

# *The importance of countercyclical income risk for the welfare costs of business cycles*

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## Income Risk & Business Cycles

**Motivation:** 3 business cycle facts, in recessions:

- i. Log earnings changes are more **negatively skewed** (Gueven, Ozkan & Song (2014))
- ii. Earnings losses after displacement are **larger** (Davis & von Wachter (2012))
- iii. Inflows into unemployment (UE rate) **increases**

**Question:** What are the **welfare consequences** of business cycles when recessions are consistent w/ these 3 business cycle facts?

## Income Risk & Business Cycles

**Quantitative contribution:** Bewley model + directed search in labor market w/

- a. human capital losses in unemployment
- b. counter-cyclical employment risk
- c. aggregate productivity shocks

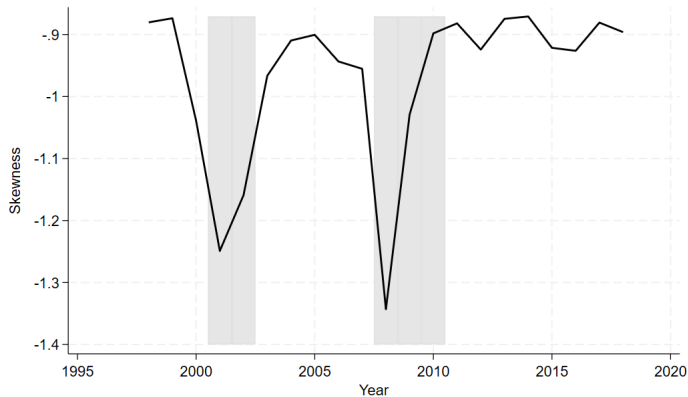
### Findings:

- i. Model ingredients (a.) & (b.) generate empirically relevant amount of excess negative skewness in recessions
- ii. Welfare gain from eliminating business cycles is **large** in baseline model, **2.7%** of lifetime consumption
- iii. Welfare gain from eliminating business cycles in model w/o (a.) & (b.) is **small**, **0.03%** of lifetime consumption

## Empirical Motivation

**Empirical Fact 1:** Log earnings changes income are more **negatively skewed** in recessions (Guvenen, Ozkan & Song, 2014)

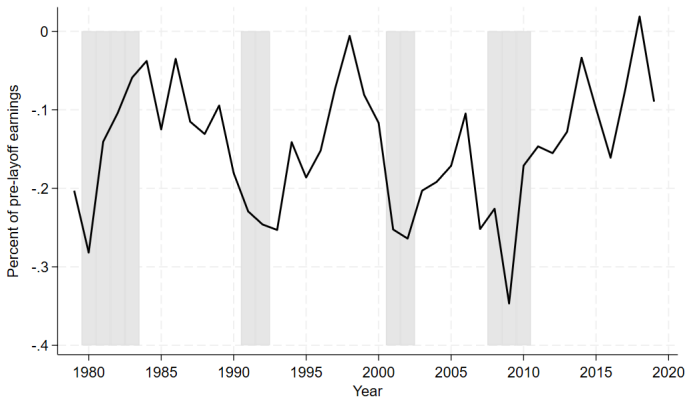
- GRID database: Skewness in expansions: -0.915; recessions: -1.135



## Empirical Motivation

**Empirical Fact 2:** Earnings losses after displace are **larger** negatively skewed in recessions (Davis & von Wachter, 2012)

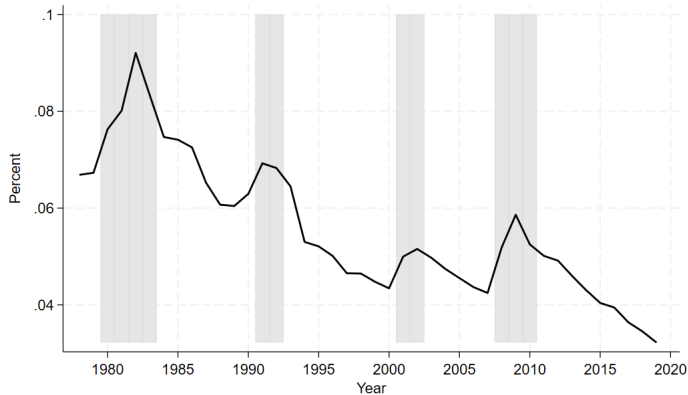
- CPS DWS: Avg. earnings decline in expansions: -12.0%; recessions: -16.7%



## Empirical Motivation

**Empirical Fact 3:** Inflows into unemployment increase in recessions (Davis & von Wachter, 2012)

- CPS: Quarterly EU rate in expansions: -5.2%; recessions: 6.7%



# Model Environment

## Overview

- Directed search
- Discrete time
- Finite horizon: agents live for  $T$  periods
- Aggregate labor productivity shocks,  $y$

# Model Environment: Agents

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## Workers

- Either unemployed or employed
- Age  $t$
- Heterogeneous in human capital level,  $h$
- Direct search over piece rates,  $\omega$
- Self insure through saving and borrowing,  $a$ , at price  $Q$ 
  - $a \geq \underline{a}$
- Unemployed workers receive unemployment benefit,  $b$ , and home production,  $g$
- Employed workers search on the job with probability  $\lambda_e$
- Employed workers exogenously separate from firm with probability  $\delta$

## Firms

- Post wage contracts

# Model Environment: Human Capital Evolution

## Unemployed Workers - Two Sources of Human Capital Risk

1. Gradual skill decline - human capital declines by  $\Delta_h$  w/ probability  $p_h$
2. Obsolescence - draw  $h_{obs}$  from lower human capital distribution w/ probability  $\psi(y)$

$$h' = \begin{cases} h & (1 - p_{h,u})(1 - \psi(y)) \\ h - \Delta_h & p_{h,u}(1 - \psi(y)) \\ h_{obs} & (1 - p_{h,u})\psi(y) \\ h_{obs} - \Delta_h & p_{h,u}\psi(y) \end{cases}$$

## Employed workers

- Gain  $\Delta_h$  amount of skills with probability  $p_{h,e}$

$$h' = \begin{cases} h + \Delta_h & p_{h,w} \\ h & (1 - p_{h,w}) \end{cases}$$

# Model Environment: Aggregate Productivity and Separation Shock

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## Aggregate Productivity

$$y' = \rho_y \cdot y + \epsilon_y, \quad \epsilon_y \sim N(0, \sigma_Y)$$

## Separation Shock

- Workers separate in unemployment with probability  $\delta(y)$

$$\delta(y) = \delta \exp(\eta_y^\delta (y - \bar{y}))$$

## Model Timing

- i. Aggregate state,  $y$ , and human capital,  $h$  is realized
- ii. Firms post vacancies according to free entry:  $\kappa \geq q(\theta(\cdot))J(\cdot)$
- iii. Exogenous separation shock  $\delta(y)$  is realized by employed workers
- iv. Employed workers search on-the-job if draw  $\lambda_e$
- v. Agents enter labor market
- vi. Search and match occurs
- vii. Production, consumption and savings occurs

## Unemployed Bellman

Today: Age  $t$  and make savings/consumption decisions,  $a'$

$$U_t(a, h, y) = \max_{a' \geq \underline{a}} u(c) + \beta \mathbb{E}_{y'|y, h'|h} \left[ \hat{U}_{t+1}(a', h', y') \right]$$

$$s.t. \quad c + Qa' \leq a + b + g$$

Tomorrow: Search in labor market over wage piece rates  $\omega'$

$$\begin{aligned} \hat{U}_{t+1}(a', h', y') = & \max_{\omega' \in [0,1]} p(\theta_{t+1}(a', h', \omega', y')) W_{t+1}(a', h', \omega', y') \\ & + (1 - p(\theta_{t+1}(a', h', \omega', y'))) U_{t+1}(a', h', y') \end{aligned}$$

## Employed Bellman

Today: Receive earnings  $\omega f(y, h)$ , make savings/consumption decisions  $a'$

$$W_t(a, h, \omega, y) = \max_{a' \geq \underline{a}'} u(c) + \beta \mathbb{E}_{y'|y, h'|h} \left[ \delta(y') \hat{U}_{t+1}(a', h', y') + (1 - \delta(y')) \hat{W}_{t+1}(a', h', \omega, y') \right]$$

$$s.t. \quad c + Qa' \leq a + \omega f(y, h)$$

Tomorrow: Separate with probability  $\delta(y')$ . If avoid separation, search w/ prob.  $\lambda_e$

$$\begin{aligned} \hat{W}_{t+1}(a', h', \omega, y') = & \max_{\omega' \in [0,1]} \lambda_e p(\theta_{t+1}(a', h', \omega', y')) (W_{t+1}(a', h', \omega', y') \\ & + (1 - \lambda_e p(\theta_{t+1}(a', h', \omega', y'))) W_{t+1}(a', h', \omega, y') \end{aligned}$$

## Firm Bellman

### Matched Firms:

$$J_t(a, h, \omega, y) = (1 - \omega)f(y, h) + \beta \mathbb{E}_{y'|y, h'} \left[ (1 - \lambda_e p(\theta_{t+1}(a', h', \omega', y')))(1 - \delta(y')) J_{t+1}(a', h', \omega, y') \right]$$

- **Today:** Produce  $f(y, h)$  w/ worker, keep  $(1 - \omega)$  of output
- **Tomorrow:** Continue match if avoid exogenous separation  $\delta(y')$  or OJS  $(1 - \lambda_e p(\cdot))$

### New Firms: Pay $\kappa$ to post new vacancies:

$$\kappa \geq q(\theta_t(a, h, \omega, y)) J_t(a, h, \omega, y)$$

# Equilibrium

## Recursive Competitive Equilibrium

- i. Individual decision rules are optimal
- ii. Firms satisfy free entry into each submarket
- iii. The aggregate distribution of agents across states is consistent with optimal decision rules

## *Block Recursive*

- As in Menzio and Shi (2011), individual optimal decision rules are independent of aggregate distribution of agents across states

# Calibration

## Externally Calibrated Parameters

- Model calibrated at quarterly frequency
- Aggregate productivity process ( $\rho_y$  &  $\sigma_y$ ) from Herkenhoff (2019)

| Parameter  | Value  | Description                              |
|------------|--------|------------------------------------------|
| $\sigma$   | 2      | Risk Aversion                            |
| $r$        | 0.985% | (Quarterly) Risk free rate               |
| $\delta$   | 5%     | Average job destruction rate             |
| $\rho_y$   | 0.8961 | Auto Correlation of Labor Productivity   |
| $\sigma_y$ | 0.0055 | Standard Deviation of Labor Productivity |
| $\gamma$   | 1.6    | Labor Matching Elasticity                |
| $T$        | 120    | Lifespan in quarters                     |

## Calibration

Exogenous separation probability:  $\delta(y) = \delta \exp(\eta_y^\delta (y - \bar{y}))$

Obsolescence shock:  $\psi(y) = \psi$  if  $y < \bar{y}$

| Parameter       | Target                                       | Model  | Data   | Source              |
|-----------------|----------------------------------------------|--------|--------|---------------------|
| $\beta$         | Fraction of agents borrowing                 | 34.9%  | 30.5%  | SCF, 1999-2013      |
| $b$             | Replacement rate of unemployment benefit     | 41.1%  | 41.2%  | PSID                |
| $\kappa$        | Unemployment Rate in Expansions              | 5.7%   | 5.5%   | BLS                 |
| $\lambda_e$     | E-E Transition Rate                          | 6.7%   | 7.3%   | Fujita et al (2021) |
| $p_{h,e}$       | Elasticity of earnings gain with age         | 1.4    | 0.95   | LEHD-TU             |
| $p_{h,u}$       | Earnings loss 4Q after job loss (expansions) | -11.8% | -12%   | DWS                 |
| $\psi$          | Earnings loss from job loss (recessions)     | -15.2% | -16.7% | DWS                 |
| $\lambda_H$     | Dispersion of initial earnings among young   | 0.63   | 0.66   | LEHD-TU             |
| $\eta_y^\delta$ | Unemployment rate in recessions              | 7.4%   | 7.6%   | BLS                 |

## Model Validation: Non-targeted moments

### Higher order moments of log earnings changes

- Difference between recession and expansions

| Moment                 | Model | Data  | Source                  |
|------------------------|-------|-------|-------------------------|
| Difference in Variance | 0.020 | 0.024 | GRID - LEHD 1998 - 2018 |
| Difference in Skewness | -0.23 | -0.22 | GRID - LEHD 1998 - 2018 |

### EU transition rate over business cycle

| Moment     | Model | Data | Source        |
|------------|-------|------|---------------|
| Expansions | 4.6%  | 5.2% | CPS 1978-2019 |
| Recessions | 6.4%  | 6.7% | CPS 1978-2019 |

# Generating Negatively Skewed Income Risk Across the Business Cycle

## Two Sources

1. Increase in unemployment during recessions
  - $\delta(y)$  decreasing in  $y$
2. Size of earnings losses larger during recessions
  - Obsolescence shock  $\psi(y)$  decreasing in  $y$

| Moment                 | Data  | Baseline Model | No $\psi(y)$ , $\delta(y)$ |
|------------------------|-------|----------------|----------------------------|
| Difference in Variance | 0.024 | 0.020          | 0.002                      |
| Difference in Skewness | -0.22 | -0.23          | -0.03                      |

## Welfare Experiment

**Eliminate** business cycles ( $y_t = 1 \forall t$ ) in:

- i. Baseline Model
- ii. Model without  $\psi(y)$ ,  $\delta(y)$

**Measure** welfare cost of business cycles using lifetime consumption equivalents

|                                        | Baseline Model | No $\psi(y)$ , $\delta(y)$ |
|----------------------------------------|----------------|----------------------------|
| Welfare Change Removing Business Cycle | 2.71%          | 0.03%                      |
| Difference in Variance                 | 0.020          | 0.002                      |
| Difference in Skewness                 | -0.23          | -0.03                      |

## Conclusion

- Use a quantitative model of the labor to understand sources of negatively skewed income risk
- Find negatively skewed income risk amplifies larger welfare losses from business cycle

## Future Work

- Decompose the welfare losses from negatively skewed income shocks
- Match the time series of earnings across the business cycle to data

## Model Environment: Firms

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- Produce  $f(y, h)$  when matched with worker
- Post piece rate contracts,  $\omega$
- Exogenously separate from worker with probability  $\delta$
- Submarket indexed by  $(t, \omega, a, y, h)$
- Pay  $\kappa$  to enter competitively into each submarket

Free entry condition

$$\kappa \geq q(\theta_t(\omega, a, y, h))J_t(\omega, a, y, h)$$