2}$ (2pt). Larger than EV, because I am risk loving (2pt). **c)** $U(C_T, C_N) = 0.5\ln C_T + 0.5\ln C_N$ (1pt). Yes, I'm risk averse (2pt). **d)** $C_T + C_N = 2$ (2pt). Graph is needed on the C_T-C_N plane and its position must be clarified with slope and intercepts (2pt). Plot a point on $(C_T, C_N) = (0, 2)$ for endowment (1pt). **e)** $C_T = 1$ (1pt). $C_N = 1$ (1pt). $x = 2$ (2pt). Plot a point on $(1, 1)$ (1pt). Yes, fully insured (1pt). **f)** e.g. $\gamma = 1$ (2pt). [Any number larger than 0.5 because we need $C_N > C_T$.]

Problem 2. [Here I denote apple by 1, orange by 2, Andy by A and Bob by B. You could use another notation, as long as you clarified it.] **a)** The Edgeworth box should have length of 10 on each axis (1pt). The endowment is $(10, 0)$ looked from A's origin, i.e. $(0, 10)$ from B's origin (1pt). [This is a single point in an Edgeworth box. And, it cannot be an origin.] **b)** ... if none could not be better off (by another feasible allocation) unless anyone is worth off (2pt). [$MRS^A = MRS^B$: no point since it is just a mathematical equivalent property and not the definition.[1]]

c) [Need to prove the claim directly from the definition of Pareto efficiency. Arguing only slopes or tangency is wrong. Besides, logical sequence (especially starting assumptions and ending conclusions) must be clarified. The curve's name, namely "an indifference curve", should be clarified.] **Necessity** (4pt): If $MRS^A \neq MRS^B$ at an allocation x, both people's indifference curves should cross each other at x and thus we can find a point between them. Because this point is above each indifference curve looked from the people's origin, this allocation is better than x for both and thus the allocation x is not Pareto efficient. [The proof should start with $MRS^A \neq MRS^B$ and end with Pareto inefficiency of x. Graph is needed. On the graph, you need to *specify* another allocation that improves their utilities. If you wrote two separate points and two separate curves for a single allocation, you misunderstand an Edgeworth box.]

Sufficiency (4pt): If $MRS^A = MRS^B$ at an allocation x, both people's indifference curves should be tangent to each other at x and thus no point is below A's indifferent curve looked from A's origin, i.e. worse for A than x, or below B's indifferent curve looked from B's origin, i.e. worse for B, or below both. So any point (allocation) cannot be better than x for both people and x is Pareto efficient. [The proof should start with $MRS^A = MRS^B$ at x and end with Pareto efficiency of x. Graph is needed. On the graph, you need to clarify who is worse off than x in each region defined by the two indifference curves.]

d) $x_1^A = x_2^A$ [or $x_1^B = x_2^B$] (3pt). [You need to clarify whose consumption it is.] The diagonal line connecting the two origins of the Edgeworth box (1pt). **e)** $x_1^A = 5$ (2pt). $x_2^A = 5$ (2pt). $x_1^B = 5$ (2pt). $x_2^B = 5$ (2pt) $p_1 = 1, p_2 = 1$ (2pt). [p_1, p_2 can be any pair of two positive numbers as long as $p_1 = p_2$. No partial credit for only p_1 or p_2.] **f)** $p_1 = 2, p_2 = 2$ (2pt). [p_1, p_2 can be any pair of two positive numbers as long as $p_1 = p_2$ and different from your answer in e).] **g)** $MRS^A = -1 = MRS^B$ and thus this equilibrium allocation is Pareto efficient (2pt). [MRS must be calculated.]

Problem 3. **a)** \$4000 (4pt). **b)** $C_1 = 500$ (2pt). $C_2 = 500$ (2pt). $S = 500$ (2pt). Yes, he's smoothing (1pt). No, he's not tilting (1pt). [If you answered only either one question and did not clarify which question you answered, you get no point.] **c)** Demand: $L^D = (w/p)^{-2}$ where p is the product's price and w is wage (4pt). [Thus w/p is the real wage rate.] Equilibrium real wage: $w/p = 1/4$ (2pt). The point $(L, w/p) = (16, 1/4)$ must be plotted on a graph (1pt). **d)** (6pt.) The annual consumption C (thousand dollars) is determined from $\left\{1 - (1.05)^{-40}\right\} \cdot 60/1.05 = \left\{1 - (1.05)^{-60}\right\} C/1.05$. [Further simplification gets full points.]

Problem 4. **a)** DRS (1pt). This is because $F(tK, tL) = t^{1/2}K^{1/4}L^{1/4} = t^{1/2}F(K, L) < tF(K, L)$ [if $t > 1$] (4pt). [Here $F(k, l)$ is the output from $K = k$ and $L = l$.] **b)** $C = 4y^2$ (4pt). Graph is needed on the y-C plane (1pt). **c)** $y^{MES} = 1/\sqrt{2}$ (2pt). $ATC^{MES} = 4\sqrt{2}$ (2pt). **d)** (6pt for giving both the function and the graph.) The supply function $S(p)$ is $p/8$ for $p \geq 4\sqrt{2}$, and 0 for $p \leq 4\sqrt{2}$. On the y-p plane, the graph is $y = p/8$ (i.e. $p = 8y$) for $p \geq 4\sqrt{2}$ and $y = 0$ (a part of the vertical axis) for $p \leq 4\sqrt{2}$.

[1] A Pareto efficient allocation may not hold this equation, unless the preference is convex, monotone, and smooth.

Econ 301
Intermediate Microeconomics
Prof. Marek Weretka

Midterm 2 (Group A)

You have 70 minutes to complete the exam. The midterm consists of 4 questions (25+35+15+25=100 points) + a bonus (10 "extra" points). Make sure you answer the first four questions before working on the bonus one!

Problem 1 (25p). (Labor supply)

Eric's total available time is $24h$ (per day). He works as a waiter with the wage rate w and he spends his money on consuming New York Steaks C, that cost $\$p$ each.

a) on a graph with leisure time (R) measured on the horizontal axis and consumption (C) on the vertical one plot Eric's budget set assuming $w = 10, p = 2$. Provide some economic interpretation of the slope of the budget line.

b) suppose his utility is given by

$$U(C; R) = R^2 \times C$$

where R is leisure and C is consumption of New York Steaks. Find his optimal time at work (labor supply LS), the relaxation time R and the steak consumption C as a function of w and p (parameters). Calculate the values of the three variables for $w = 10$, and $p = 2$.

c) on a graph with labor supply LS measured on the horizontal axis and real wage w/p on the vertical one plot the entire labor supply curve (marking the three points that you have found analytically); what can you say about the sensitivity (elasticity) of labor supply to changes in real wage rate? explain in 2 short sentences.

Problem 2 (35p). (Edgeworth box - Irving Fisher interest rate determination)

Consumption can take place in two periods: today (C_1) and tomorrow (C_2). Peter has income of \$100 today and tomorrow. (hence his endowment is $\omega^P = (100, 100)$). Amanda today's income is \$100 and tomorrow is \$300 ($\omega^A = (100, 300)$). They both have the same utility function

$$U^i (C_1, C_2) = \ln (C_1) + \ln (C_2)$$

a) mark the allocation corresponding to the endowment point in the Edgeworth box

b) argue whether the endowment allocation is (or is not) Pareto efficient (use values of MRS at the endowment point in your argument). Illustrate your argument geometrically in the Edgeworth Box from a)

c) find analytically the equilibrium interest rate and allocation and show it in the Edgeworth box. (Hint: Instead of working with "intertemporal" model, you can first find equilibrium prices p_1 and p_2, and then use the formula:

$$\frac{p_1}{p_2} = 1 + r$$

d) who among the two traders is borrowing and who is lending? How much? (one sentence + two numbers)

e) argue that the "invisible hand of financial markets" works perfectly, that is, the equilibrium outcome is Pareto efficient. (one sentence, two numbers, use values of MRS)

f) find PV (in today's \$) , and FV (in tomorrow's \$) of Amanda's income, given the equilibrium interest rate. (give two numbers)

Problem 3 (15p). (Short questions)

Answer the following three questions a), b) and c)

a) Consider a lottery that pays \$100 when it rains and \$36 when it does not, and both states are equally likely $\left(\pi_R = \pi_{NR} = \frac{1}{2}\right)$. Find the expected value of the lottery and the certainly equivalent of the lottery, given Bernoulli utility function $u(c) = \sqrt{c}$. Which is bigger? Explain why. (two numbers+ one sentence)

b) Consider a pineapple tree that every year produces fruits worth $5000 (starting next year), forever. How much are you willing to pay for such a tree now, given the interest rate of 20%? (one number)

c) Find the constant payment x you have to make in three consecutive periods (one, two, and three), in order to pay back a loan worth $1400 taken in period zero, given that the interest rate is 100%? (one number)

Problem 4 (25p). (Producers)

Consider a producer that has the following technology

$$y = K^{\frac{1}{4}} L^{\frac{1}{4}}$$

a) what returns to scale are represented by this production function? (choose: CRS, IRS or DRS and support your choice with a mathematical argument).

b) find analytically the level of capital (K), labor (L) and output (y) that maximizes profit, and the value of maximal profit, given $p = 4$ and $w_K = w_L = 1$.

c) find the average cost function $AC(y)$, and plot it on a graph (prices are as in b). On the same graph show geometrically the level of maximal profit from b) (Hint: for the second part, take the value y from b)).

Bonus Problem. (extra 10 points)

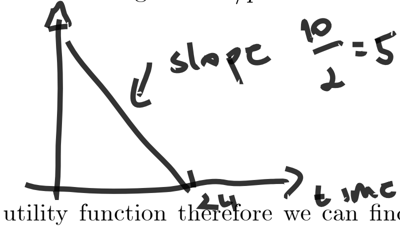
Derive (not just give!) the formula for PV of annuity (explain each step, starting with deriving PV for perpetuity).

Econ 301
Intermediate Microeconomics
Prof. Marek Weretka

Solutions to midterm 2 (Group A)

Problem 1 (25p). (Labor supply)

a) The slope of the budget set is a real wage rate w/p that tells how many steaks Peter can get for every hour he works.



b) This is a Cobb-Douglass utility function therefore we can find his optimal choice R, C using our "magic" formula, we have derived earlier in our class. The values of parameters are:

$$a = 2, b = 1,$$

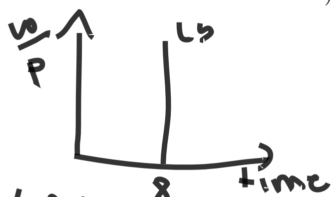
and hence the relaxation time and consumption is

$$R = \frac{a}{a+b}\frac{m}{p_1} = \frac{2}{3}\frac{24w}{w} = 16$$
$$C = \frac{b}{a+b}\frac{m}{p_1} = \frac{1}{3}\frac{24w}{p} = 8\frac{w}{p}$$

For $w = 10$ and $p = 2$ we have $R = 16$ and C=40. In such case the labor supply is given by

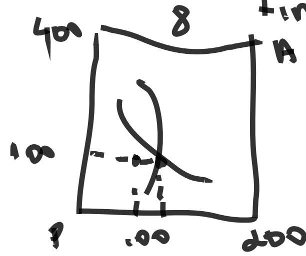
$$LS = 24 - R = 8$$

c) The labor supply function is inelastic with respect to $\frac{w}{p}$. The reason for that is that the substitution effect (higher real wage makes leisure more expensive relative to consumption encouraging work) is offset by income effect (the higher income makes leisure more attractive)



Problem 2 (35p).

a)



b) The endowment allocation is not Pareto efficient, as at this allocation the slopes of indifference curves

$$MRS^P = \frac{C_2^P}{C_1^P} = \frac{100}{100} = 1$$
$$MRS^A = \frac{C_2^A}{C_1^A} = \frac{300}{100} = 3$$

and hence they do not coincide (see graph above).

c) We normalize $p_2 = 1$. The optimal consumption today is

$$\begin{aligned} C_1^P &= \frac{1}{2}\frac{100p_1 + 100}{p_1} \\ C_1^P &= \frac{1}{2}\frac{100p_1 + 300}{p_1} \end{aligned}$$

Market clearing condition implies that

$$\frac{1}{2}\frac{100p_1 + 100}{p_1} + \frac{1}{2}\frac{100p_1 + 300}{p_1} = 200$$

or

$$p_1 = 2$$

and hence

$$r = 100\%$$

At this price consumption is given by

$$C_1^P = \frac{1}{2}\frac{2 \times 100 + 100}{2} = 75 \text{ and } C_1^A = 200 - 75 = 125$$

and

$$C_2^P = \frac{1}{2}\frac{2 \times 100 + 100}{1} = 150 \text{ and } C_2^A = 400 - 150 = 250$$

Hence allocation $C^P = (75, 150)$, $C^A = (125, 250)$ and interest rate $r = 100\%$ is an equilibrium.

d) Savings are given by

$$s^P = \omega_1^P - C_1^P = 100 - \frac{1}{2}\frac{200 + 100}{2} = 25$$

hence Peter is saving \$25

$$s^A = \omega_1^A - C_1^A = 100 - \frac{1}{2}\frac{500}{2} = -25$$

and Amanda is borrowing \$25

e)

$$\begin{aligned} MRS^P &= \frac{C_2^P}{C_1^P} = \frac{150}{75} = 2 \\ MRS^A &= \frac{C_2^A}{C_1^A} = \frac{250}{125} = 2 \end{aligned}$$

The equilibrium allocation is Pareto efficient as the indifference curves are tangent (they have the same slope MRS)

f)

$$\begin{aligned} PV &= 100 + \frac{300}{1 + 100\%} = 100 + \frac{300}{2} = 250 \\ FV &= 100 \times (1 + 100\%) + 300 = 200 + 300 = 500 \end{aligned}$$

Problem 3 (15p). (Short questions)

a) Expected value of the lottery is

$$E(L) = \frac{1}{2} \times 100 + \frac{1}{2} \times 36 = 68$$

The von Neuman Morgenstern lottery is

$$U = \frac{1}{2}\sqrt{100} + \frac{1}{2}\sqrt{36} = \frac{1}{2} \times 10 + \frac{1}{2} \times 6 = 8$$

the Certainty equivalent is

$$\sqrt{CE} = 8 \Rightarrow CE = 64$$

$CE < E(L)$ because the agent is risk averse, and hence is willing to accept lower payment for sure.

b) You are willing to pay PV

$$PV = \frac{5000}{0.2} = 25000$$

c) Using annuity formula

$$1400 = \frac{x}{1}\left(1 - \left(\frac{1}{2}\right)^3\right) = \frac{7}{8}x \Rightarrow x = \frac{8}{7}1400 = 8 \times 200 = 1600$$

Problem 4 (25p). (Producers)

a) Suppose $\lambda > 1$. Then

$$F(\lambda K, \lambda L) = (\lambda K)^{\frac{1}{4}} (\lambda L)^{\frac{1}{4}} = \lambda^{\frac{1}{2}} K^{\frac{1}{4}} L^{\frac{1}{4}} < \lambda K^{\frac{1}{4}} L^{\frac{1}{4}} = \lambda F(K, L)$$

hence we have DRS.

b) We use two conditions

$$\begin{aligned} MPK &= \frac{w_K}{p} \\ MPL &= \frac{w_L}{p} \end{aligned}$$

which become $\frac{1}{4}$

$$\begin{aligned} \frac{1}{4}K^{-\frac{3}{4}}L^{\frac{1}{4}} &= \frac{1}{4} \\ \frac{1}{4}K^{\frac{1}{4}}L^{-\frac{3}{4}} &= \frac{1}{4} \end{aligned}$$

Implying

$$\frac{K}{L} = 1 \Rightarrow K = L$$

Plugging back in the two secrets of happiness

$$\begin{aligned} K^{-\frac{3}{4}}K^{\frac{1}{4}} &= K^{-\frac{1}{2}} = 1 \Rightarrow K = 1 \\ L^{\frac{1}{4}}L^{-\frac{3}{4}} &= L^{-\frac{1}{2}} = 1 \Rightarrow L = 1 \end{aligned}$$

The optimal level of production is

$$y = K^{\frac{1}{4}}L^{\frac{1}{4}} = 1^{\frac{1}{4}}1^{\frac{1}{4}} = 1$$

and profit

$$\pi = 4 \times 1 - 1 \times 1 - 1 \times 1 = 2$$

c) Secret of happiness for cost minimization is

$$TRS = \frac{L}{K} = \frac{w_K}{w_L} = 1 \Rightarrow K = L$$

hence

$$y = K^{\frac{1}{4}}L^{\frac{1}{4}} = K^{\frac{1}{2}}$$

hence

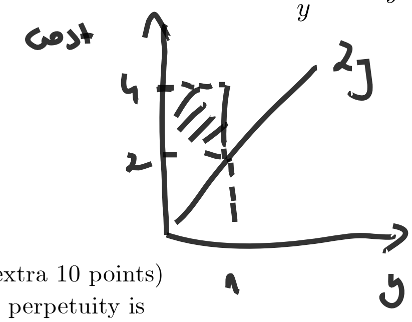
$$K = L = y^2$$

hence

$$C(y) = 2y^2$$

and

$$AC = \frac{C(y)}{y} = 2y$$



Bonus Problem. (extra 10 points)
The present value of a perpetuity is

$$\begin{aligned} PV &= \frac{x}{1+r} + \frac{x}{(1+r)^2} + \frac{x}{(1+r)^3} + ... \\ &= \frac{1}{1+r}[x + PV] \end{aligned}$$

Solving for PV gives

$$PV = \frac{x}{r}$$

For the asset that pays x up to period T we have to decrease the PV by the PV of a "missing payment" after T. The present value of this payment in \$ from period T is $\frac{x}{r}$ and hence in \$ from period zero it is $\left(\frac{1}{1+r}\right)^T \frac{x}{r}$. Subtracting this number from PV for perpetuity gives

$$PV = \frac{x}{r} - \left(\frac{1}{1+r}\right)^T \frac{x}{r} = \frac{x}{r}\left[1 - \left(\frac{1}{1+r}\right)^T\right]$$

which is the formula of the PV of annuity.

Econ 301
Intermediate Microeconomics
Prof. Marek Weretka

Midterm 2 (Group A)

You have 70 minutes to complete the exam. The midterm consists of 4 questions (20+30+20+30=100 points) + a bonus (10 "extra" points). Make sure you answer the first four questions before working on the bonus one!

Problem 1 (20p). (Intertemporal choice)

Frank works as a consultant. His income when young is \$2000 (period 1) and \$8000 when old (period 2), the interest rate is $r = 100\%$.

a) In the graph depict Frank's budget set. Mark all the bundles on the budget line that involve saving and the ones that involve borrowing. Find analytically PV and FV of income and show it in the graph

b) Frank's intertemporal preferences are given by

$$U(C_1; C_2) = \ln C_1 + \frac{1}{1+\delta} \ln C_2$$

where the discount factor is $\delta = 100\%$. Using the magic formula, find the optimal consumption plan (C_1, C_2) and how much Frank borrows or saves (three numbers).

c) Is Frank smoothing his consumption? (yes or no answer + one sentence)

Problem 2 (30p). (Edgeworth box, and equilibrium)

Consider an economy with apples and oranges. Peter is initially endowed with ten apples and 30 oranges ($\omega^P = (10, 30)$). Amanda's endowment is $\omega^A = (30, 10)$.

a) Plot the Edgeworth box and mark the allocation representing the initial endowment.

b) Describe the concept of Pareto efficiency (one intuitive sentence). Peter and Amanda have the same utility function

$$U^i(C_1, C_2) = 3\ln(C_1) + 3\ln(C_2).$$

Verify whether the endowment allocation is (or is not) Pareto efficient (use values of MRS in your argument). Illustrate your argument geometrically in the Edgeworth Box from a).

c) Find analytically the competitive equilibrium (six numbers) and show it in the Edgeworth box. Find some other prices that define competitive equilibrium (two numbers).

d) Argue that competitive markets allocate resources efficiently (give two numbers and compare them).

Problem 3 (20p). (Short questions)

Answer the following three questions.

a) The Bernoulli utility function is given by $u(c) = c^2$ and two states of the world are equally likely. Find the corresponding von Neuman Morgenstern (expected) utility function (give formula). Is such agent risk neutral, risk loving or risk averse? (one sentence). Find the expected value and the certainty equivalent of a lottery $(10, \sqrt{28})$.(two numbers). Which is bigger and why (one sentence) (Hint: when calculating expected value of the lottery, use that $\sqrt{28} \simeq 5.3$).

b) Derive the formula for perpetuity.

c) You will live for 4 periods. You would like to maintain the constant level consumption throughout your life C. How much can you consume if in the first three periods you earn \$1500? The interest rate is $r = 100\%$? (one number)

Problem 4 (30p). (Producers)

Consider a producer that has the following technology

$$y = 8K^{\frac{1}{4}}L^{\frac{1}{2}}$$

a) What returns to scale are represented by this production function? (choose between CRS, IRS or DRS; you do not have to prove it).

b) (Short run) Assume that $\bar{K} = 1$ and the firm cannot change it in a short run. Derive a condition for optimal demand for labor. Explain intuitively its economic meaning. (one sentence).

c) Suppose that labor supply is inelastic and given by $L^s = 16h$. Find analytically and on the graph the equilibrium wage rate.

d) Find the unemployment rate with the minimal (real) wage given by $w_L/p = 4/3$.(one number)

e) Suppose $w_L = 1, w_K = 2$. Derive the cost function $C(y)$, assuming that you can adjust both K and L, and plot it on the graph. Relate the shape of your cost function to the returns to scale. (Hint: the constants in this last questions are not round numbers)

Bonus Problem. (extra 10 points)

Give examples of production functions with perfect complements and perfect substitutes that are characterized by increasing and decreasing returns to scale.

Econ 301
Intermediate Microeconomics
Prof. Marek Weretka

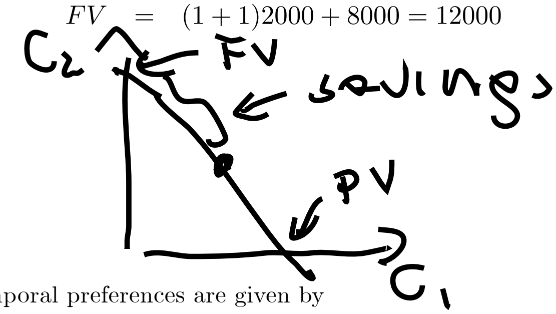
Solutions to midterm 2 (Group A)

Problem 1 (20p). (Intertemporal choice)

Frank works as a consultant. His income when young is \$2000 (period 1) and \$8000 when old (period 2), the interest rate is $r = 100\%$.

a) In the graph depict Frank's budget set. Mark all the bundles on the budget line that involve saving and the ones that involve borrowing. Find analytically PV and FV of income and show it in the graph

$$
\begin{aligned}
PV &= 2000 + \frac{8000}{1+1} = 2000 + 4000 = 6000 \\
FV &= (1+1)2000 + 8000 = 12000
\end{aligned}
$$



b) Frank's intertemporal preferences are given by

$$U(C_1; C_2) = \ln C_1 + \frac{1}{1+\delta} \ln C_2$$

where the discount factor is $\delta = 100\%$. Using the magic formula, find the optimal consumption plan (C_1, C_2) and how much Frank borrows or saves (three numbers).

$$
\begin{aligned}
C_1 &= \frac{1}{1+\frac{1}{2}} \frac{12000}{2} = \frac{2}{3} 6000 = 4000 \\
C_2 &= \frac{\frac{1}{2}}{1+\frac{1}{2}} \frac{12000}{1} = \frac{1}{3} 12000 = 4000 \\
S &= 2000 - 4000 = -2000
\end{aligned}
$$

Frank borrows $-\$2000$

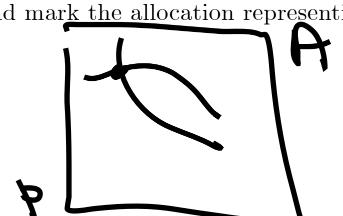
c) Is Frank smoothing his consumption? (yes or no answer + one sentence).

Yes, because $C_1 = C_2$. This is because $\delta = r$.

Problem 2 (30p). (Edgeworth box, and equilibrium)

Consider an economy with apples and oranges. Peter is initially endowed with five apples and ten oranges $\omega^P = (10, 30)$. Amanda's endowment is $\omega^A = (30, 10)$.

a) Plot the Edgeworth box and mark the allocation representing the initial endowment.



b) Describe the concept of Pareto efficiency (one intuitive sentence). Peter and Amanda have the same utility function

$$U^i(C_1, C_2) = 2\ln(C_1) + 2\ln(C_2).$$

Verify whether the endowment allocation is (or is not) Pareto efficient (use values of MRS in your argument). Illustrate your argument geometrically in the Edgeworth Box from a).

The endowment allocation is not Pareto efficient, as at this allocation the slopes of indifference curves are not tangent to each other

$$MRS^P = \frac{C_2^P}{C_1^P} = \frac{30}{10} = 3$$
$$MRS^A = \frac{C_2^A}{C_1^A} = \frac{10}{30} = \frac{1}{3}$$

and hence they do not coincide (see graph above).

c) Find analytically the competitive equilibrium (six numbers) and show it in the Edgeworth box. Find some other prices that define competitive equilibrium (two numbers).

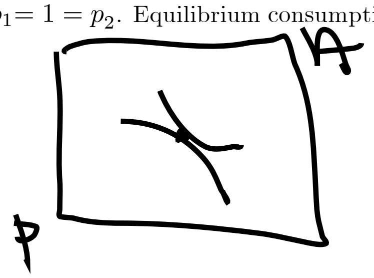
We normalize $p_2 = 1$

$$C_1^P = \frac{1}{2}\frac{10p_1 + 30}{p_1}$$
$$C_1^A = \frac{1}{2}\frac{30p_1 + 10}{p_1}$$

and market clearing condition gives

$$\frac{1}{2}\frac{10p_1 + 30}{p_1} + \frac{1}{2}\frac{30p_1 + 10}{p_1} = 40$$

From which one can find price $p_1 = 1 = p_2$. Equilibrium consumption is $C_1^A = C_1^B = 20$ and $C_2^A = C_2^B = 20$.



Other prices: $p_1 = p_2 = 2$

d) Argue that competitive markets allocate resources efficiently (give two numbers and compare them).

Allocation in competitive equilibrium is Pareto efficient as MRS of both agents are the same

$$MRS^P = \frac{C_2^P}{C_1^P} = \frac{20}{20} = 1$$
$$MRS^A = \frac{C_2^A}{C_1^A} = \frac{20}{20} = 1$$

Problem 3 (20p). (Short questions)

a) The Bernoulli utility function is given by $u(c) = c^2$ and two states of the world are equally likely. Find the corresponding von Neuman Morgenstern (expected) utility function (give formula). Is such agent risk neutral, risk loving or risk averse? (one sentence). Find the expected value and the certainty equivalent of a lottery $\left(10, \sqrt{28}\right)$.(two numbers). Which is bigger and why (one sentence) (Hint: when calculating expected value of the lottery, use that $\sqrt{28} \simeq 5.3$).

Expected Utility function is given by

$$U(c_1, c_2) = \frac{1}{2}c_1^2 + \frac{1}{2}c_2^2$$

Agent is risk loving as Bernouli utility function is convex.

$$E(L) = \frac{1}{2}10 + \frac{1}{2}5.3 = 7.6$$

. Certainty equivalent can be found as

$$(CE)^2 = U\left(10, \sqrt{28}\right) = 50 + 14 = 64$$

and hence

$$CE = 8 > E(L)$$

This is because risk loving agent derives extra utility from uncertainty regarding the outcome.

b) Derive the formula for perpetuity

$$\begin{aligned} PV &= \frac{x}{1+r} + \frac{x}{(1+r)^2} + \frac{x}{(1+r)^2} + ... = \\ &= \frac{x}{1+r} + \frac{1}{1+r}\left(\frac{x}{(1+r)} + \frac{x}{(1+r)} + ...\right) = \\ &= \frac{x}{1+r} + \frac{1}{1+r}PV \end{aligned}$$

Solving for PV gives

$$PV = \frac{x}{r}$$

.c) You will live for 4 periods. You would like to maintain the constant level consumption throughout your life C. How much can you consume if in the first three periods you earn \$1500? The interest rate is r=100%? (one number)

$$\begin{aligned} \frac{C}{r}\left(1 - \left(\frac{1}{1+r}\right)^4\right) &= \frac{1500}{r}\left(1 - \left(\frac{1}{1+r}\right)^3\right) \\ \frac{15}{16}C &= 1500\frac{7}{8} \\ C &= 1400 \end{aligned}$$

Problem 4 (30p). (Producers)

Consider a producer that has the following technology

$$y = 8K^{\frac{1}{4}}L^{\frac{1}{2}}$$

a) What returns to scale are represented by this production function? (choose between CRS, IRS or DRS; you do not have to prove it).

DRS

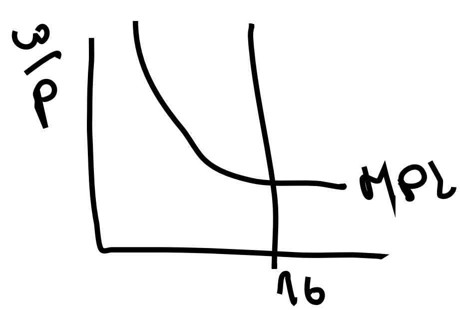
b) (Short run) Assume that $\bar{K} = 1$ and the firm cannot change it in a short run. Derive a condition for optimal demand for labor. Explain intuitively its economic meaning. (one sentence). ($MPL = \frac{w_k}{p}$)

$$\frac{w_k}{p} = 4L^{-\frac{1}{2}}$$

Last worker produces as much as he gets in terms of wage.

c) Suppose that labor supply is inelastic and given by $L^s = 16h$. Find analytically and on the graph the equilibrium wage rate.

$$\frac{w_k}{p} = 4(16)^{-\frac{1}{2}} = 1$$



d) Find the unemployment rate with the minimal (real) wage given by $w_L/p = 4/3$.(one number)
With minimal wage rate the demand for labor is

$$4/3 = 4L^{-\frac{1}{2}} \Rightarrow L = 9$$

and hence unemployment rate is

$$UR = \frac{16-9}{16} = \frac{7}{16}$$

e) Suppose $w_L = 1, w_K = 2$. Derive the cost function C(y), assuming that you can adjust both K and L, and plot it on the graph. Relate the shape of your cost function to the returns to scale. (Hint: the constants in this last questions are not round numbers)

$$8K^{\frac{1}{4}}L^{\frac{1}{2}}$$

$$TRS = -\frac{1}{2}\frac{L}{K} = -\frac{2}{1}$$

and hence

$$L = 4K$$

It follows that

$$K = \left(\frac{1}{16}y\right)^{\frac{4}{3}}$$

and

$$L = 4\left(\frac{1}{16}y\right)^{\frac{4}{3}}$$

It follows that

$$c(y) = 4\left(\frac{1}{16}y\right)^{\frac{4}{3}} + 2\left(\frac{1}{16}y\right)^{\frac{4}{3}} = 6\left(\frac{1}{16}y\right)^{\frac{4}{3}}$$

The function is convex as we have DRS.

Bonus Problem. (extra 10 points)

Give examples of production functions with perfect complements and perfect substitutes that are characterized by increasing and decreasing returns to scale.

Perfect complements

$$\begin{aligned} y &= [\min(2K, 7L)]^2 \text{ (IRS)} \\ y &= [\min(2K, 7L)]^{\frac{1}{2}} \text{ (DRS)} \end{aligned}$$

Perfect substitutes

$$\begin{aligned} y &= (2K + 7L)^2 \text{ (IRS)} \\ y &= [2K + 7L]^{\frac{1}{2}} \text{ (DRS)} \end{aligned}$$

Econ 301
Intermediate Microeconomics
Prof. Marek Weretka

Midterm 2 (Group B)

You have 70 minutes to complete the exam. The midterm consists of 4 questions (25+35+15+25=100 points) + a bonus (10 "extra" points). Make sure you answer the first four questions before working on the bonus one!

Problem 1 (25p). (Labor supply)

Eric's total available time is $24h$ (per day). He works as a waiter with the wage rate w and he spends his money on consuming New York Steaks C, that cost $\$p$ each.

a) on a graph with leisure time (R) measured on the horizontal axis and consumption (C) on the vertical one plot Eric's budget set assuming $w = 100, p = 5$. Provide some economic interpretation of the slope of the budget line.

b) suppose his utility is given by

$$U(C; R) = R^3 \times C$$

where R is leisure and C is consumption of New York Steaks. Find his optimal time at work (labor supply LS), the relaxation time R and the steak consumption C as a function of w and p (parameters). Calculate the values of the three variables for $w = 100$, and $p = 5$.

c) on a graph with labor supply LS measured on the horizontal axis and real wage w/p on the vertical one plot the entire labor supply curve (marking the three points that you have found analytically); what can you say about the sensitivity (elasticity) of labor supply to changes in real wage rate? explain in 2 short sentences.

Problem 2 (35p). (Edgeworth box - Irving Fisher interest rate determination)

Consumption can take place in two periods: today (C_1) and tomorrow (C_2). Peter has income of $100 today and $300 tomorrow. (hence his endowment is $\omega^P = (100, 300)$). Amanda today's income is $100 in both periods ($\omega^A = (100, 100)$). They both have the same utility function

$$U^i (C_1, C_2) = \frac{1}{2} \ln (C_1) + \frac{1}{2} \ln (C_2)$$

a) mark the allocation corresponding to the endowment point in the Edgeworth box

b) argue whether the endowment allocation is (or is not) Pareto efficient (use values of MRS at the endowment point in your argument). Illustrate your argument geometrically in the Edgeworth Box from a)

c) find analytically the equilibrium interest rate and allocation and show it in the Edgeworth box. (Hint: Instead of working with "intertemporal" model, you can first find equilibrium prices p_1 and p_2, and then use the formula:

$$\frac{p_1}{p_2} = 1 + r$$

d) who among the two traders is borrowing and who is lending? How much? (one sentence + two numbers)

e) argue that the "invisible hand of financial markets" works perfectly, that is, the equilibrium outcome is Pareto efficient. (one sentence, two numbers, use values of MRS)

f) find PV (in today's $) , and FV (in tomorrow's $) of Amanda's income, given the equilibrium interest rate. (give two numbers)

Problem 3 (15p). (Short questions)

Answer the following three questions a), b) and c)

a) Find the constant payment x you have to make in three consecutive periods (one, two, and three), in order to pay back a loan worth $2800 taken in period zero, given that the interest rate is 100%? (one number)

b) Consider a lottery that pays $\$36$ when it rains and $\$25$ when it does not, and both states are equally likely $\left(\pi_R = \pi_{NR} = \frac{1}{2}\right)$. Find the expected value of the lottery and the certainly equivalent of the lottery, given Bernoulli utility function $u(c) = \sqrt{c}$. Which is bigger? Explain why. (two numbers+ one sentence)

c) Consider a pineapple tree that every year produces fruits worth $\$500$ (starting next year), forever. How much are you willing to pay for such a tree now, given the interest rate of 10%? (one number)

Problem 4 (25p). (Producers)

Consider a producer that has the following technology

$$y = K^{\frac{1}{6}} L^{\frac{1}{6}}$$

a) what returns to scale are represented by this production function? (choose: CRS, IRS or DRS and support your choice with a mathematical argument).

b) find analytically the level of capital (K), labor (L) and output (y) that maximizes profit, and the value of maximal profit, given $p = 6$ and $w_K = w_L = 1$.

c) find the average cost function $AC(y)$, and plot it on a graph (prices are as in b). On the same graph show geometrically the level of maximal profit from b) (Hint: for the second part, take the value y from b)).

Bonus Problem. (extra 10 points)

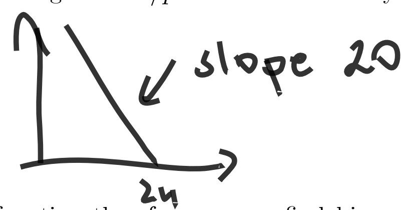
Derive (not just give!) the formula for PV of annuity (explain each step, starting with deriving PV for perpetuity).

Econ 301
Intermediate Microeconomics
Prof. Marek Weretka

Solutions to midterm 2 (Group B)

Problem 1 (25p). (Labor supply)

a) The slope of the budget set is a real wage rate w/p that tells how many steaks Peter can get for every hour he works.



b) This is a Cobb-Douglass utility function therefore we can find his optimal choice R, C using our "magic" formula, we have derived earlier in our class. The values of parameters are:

$$a = 3, b = 1,$$

and hence the relaxation time and consumption is

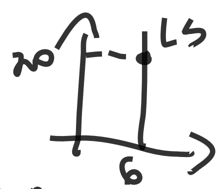
$$R = \frac{a}{a+b}\frac{m}{p_1} = \frac{3}{4}\frac{24w}{w} = 18$$
$$C = \frac{b}{a+b}\frac{m}{p_1} = \frac{1}{4}\frac{24w}{p} = 6\frac{w}{p}$$

For $w = 100$, and $p = 5$ we have $R = 18$ and $C = 120$

In such case the labor supply is given by

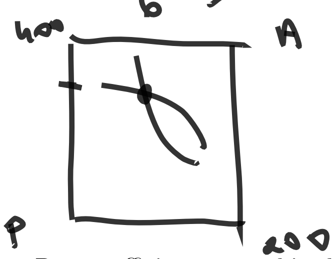
$$LS = 24 - R = 6$$

c) The labor supply function is inelastic with respect to $\frac{w}{p}$. The reason for that is that the substitution effect (higher real wage makes leisure more expensive relative to consumption encouraging work) is offset by income effect (the higher income makes leisure more attractive)



Problem 2 (35p).

a)



b) The endowment allocation is not Pareto efficient, as at this allocation the slopes of indifference curves

$$MRS^P = \frac{C_2^P}{C_1^P} = \frac{300}{100} = 3$$
$$MRS^A = \frac{C_2^A}{C_1^A} = \frac{100}{100} = 1$$

and hence they do not coincide (see graph above).

c) We normalize $p_2 = 1$. The optimal consumption today is

$$\begin{aligned} C_1^P &= \frac{1}{2}\frac{100p_1 + 300}{p_1} \\ C_1^A &= \frac{1}{2}\frac{100p_1 + 100}{p_1} \end{aligned}$$

Market clearing condition implies that

$$\frac{1}{2}\frac{100p_1 + 300}{p_1} + \frac{1}{2}\frac{100p_1 + 100}{p_1} = 200$$

or

$$p_1 = 2$$

and hence

$$r = 100\%$$

At this price consumption is given by

$$C_1^P = \frac{1}{2}\frac{2 \times 100 + 300}{2} = 125 \text{ and } C_1^A = 200 - 125 = 75$$

and

$$C_2^P = \frac{1}{2}\frac{2 \times 100 + 300}{1} = 250 \text{ and } C_2^A = 400 - 250 = 150$$

Hence allocation $C^P = (125, 250)$, $C^A = (75, 125)$ and interest rate $r = 100\%$ is an equilibrium.

d) Savings are given by

$$s^P = \omega_1^P - C_1^P = 100 - 125 = -25$$

hence Peter is borrowing \$25

$$s^A = \omega_1^A - C_1^A = 100 - 75 = 25$$

and Amanda is saving \$25

e)

$$\begin{aligned} MRS^P &= \frac{C_2^P}{C_1^P} = \frac{250}{125} = 2 \\ MRS^A &= \frac{C_2^A}{C_1^A} = \frac{150}{75} = 2 \end{aligned}$$

The equilibrium allocation is Pareto efficient as the indifference curves are tangent (they have the same slope MRS)

f)

$$\begin{aligned} PV &= 100 + \frac{100}{1 + 100\%} = 100 + \frac{100}{2} = 150 \\ FV &= 100 \times (1 + 100\%) + 100 = 200 + 100 = 300 \end{aligned}$$

Problem 3 (15p). (Short questions)

a) Using annuity formula

$$2800 = \frac{x}{1}\left(1 - \left(\frac{1}{2}\right)^3\right) = \frac{7}{8}x \Rightarrow x = \frac{8}{7}2800 = 8 \times 400 = 3200$$

b) Expected value of the lottery is

$$E(L) = \frac{1}{2} \times 36 + \frac{1}{2} \times 25 = 18 + 12\frac{1}{2} = 30\frac{1}{2}$$

The von Neuman Morgenstern lottery is

$$U = \frac{1}{2}\sqrt{36} + \frac{1}{2}\sqrt{25} = \frac{1}{2} \times 6 + \frac{1}{2} \times 5 = \frac{11}{2}$$

the Certainty equivalent is

$$\sqrt{CE} = \frac{11}{2} \Rightarrow CE = \frac{(11)^2}{4} = 30\frac{1}{4}$$

$CE < E(L)$ because the agent is risk averse, and hence is willing to accept lower payment for sure.

c) You are willing to pay PV

$$PV = \frac{500}{0.1} = 5000$$

Problem 4 (25p). (Producers)

a) Suppose $\lambda > 1$. Then

$$F(\lambda K, \lambda L) = (\lambda K)^{\frac{1}{6}} (\lambda L)^{\frac{1}{6}} = \lambda^{\frac{1}{3}} K^{\frac{1}{6}} L^{\frac{1}{6}} < \lambda K^{\frac{1}{6}} L^{\frac{1}{6}} = \lambda F(K, L)$$

hence we have DRS.

b) We use two conditions

$$\begin{aligned} MPK &= \frac{w_K}{p} \\ MPL &= \frac{w_L}{p} \end{aligned}$$

which become $\frac{1}{4}$

$$\begin{aligned} \frac{1}{6} K^{-\frac{5}{6}} L^{\frac{1}{6}} &= \frac{1}{6} \\ \frac{1}{6} K^{\frac{1}{6}} L^{-\frac{5}{6}} &= \frac{1}{6} \end{aligned}$$

Implying

$$\frac{K}{L} = 1 \Rightarrow K = L$$

Plugging back in the two secrets of happiness

$$\begin{aligned} K^{-\frac{5}{6}} K^{\frac{1}{6}} &= K^{-\frac{2}{3}} = 1 \Rightarrow K = 1 \\ L^{\frac{1}{6}} L^{-\frac{5}{6}} &= L^{-\frac{2}{3}} = 1 \Rightarrow L = 1 \end{aligned}$$

The optimal level of production is

$$y = K^{\frac{1}{6}} L^{\frac{1}{6}} = 1^{\frac{1}{6}} 1^{\frac{1}{6}} = 1$$

and profit

$$\pi = 6 \times 1 - 1 \times 1 - 1 \times 1 = 4$$

c) Secret of happiness for cost minimization is

$$TRS = \frac{L}{K} = \frac{w_K}{w_L} = 1 \Rightarrow K = L$$

hence

$$y = K^{\frac{1}{6}} L^{\frac{1}{6}} = K^{\frac{1}{3}}$$

hence

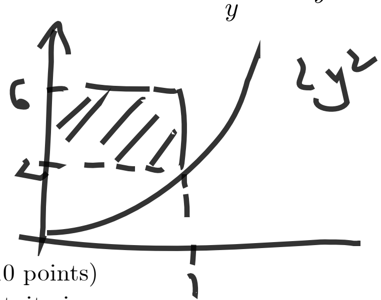
$$K = L = y^3$$

hence

$$C(y) = 2y^3$$

and

$$AC = \frac{C(y)}{y} = 2y^2$$



Bonus Problem. (extra 10 points)
The present value of a perpetuity is

$$\begin{aligned} PV &= \frac{x}{1+r} + \frac{x}{(1+r)^2} + \frac{x}{(1+r)^3} + ... \\ &= \frac{1}{1+r}[x + PV] \end{aligned}$$

Solving for PV gives

$$PV = \frac{x}{r}$$

For the asset that pays x up to period T we have to decrease the PV by the PV of a "missing payment" after T. The present value of this payment in \$ from period T is $\frac{x}{r}$ and hence in \$ from period zero it is $\left(\frac{1}{1+r}\right)^T \frac{x}{r}$. Subtracting this number from PV for perpetuity gives

$$PV = \frac{x}{r} - \left(\frac{1}{1+r}\right)^T \frac{x}{r} = \frac{x}{r}\left[1 - \left(\frac{1}{1+r}\right)^T\right]$$

which is the formula of the PV of annuity.

Econ 301
Intermediate Microeconomics
Prof. Marek Weretka

Midterm 2 (Group B)

You have 70 minutes to complete the exam. The midterm consists of 4 questions (20+30+20+30=100 points) + a bonus (10 "extra" points). Make sure you answer the first four questions before working on the bonus one!

Problem 1 (20p). (Intertemporal choice)

Frank works as a consultant. His income when young is \$4000 (period 1) and \$16000 when old (period 2), the interest rate is $r = 100\%$.

a) In the graph depict Frank's budget set. Mark all the bundles on the budget line that involve saving and the ones that involve borrowing. Find analytically PV and FV of income and show it in the graph

b) Frank's intertemporal preferences are given by

$$U(C_1; C_2) = \ln C_1 + \frac{1}{1+\delta} \ln C_2$$

where the discount factor is $\delta = 100\%$. Using the magic formula, find the optimal consumption plan (C_1, C_2) and how much Frank borrows or saves (three numbers).

c) Is Frank smoothing his consumption? (yes or no answer + one sentence)

Problem 2 (30p). (Edgeworth box, and equilibrium)

Consider an economy with apples and oranges. Peter is initially endowed with five apples and ten oranges $\omega^P = (5, 10)$. Amanda's endowment is $\omega^A = (10, 5)$.

a) Plot the Edgeworth box and mark the allocation representing the initial endowment.

b) Describe the concept of Pareto efficiency (one intuitive sentence). Peter and Amanda have the same utility function

$$U^i(C_1, C_2) = 2\ln(C_1) + 2\ln(C_2).$$

Verify whether the endowment allocation is (or is not) Pareto efficient (use values of MRS in your argument). Illustrate your argument geometrically in the Edgeworth Box from a).

c) Find analytically the competitive equilibrium (six numbers) and show it in the Edgeworth box. Find some other prices that define competitive equilibrium (two numbers).

d) Argue that competitive markets allocate resources efficiently (give two numbers and compare them).

Problem 3 (20p). (Short questions)

Answer the following three questions.

a) The Bernoulli utility function is given by $u(c) = c^2$ and two states of the world are equally likely. Find the corresponding von Neuman Morgenstern (expected) utility function (give formula). Is such agent risk neutral, risk loving or risk averse? (one sentence). Find the expected value and the certainty equivalent of a lottery $\left(2, \sqrt{28}\right)$.(two numbers). Which is bigger and why (one sentence) (Hint: when calculating expected value of the lottery, use that $\sqrt{28} \simeq 5.3$).

b) Derive the formula for perpetuity.

c) You will live for 4 periods. You would like to maintain the constant level consumption throughout your life C. How much can you consume if in the first three periods you earn \$3000? The interest rate is $r = 100\%$? (one number)

Problem 4 (30p). (Producers)

Consider a producer that has the following technology

$$y = 8K^{\frac{1}{4}}L^{\frac{1}{2}}$$

a) What returns to scale are represented by this production function? (choose between CRS, IRS or DRS; you do not have to prove it).

b) (Short run) Assume that $\bar{K} = 1$ and the firm cannot change it in a short run. Derive a condition for optimal demand for labor. Explain intuitively its economic meaning. (one sentence).

c) Suppose that labor supply is inelastic and given by $L^s = 16h$. Find analytically and on the graph the equilibrium wage rate.

d) Find the unemployment rate with the minimal (real) wage given by $w_L/p = 4/3$.(one number)

e) Suppose $w_L = 1, w_K = 2$. Derive the cost function $C(y)$, assuming that you can adjust both K and L, and plot it on the graph. Relate the shape of your cost function to the returns to scale. (Hint: the constants in this last questions are not round numbers)

Bonus Problem. (extra 10 points)

Give examples of production functions with perfect complements and perfect substitutes that are characterized by increasing and decreasing returns to scale.

Econ 301
Intermediate Microeconomics
Prof. Marek Weretka

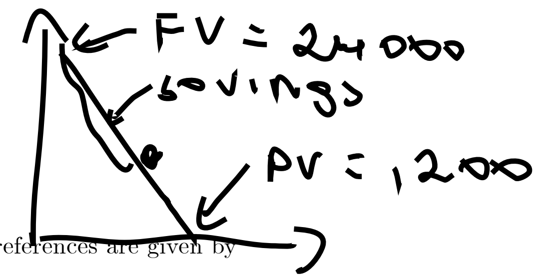
Solutions to midterm 2 (Group B)

Problem 1 (20p). (Intertemporal choice)

Frank works as a consultant. His income when young is \$4000 (period 1) and \$16000 when old (period 2), the interest rate is $r = 100\%$.

a) In the graph depict Frank's budget set. Mark all the bundles on the budget line that involve saving and the ones that involve borrowing. Find analytically PV and FV of income and show it in the graph

$$
\begin{aligned}
PV &= 4000 + \frac{16000}{1+1} = 4000 + 8000 = 12000 \\
FV &= (1+1)4000 + 16000 = 24000
\end{aligned}
$$



b) Frank's intertemporal preferences are given by

$$U(C_1; C_2) = \ln C_1 + \frac{1}{1+\delta} \ln C_2$$

where the discount factor is $\delta = 100\%$. Using the magic formula, find the optimal consumption plan (C_1, C_2) and how much Frank borrows or saves (three numbers).

$$
\begin{aligned}
C_1 &= \frac{1}{1+\frac{1}{2}} \frac{24000}{2} = \frac{2}{3} 12000 = 8000 \\
C_2 &= \frac{\frac{1}{2}}{1+\frac{1}{2}} \frac{24000}{1} = \frac{1}{3} 24000 = 8000 \\
S &= 4000 - 8000 = -4000
\end{aligned}
$$

Frank borrows $-\$4000$

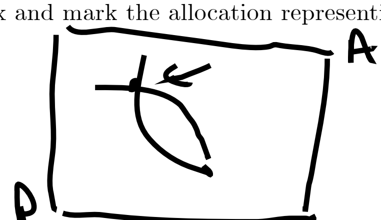
c) Is Frank smoothing his consumption? (yes or no answer + one sentence).

Yes, because $C_1 = C_2$. This is because $\delta = r$.

Problem 2 (30p). (Edgeworth box, and equilibrium)

Consider an economy with apples and oranges. Peter is initially endowed with five apples and ten oranges $\omega^P = (5, 10)$. Amanda's endowment is $\omega^A = (10, 5)$.

a) Plot the Edgeworth box and mark the allocation representing the initial endowment.



b) Describe the concept of Pareto efficiency (one intuitive sentence). Peter and Amanda have the same utility function

$$U^i(C_1, C_2) = 2\ln(C_1) + 2\ln(C_2).$$

Verify whether the endowment allocation is (or is not) Pareto efficient (use values of MRS in your argument). Illustrate your argument geometrically in the Edgeworth Box from a).

The endowment allocation is not Pareto efficient, as at this allocation the slopes of indifference curves are not tangent to each other

$$MRS^P = \frac{C_2^P}{C_1^P} = \frac{10}{5} = 2$$
$$MRS^A = \frac{C_2^A}{C_1^A} = \frac{5}{10} = \frac{1}{2}$$

and hence they do not coincide (see graph above).

c) Find analytically the competitive equilibrium (six numbers) and show it in the Edgeworth box. Find some other prices that define competitive equilibrium (two numbers).

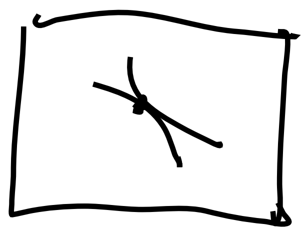
We normalize $p_2 = 1$

$$C_1^P = \frac{1}{2}\frac{5p_1 + 10}{p_1}$$
$$C_1^A = \frac{1}{2}\frac{10p_1 + 5}{p_1}$$

and market clearing condition gives

$$\frac{1}{2}\frac{5p_1 + 10}{p_1} + \frac{1}{2}\frac{10p_1 + 5}{p_1} = 15$$

From which one can find price $p_1 = 1 = p_2$. Equilibrium consumption is $C_1^A = C_1^B = 7.5$ and $C_2^A = C_2^B = 7.5$.



Other prices: $p_1 = p_2 = 2$

d) Argue that competitive markets allocate resources efficiently (give two numbers and compare them).

Allocation in competitive equilibrium is Pareto efficient as MRS of both agents are the same

$$MRS^P = \frac{C_2^P}{C_1^P} = \frac{7.5}{7.5} = 1$$
$$MRS^A = \frac{C_2^A}{C_1^A} = \frac{7.5}{7.5} = 1$$

Problem 3 (20p). (Short questions)

a) The Bernoulli utility function is given by $u(c) = c^2$ and two states of the world are equally likely. Find the corresponding von Neuman Morgenstern (expected) utility function (give formula). Is such agent risk neutral, risk loving or risk averse? (one sentence). Find the expected value and the certainty equivalent of a lottery $\left(2, \sqrt{28}\right)$.(two numbers). Which is bigger and why (one sentence) (Hint: when calculating expected value of the lottery, use that $\sqrt{28} \simeq 5.3$).

Expected Utility function is given by

$$U(c_1, c_2) = \frac{1}{2}c_1^2 + \frac{1}{2}c_2^2$$

Agent is risk loving as Bernouli utility function is convex.

$$E(L) = \frac{1}{2}2 + \frac{1}{2}5.3 = 3.6$$

. Certainty equivalent can be found as

$$(CE)^2 = U\left(2, \sqrt{28}\right) = 2 + 14 = 16$$

and hence

$$CE = 4 > E(L)$$

This is because risk loving agent derives extra utility from uncertainty regarding the outcome.

b) Derive the formula for perpetuity

$$\begin{aligned} PV &= \frac{x}{1+r} + \frac{x}{(1+r)^2} + \frac{x}{(1+r)^2} + ... = \\ &= \frac{x}{1+r} + \frac{1}{1+r}\left(\frac{x}{(1+r)} + \frac{x}{(1+r)} + ...\right) = \\ &= \frac{x}{1+r} + \frac{1}{1+r}PV \end{aligned}$$

Solving for PV gives

$$PV = \frac{x}{r}$$

.c) You will live for 4 periods. You would like to maintain the constant level consumption throughout your life C. How much can you consume if in the first three periods you earn \$1500? The interest rate is r=100%? (one number)

$$\begin{aligned} \frac{C}{r}\left(1 - \left(\frac{1}{1+r}\right)^4\right) &= \frac{3000}{r}\left(1 - \left(\frac{1}{1+r}\right)^3\right) \\ \frac{15}{16}C &= 3000\frac{7}{8} \end{aligned}$$

[redacted] $C = 2800$

Problem 4 (30p). (Producers)

Consider a producer that has the following technology

$$y = 8K^{\frac{1}{4}}L^{\frac{1}{2}}$$

a) What returns to scale are represented by this production function? (choose between CRS, IRS or DRS; you do not have to prove it).

DRS

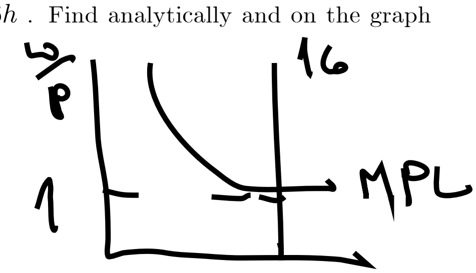
b) (Short run) Assume that $\bar{K} = 1$ and the firm cannot change it in a short run. Derive a condition for optimal demand for labor. Explain intuitively its economic meaning. (one sentence). ($MPL = \frac{w_k}{p}$)

$$\frac{w_k}{p} = 4L^{-\frac{1}{2}}$$

Last worker produces as much as he gets in terms of wage.

c) Suppose that labor supply is inelastic and given by $L^s = 16h$. Find analytically and on the graph the equilibrium wage rate.

$$\frac{w_k}{p} = 4(16)^{-\frac{1}{2}} = 1$$



d) Find the unemployment rate with the minimal (real) wage given by $w_L/p = 4/3$.(one number)
With minimal wage rate the demand for labor is

$$4/3 = 4L^{-\frac{1}{2}} \Rightarrow L = 9$$

and hence unemployment rate is

$$UR = \frac{16 - 9}{16} = \frac{7}{16}$$

e) Suppose w_L= 1, w_K= 2. Derive the cost function C(y), assuming that you can adjust both K and L, and plot it on the graph. Relate the shape of your cost function to the returns to scale. (Hint: the constants in this last questions are not round numbers)

$$8K^{\frac{1}{4}}L^{\frac{1}{2}}$$

$$TRS = -\frac{1}{2}\frac{L}{K} = -\frac{2}{1}$$

and hence

$$L = 4K$$

It follows that

$$K = \left(\frac{1}{16}y\right)^{\frac{4}{3}}$$

and

$$L = 4\left(\frac{1}{16}y\right)^{\frac{4}{3}}$$

It follows that

$$c(y) = 4\left(\frac{1}{16}y\right)^{\frac{4}{3}} + 2\left(\frac{1}{16}y\right)^{\frac{4}{3}} = 6\left(\frac{1}{16}y\right)^{\frac{4}{3}}$$

The function is convex as we have DRS.

Bonus Problem. (extra 10 points)

Give examples of production functions with perfect complements and perfect substitutes that are characterized by increasing and decreasing returns to scale.

Perfect complements

$$\begin{aligned} y &= [\min(2K, 7L)]^2 \text{ (IRS)} \\ y &= [\min(2K, 7L)]^{\frac{1}{2}} \text{ (DRS)} \end{aligned}$$

Perfect substitutes

$$\begin{aligned} y &= (2K + 7L)^2 \text{ (IRS)} \\ y &= [2K + 7L]^{\frac{1}{2}} \text{ (DRS)} \end{aligned}$$

Econ 301
Intermediate Microeconomics
Prof. Marek Weretka

Midterm 2 (Group A)

You have 70 minutes to complete the exam. The midterm consists of 4 questions (25,30,25 and 20 points).

Problem 1 (25p). (Uncertainty and insurance)

You are an owner of a luxurious sailing boat, worth \$10, that you use for recreation on Mendota lake. Unfortunately, there is a good (50%) chance of a tornado in Madison (probability is equal to $\frac{1}{2}$) that completely destroys it. Thus, your boat is in fact a lottery with payment $(0, 10)$.

a) What is the expected value of the "boat" lottery? (give one number)

b) Suppose your Bernoulli utility function is given by $u(c) = c^2$. Give von Neuman-Morgenstern utility function over lotteries $U(C_1; C_2)$. (formula) Are you risk averse, neutral or risk loving? (two words). Find the certainty equivalent (CE) of the "boat lottery" (one number). Which is bigger, CE or the expected value of a lottery from a)? Why? (one sentence)

c) Your Bernoulli utility function changes to $u(c) = \ln c$. Give von Neuman-Morgenstern utility function. (give a formula). Are you risk averse now?

d) You can insure your boat by buying insurance policy in which you specify coverage x. The insurance contract costs $\gamma \cdot x$ where the premium rate is equal to $\gamma = \frac{1}{2}$. Find analytically and depict in the graph your budget constraint. Mark the point that corresponds to no insurance.

e) Find optimal level of coverage x. Are you going to fully insure your boat? (one number and yes-no answer). Depict optimal consumption plan on the graph.

f) Propose a premium rate γ for which you will only partially insure your boat. (one number)

Problem 2 (30p). (Edgeworth box, and equilibrium)

Consider an economy with apples and oranges. Andy is initially endowed with $\omega^A = (0, 50)$ and Bob's endowment is $\omega^B = (50, 0)$.

The utility function of both Andy and Bob is the same and given by

$$U(x_1, x_2) = 3 \ln x_1 + 3 \ln x_2$$

a) Plot the Edgeworth box and mark the allocation representing the initial endowment.

b) Provide general definition of Pareto efficiency (one sentence starting with: Allocation is Pareto efficient if ...).

c) Prove, that an allocation is Pareto efficient if and only in such allocation satisfies $MRS^A = MRS^B$. Start with necessity by showing that if the MRS condition does not hold then allocation is not Pareto efficient. Then proceed to sufficiency by showing that if the condition MRS is satisfied then indeed allocation is efficient (use a graph and write two sentences for each of the two conditiions).

d) Find analytically a collection of all Pareto efficient allocations (contract curve) and depict it in the graph.

e) Find the competitive equilibrium (give six numbers).

g) Give some other prices that are consistent with competitive equilibrium (give two numbers).

f) Using MRS condition verify that equilibrium allocation is Pareto efficient and hence an invisible hand of a free (and competitive) market guides selfish Andy and Bob to a socially optimal outcome.

Problem 3 (25p). (Short questions)

a) Your sister has just promised to send you pocket money of \$500 each month starting next month and she will keep doing it forever. What is the present value of "having such sister" if monthly interest rate is equal to 5% (one number).

b) Sam is a hockey player who earns \$100 when young and \$0 when old. Sam's intertemporal utility is given by $U(C_1, C_2) = \ln(c_1) + \frac{1}{1+\delta}\ln(c_1)$. Assuming $\delta = r = 0$ and using magic formulas find optimal consumption plan and optimal saving strategy (give three numbers C_1, C_2, S). Does Sam smooth his consumption? (yes/ no + one sentence) Is Sam tilting his consumption? (yes/ no + one sentence)

c) A production function is given by $y = 2\bar{K}^3 L^{\frac{1}{2}}$. Find analytically a short-run demand for labor (assume $\bar{K} = 1$). Find analytically equilibrium real wage rate if labor supply is given by $L^s = 16$. Depict it in a gaph.

d) You start you first job at the age of 21 and you work till 60, and then your retire. You live till 80. Your annual earnings between $21 - 60$ are \$100,000 and interest rate is $r = 5\%$. You want to maintain a constant level of consumption. Write down an equation that allows to determine C (write down the equation but you **do not need** to solve for C).

Problem 4 (20p). (Producers)

Consider a producer that has the following technology

$$y = K^{\frac{1}{4}} L^{\frac{1}{4}}.$$

a) What returns to scale are represented by this production function? (choose between CRS, IRS or DRS; prove your statement with λ argument).

b) Find analytically a (variable) cost function given $w_K = w_L = 2$. Plot it in the graph.

c) find y^{MES} and ATC^{MES} if a fixed cost is $F = 2$.

d) Find analitically a supply function of the firm and show it in the graph.

Just for fun

Using "secrets of happiness" show that if a firm is maximizing profit by producing y^*, it necessarily minimizes the cost of production of y^* (give two conditions for profit maximization and show that they imply condition for cost minimization).

Econ 301
Intermediate Microeconomics
Prof. Marek Weretka

Midterm 2 (Group B)

You have 70 minutes to complete the exam. The midterm consists of 4 questions (25,30,25 and 20 points).

Problem 1 (25p). (Uncertainty and insurance)

You are an owner of a luxurious sailing boat, worth \$4, that you use for recreation on Mendota lake. Unfortunately, there is a good (50%) chance of a tornado in Madison (probability is equal to $\frac{1}{2}$) that completely destroys it. Thus, your boat is in fact a lottery with payment $(0, 4)$.

a) What is the expected value of the "boat" lottery? (give one number)

b) Suppose your Bernoulli utility function is given by $u(c) = c^2$. Give von Neuman-Morgenstern utility function over lotteries $U(C_1; C_2)$. (formula) Are you risk averse, neutral or risk loving? (two words). Find the certainty equivalent (CE) of the "boat lottery" (one number). Which is bigger, CE or the expected value of a lottery from a)? Why? (one sentence)

c) Your Bernoulli utility function changes to $u(c) = \ln c$. Give von Neuman-Morgenstern utility function. (give a formula). Are you risk averse now?

d) You can insure your boat by buying insurance policy in which you specify coverage x. The insurance contract costs $\gamma \cdot x$ where the premium rate is equal to $\gamma = \frac{1}{2}$. Find analytically and depict in the graph your budget constraint. Mark the point that corresponds to no insurance.

e) Find optimal level of coverage x. Are you going to fully insure your boat? (one number and yes-no answer). Depict optimal consumption plan on the graph.

f) Propose a premium rate γ for which you will only partially insure your boat. (one number)

Problem 2 (30p). (Edgeworth box, and equilibrium)

Consider an economy with apples and oranges. Andy is initially endowed with $\omega^A = (20, 0)$ and Bob's endowment is $\omega^B = (0, 20)$.

The utility function of both Andy and Bob is the same and given by

$$U(x_1, x_2) = 5 \ln x_1 + 5 \ln x_2$$

a) Plot the Edgeworth box and mark the allocation representing the initial endowment.

b) Provide general definition of Pareto efficiency (one sentence starting with: Allocation is Pareto efficient if ...).

c) Prove, that an allocation is Pareto efficient if and only in such allocation satisfies $MRS^A = MRS^B$. Start with necessity by showing that if the MRS condition does not hold then allocation is not Pareto efficient. Then proceed to sufficiency by showing that if the condition MRS is satisfied then indeed allocation is efficient (use a graph and write two sentences for each of the two conditiions).

d) Find analytically a collection of all Pareto efficient allocations (contract curve) and depict it in the graph.

e) Find the competitive equilibrium (give six numbers).

g) Give some other prices that are consistent with competitive equilibrium (give two numbers).

f) Using MRS condition verify that equilibrium allocation is Pareto efficient and hence an invisible hand of a free (and competitive) market guides selfish Andy and Bob to a socially optimal outcome.

Problem 3 (25p). (Short questions)

a) Your sister has just promised to send you pocket money of \$100 each month starting next month and she will keep doing it forever. What is the present value of "having such sister" if monthly interest rate is equal to 5% (one number).

b) Sam is a hockey player who earns \$200 when young and \$0 when old. Sam's intertemporal utility is given by $U(C_1, C_2) = \ln(c_1) + \frac{1}{1+\delta}\ln(c_1)$. Assuming $\delta = r = 0$ and using magic formulas find optimal consumption plan and optimal saving strategy (give three numbers C_1, C_2, S). Does Sam smooth his consumption? (yes/ no + one sentence) Is Sam tilting his consumption? (yes/ no + one sentence)

c) A production function is given by $y = 2\bar{K}^3 L^{\frac{1}{2}}$. Find analytically a short-run demand for labor (assume $\bar{K} = 1$). Find analytically equilibrium real wage rate if labor supply is given by $L^s = 16$. Depict it in a gaph.

d) You start you first job at the age of 21 and you work till 60, and then your retire. You live till 80. Your annual earnings between $21 - 60$ are \$50,000 and interest rate is $r = 5\%$. You want to maintain a constant level of consumption. Write down an equation that allows to determine C (write down the equation but you **do not need** to solve for C).

Problem 4 (20p). (Producers)

Consider a producer that has the following technology

$$y = K^{\frac{1}{4}} L^{\frac{1}{4}}.$$

a) What returns to scale are represented by this production function? (choose between CRS, IRS or DRS; prove your statement with λ argument).

b) Find analytically a (variable) cost function given $w_K = w_L = 2$. Plot it in the graph.

c) find y^{MES} and ATC^{MES} if a fixed cost is $F = 2$.

d) Find analitically a supply function of the firm and show it in the graph.

Just for fun

Using "secrets of happiness" show that if a firm is maximizing profit by producing y^*, it necessarily minimizes the cost of production of y^* (give two conditions for profit maximization and show that they imply condition for cost minimization).

Econ 301
Intermediate Microeconomics
Prof. Marek Weretka

Midterm 2 (Group C)

You have 70 minutes to complete the exam. The midterm consists of 4 questions (25,30,25 and 20 points).

Problem 1 (25p). (Uncertainty and insurance)

You are an owner of a luxurious sailing boat, worth \$6, that you use for recreation on Mendota lake. Unfortunately, there is a good (50%) chance of a tornado in Madison (probability is equal to $\frac{1}{2}$) that completely destroys it. Thus, your boat is in fact a lottery with payment $(6, 0)$.

a) What is the expected value of the "boat" lottery? (give one number)

b) Suppose your Bernoulli utility function is given by $u(c) = c^2$. Give von Neuman-Morgenstern utility function over lotteries $U(C_1; C_2)$. (formula) Are you risk averse, neutral or risk loving? (two words). Find the certainty equivalent (CE) of the "boat lottery" (one number). Which is bigger, CE or the expected value of a lottery from a)? Why? (one sentence)

c) Your Bernoulli utility function changes to $u(c) = \ln c$. Give von Neuman-Morgenstern utility function. (give a formula). Are you risk averse now?

d) You can insure your boat by buying insurance policy in which you specify coverage x. The insurance contract costs $\gamma \cdot x$ where the premium rate is equal to $\gamma = \frac{1}{2}$. Find analytically and depict in the graph your budget constraint. Mark the point that corresponds to no insurance.

e) Find optimal level of coverage x. Are you going to fully insure your boat? (one number and yes-no answer). Depict optimal consumption plan on the graph.

f) Propose a premium rate γ for which you will only partially insure your boat. (one number)

Problem 2 (30p). (Edgeworth box, and equilibrium)

Consider an economy with apples and oranges. Andy is initially endowed with $\omega^A = (40, 0)$ and Bob's endowment is $\omega^B = (0, 40)$.

The utility function of both Andy and Bob is the same and given by

$$U(x_1, x_2) = 2\ln x_1 + 2\ln x_2$$

a) Plot the Edgeworth box and mark the allocation representing the initial endowment.

b) Provide general definition of Pareto efficiency (one sentence starting with: Allocation is Pareto efficient if ...).

c) Prove, that an allocation is Pareto efficient if and only in such allocation satisfies $MRS^A = MRS^B$. Start with necessity by showing that if the MRS condition does not hold then allocation is not Pareto efficient. Then proceed to sufficiency by showing that if the condition MRS is satisfied then indeed allocation is efficient (use a graph and write two sentences for each of the two conditiions).

d) Find analytically a collection of all Pareto efficient allocations (contract curve) and depict it in the graph.

e) Find the competitive equilibrium (give six numbers).

g) Give some other prices that are consistent with competitive equilibrium (give two numbers).

f) Using MRS condition verify that equilibrium allocation is Pareto efficient and hence an invisible hand of a free (and competitive) market guides selfish Andy and Bob to a socially optimal outcome.

Problem 3 (25p). (Short questions)

a) Your sister has just promised to send you pocket money of \$50 each month starting next month and she will keep doing it forever. What is the present value of "having such sister" if monthly interest rate is equal to 5% (one number).

b) Sam is a hockey player who earns \$1000 when young and \$0 when old. Sam's intertemporal utility is given by $U(C_1, C_2) = \ln(c_1) + \frac{1}{1+\delta}\ln(c_1)$. Assuming $\delta = r = 0$ and using magic formulas find optimal consumption plan and optimal saving strategy (give three numbers C_1, C_2, S). Does Sam smooth his consumption? (yes/ no + one sentence) Is Sam tilting his consumption? (yes/ no + one sentence)

c) A production function is given by $y = 2\bar{K}^3 L^{\frac{1}{2}}$. Find analytically a short-run demand for labor (assume $\bar{K} = 1$). Find analytically equilibrium real wage rate if labor supply is given by $L^s = 16$. Depict it in a gaph.

d) You start you first job at the age of 21 and you work till 60, and then your retire. You live till 80. Your annual earnings between $21 - 60$ are \$40,000 and interest rate is $r = 5\%$. You want to maintain a constant level of consumption. Write down an equation that allows to determine C (write down the equation but you **do not need** to solve for C).

Problem 4 (20p). (Producers)

Consider a producer that has the following technology

$$y = K^{\frac{1}{4}} L^{\frac{1}{4}}.$$

a) What returns to scale are represented by this production function? (choose between CRS, IRS or DRS; prove your statement with λ argument).

b) Find analytically a (variable) cost function given $w_K = w_L = 2$. Plot it in the graph.

c) find y^{MES} and ATC^{MES} if a fixed cost is $F = 2$.

d) Find analitically a supply function of the firm and show it in the graph.

Just for fun

Using "secrets of happiness" show that if a firm is maximizing profit by producing y^*, it necessarily minimizes the cost of production of y^* (give two conditions for profit maximization and show that they imply condition for cost minimization).

Econ 301
Intermediate Microeconomics
Prof. Marek Weretka

Midterm 2 (Group D)

You have 70 minutes to complete the exam. The midterm consists of 4 questions (25,30,25 and 20 points).

Problem 1 (25p). (Uncertainty and insurance)

You are an owner of a luxurious sailing boat, worth \$2, that you use for recreation on Mendota lake. Unfortunately, there is a good (50%) chance of a tornado in Madison (probability is equal to $\frac{1}{2}$) that completely destroys it. Thus, your boat is in fact a lottery with payment $(2, 0)$.

a) What is the expected value of the "boat" lottery? (give one number)

b) Suppose your Bernoulli utility function is given by $u(c) = c^2$. Give von Neuman-Morgenstern utility function over lotteries $U(C_1; C_2)$. (formula) Are you risk averse, neutral or risk loving? (two words). Find the certainty equivalent (CE) of the "boat lottery" (one number). Which is bigger, CE or the expected value of a lottery from a)? Why? (one sentence)

c) Your Bernoulli utility function changes to $u(c) = \ln c$. Give von Neuman-Morgenstern utility function. (give a formula). Are you risk averse now?

d) You can insure your boat by buying insurance policy in which you specify coverage x. The insurance contract costs $\gamma \cdot x$ where the premium rate is equal to $\gamma = \frac{1}{2}$. Find analytically and depict in the graph your budget constraint. Mark the point that corresponds to no insurance.

e) Find optimal level of coverage x. Are you going to fully insure your boat? (one number and yes-no answer). Depict optimal consumption plan on the graph.

f) Propose a premium rate γ for which you will only partially insure your boat. (one number)

Problem 2 (30p). (Edgeworth box, and equilibrium)

Consider an economy with apples and oranges. Andy is initially endowed with $\omega^A = (10, 0)$ and Bob's endowment is $\omega^B = (0, 10)$.

The utility function of both Andy and Bob is the same and given by

$$U(x_1, x_2) = 8 \ln x_1 + 8 \ln x_2$$

a) Plot the Edgeworth box and mark the allocation representing the initial endowment.

b) Provide general definition of Pareto efficiency (one sentence starting with: Allocation is Pareto efficient if ...).

c) Prove, that an allocation is Pareto efficient if and only in such allocation satisfies $MRS^A = MRS^B$. Start with necessity by showing that if the MRS condition does not hold then allocation is not Pareto efficient. Then proceed to sufficiency by showing that if the condition MRS is satisfied then indeed allocation is efficient (use a graph and write two sentences for each of the two conditiions).

d) Find analytically a collection of all Pareto efficient allocations (contract curve) and depict it in the graph.

e) Find the competitive equilibrium (give six numbers).

g) Give some other prices that are consistent with competitive equilibrium (give two numbers).

f) Using MRS condition verify that equilibrium allocation is Pareto efficient and hence an invisible hand of a free (and competitive) market guides selfish Andy and Bob to a socially optimal outcome.

Problem 3 (25p). (Short questions)

a) Your sister has just promised to send you pocket money of \$200 each month starting next month and she will keep doing it forever. What is the present value of "having such sister" if monthly interest rate is equal to 5% (one number).

b) Sam is a hockey player who earns \$1000 when young and \$0 when old. Sam's intertemporal utility is given by $U(C_1, C_2) = \ln(c_1) + \frac{1}{1+\delta}\ln(c_1)$. Assuming $\delta = r = 0$ and using magic formulas find optimal consumption plan and optimal saving strategy (give three numbers C_1, C_2, S). Does Sam smooth his consumption? (yes/ no + one sentence) Is Sam tilting his consumption? (yes/ no + one sentence)

c) A production function is given by $y = 2\bar{K}^3 L^{\frac{1}{2}}$. Find analytically a short-run demand for labor (assume $\bar{K} = 1$). Find analytically equilibrium real wage rate if labor supply is given by $L^s = 16$. Depict it in a gaph.

d) You start you first job at the age of 21 and you work till 60, and then your retire. You live till 80. Your annual earnings between $21 - 60$ are \$60,000 and interest rate is $r = 5\%$. You want to maintain a constant level of consumption. Write down an equation that allows to determine C (write down the equation but you **do not need** to solve for C).

Problem 4 (20p). (Producers)

Consider a producer that has the following technology

$$y = K^{\frac{1}{4}} L^{\frac{1}{4}}.$$

a) What returns to scale are represented by this production function? (choose between CRS, IRS or DRS; prove your statement with λ argument).

b) Find analytically a (variable) cost function given $w_K = w_L = 2$. Plot it in the graph.

c) find y^{MES} and ATC^{MES} if a fixed cost is $F = 2$.

d) Find analitically a supply function of the firm and show it in the graph.

Just for fun

Using "secrets of happiness" show that if a firm is maximizing profit by producing y^*, it necessarily minimizes the cost of production of y^* (give two conditions for profit maximization and show that they imply condition for cost minimization).

Econ 703
Intermediate Microeconomics
Prof. Marek Weretka

Answer Keys to midterm 2 (Group A)

"X and Y (2pt)." means that you get 2 pts if you answered both X and Y, and no pts if you missed either (or both).

Problem 1. [Here I denote by T the state with tornado and by N the state without tornado, instead of 1 and 2.]

a) $0.5 \cdot \$10 + 0.5 \cdot \$0 = \$5$(2pt).

b) With the Bernoulli utility function $u(c) = c^2$, the v.N.M. expected utility *function* is $U(C_T, C_N) = 0.5C_T^2 + 0.5C_N^2$ (1pt). Since $u(c) = c^2$ is a convex function, I am risk loving (2pt). The certainty equivalent CE is the amount of sure money s.t. $U(CE, CE) = CE^2 = U(0, 10) = 50$, i.e. $CE = 5\sqrt{2}$ (2pt). CE is larger than EV, because I am risk loving (2pt).

c) With the Bernoulli utility function $u(c) = c^2$, the v.N.M. expected utility *function* is $U(C_T, C_N) = 0.5 \ln C_T + 0.5 \ln C_N$ (1pt). Yes, I'm risk averse (2pt), since $u(c) = \ln c$ is a concave function.

d) As $C_T = (1-\gamma)x$ and $C_N = 4 - \gamma x$ with $\gamma = .5$, we obtain the budget constraint $C_T + C_N = 10$ (2pt). Its graph has the C_T intercept on $(C_T, C_N) = (10, 0)$, the C_N intercept on $(C_T, C_N) = (0, 10)$, and the slope -1 on the C_T-C_N plane (2pt). The endowment point should be plotted on $(C_T, C_N) = (0, 10)$ (1pt).

e) Now I should maximize the utility $U(C_T, C_N) = 0.5C_T^2 + 0.5C_N^2$ on the constraint $C_T + C_N = 10$. The magic formula yields $C_T = (1/2) \cdot (10/1) = 5$ (1pt) and $C_N = (1/2) \cdot (10/1) = 5$ (1pt). Plugging this into $C_N = 4 - \gamma x$, we obtain $x = 10$ (2pt). The optimal point should be plotted on $(5, 5)$ (1pt). Yes, I am fully insured (1pt) since $C_T = C_N$.

f) e.g. $\gamma = 1$ (2pt). Actually I would be partially insured, i.e. $C_T < C_N$ under any premium rate larger than 0.5.

Problem 2. [Here I denote apple by 1, orange by 2, Andy by A and Bob by B. You could use another notation, as long as you clarified it.]

a) The Edgeworth box should have length of 50 on each axis (1pt). The endowment is $(50, 0)$ looked from A's origin, i.e. $(0, 50)$ from B's origin (1pt). [This is a single point in an Edgeworth box. And, it cannot be an origin.]

b) ... if none could not be better off (by another feasible allocation) unless anyone is worth off (2pt). [$MRS^A = MRS^B$: no point since it is just a mathematical equivalent property and not the definition.[1]]

c) [Need to prove the claim directly from the definition of Pareto efficiency. Arguing only slopes or tangency is wrong. Besides, logical sequence (especially starting assumptions and ending conclusions) must be clarified. The curve's name, namely "an indifference curve", should be clarified.]

Necessity (4pt): If $MRS^A \neq MRS^B$ at an allocation x, both people's indifference curves should cross each other at x and thus we can find a point between them. Because this point is above each indifference curve looked from the people's origin, this allocation is better than x for both and thus the allocation x is not Pareto efficient. [The proof should start with $MRS^A \neq MRS^B$ and end with Pareto inefficiency of x. Graph is needed. On the graph, you need to *specify* another allocation that improves their utilities. If you wrote two separate points and two separate curves for a single allocation, you misunderstand an Edgeworth box.]

Sufficiency (4pt): If $MRS^A = MRS^B$ at an allocation x, both people's indifference curves should be tangent to each other at x and thus no point is below A's indifferent curve looked from A's origin, i.e. worse for A than x, or below B's indifferent curve looked from B's origin, i.e. worse for B, or below both. So any point (allocation) cannot be better than x for both people and x is Pareto efficient. [The proof should start with $MRS^A = MRS^B$ at x and end with Pareto efficiency of x. Graph is needed. On the graph, you need to clarify who is worse off than x in each region defined by the two indifference curves.]

d) As we proved above, the Pareto efficiency is equivalent to $MRS^A = MRS^B$, given the feasibility of the allocation $x_1^A + x_1^B = 50, x_2^A + x_2^B = 50$. So we solve

$$MRS^A(x_1^A, x_2^A) = \frac{3/x_1^A}{3/x_2^A} = \frac{3/(50 - x_1^A)}{3/(50 - x_2^A)} = MRS^B(50 - x_1^A, 50 - x_2^A).$$

[1] A Pareto efficient allocation may not hold this equation, unless the preference is convex, monotone, and smooth.

Then we obtain $x_1^A = x_2^A$ [or $x_1^B = x_2^B$] (3pt). This is the equation for the contract curve. [You need to clarify whose consumption it is.] Graphically it is the line starting from the origin of A with slope 1, i.e. the diagonal line connecting the two origins of the Edgeworth box (1pt).

e) Let the equilibrium price be (p_1, p_2). Then, Andy should maximize his utility $U^A(x_1^A, x_2^A) = 3\ln x_1^A + 3\ln x_2^A$ on the budget constraint $p_1 x_1^A + p_2 x_2^A = 50p_1$. The magic formula yields his optimal consumption bundle

$$x_1^A = \frac{1}{2}\frac{50p_1}{p_1} = 25, \quad x_2^A = \frac{1}{2}\frac{50p_1}{p_2} = 25\frac{p_1}{p_2}.$$

Bob should maximize his utility $U^B(x_1^B, x_2^B) = 3\ln x_1^B + 3\ln x_2^B$ on the budget constraint $p_1 x_1^B + p_2 x_2^B = 50p_2$. The magic formula yields his optimal consumption bundle

$$x_1^B = \frac{1}{2}\frac{50p_2}{p_1} = 25\frac{p_2}{p_1}, \quad x_2^B = \frac{1}{2}\frac{50p_2}{p_2} = 25.$$

The feasibility (a.k.a. market clearing) of the allocation requires[2]

$$x_1^A + x_1^B = 25 + 25\frac{p_2}{p_1} = 50, \quad \therefore\ p_2 = p_1 \neq 0.$$

Plugging this into the above optimal bundles, we obtain $x_1^A = 25$ (2pt), $x_2^A = 25$ (2pt), $x_1^B = 25$ (2pt) and $x_2^B = 25$ (2pt). The equilibrium price (p_1, p_2) can be any pair of two positive numbers as long as $p_1 = p_2$: for example, $p_1 = 1, p_2 = 1$ (2pt). [No partial credit for only p_1 or p_2.]

f) As we argued, p_1, p_2 can be any pair of two positive numbers as long as $p_1 = p_2$ and different from the answer in e): for example, $p_1 = 2, p_2 = 2$ (2pt).

g) At the equilibrium allocation $((x_1^A, x_2^A), (x_1^B, x_2^B)) = ((25, 25), (25, 25))$, the two's MRSs are

$$MRS^A(25, 25) = \frac{3/25}{3/25} = 1, \quad MRS^B(25, 25) = \frac{3/25}{3/25} = 1.$$

So we have $MRS^A = -1 = MRS^B$ and thus this equilibrium allocation is Pareto efficient (2pt). [MRS must be calculated.]

Problem 3. **a)** $PV = 100/(1.05) + 100/(1.05)^2 + \ldots = 10000$ (dollars, 4pt).

b) Sam should maximize his utility $U = \ln C_1 + \ln C_2$ on the budget constraint $C_1 + C_2 = 200$ (as $C_1 + S = 200, C_2 = S$.) The magic formula yields his optimal consumption bundle $C_1 = (1/2) \cdot (200/1) = 50$ (2pt), $C_2 = (1/2) \cdot (200/1) = 50$ (2pt). Plugging this into $C_2 = S$, we have $S = 50$ (2pt). Yes, he's smoothing (1pt) as $C_1 = C_2$. No, he's not tilting (1pt) as $C_1 = C_2$. [If you answered only either one question and did not clarify which question you answered, you get no point.]

c) The production function $y = 2K^3L^{1/2}$ implies the marginal productivity of labor $MP_L = (1/2) \cdot 2K^3L^{-1/2} = K^3L^{-1/2}$. In particular, $MP_L = L^{-1/2}$ at $K = \bar{K} = 1$. Solving the secret of happiness $MP_L = L^{-1/2} = w/p$, we find the short-run labor demand $L^D = (w/p)^{-2}$ where p is the product's price and w is wage (4pt). [Thus w/p is the real wage rate. It is not enough to state only the secret of happiness; the demand L^D should be explicitly determined.[3]] Solving the demand-supply equality $L^D = (w/p)^{-2} = 16 = L^S$, we obtain the equilibrium real wage $w/p = 1/4$ (2pt). The equilibrium point $(L, w/p) = (16, 1/4)$ must be plotted on a graph (1pt).

d) (6pt.) The annual consumption C (thousand dollars) is determined from

$$\frac{100}{1.05} + \cdots + \frac{100}{1.05^{40}} = \frac{C}{1.05} + \cdots + \frac{C}{1.05^{60}} \quad \therefore \left(1 - \frac{1}{1.05^{40}}\right)\frac{100}{1.05} = \left(1 - \frac{1}{1.05^{-60}}\right)\frac{C}{1.05}.$$

[Further simplification gets full points.]

[2] We do not have to consider the market clearing of the other good 2: Walras's theorem. Notice that if $p_1 = 0$ then $p_2/p_1 = \infty$ and the equation does not hold; so we need $p_1 \neq 0$ too.

[3] Also I saw so manyy answers "$L^D = L^{-1/2}$"; this does not make sense at all, as it is read as the short-run labor demand L^D is the inverse of the square root of L and we must ask what is L. $L = L^D$ is the solution of $MP_L = L^{-1/2} = w/p$, but not a number on either side of this equation.

Problem 4. **a)** DRS (1pt). This is because $F(\lambda K, \lambda L) = (\lambda^{1/4}K^{1/4})(\lambda^{1/4}L^{1/4}) = \lambda^{1/2}K^{1/4}L^{1/4} = \lambda^{1/2}F(K, L) < \lambda^{1/2}F(K, L)$ [if $\lambda > 1$] (4pt). [Here $F(k, l)$ is the output from $K = k$ and $L = l$.]

b) The secret of happiness is

$$\frac{MP_K}{MP_L} = \frac{0.25K^{-3/4}L^{1/4}}{0.25K^{1/4}L^{-3/4}} = \frac{2}{2} = \frac{w_K}{w_L}, \quad \therefore\ K = L$$

To achieve the production of $y = F(K, L)$, we need

$$y = F(K, K) = K^{1/2}, \quad \therefore\ K = L = y^2$$

So the cost function is $C = 2K + 2L = 2y^2 + 2y^2 = 4y^2$ (4pt).[4] Graph should be drawn on the y-C plane (1pt).

c) Solving $MC(y) = 8y = (4y^2+2)/y = ATC(y)$, we obtain $y^{MES} = 1/\sqrt{2}$ (2pt) and $ATC^{MES} = ATC(y^{MES}) = MC(y^{MES}) = 4\sqrt{2}$ (2pt).[5]

d) (6pt for giving both the function and the graph.) The optimal supply should satisfy $p = 8y^* = MC(y^*)$, i.e. $y^* = p/8$. But when $p < ATC^{MES} = 4\sqrt{2}$, the firm cannot get positive profit even from the optimal supply and thus should quit the production.

The supply function $S(p)$ is therefore

$$S(p) = \begin{cases} p/8 & \text{if } p \geq 4\sqrt{2} \\ 0 & \text{if } p \leq 4\sqrt{2}. \end{cases}$$

On the y-p plane, the graph is $y = p/8$ (i.e. $p = 8y$) for $p \geq 4\sqrt{2}$ and $y = 0$ (a part of the vertical axis) for $p \leq 4\sqrt{2}$.

Just for fun The secret of happiness for profit maximization is

$$MP_K = pw_K, \quad MP_L = pw_L.$$

Here p is the product price, MP_i is the marginal productivity of factor i, and w_i is the price of factor i. These two equations imply

$$\frac{MP_K}{MP_L} = \frac{w_K}{w_L};$$

i.e. the secret of happiness for cost minimization.[6]

[4] Or, you can think of maximization of $Y = F(K, L) = K^{1/4}L^{1/4}$ on the constraint $2K + 2L = c$, thinking Y as a variable and c as a constant. Then the magic formula of Cobb-Douglas (utility) maximization implies $K = (1/2)(c/2) = c/4$ and $L = (1/2)(c/2) = c/4$. Then we obtain at the maximum $Y = (c/4)^{1/4}(c/4)^{1/4} = (c/4)^{1/2}$, i.e. $c = 4Y^2$. That is, when $Y = y$ is given, the budget/cost $C = 4y^2$ is needed to achieve this y at the optimum.

[5] Maybe ATC^{MES} is easier to calculate from $MC(y^{MES})$ than from $ATC(y^{MES})$, though they should yield the same number.

[6] So there's a close link between maximization and minimization. This link is called duality and was a driving force of mathematical economic theory during 1970s-80s: see Varian's textbook for graduate and advanced undergraduate, *Microeconomic Analysis*. And, you will use it in undergraduate linear programming, like Computer Science 525: see Ferris, Mangasarian, and Wright, *Linear Programming with MATLAB*, SIAM-MPS, 2007.

Econ 703
Intermediate Microeconomics
Prof. Marek Weretka

Solutions to midterm 2 (Group B)

"X and Y (2pt)." means that you get 2 pts if you answered both X and Y, and no pts if you missed either (or both).

Problem 1. [Here I denote by T the state with tornado and by N the state without tornado, instead of 1 and 2.] **a)** \$2 (2pt). **b)** $U(C_T, C_N) = 0.5C_T^2 + 0.5C_N^2$ (1pt). Risk loving (2pt). $CE = 2\sqrt{2}$ (2pt). Larger than EV, because I am risk loving (2pt). **c)** $U(C_T, C_N) = 0.5 \ln C_T + 0.5 \ln C_N$ (1pt). Yes, I'm risk averse (2pt). **d)** $C_T + C_N = 4$ (2pt). Graph is needed on the C_T-C_N plane and its position must be clarified with slope and intercepts (2pt). Plot a point on $(C_T, C_N) = (0, 4)$ for endowment (1pt). **e)** $C_T = 2$ (1pt). $C_N = 2$ (1pt). $x = 4$ (2pt). Plot a point on $(2, 2)$ (1pt). Yes, fully insured (1pt). **f)** e.g. $\gamma = 1$ (2pt). [Any number larger than 0.5 because we need $C_N > C_T$.]

Problem 2. [Here I denote apple by 1, orange by 2, Andy by A and Bob by B. You could use another notation, as long as you clarified it.] **a)** The Edgeworth box should have length of 20 on each axis (1pt). The endowment is $(20, 0)$ looked from A's origin, i.e. $(0, 20)$ from B's origin (1pt). [This is a single point in an Edgeworth box. And, it cannot be an origin.] **b)** ... if none could not be better off (by another feasible allocation) unless anyone is worth off (2pt). [$MRS^A = MRS^B$: no point since it is just a mathematical equivalent property and not the definition.[1]]

c) [Need to prove the claim directly from the definition of Pareto efficiency. Arguing only slopes or tangency is wrong. Besides, logical sequence (especially starting assumptions and ending conclusions) must be clarified. The curve's name, namely "an indifference curve", should be clarified.] **Necessity** (4pt): If $MRS^A \neq MRS^B$ at an allocation x, both people's indifference curves should cross each other at x and thus we can find a point between them. Because this point is above each indifference curve looked from the people's origin, this allocation is better than x for both and thus the allocation x is not Pareto efficient. [The proof should start with $MRS^A \neq MRS^B$ and end with Pareto inefficiency of x. Graph is needed. On the graph, you need to *specify* another allocation that improves their utilities. If you wrote two separate points and two separate curves for a single allocation, you misunderstand an Edgeworth box.]

Sufficiency (4pt): If $MRS^A = MRS^B$ at an allocation x, both people's indifference curves should be tangent to each other at x and thus no point is below A's indifferent curve looked from A's origin, i.e. worse for A than x, or below B's indifferent curve looked from B's origin, i.e. worse for B, or below both. So any point (allocation) cannot be better than x for both people and x is Pareto efficient. [The proof should start with $MRS^A = MRS^B$ at x and end with Pareto efficiency of x. Graph is needed. On the graph, you need to clarify who is worse off than x in each region defined by the two indifference curves.]

d) $x_1^A = x_2^A$ [or $x_1^B = x_2^B$] (3pt). [You need to clarify whose consumption it is.] The diagonal line connecting the two origins of the Edgeworth box (1pt). **e)** $x_1^A = 10$ (2pt). $x_2^A = 10$ (2pt). $x_1^B = 10$ (2pt). $x_2^B = 10$ (2pt) $p_1 = 1, p_2 = 1$ (2pt). [p_1, p_2 can be any pair of two positive numbers as long as $p_1 = p_2$. No partial credit for only p_1 or p_2.] **f)** $p_1 = 2, p_2 = 2$ (2pt). [p_1, p_2 can be any pair of two positive numbers as long as $p_1 = p_2$ and different from your answer in e).] **g)** $MRS^A = -1 = MRS^B$ and thus this equilibrium allocation is Pareto efficient (2pt). [MRS must be calculated.]

Problem 3. **a)** \$2000 (4pt). **b)** $C_1 = 100$ (2pt). $C_2 = 100$ (2pt). $S = 100$ (2pt). Yes, he's smoothing (1pt). No, he's not tilting (1pt). [If you answered only either one question and did not clarify which question you answered, you get no point.] **c)** Demand: $L^D = (w/p)^{-2}$ where p is the product's price and w is wage (4pt). [Thus w/p is the real wage rate.] Equilibrium real wage: $w/p = 1/4$ (2pt). The point $(L, w/p) = (16, 1/4)$ must be plotted on a graph (1pt). **d)** (6pt.) The annual consumption C (thousand dollars) is determined from $\left\{1 - (1.05)^{-40}\right\} \cdot 50/1.05 = \left\{1 - (1.05)^{-60}\right\} C/1.05$. [Further simplification gets full points.]

Problem 4. **a)** DRS (1pt). This is because $F(tK, tL) = t^{1/2}K^{1/4}L^{1/4} = t^{1/2}F(K, L) < tF(K, L)$ [if $t > 1$] (4pt). [Here $F(k, l)$ is the output from $K = k$ and $L = l$.] **b)** $C = 4y^2$ (4pt). Graph is needed on the y-C plane (1pt). **c)** $y^{MES} = 1/\sqrt{2}$ (2pt). $ATC^{MES} = 4\sqrt{2}$ (2pt). **d)** (6pt for giving both the function and the graph.) The supply function $S(p)$ is $p/8$ for $p \geq 4\sqrt{2}$, and 0 for $p \leq 4\sqrt{2}$. On the y-p plane, the graph is $y = p/8$ (i.e. $p = 8y$) for $p \geq 4\sqrt{2}$ and $y = 0$ (a part of the vertical axis) for $p \leq 4\sqrt{2}$.

[1] A Pareto efficient allocation may not hold this equation, unless the preference is convex, monotone, and smooth.

Econ 703
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Solutions to midterm 2 (Group C)

"X and Y (2pt)." means that you get 2 pts if you answered both X and Y, and no pts if you missed either (or both).

Problem 1. [Here I denote by T the state with tornado and by N the state without tornado, instead of 1 and 2.] **a)** \$3 (2pt). **b)** $U(C_T, C_N) = 0.5C_T^2 + 0.5C_N^2$ (1pt). Risk loving (2pt). $CE = 3\sqrt{2}$ (2pt). Larger than EV, because I am risk loving (2pt). **c)** $U(C_T, C_N) = 0.5 \ln C_T + 0.5 \ln C_N$ (1pt). Yes, I'm risk averse (2pt). **d)** $C_T + C_N = 6$ (2pt). Graph is needed on the C_T-C_N plane and its position must be clarified with slope and intercepts (2pt). Plot a point on $(C_T, C_N) = (0, 6)$ for endowment (1pt). **e)** $C_T = 3$ (1pt). $C_N = 3$ (1pt). $x = 6$ (2pt). Plot a point on $(3, 3)$ (1pt). Yes, fully insured (1pt). **f)** e.g. $\gamma = 1$ (2pt). [Any number larger than 0.5 because we need $C_N > C_T$.]

Problem 2. [Here I denote apple by 1, orange by 2, Andy by A and Bob by B. You could use another notation, as long as you clarified it.] **a)** The Edgeworth box should have length of 40 on each axis (1pt). The endowment is $(40, 0)$ looked from A's origin, i.e. $(0, 40)$ from B's origin (1pt). [This is a single point in an Edgeworth box. And, it cannot be an origin.] **b)** ... if none could not be better off (by another feasible allocation) unless anyone is worth off (2pt). [$MRS^A = MRS^B$: no point since it is just a mathematical equivalent property and not the definition.[1]]

c) [Need to prove the claim directly from the definition of Pareto efficiency. Arguing only slopes or tangency is wrong. Besides, logical sequence (especially starting assumptions and ending conclusions) must be clarified. The curve's name, namely "an indifference curve", should be clarified.] **Necessity** (4pt): If $MRS^A \neq MRS^B$ at an allocation x, both people's indifference curves should cross each other at x and thus we can find a point between them. Because this point is above each indifference curve looked from the people's origin, this allocation is better than x for both and thus the allocation x is not Pareto efficient. [The proof should start with $MRS^A \neq MRS^B$ and end with Pareto inefficiency of x. Graph is needed. On the graph, you need to *specify* another allocation that improves their utilities. If you wrote two separate points and two separate curves for a single allocation, you misunderstand an Edgeworth box.]

Sufficiency (4pt): If $MRS^A = MRS^B$ at an allocation x, both people's indifference curves should be tangent to each other at x and thus no point is below A's indifferent curve looked from A's origin, i.e. worse for A than x, or below B's indifferent curve looked from B's origin, i.e. worse for B, or below both. So any point (allocation) cannot be better than x for both people and x is Pareto efficient. [The proof should start with $MRS^A = MRS^B$ at x and end with Pareto efficiency of x. Graph is needed. On the graph, you need to clarify who is worse off than x in each region defined by the two indifference curves.]

d) $x_1^A = x_2^A$ [or $x_1^B = x_2^B$] (3pt). [You need to clarify whose consumption it is.] The diagonal line connecting the two origins of the Edgeworth box (1pt). **e)** $x_1^A = 20$ (2pt). $x_2^A = 20$ (2pt). $x_1^B = 20$ (2pt). $x_2^B = 20$ (2pt) $p_1 = 1, p_2 = 1$ (2pt). [p_1, p_2 can be any pair of two positive numbers as long as $p_1 = p_2$. No partial credit for only p_1 or p_2.] **f)** $p_1 = 2, p_2 = 2$ (2pt). [p_1, p_2 can be any pair of two positive numbers as long as $p_1 = p_2$ and different from your answer in e).] **g)** $MRS^A = -1 = MRS^B$ and thus this equilibrium allocation is Pareto efficient (2pt). [MRS must be calculated.]

Problem 3. **a)** \$1000 (4pt). **b)** $C_1 = 500$ (2pt). $C_2 = 500$ (2pt). $S = 500$ (2pt). Yes, he's smoothing (1pt). No, he's not tilting (1pt). [If you answered only either one question and did not clarify which question you answered, you get no point.] **c)** Demand: $L^D = (w/p)^{-2}$ where p is the product's price and w is wage (4pt). [Thus w/p is the real wage rate.] Equilibrium real wage: $w/p = 1/4$ (2pt). The point $(L, w/p) = (16, 1/4)$ must be plotted on a graph (1pt). **d)** (6pt.) The annual consumption C (thousand dollars) is determined from $\left\{1 - (1.05)^{-40}\right\} \cdot 40/1.05 = \left\{1 - (1.05)^{-60}\right\} C/1.05$. [Further simplification gets full points.]

Problem 4. **a)** DRS (1pt). This is because $F(tK, tL) = t^{1/2}K^{1/4}L^{1/4} = t^{1/2}F(K, L) < tF(K, L)$ [if $t > 1$] (4pt). [Here $F(k, l)$ is the output from $K = k$ and $L = l$.] **b)** $C = 4y^2$ (4pt). Graph is needed on the y-C plane (1pt). **c)** $y^{MES} = 1/\sqrt{2}$ (2pt). $ATC^{MES} = 4\sqrt{2}$ (2pt). **d)** (6pt for giving both the function and the graph.) The supply function $S(p)$ is $p/8$ for $p \geq 4\sqrt{2}$, and 0 for $p \leq 4\sqrt{2}$. On the y-p plane, the graph is $y = p/8$ (i.e. $p = 8y$) for $p \geq 4\sqrt{2}$ and $y = 0$ (a part of the vertical axis) for $p \leq 4\sqrt{2}$.

[1] A Pareto efficient allocation may not hold this equation, unless the preference is convex, monotone, and smooth.

Econ 703
Intermediate Microeconomics
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Solutions to midterm 2 (Group D)

"X and Y (2pt)." means that you get 2 pts if you answered both X and Y, and no pts if you missed either (or both).

Problem 1. [Here I denote by T the state with tornado and by N the state without tornado, instead of 1 and 2.] **a)** \$1 (2pt). **b)** $U(C_T, C_N) = 0.5C_T^2 + 0.5C_N^2$ (1pt). Risk loving (2pt). $CE = \sqrt{2}$ (2pt). Larger than EV, because I am risk loving (2pt). **c)** $U(C_T, C_N) = 0.5 \ln C_T + 0.5 \ln C_N$ (1pt). Yes, I'm risk averse (2pt). **d)** $C_T + C_N = 2$ (2pt). Graph is needed on the C_T-C_N plane and its position must be clarified with slope and intercepts (2pt). Plot a point on $(C_T, C_N) = (0, 2)$ for endowment (1pt). **e)** $C_T = 1$ (1pt). $C_N = 1$ (1pt). $x = 2$ (2pt). Plot a point on $(1, 1)$ (1pt). Yes, fully insured (1pt). **f)** e.g. $\gamma = 1$ (2pt). [Any number larger than 0.5 because we need $C_N > C_T$.]

Problem 2. [Here I denote apple by 1, orange by 2, Andy by A and Bob by B. You could use another notation, as long as you clarified it.] **a)** The Edgeworth box should have length of 10 on each axis (1pt). The endowment is $(10, 0)$ looked from A's origin, i.e. $(0, 10)$ from B's origin (1pt). [This is a single point in an Edgeworth box. And, it cannot be an origin.] **b)** ... if none could not be better off (by another feasible allocation) unless anyone is worth off (2pt). [$MRS^A = MRS^B$: no point since it is just a mathematical equivalent property and not the definition.[1]]

c) [Need to prove the claim directly from the definition of Pareto efficiency. Arguing only slopes or tangency is wrong. Besides, logical sequence (especially starting assumptions and ending conclusions) must be clarified. The curve's name, namely "an indifference curve", should be clarified.] **Necessity** (4pt): If $MRS^A \neq MRS^B$ at an allocation x, both people's indifference curves should cross each other at x and thus we can find a point between them. Because this point is above each indifference curve looked from the people's origin, this allocation is better than x for both and thus the allocation x is not Pareto efficient. [The proof should start with $MRS^A \neq MRS^B$ and end with Pareto inefficiency of x. Graph is needed. On the graph, you need to *specify* another allocation that improves their utilities. If you wrote two separate points and two separate curves for a single allocation, you misunderstand an Edgeworth box.]

Sufficiency (4pt): If $MRS^A = MRS^B$ at an allocation x, both people's indifference curves should be tangent to each other at x and thus no point is below A's indifferent curve looked from A's origin, i.e. worse for A than x, or below B's indifferent curve looked from B's origin, i.e. worse for B, or below both. So any point (allocation) cannot be better than x for both people and x is Pareto efficient. [The proof should start with $MRS^A = MRS^B$ at x and end with Pareto efficiency of x. Graph is needed. On the graph, you need to clarify who is worse off than x in each region defined by the two indifference curves.]

d) $x_1^A = x_2^A$ [or $x_1^B = x_2^B$] (3pt). [You need to clarify whose consumption it is.] The diagonal line connecting the two origins of the Edgeworth box (1pt). **e)** $x_1^A = 5$ (2pt). $x_2^A = 5$ (2pt). $x_1^B = 5$ (2pt). $x_2^B = 5$ (2pt) $p_1 = 1, p_2 = 1$ (2pt). [p_1, p_2 can be any pair of two positive numbers as long as $p_1 = p_2$. No partial credit for only p_1 or p_2.] **f)** $p_1 = 2, p_2 = 2$ (2pt). [p_1, p_2 can be any pair of two positive numbers as long as $p_1 = p_2$ and different from your answer in e).] **g)** $MRS^A = -1 = MRS^B$ and thus this equilibrium allocation is Pareto efficient (2pt). [MRS must be calculated.]

Problem 3. **a)** \$4000 (4pt). **b)** $C_1 = 500$ (2pt). $C_2 = 500$ (2pt). $S = 500$ (2pt). Yes, he's smoothing (1pt). No, he's not tilting (1pt). [If you answered only either one question and did not clarify which question you answered, you get no point.] **c)** Demand: $L^D = (w/p)^{-2}$ where p is the product's price and w is wage (4pt). [Thus w/p is the real wage rate.] Equilibrium real wage: $w/p = 1/4$ (2pt). The point $(L, w/p) = (16, 1/4)$ must be plotted on a graph (1pt). **d)** (6pt.) The annual consumption C (thousand dollars) is determined from $\left\{1 - (1.05)^{-40}\right\} \cdot 60/1.05 = \left\{1 - (1.05)^{-60}\right\} C/1.05$. [Further simplification gets full points.]

Problem 4. **a)** DRS (1pt). This is because $F(tK, tL) = t^{1/2}K^{1/4}L^{1/4} = t^{1/2}F(K, L) < tF(K, L)$ [if $t > 1$] (4pt). [Here $F(k, l)$ is the output from $K = k$ and $L = l$.] **b)** $C = 4y^2$ (4pt). Graph is needed on the y-C plane (1pt). **c)** $y^{MES} = 1/\sqrt{2}$ (2pt). $ATC^{MES} = 4\sqrt{2}$ (2pt). **d)** (6pt for giving both the function and the graph.) The supply function $S(p)$ is $p/8$ for $p \geq 4\sqrt{2}$, and 0 for $p \leq 4\sqrt{2}$. On the y-p plane, the graph is $y = p/8$ (i.e. $p = 8y$) for $p \geq 4\sqrt{2}$ and $y = 0$ (a part of the vertical axis) for $p \leq 4\sqrt{2}$.

[1] A Pareto efficient allocation may not hold this equation, unless the preference is convex, monotone, and smooth.