Midterm Exam 1 Solutions
(D) The Blue

Q1) *(55 points)*

a) Check figure 1 for the budget set *(3 p)*
If there is 100% inflation, the prices double.
The new budget set is also shown on Figure 1. *(3 p)*

b) $U(X_1, X_2) = X_1^{1/3} X_2^{1/3}$

$MRS^{12} = ((MU_1)/(MU_2)) = -(\partial U/\partial X_1)/(\partial U/\partial X_2) = (-(\frac{1}{3}X_1^{-1/3}X_2^{1/3})/(\frac{1}{3}X_1^{1/3}X_2^{-1/3})) = -X_2/X_1 = MRS^{12}$
(4 p)
at the point $x_1 = 3$ & $x_2 = 6$: $MRS^{12} = (-6/3) = -2$ *(2 p)*

which means:
$MU_1 > MU_2$ So 1st good, ribeye, is more valuable than top sirleon to Ava at the consumption level of
$x_1 = 3$ & $x_2 = 6$ *(4 p)*

d)

secret 1 : $p_1x_1 + p_2x_2 = m$ *(3p)*
so $4x_1 + 2x_2 = 80$
This means "spend all of your money". If the consumer does not spend all of his/her money s/he is wasting his/her opportunity to increase his/her utility since money does not have an effect on utility itself. *(2p)*

secret 2 : $MRS^{12} = (MU_1)/(MU_2) = P_1/P_2$ *(3p)*
$\rightarrow \quad X_2/X_1 = P_1/P_2$

"the last spent on each good should give the same utility" OR "marginal utility of a \$ spent on each good should be equal " . If this condition does not hold, lets say last \$ spent on good 1 brings more utility than the last \$ spent on good 2 , then the consumer should buy less of the second good and buy more of the first good to increase his/her utility. *(2p)*

start with secret 2 : $X_2/X_1 = P_1/P_2 \rightarrow X_2 = ((X_1P_1)/(P_2))$ *(1p)*
now plug this in secret 1 :
$p_1x_1 + p_2x_2 = m = P_1x_1 + P_2(X_1P_1)/(P_2) = m = 2P_1x_1$ *(2p)*
$\rightarrow$ Then $X_1 = (m/(2p_1)) = (1/2)(m/p_1)$
Since $X_2 = ((X_1P_1)/(P_2)) = ((mP_1)/(2P_1P_2)) = (m/(2p_2))$
$= (1/2)(m/(p_2)) = X_2$

$X_1 = (m/(2p_1))$

$X_2 = (m/(2p_2))$ *(2p)*

The solution is interior since none of good's optimal consumption level is zero. Furthermore in a Cobb-Douglas case the solution has to be interior as long as income level (m) is larger than zero ! (we also assume positive prices) *(1p)*

Using the magic formula $X_2 = (m/(2p_2)) \rightarrow X_2 p_2 = m/2$
note that the left hand side is the total money spent on good 2 (sirleon). So half of the income is spent on sirleon. *(2p)*

e) $X_1 = (m/(2p_1)) = ((80)/(2*4)) = 10$ *(1p)*

$X_2 = (m/(2p_2)) = ((80)/(2*2)) = 20$ *(1p)*

$X_1^{'} = (m/(2p_1^{'})) = ((80)/(2*2)) = 20$ *(1p)*

$X_2^{'} = (m/(2p_2)) = ((80)/(2*2)) = 20$ *(1p)*

Total change in consumption of ribeye is $X_1^{'} - X_1 = 20 - 10 = 10$ *(1p)*

Check figure 2 for the illustration of the change. *(3p)*

Ribeye is an ordinary good since it's consumption increases as it's price decrease. We can also see this fact from the magic formula . Since the price of ribeye is in the denominator in optimal consumption formula of ribeye, as the price goes down the consumption of it will increase ! *(2p)*

f) How much money does Ava need to consume the original bundle with the new prices ?

$p_1^{'}x_1 + p_2x_2 = m^{'} = (\$2*10) + (\$2*20) = \$60$ is enough *(3p)*

Now calculating the optimal bundle with this imaginary income :
$X_1^{s} = (m^{'}/(2p_1^{'})) = ((60)/(2*2)) = 15$ *(2p)*

$X_2^{s} = (m^{'}/(2p_2)) = ((60)/(2*2)) = 15$
So Substitution Effect (S.E.) is : $15 - 10 = 5$ *(2p)*

$T.E = S.E. + I.E.$ then : $I.E. = 10 - 5 = 5$ *(1p)*

Check figure 2 for the illustration. *(3p)*

Q 2) *(15 points)*
a) the bundle : $(1x_1 + 2x_2)$
A suitable utility function would be : $min(2x_1, 1x_2)$ *(1p)*

b) Check figure 3 *(2p)*
c) Two secrets are : *(1.5p + 1.5p)*

$$\begin{aligned} 2x_1 &= x_2 \\ p_1x_1 + p_2x_2 &= m \end{aligned}$$

hence

$$2x_1 + 2(2x_1) = 30$$

$$x_1^* = \frac{m}{p_1 + 2p_2} \text{ and } x_2^* = 2\frac{m}{p_1 + 2p_2}$$

$$\begin{aligned} x_1^* &= \frac{30}{2 + (2 * 2)} = 5 \\ \text{and } x_2^* &= 2\frac{30}{2 + (2 * 2)} = 10 \end{aligned}$$

(1.5p + 1.5p)
The solution is interior since none of the optimal consumption levels are zero. (1p)

d) Using the magic formulas :

$$\begin{aligned} x_1^* &= \frac{30}{1 + (2 * 2)} = 6 \\ \text{and } x_2^* &= 2\frac{30}{1 + (2 * 2)} = 12 \end{aligned}$$

(1.5p + 1.5p)
S.E. is zero. In a perfect complements case there is no substitution effect related to a price change. *(2p)*

Q.3) *(15 points)*
a) $U(X_1, X_2) = 2X_1 + 2X_2$ can be a utility function for this kind of preferences. *(2p)*
b) Check the figure 4 *(2p)*
c) $MRS^{12} = ((MU_1)/(MU_2)) = 1/1 = 1$ given the utility function. *(2p)*
comparing MRS to the price ratio : $1 > \frac{2}{6} = \frac{P_1}{P_2}$

So only good 1 is consumed : $X_1^* = m/P_1 = 12/2 = 6$ and $X_2^* = 0$ *(3p)*

The solution is a corner one. *(1p)*

d) check figure 5 & 6 for the income-offer curve and the engel curve. *(2p+2p)*

Pepsi is normal as the consumption of it increases as the income increases. *(1p)*

Q.4)

a) $\frac{10}{2} = 5$ is her real wage. *(1p)*

This number is her purchasing power in terms of consumption good. *(1p)*

b) check figure 7 *(3p)*

c)

secret 1: $w * R + p_c C = 24 * w$ *(2p)*

or $10R + 2C = 24 * 10 = 240$

secret 2 : $MRS^{RC} = ((MU_R)/(MU_C)) = w/P_c \rightarrow 1/R = w/P_c$ *(2p)*

$\rightarrow$ $R = \frac{P_c}{w}$ *(2p)*

d) If $w = \$1$ and $P_c = \$10$ the labor supply : $R = \frac{P_c}{w} = 10hr$ *(1p)*

If $w = \$2$ and $P_c = \$10$ the labor supply : $R = \frac{P_c}{w} = 5hr$ *(1p)*

The labor supply (24-R) is increasing in real wage ! *(2p)*

Bonus Question

$V(x_1, x_2) = f[U(x_1, x_2)]$

$MRS^U = ((MU_1)/(MU_2)) = (\partial U/\partial X_1)/(\partial U/\partial X_2)$

$MRS^V = ((MV_1)/(MV_2)) = (\partial V/\partial X_1)/(\partial V/\partial X_2)$

$= \frac{\partial f[U(x_1,x_2)]/\partial X_1}{\partial f[U(x_1,x_2)]/\partial X_2}$ by chain rule :

$$= \frac{f^{'}(.) * \partial U(x_1,x_2)/\partial X_1}{f^{'}(.) * \partial U(x_1,x_2)/\partial X_2} = \frac{\partial U(x_1,x_2)/\partial X_1}{\partial U(x_1,x_2)/\partial X_2} = MRS^U$$