

## Midterm Exam 1 Solutions

### (A) The Pink

**Q1)** (55 points)

**a)** Check figure 1 for the budget set (3 p)

If there is 100% inflation, the prices double.

The new budget set is also shown on Figure 1. (3 p)

**b)**  $U(X_1, X_2) = X_1^5 X_2^5$

$$\frac{MRS^{12}}{X_2/X_1} = ((MU_1)/(MU_2)) = (\partial U/\partial X_1)/(\partial U/\partial X_2) = ((5X_1^4 X_2^5)/(5X_1^5 X_2^4)) =$$

(4 p)

at the point  $x_1 = 2$  &  $x_2 = 4$ :  $MRS^{12} = (4/2) = 2$  (2 p)

which means:

$MU_1 > MU_2$  So 1st good, ribeye, is more valuable than top sirloin to Ava at the consumption level of

$x_1 = 2$  &  $x_2 = 4$  (4 p)

**d)**

secret 1 :  $p_1 x_1 + p_2 x_2 = m$  (3p)

so  $50x_1 + 25x_2 = 500$

This means "spend all of your money". If the consumer does not spend all of his/her money s/he is wasting his/her opportunity to increase his/her utility since money does not have an effect on utility itself. (2p)

secret 2 :  $MRS^{12} = (MU_1)/(MU_2) = P_1/P_2$  (3p)

$\rightarrow X_2/X_1 = P_1/P_2$

"the last spent on each good should give the same utility" OR "marginal utility of a \$ spent on each good should be equal". If this condition does not hold, let's say last \$ spent on good 1 brings more utility than the last \$ spent on good 2, then the consumer should buy less of the second good and buy more of the first good to increase his/her utility. (2p)

start with secret 2 :  $X_2/X_1 = P_1/P_2 \rightarrow X_2 = ((X_1 P_1)/(P_2))$  (1p)

now plug this in secret 1 :

$p_1 x_1 + p_2 x_2 = m = P_1 x_1 + P_2 (X_1 P_1)/(P_2) = m = 2P_1 x_1$  (2p)

$\rightarrow$  Then  $X_1 = (m/(2p_1)) = (1/2)(m/p_1)$

Since  $X_2 = ((X_1 P_1)/(P_2)) = ((m P_1)/(2P_1 P_2)) = (m/(2p_2))$   
 $= (1/2)(m/(p_2)) = X_2$

$$X_1 = (m/(2p_1))$$

$$X_2 = (m/(2p_2)) \quad (2p)$$

The solution is interior since none of good's optimal consumption level is zero. Furthermore in a Cobb-Douglas case the solution has to be interior as long as income level (m) is larger than zero ! (we also assume positive prices)  
(1p)

$$\text{Using the magic formula } X_2 = (m/(2p_2)) \rightarrow X_2 p_2 = m/2$$

note that the left hand side is the total money spent on good 2 (sirleon).  
So half of the income is spent on sirleon. (2p)

$$\text{e) } X_1 = (m/(2p_1)) = ((500)/(2 * 50)) = 5 \quad (1p)$$

$$X_2 = (m/(2p_2)) = ((500)/(2 * 25)) = 10 \quad (1p)$$

$$X'_1 = (m/(2p'_1)) = ((500)/(2 * 25)) = 10 \quad (1p)$$

$$X'_2 = (m/(2p_2)) = ((500)/(2 * 25)) = 10 \quad (1p)$$

$$\text{Total change in consumption of ribeye is } X'_1 - X_1 = 10 - 5 = 5 \quad (1p)$$

Check figure 2 for the illustration of the change. (3p)

Ribeye is an ordinary good since it's consumption increases as it's price decrease. We can also see this fact from the magic formula . Since the price of ribeye is in the denominator in optimal consumption formula of ribeye, as the price goes down the consumption of it will increase ! (2p)

f) How much money does Ava need to consume the original bundle with the new prices ?

$$p'_1 x_1 + p_2 x_2 = m' = (\$25 * 5) + (\$25 * 10) = \$375 \text{ is enough} \quad (3p)$$

Now calculating the optimal bundle with this imaginary income :

$$X_1^s = (m'/(2p_1)) = ((375)/(2 * 25)) = 15/2 \quad (2p)$$

$$X_2^s = (m'/(2p_2)) = ((375)/(2 * 25)) = 15/2$$

$$\text{So Substitution Effect (S.E.) is : } 15/2 - 5 = 5/2 \quad (2p)$$

$$T.E = S.E. + I.E. \quad \text{then : } I.E. = 5 - (5/2) = 5/2 \quad (1p)$$

Check figure 2 for the illustration. (3p)

**Q 2)** (15 points)

a) the bundle :  $(1x_1 + 2x_2)$

A suitable utility function would be :  $\min(2x_1, 1x_2)$  (1p)

b) Check figure 3 (2p)

c) Two secrets are :  $(1.5p + 1.5p)$

$$\begin{aligned}2x_1 &= x_2 \\ p_1x_1 + p_2x_2 &= m\end{aligned}$$

hence

$$4x_1 + 3(2x_1) = 40$$

$$x_1^* = \frac{m}{p_1 + 2p_2} \text{ and } x_2^* = 2\frac{m}{p_1 + 2p_2}$$

$$\begin{aligned}x_1^* &= \frac{40}{4 + (2 * 3)} = 4 \\ \text{and } x_2^* &= 2\frac{40}{4 + (2 * 3)} = 8\end{aligned}$$

(1.5p + 1.5p)

The solution is interior since none of the optimal consumption levels are zero. (1p)

d) Using the magic formulas :

$$\begin{aligned}x_1^* &= \frac{40}{2 + (2 * 3)} = 5 \\ \text{and } x_2^* &= 2\frac{40}{2 + (2 * 3)} = 10\end{aligned}$$

(1.5p + 1.5p)

S.E. is zero. In a perfect complements case there is no substitution effect related to a price change. (2p)

**Q.3)** (15 points)

a)  $U(X_1, X_2) = 2X_1 + 2X_2$  can be a utility function for this kind of preferences. (2p)

b) Check the figure 4 (2p)

c)  $MRS^{12} = ((MU_1)/(MU_2)) = 1/1 = 1$  given the utility function. (2p)  
comparing MRS to the price ratio :  $1 > \frac{2}{3} = \frac{P_1}{P_2}$

So only good 1 is consumed :  $X_1^* = m/P_1 = 12/2 = 6$  and  $X_2^* = 0$  (3p)

The solution is a corner one. (1p)

d) check figure 5 for the income-offer curve and the engel curve. (2p+2p)

Pepsi is normal as the consumption of it increases as the income increases.

(1p)

**Q.4)**

a)  $\frac{10}{2} = 5$  is her real wage. (1p)

This number is her purchasing power in terms of consumption good. (1p)

b) check figure 6 (3p)

c)

$$\text{secret 1: } w * R + p_c C = 24 * w \quad (2p)$$

$$\text{or } 10R + 2C = 24 * 10 = 240$$

$$\text{secret 2 : } MRS^{RC} = ((MU_R)/(MU_C)) = w/P_c \rightarrow 1/R = w/P_c \quad (2p)$$

$$\rightarrow R = \frac{P_c}{w} \quad (2p)$$

d) If  $w = \$1$  and  $P_c = \$10$  the labor supply :  $R = \frac{P_c}{w} = 10hr$  (1p)

If  $w = \$2$  and  $P_c = \$10$  the labor supply :  $R = \frac{P_c}{w} = 5hr$  (1p)

The labor supply (24-R) is increasing in real wage ! (2p)

**Bonus Question**

$$V(x_1, x_2) = f[U(x_1, x_2)]$$

$$MRS^U = ((MU_1)/(MU_2)) = (\partial U/\partial X_1)/(\partial U/\partial X_2)$$

$$MRS^V = ((MV_1)/(MV_2)) = (\partial V/\partial X_1)/(\partial V/\partial X_2)$$

$$= \frac{\partial f[U(x_1, x_2)]/\partial X_1}{\partial f[U(x_1, x_2)]/\partial X_2} \quad \text{by chain rule :}$$

$$= \frac{f'(\cdot) * \partial U(x_1, x_2)/\partial X_1}{f'(\cdot) * \partial U(x_1, x_2)/\partial X_2} = \frac{\partial U(x_1, x_2)/\partial X_1}{\partial U(x_1, x_2)/\partial X_2} = MRS^U$$