

Econ 301
Intermediate Microeconomics
Prof. Marek Weretka

Midterm 1 (A)

You have 70 minutes to complete the exam. The midterm consists of 4 questions (45+15+15+25=100 points) + bonus (just for fun). Make sure you answer the first four questions before working on the bonus one!

Problem 1 (45p) (Well-behaved preferences)

Freddie Frolic consumes only two types of commodities: cheese curds, x_1 , and diet coke, x_2 .

a) The price of one portion of cheese curds is $p_1 = 5$ and one diet coke is $p_2 = 2$. Freddie's income spent entirely on the two commodities is $m = 60$. Show geometrically Freddie's budget set. Find relative price of cheese curds in terms of coke (number). Give economic interpretation of the relative price (one sentence). Where can the relative price be seen in the graph of a budget set? (one sentence)

b) Suppose that due to shortages in cheese supply, cheese curds are rationed, i.e., each consumer can buy at most five portions. Show the new budget set on the graph.

c) Freddie's utility is

$$U(x_1, x_2) = a \ln x_1 + b \ln x_2.$$

-Find Marginal Rate of Substitution (MRS) as a function of parameters a, b and x_1, x_2 (derive formula). For parameters $a = 4, b = 2$ and bundle $(7, 7)$ find value of MRS (one number). Give economic interpretation of MRS (one sentence). Which of the goods is more valuable at the bundle $(7, 7)$? What is the geometric interpretation of MRS? (one sentence)

-Write down two secrets of happiness that determine optimal choice. Explain economic intuition behind the two conditions (two sentences for each).

-Derive optimal choice of x_1 and x_2 as a function of a, b, p_1, p_2 and m (show the derivation of magic formulas). Is your solution corner or interior (choose one and provide mathematical argument)

d) Assume $a = 4, b = 2$ and $p_2 = 2, m = 60$. Find geometrically and determine analytically price offer curve and demand curve (give two functions). Is x_1 an ordinary or Giffen good and why (one sentence)?

e) Show that utility functions $V(x_1, x_2) = x_1^a x_2^b$ and $U(x_1, x_2) = a \ln x_1 + b \ln x_2$ represent the same preferences (derive one function from the other).

Problem 2 (15p) (Quasilinear Preferences)

You are asked to plan a budget of University of Wisconsin, Madison for the next year. The two major expenses involve computers, x_1 and classroom furniture, x_2 . The university's utility function is given by

$$U(x_1, x_2) = 2 \ln x_1 + x_2.$$

a) Find marginal rate of substitution as a function of (x_1, x_2) (give formula).

b) Using two secrets of happiness find optimal "consumption" of computers and furniture if corresponding prices are $p_1 = 2$ and $p_2 = 4$ and the available funds are $m = 40$ (give two numbers).

c) Suppose the price of a computer goes down to $p_1 = 1$. Find optimal choice after the price change (two numbers). Decompose the change in x_1 into a substitution and income effect (two numbers).

d) Find optimal consumption for $p_1 = 2, p_2 = 4$ and $m = 4$ (give two numbers). Is your solution interior? (yes-no answer). Is marginal utility of a dollar equalized? (give two numbers and yes-no answer)

Problem 3 (15p) (Perfect complements, Intertemporal choice)

Casper is a manager in a small startup firm. His income today is relatively small ($m_1 = 50$) but in the future (period two) he expects to become very rich ($m_2 = 200$)

a) Depict Casper's budget set assuming that he can borrow and save at the interest rate $r = 100\%$. Mark consumption plans on the budget line that involve savings and the plans that require borrowing. Find Present and Future Value of Casper's income (two numbers) and show the two in the graph.

b) Casper's utility function is $U(C_1, C_2) = \min(C_1, C_2)$. In the commodity space plot Casper's indifference curves.

c) Find optimal consumption plan (C_1, C_2) (give two numbers: use two secrets of happiness for perfect complements and the fact that $p_1 = 1$ and $p_2 = 1/(1+r)$). Find the level of savings/borrowing in equilibrium (one number). Is Casper smoothing his consumption over time? (yes-no answer)

Problem 4 (25p) (Short questions)

a) Assume utility function $U(C, R) = C \times R$ and the daily endowment of time equal to $24h$. Find optimal choice of consumption C , relaxation time R and labor supply L as a function of real wage rate w/p_c . (three numbers) use magic formula. Is labor supply elastic or inelastic (one sentence)?

b) Find optimal choice given utility function $U(x_1, x_2) = 3x_1 + x_2$, prices $p_1 = \$8, p_2 = \2 and income $m = 100$. Is your solution corner or interior?

c) You are going to save \$10,000 when working (age 21-70) and then you are going to live for the next 30 years. Write down equation that determines constant (maximal) level of consumption during retirement age given your savings. Assume annual interest rate $r = 3\%$.

d) Derive Present Value formula for perpetuity.

Bonus question (Just for fun)

a) Prove that for perfect complements $U(x_1, x_2) = \min(ax_1, bx_2)$, MRS is equal to zero for all bundles below the optimal proportion line and equal to $-\infty$ for bundles above it.

b) Explain in words why the solution to a linear optimization problem such as with perfect substitutes is called a bang bang solution.

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Midterm 1 (B)

You have 70 minutes to complete the exam. The midterm consists of 4 questions (45+15+15+25=100 points) + bonus (just for fun). Make sure you answer the first four questions before working on the bonus one!

Problem 1 (45p) (Well-behaved preferences)

Freddie Frolic consumes only two types of commodities: cheese curds, x_1 , and diet coke, x_2 .

a) The price of one portion of cheese curds is $p_1 = 10$ and one diet coke is $p_2 = 2$. Freddie's income spent entirely on the two commodities is $m = 120$. Show geometrically Freddie's budget set. Find relative price of cheese curds in terms of coke (number). Give economic interpretation of the relative price (one sentence). Where can the relative price be seen in the graph of a budget set? (one sentence)

b) Suppose that due to shortages in cheese supply, cheese curds are rationed, i.e., each consumer can buy at most five portions. Show the new budget set on the graph.

c) Freddie's utility is

$$U(x_1, x_2) = a \ln x_1 + b \ln x_2.$$

-Find Marginal Rate of Substitution (MRS) as a function of parameters a, b and x_1, x_2 (derive formula). For parameters $a = 4, b = 2$ and bundle $(7, 14)$ find value of MRS (one number). Give economic interpretation of MRS (one sentence). Which of the goods is more valuable at the bundle $(7, 7)$? What is the geometric interpretation of MRS? (one sentence)

-Write down two secrets of happiness that determine optimal choice. Explain economic intuition behind the two conditions (two sentences for each).

-Derive optimal choice of x_1 and x_2 as a function of a, b, p_1, p_2 and m (show the derivation of magic formulas). Is your solution corner or interior (choose one and provide mathematical argument)

d) Assume $a = 4, b = 2$ and $p_2 = 2, m = 120$. Find geometrically and determine analytically price offer curve and demand curve (give two functions). Is x_1 an ordinary or Giffen good and why (one sentence)?

e) Show that utility functions $V(x_1, x_2) = x_1^a x_2^b$ and $U(x_1, x_2) = a \ln x_1 + b \ln x_2$ represent the same preferences (derive one function from the other).

Problem 2 (15p) (Quasilinear Preferences)

You are asked to plan a budget of University of Wisconsin, Madison for the next year. The two major expenses involve computers, x_1 and classroom furniture, x_2 . The university's utility function is given by

$$U(x_1, x_2) = 2 \ln x_1 + x_2.$$

a) Find marginal rate of substitution as a function of (x_1, x_2) (give formula).

b) Using two secrets of happiness find optimal "consumption" of computers and furniture if corresponding prices are $p_1 = 4$ and $p_2 = 8$ and the available funds are $m = 80$ (give two numbers).

c) Suppose the price of a computer goes down to $p_1 = 2$. Find optimal choice after the price change (two numbers). Decompose the change in x_1 into a substitution and income effect (two numbers).

d) Find optimal consumption for $p_1 = 4, p_2 = 8$ and $m = 8$ (give two numbers). Is your solution interior? (yes-no answer). Is marginal utility of a dollar equalized? (give two numbers and yes-no answer)

Problem 3 (15p) (Perfect complements, Intertemporal choice)

Casper is a manager in a small startup firm. His income today is relatively small ($m_1 = 100$) but in the future (period two) he expects to become very rich ($m_2 = 300$)

a) Depict Casper's budget set assuming that he can borrow and save at the interest rate $r = 200\%$. Mark consumption plans on the budget line that involve savings and the plans that require borrowing. Find Present and Future Value of Casper's income (two numbers) and show the two in the graph.

b) Casper's utility function is $U(C_1, C_2) = \min(C_1, C_2)$. In the commodity space plot Casper's indifference curves.

c) Find optimal consumption plan (C_1, C_2) (give two numbers: use two secrets of happiness for perfect complements and the fact that $p_1 = 1$ and $p_2 = 1/(1+r)$). Find the level of savings/borrowing in equilibrium (one number). Is Casper smoothing his consumption over time? (yes-no answer)

Problem 4 (25p) (Short questions)

a) Assume utility function $U(C, R) = C \times R$ and the daily endowment of time equal to $24h$. Find optimal choice of consumption C , relaxation time R and labor supply L as a function of real wage rate w/p_c . (three numbers) use magic formula. Is labor supply elastic or inelastic (one sentence)?

b) Find optimal choice given utility function $U(x_1, x_2) = 5x_1 + x_2$, prices $p_1 = \$10, p_2 = \1 and income $m = 100$. Is your solution corner or interior?

c) You are going to save $\$30,000$ when working (age 21-50) and then you are going to live for the next 40 years. Write down equation that determines constant (maximal) level of consumption during retirement age given your savings. Assume annual interest rate $r = 2\%$.

d) Derive Present Value formula for perpetuity.

Bonus question (Just for fun)

a) Prove that for perfect complements $U(x_1, x_2) = \min(ax_1, bx_2)$, MRS is equal to zero for all bundles below the optimal proportion line and equal to $-\infty$ for bundles above it.

b) Explain in words why the solution to a linear optimization problem such as with perfect substitutes is called a bang bang solution.

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Midterm 1 (C)

You have 70 minutes to complete the exam. The midterm consists of 4 questions (45+15+15+25=100 points) + bonus (just for fun). Make sure you answer the first four questions before working on the bonus one!

Problem 1 (45p) (Well-behaved preferences)

Freddie Frolic consumes only two types of commodities: cheese curds, x_1 , and diet coke, x_2 .

a) The price of one portion of cheese curds is $p_1 = 3$ and one diet coke is $p_2 = 1$. Freddie's income spent entirely on the two commodities is $m = 60$. Show geometrically Freddie's budget set. Find relative price of cheese curds in terms of coke (number). Give economic interpretation of the relative price (one sentence). Where can the relative price be seen in the graph of a budget set? (one sentence)

b) Suppose that due to shortages in cheese supply, cheese curds are rationed, i.e., each consumer can buy at most five portions. Show the new budget set on the graph.

c) Freddie's utility is

$$U(x_1, x_2) = a \ln x_1 + b \ln x_2.$$

-Find Marginal Rate of Substitution (MRS) as a function of parameters a, b and x_1, x_2 (derive formula). For parameters $a = 4, b = 2$ and bundle $(3, 9)$ find value of MRS (one number). Give economic interpretation of MRS (one sentence). Which of the goods is more valuable at the bundle $(7, 7)$? What is the geometric interpretation of MRS? (one sentence)

-Write down two secrets of happiness that determine optimal choice. Explain economic intuition behind the two conditions (two sentences for each).

-Derive optimal choice of x_1 and x_2 as a function of a, b, p_1, p_2 and m (show the derivation of magic formulas). Is your solution corner or interior (choose one and provide mathematical argument)

d) Assume $a = 4, b = 2$ and $p_2 = 1, m = 60$. Find geometrically and determine analytically price offer curve and demand curve (give two functions). Is x_1 an ordinary or Giffen good and why (one sentence)?

e) Show that utility functions $V(x_1, x_2) = x_1^a x_2^b$ and $U(x_1, x_2) = a \ln x_1 + b \ln x_2$ represent the same preferences (derive one function from the other).

Problem 2 (15p) (Quasilinear Preferences)

You are asked to plan a budget of University of Wisconsin, Madison for the next year. The two major expenses involve computers, x_1 and classroom furniture, x_2 . The university's utility function is given by

$$U(x_1, x_2) = 4 \ln x_1 + x_2.$$

a) Find marginal rate of substitution as a function of (x_1, x_2) (give formula).

b) Using two secrets of happiness find optimal "consumption" of computers and furniture if corresponding prices are $p_1 = 4$ and $p_2 = 2$ and the available funds are $m = 20$ (give two numbers).

c) Suppose the price of a computer goes down to $p_1 = 2$. Find optimal choice after the price change (two numbers). Decompose the change in x_1 into a substitution and income effect (two numbers).

d) Find optimal consumption for $p_1 = 4, p_2 = 2$ and $m = 4$ (give two numbers). Is your solution interior? (yes-no answer). Is marginal utility of a dollar equalized? (give two numbers and yes-no answer)

Problem 3 (15p) (Perfect complements, Intertemporal choice)

Casper is a manager in a small startup firm. His income today is relatively small ($m_1 = 50$) but in the future (period two) he expects to become very rich ($m_2 = 150$)

a) Depict Casper's budget set assuming that he can borrow and save at the interest rate $r = 100\%$. Mark consumption plans on the budget line that involve savings and the plans that require borrowing. Find Present and Future Value of Casper's income (two numbers) and show the two in the graph.

b) Casper's utility function is $U(C_1, C_2) = \min(C_1, C_2)$. In the commodity space plot Casper's indifference curves.

c) Find optimal consumption plan (C_1, C_2) (give two numbers: use two secrets of happiness for perfect complements and the fact that $p_1 = 1$ and $p_2 = 1/(1+r)$). Find the level of savings/borrowing in equilibrium (one number). Is Casper smoothing his consumption over time? (yes-no answer)

Problem 4 (25p) (Short questions)

a) Assume utility function $U(C, R) = C \times R$ and the daily endowment of time equal to $24h$. Find optimal choice of consumption C , relaxation time R and labor supply L as a function of real wage rate w/p_c . (three numbers) use magic formula. Is labor supply elastic or inelastic (one sentence)?

b) Find optimal choice given utility function $U(x_1, x_2) = 5x_1 + x_2$, prices $p_1 = \$8, p_2 = \2 and income $m = 100$. Is your solution corner or interior?

c) You are going to save \$50,000 when working (age 21-80) and then you are going to live for the next 20 years. Write down equation that determines constant (maximal) level of consumption during retirement age given your savings. Assume annual interest rate $r = 2\%$.

d) Derive Present Value formula for perpetuity.

Bonus question (Just for fun)

a) Prove that for perfect complements $U(x_1, x_2) = \min(ax_1, bx_2)$, MRS is equal to zero for all bundles below the optimal proportion line and equal to $-\infty$ for bundles above it.

b) Explain in words why the solution to a linear optimization problem such as with perfect substitutes is called a bang bang solution.

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Midterm 1 (D)

You have 70 minutes to complete the exam. The midterm consists of 4 questions (45+15+15+25=100 points) + bonus (just for fun). Make sure you answer the first four questions before working on the bonus one!

Problem 1 (45p) (Well-behaved preferences)

Freddie Frolic consumes only two types of commodities: cheese curds, x_1 , and diet coke, x_2 .

a) The price of one portion of cheese curds is $p_1 = 3$ and one diet coke is $p_2 = 3$. Freddie's income spent entirely on the two commodities is $m = 90$. Show geometrically Freddie's budget set. Find relative price of cheese curds in terms of coke (number). Give economic interpretation of the relative price (one sentence). Where can the relative price be seen in the graph of a budget set? (one sentence)

b) Suppose that due to shortages in cheese supply, cheese curds are rationed, i.e., each consumer can buy at most five portions. Show the new budget set on the graph.

c) Freddie's utility is

$$U(x_1, x_2) = a \ln x_1 + b \ln x_2.$$

-Find Marginal Rate of Substitution (MRS) as a function of parameters a, b and x_1, x_2 (derive formula). For parameters $a = 4, b = 2$ and bundle $(3, 9)$ find value of MRS (one number). Give economic interpretation of MRS (one sentence). Which of the goods is more valuable at the bundle $(7, 7)$? What is the geometric interpretation of MRS? (one sentence)

-Write down two secrets of happiness that determine optimal choice. Explain economic intuition behind the two conditions (two sentences for each).

-Derive optimal choice of x_1 and x_2 as a function of a, b, p_1, p_2 and m (show the derivation of magic formulas). Is your solution corner or interior (choose one and provide mathematical argument)

d) Assume $a = 4, b = 2$ and $p_2 = 3, m = 90$. Find geometrically and determine analytically price offer curve and demand curve (give two functions). Is x_1 an ordinary or Giffen good and why (one sentence)?

e) Show that utility functions $V(x_1, x_2) = x_1^a x_2^b$ and $U(x_1, x_2) = a \ln x_1 + b \ln x_2$ represent the same preferences (derive one function from the other).

Problem 2 (15p) (Quasilinear Preferences)

You are asked to plan a budget of University of Wisconsin, Madison for the next year. The two major expenses involve computers, x_1 and classroom furniture, x_2 . The university's utility function is given by

$$U(x_1, x_2) = 4 \ln x_1 + x_2.$$

a) Find marginal rate of substitution as a function of (x_1, x_2) (give formula).

b) Using two secrets of happiness find optimal "consumption" of computers and furniture if corresponding prices are $p_1 = 4$ and $p_2 = 4$ and the available funds are $m = 80$ (give two numbers).

c) Suppose the price of a computer goes down to $p_1 = 2$. Find optimal choice after the price change (two numbers). Decompose the change in x_1 into a substitution and income effect (two numbers).

d) Find optimal consumption for $p_1 = 4, p_2 = 4$ and $m = 4$ (give two numbers). Is your solution interior? (yes-no answer). Is marginal utility of a dollar equalized? (give two numbers and yes-no answer)

Problem 3 (15p) (Perfect complements, Intertemporal choice)

Casper is a manager in a small startup firm. His income today is relatively small ($m_1 = 100$) but in the future (period two) he expects to become very rich ($m_2 = 400$)

a) Depict Casper's budget set assuming that he can borrow and save at the interest rate $r = 100\%$. Mark consumption plans on the budget line that involve savings and the plans that require borrowing. Find Present and Future Value of Casper's income (two numbers) and show the two in the graph.

b) Casper's utility function is $U(C_1, C_2) = \min(C_1, C_2)$. In the commodity space plot Casper's indifference curves.

c) Find optimal consumption plan (C_1, C_2) (give two numbers: use two secrets of happiness for perfect complements and the fact that $p_1 = 1$ and $p_2 = 1/(1+r)$). Find the level of savings/borrowing in equilibrium (one number). Is Casper smoothing his consumption over time? (yes-no answer)

Problem 4 (25p) (Short questions)

a) Assume utility function $U(C, R) = C \times R$ and the daily endowment of time equal to $24h$. Find optimal choice of consumption C , relaxation time R and labor supply L as a function of real wage rate w/p_c . (three numbers) use magic formula). Is labor supply elastic or inelastic (one sentence)?

b) Find optimal choice given utility function $U(x_1, x_2) = 5x_1 + x_2$, prices $p_1 = \$8, p_2 = \2 and income $m = 40$. Is your solution corner or interior?

c) You are going to save \$6,000 when working (age 21-80) and then you are going to live for the next 20 years. Write down equation that determines constant (maximal) level of consumption during retirement age given your savings. Assume annual interest rate $r = 4\%$.

d) Derive Present Value formula for perpetuity.

Bonus question (Just for fun)

a) Prove that for perfect complements $U(x_1, x_2) = \min(ax_1, bx_2)$, MRS is equal to zero for all bundles below the optimal proportion line and equal to $-\infty$ for bundles above it.

b) Explain in words why the solution to a linear optimization problem such as with perfect substitutes is called a bang bang solution.

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Makeup exam

You have 70 minutes to complete the exam. The midterm consists of 4 questions (45+15+15+25=100 points) + bonus (just for fun). Make sure you answer the first four questions before working on the bonus one!

Problem 1 (45p) (Well-behaved preferences)

Freddie Frolic consumes only two types of commodities: cheese curds, x_1 , and diet coke, x_2 .

a) The price of one portion of cheese curds is $p_1 = 3$ and one diet coke is $p_2 = 3$. Freddie's income spent entirely on the two commodities is $m = 90$. Show geometrically Freddie's budget set. Find relative price of cheese curds in terms of coke (number). Give economic interpretation of the relative price (one sentence). Where can the relative price be seen in the graph of a budget set? (one sentence)

b) Suppose that due to shortages in cheese supply, cheese curds are rationed, i.e., each consumer can buy at most five portions. Show the new budget set on the graph.

c) Freddie's utility is

$$U(x_1, x_2) = a \ln x_1 + b \ln x_2.$$

-Find Marginal Rate of Substitution (MRS) as a function of parameters a, b and x_1, x_2 (derive formula). For parameters $a = 4, b = 2$ and bundle $(3, 9)$ find value of MRS (one number). Give economic interpretation of MRS (one sentence). Which of the goods is more valuable at the bundle $(7, 7)$? What is the geometric interpretation of MRS? (one sentence)

-Write down two secrets of happiness that determine optimal choice. Explain economic intuition behind the two conditions (two sentences for each).

-Derive optimal choice of x_1 and x_2 as a function of a, b, p_1, p_2 and m (show the derivation of magic formulas). Are two goods gross substitutes, gross complements or none?

d) Assume $a = 4, b = 2$ and $p_1 = 3, p_2 = 6$. Find geometrically and determine analytically income offer curve and Engel curve (give two functions). Is x_1 normal or inferior, and why (one sentence)?

e) Show that utility functions $V(x_1, x_2) = x_1^{3a} x_2^{3b}$ and $U(x_1, x_2) = a \ln x_1 + b \ln x_2$ represent the same preferences (derive one function from the other).

Problem 2 (15p) (Quasilinear Preferences)

You are asked to plan a budget of University of Wisconsin, Madison for the next year. The two major expenses involve computers, x_1 and classroom furniture, x_2 . The university' utility function is given by

$$U(x_1, x_2) = \ln x_1 + \frac{1}{2}x_2.$$

a) Find marginal rate of substitution as a function of (x_1, x_2) (give formula).

b) Using two secrets of happiness find optimal "consumption" of computers and furniture if corresponding prices are $p_1 = 4$ and $p_2 = 4$ and the available funds are $m = 80$ (give two numbers).

c) Suppose the price of a computer goes down to $p_1 = 2$. Find optimal choice after the price change (two numbers). Decompose the change in x_1 into a substitution and income effect (two numbers).

d) Find optimal consumption for $p_1 = 4, p_2 = 4$ and $m = 4$ (give two numbers). Is your solution interior? (yes-no answer). Is marginal utility of a dollar equalized? (give two numbers and yes-no answer)

Problem 3 (15p) (Perfect complements, Intertemporal choice)

Casper is a manager in a small startup firm. His income today is relatively small ($m_1 = 100$) but in the future (period two) he expects to become very rich ($m_2 = 600$)

a) Depict Casper's budget set assuming that he can borrow and save at the interest rate $r = 100\%$. Mark consumption plans on the budget line that involve savings and the plans that require borrowing. Find Present and Future Value of Casper's income (two numbers) and show the two in the graph.

b) Casper's utility function is $U(C_1, C_2) = 2 \min(C_1, C_2)$. In the commodity space plot Casper's indifference curves.

c) Find optimal consumption plan (C_1, C_2) (give two numbers: use two secrets of happiness for perfect complements and the fact that $p_1 = 1$ and $p_2 = 1/(1+r)$). Find the level of savings/borrowing in equilibrium (one number). Is Casper smoothing his consumption over time? (yes-no answer)

Problem 4 (25p) (Short questions)

a) Assume utility function $U(C, R) = C \times R$ and the daily endowment of time equal to $24h$. Find optimal choice of consumption C , relaxation time R and labor supply L as a function of real wage rate w/p_c . (three numbers) use magic formula). Is labor supply elastic or inelastic (one sentence)?

b) Find optimal choice given utility function $U(x_1, x_2) = 10x_1 + 5x_2$, prices $p_1 = \$8, p_2 = \2 and income $m = 40$. Is your solution corner or interior?

c) You are going to save \$6,000 when working (age 21-80) and then you are going to live for the next 20 years. Write down equation that determines constant (maximal) level of consumption during retirement age given your savings. Assume annual interest rate $r = 4\%$.

d) Derive Present Value formula for perpetuity.

Bonus question (Just for fun)

a) Prove that for perfect complements $U(x_1, x_2) = \min(ax_1, bx_2)$, MRS is equal to zero for all bundles below the optimal proportion line and equal to $-\infty$ for bundles above it.

b) Explain in words why the solution to a linear optimization problem such as with perfect substitutes is called a bang bang solution.