Whom to insure - firms or workers?*

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Abstract

This paper proposes an alternative way to provide insurance to workers in the labor market through *Firm Transfers (FT)*, a payment given to businesses to prevent layoffs. I develop a frictional model of the labor market where firms make endogenous layoff decisions in the presence of idiosyncratic productivity shocks and workers face uninsurable income risk. FT prevent firm-initiated inefficient layoffs due to rigidity in wage contract by increasing the firm's value. FT improve human capital accumulation by reducing job loss scarring, but at the cost of reducing productivity and output. The Paycheck Protection Program validates the model by matching the measured employment gains in the data along the transition path. A combination of both Firm Transfers and Unemployment Insurance maximize social welfare. I find the optimal policy mitigates the scarring effect of job loss by reducing consumption losses around job loss by 4% and increasing lifetime earnings by 2%.

Keywords: *firm transfers, uninsurable risk, unemployment insurance, optimal public insurance, directed search*

JEL codes: *E21, I138, G51, G52, H4*

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1 Introduction

The U.S. traditionally insures workers against job loss through the Unemployment Insurance (UI) program. In this paper, I propose an alternative policy to provide insurance to workers through *Firm Transfers (FT)*, a payment given to businesses that prevents layoffs in times of low idiosyncratic productivity. Most firms lay off workers rather than renegotiate wage contracts to save on labor costs when experiencing low productivity shocks (Akerlof and Yellen, 1990, Bewley, 1999).[1] However, recent empirical evidence shows that on average, displaced workers can lose up to 2 years of their pre-layoff earnings that they can never fully recover throughout their lifetime (Jacobson et al., 1993, Davis and Von Wachter, 2011). Firm Transfers can potentially offset the scarring effect of job loss and provide valuable insurance to workers.

In this paper, I build a quantitative model to address three questions: i) what are the macroeconomic and welfare consequences of implementing Firm Transfers? ii) what is the combination of Unemployment Insurance and Firm Transfers that maximizes social welfare? iii) how much does introducing Firm Transfers mitigate the high cost of job loss? I find that Firm Transfers lower the steady state unemployment rate and improve human capital accumulation but at the cost of lowered aggregate productivity and output. Given the scarring effect of job loss, I find optimal transfers in the labor market are jointly given to firms and unemployed workers. Firms receive a Firm Transfer that is 80% less than the level under the Paycheck Protection Program and unemployed workers receive Unemployment Insurance that is 14% more than the current level in the US. I find under this optimal policy regime workers have 3.6% higher lifetime earnings and recover 4% more consumption after 1 year of displacement, mitigating the high cost of job loss.

The first contribution of this paper is to build a framework that quantifies the welfare and macroeconomic impacts of implementing FT. I develop a tractable directed search model, similar to Moen (1997) and Menzio and Shi (2011).[2] Workers are heterogeneous in their skill or human capital level in the spirit of Ljungqvist and Sargent (1998). They search over a menu of wage contracts, each tied to a probability of matching. Firms receive idiosyncratic match productivity shocks and decide optimally when to lay off workers and exit. The value of employing a worker to the firm can become negative due to a low temporary negative productivity shock, causing the firm to lay off its' worker. Workers and firms cannot renegotiate their wage contract, making some layoff decisions inefficient and involuntary in the eyes of

[1] Firms do not perfectly adjust their workers' earnings to productivity shocks. See Harris and Holmstrom (1982), Guiso et al. (2005), Ai and Bhandari (2021), and Balke and Lamadon (2022), for reference.

[2] In Appendix 7 of the paper, I show using a simple model without search frictions how transfers to the firm provide can increase consumption of the worker. In this set up, where firms and workers are separate agents, it does not matter who receives the transfer.

the worker. Upon layoff, the worker enters unemployment, receives UI from the government and faces reduced consumption and skill depreciation. The introduction of FT prevents firms from making layoff decisions and prevents workers from entering costly unemployment. FT are designed as a payroll subsidy program, a payment equal to a fraction of the firm's wage bill. The government sets an eligibility threshold that is based on the firms productivity level - only firms within the eligibility requirement can receive FT. The quantitative contribution of this paper is relevant and important as governments worldwide have adopted versions of FT, particularly at the onset of the COVID-19 pandemic.[3] However, there is little known about the welfare consequences of introducing payroll subsidy programs similar to FT and their interaction with UI.

The second main contribution of this paper is understanding the macroeconomic benefits and costs of Firm Transfers. In an economy with only UI, I find FT provides insurance to workers through two channels. The first one is lowered unemployment risk. FT increases the present value of the match to the firm, decreasing the probability that firms will realize a negative present discounted value. Preventing layoffs benefits the worker as they avoid entering costly unemployment and allows the worker's human capital and earnings to continue to grow over the length of the match. The second insurance channel is increased job creation. FT increases the value of employing a worker to the firm by allowing an opportunity to produce together instead of exiting the economy. This increases the expected value of employing a worker to the firm relative to the cost of posting a job vacancy under free entry. Thus, firm entry increases in the macroeconomy and leads to higher job creation. This leads to unemployed workers experiencing higher job finding rates in the labor market and transitioning to employment at a faster rate than compared to the economy with only UI. Less time spent in unemployment reduces skill depreciation leading to higher earnings and consumption over the lifetime relative to the economy with only UI. The two main benefits of Firm Transfers directly support workers, but this policy has costs for the broader economy.

Introducing FT has two costs for the macroeconomy. The first is decreased productivity resulting from an increase in lower productivity firms operating. The second cost on the macroeconomy is decreased output. However, FT improves human capital accumulation, which offsets the decline in output coming from the lowered production of firms. The government weighs the two main benefits against the two main costs of Firm Transfers when considering optimal transfers.

[3]In the U.S. under the CARES, small businesses were eligible to receive forgivable loans to subsidize payroll costs under the Paycheck Protection Program (PPP Link). Under the Canada Emergency Wage Subsidy, employers were eligible to receive payroll subsidizes depending on decreased in revenues.(Canada Wage Subsidy Link).Short-term work compensation (STW) and Short-Time Employment Aid (STEA) was implemented by 28 OECD countries (OECD Link)

The trade-offs associated with FT are relatively new compared to the long and historical debate studying trade-offs associated with UI. UI provides consumption smoothing benefits while unemployed workers search for a new job, but it also increases the option value of unemployment potentially leading to a moral hazard problem (Baily, 1978, Chetty, 2008). Unemployed workers demand higher wages that have low probability of matching, increasing average unemployment duration. Introducing FT to the economy with UI mitigates the moral hazard cost of UI through increasing job creation and thus decreasing the time spent in unemployment.

The model quantitatively reproduces the dynamics of the U.S. macroeconomy and labor market with only UI, making it a useful environment to understand the macroeconomic impacts of adding FT as an additional policy tool. Using the calibrated model, I simulate the Paycheck Protection Program (PPP) by choosing an eligibility requirement such that 37% of the labor force works at a firm that is eligible to receive a transfer and the amount of the subsidy equals 56% of the average wage in the economy.[4] Exploiting the *Block Rescursivity* nature of the model, I solve the transition path of the unexpected introduction of PPP and find that employment increases by 4%. This finding is within the range of estimates measured by Chetty et al. (2023), Autor et al. (2022b), and Hubbard and Strain (2020). This exercise validates the main mechanism of the model by showing FT can lower unemployment risk and thus increase employment. Regardless of the large employment gains of the PPP, in Section 6.2, I show PPP level transfers leads to large welfare losses and misallocation of resources.

The third main contribution of this paper is to characterize and determine optimal transfers in the labor market. I evaluate the policies using lifetime consumption equivalents and a utilitarian welfare criterion, placing equal weight on all individuals. The government finances transfers in the labor market using proportional taxes on labor income.[5] The government faces an equity-efficiency trade-off: stronger insurance (higher consumption smoothing and equity) requires higher taxes and reduces efficiency. I find that the utilitarian government sets optimal policy so unemployed workers receive Unemployment Insurance that equals 40.6% of the average wage in the economy and that 22% of firms receive Firm Transfer equal 7.5% of the average in the economy. The optimal policy provides more generous UI compared to

[4]In Section 4, I discuss how this compares to the data estimates. Using data from the Small Business Administration, eligible businesses received a transfer that was equivalent to 54.8% of the average quarterly wage in the economy prior to this policy. Using Statistics of U.S. Business provided by the U.S. Census Bureau, I find that 46.3% of the labor force was working at a firm with less than 500 employees, making them eligible to receive a PPP loan.

[5]High unemployment benefits must be financed through higher taxes. As the option value of remaining unemployed has increased, workers apply for higher after-tax wage jobs to equate their increased outside option of unemployment. These jobs have lower probability of matching, leading to workers staying in unemployment longer. For reference, the disentanglement of the increase in unemployment duration from the moral hazard verse the increased taxes is studied seriously in McKay and Reis (2021).

U.S. - 14% increase from current level where workers on average receive 36% of the average wage. The optimal transfers to firms under the optimal policy is significantly less than the amount given to firms - 86% drop from the PPP level of transfers equating to 54% of average wage. On average, an individual would be willing to give up 1% of lifetime consumption to transition from an economy with the current policy to move to an economy with optimal combination of transfers to unemployed workers and firms.

The transfers to unemployed workers and firms generate welfare gains at different time horizons. Unemployed workers receive higher transfers, which increases their consumption smoothing immediately following job loss. In the quarters following job loss, the introduction of Firm Transfers increases consumption: increased job finding rates enables unemployed workers to find jobs relatively faster, which decrease skill depreciation and increases wages. Using a welfare decomposition exercise, I find increased job creation accounts for 80% of the welfare gains and lowered unemployment risk accounts for the remained 20% of the welfare gains of adding FT to the economy with UI. In the face of idiosyncratic income risk, Firm Transfer have welfare gains and should be used with UI.

The last contribution of this paper is measuring how much Firm Transfers mitigate the scarring effect of job loss. Using simulated data from the model, I measure the consumption around job loss for an individual in the baseline version of the model and in the economy with optimal Firm Transfers. I find workers spend 3.8% less time in unemployment and recover 4% more of consumption after job loss relative to an economy with only UI. Under the optimal policy world, workers have 2% higher lifetime earnings relative to the world with only UI. This finding is substantial as it well documented that displaced workers rarely make a full recovery in earnings and consumption over their lifetime (Saporta-Eksten (2014), Chodorow-Reich and Karabarbounis (2016)).

The model used in this paper incorporates rich hetereogenity on both the firm and worker side. The equilibrium is *Block Recursive* as in Menzio and Shi (2011), meaning that the model can be solved without keeping track of the distribution of agents across states. This feature allows me to feasibly solve the transition path of the baseline economy to an economy with FT. In Appendix 7.4, I extend the model to include precautionary savings in the style of Bewley (1986), Huggett (1993), and Aiyagari (1994). In this model, the workers can self insure against idiosyncratic income risk, but still find job lost quite costly. I find the introduction of FT increases aggregate insurance in the labor market and thus decreases aggregate savings.

Contribution to literature

This paper contributes to the literature understanding the labor market effects of policies similar to Firm Transfers. Short Term Work (STW) subsidizes workers hours and is used widely by many OECD countries.[6] However, due to lack of firm level administrative data, previous empirical papers relied on cross country differences in understanding the implications of STW in the labor market (Abraham and Houseman (1993), Van Audenrode (1994), Boeri and Bruecker (2011), Cahuc and Carcillo (2011)). Other papers have attempted to use creative Instrumental Variable approaches of firm level take up rates of STW across the business cycle (Boeri and Bruecker (2011), Hijzen and Martin (2013), Cooper et al. (2017), Kopp and Siegenthaler (2021), Cahuc et al. (2021), Rodriguez et al. (2022), Salerno et al. (2024)). However, even with the availability of firm level administrative data, there is little exogenous variation in firms' take up rates of STW across countries, making it difficult to have reliable and clean estimates. Giupponi et al. (2022) and Giupponi and Landais (2023) are able to overcome this by exploiting the variation in STW used in Europe at the onset of the COVID-19 recession to measure welfare. The results of this paper are most closely related to their empirical results, which show STW is welfare enhancing in the face of temporary shocks. However, they do not study the interaction of this policy with UI nor the welfare benefits in the presence of idiosyncratic risk.

This paper also contributes to the small but growing literature that uses structural models to understand the general equilibrium and welfare effects of Firm Transfers (Burdett and Wright (1989), Tilly and Niedermayer (2016), Birinci et al. (2021)). Of these papers, the one that is most similar to the present paper is Burdett and Wright (1989), who were one of the first to explore an optimal contracting theoretical framework of STW. Aligned with my results, they find STW can distort the allocation of resources and lead to reduced output. The first contribution of my paper relative to this prior literature is to incorporate idiosyncratic income risk which allows this paper to make realistic predictions over when it is socially optimal to use policies similar to Firm Transfers. The second contribution to the structural literature is the incorporation of human capital risk, which enhances our understanding of the types of workers who will benefit from, or be hurt by, the introduction of Firm Transfers in the U.S. labor market.

This paper also contributes to the literature studying the benefits and costs of Paycheck Protection Program. The policy spurred many empirical papers attempting to understanding the employment gains from the PPP (Hubbard and Strain (2020), Bartik et al. (2020) Chetty et al. (2023), Autor et al. (2022a), Granja et al. (2022)). Of this literature, my paper is closest to Gertler et al. (2022), who develop a labor search model to understand the employment gains from recall and temporary layoffs under the PPP. The contribution of my paper relative to this paper is to incorporate a government to fund PPP level of transfers in the labor

[6] See the Appendix A of Van Audenrode (1994) to see variations of STW across OECD countries. Other policies similar to Firm Transfers are Short-Time Employment Aid (STEA).

market and show the large welfare loss from this policy due to high taxes. To the best of my knowledge, this is the first paper to make welfare and general equilibrium statements about the Paycheck Protection Protection Program.

There is a large literature studying how to insurance workers from labor market risk. Much of that literature has focused on optimal Unemployment Insurance (Landais et al. (2010), Krusell et al. (2010), Nakajima (2012), Chetty and Finkelstein (2013), Mitman and Rabinovich (2015), Jung and Kuester (2015), McKay and Reis (2016), Herkenhoff (2019), Birinci and See (2020), McKay and Reis (2021), Birinci et al. (2021), Chaumont and Shi (2022), Braxton et al. (2023)). The novel features of this paper relative to previous literature is to solve jointly for the optimal levels of Firm Transfers and Unemployment Insurance. My paper is closest to Birinci et al. (2021), who find that optimal policy in the labor market include both Unemployment Insurance and payroll subsidies to firms. More broadly, this paper contributes to the literature studying consumption inequality and optimal welfare in the presence of idiosyncratic income risk as in Davila et al. (2012) and Dyrda and Pedroni (2023). I contribute to this literature by using a model with search frictions that induce interesting tradeoffs with optimal insurance policies not deeply explored with idiosyncratic income risk.

Lastly, this paper contributes to the literature proposing different policies to mitigate the scaring effect of job loss. It is well documented that workers face high cost of job loss, particularly if they become displaced during a recession (Jacobson et al. (2011), Davis and Von Wachter (2011), Couch and Placzek (2010)).[7] The previous literature has focused on retraining subsidies (Chari et al. (2005), Jacobson et al. (2011), Hawkins and Mustre-del Rio (2016), Hyman (2018), Jung and Kuhn (2019), Macaluso (2023), Braxton and Taska (2023)). Other papers have studied a policy called Wage Insurance to help prevent large earnings losses after job loss (Kling (2006), Wandner (2016), and Hyman et al. (2024)). The novel contribution of this paper is to show Firm Transfers can directly prevent workers from entering costly unemployment by preventing layoffs and they can indirectly mitigate earnings losses around job loss by increasing job creation and lowering time spent in unemployment.

Roadmap

The rest of this paper is organized as followed: Section 2 shows the quantitative model, Section 3 discusses calibration and estimation strategies. Section 4 simulates the Paycheck Protection Program, Section 5 shows the macroeconomic implications of Firm Transfers. Section 6 defines welfare and solves for optimal transfers in the labor market and lastly Section

[7]This cost can be larger if they worker becomes displaced through firm level bankruptcy as studied in Graham et al. (2023).

7 concludes.

2 Quantitative Model

To understand the macroeconomic benefits and costs of providing liquidity to firms before they layoff workers, I build a new dynamic quantitative model of a frictional labor market. Search for jobs is directed as in Menzio and Shi (2011). The model features human capital accumulation and depreciation as in Ljungqvist and Sargent (1998).[8] The two key agents in the model are workers and firms - both of which are eligible to receive a public transfer. Firms face idiosyncratic match productivity shocks each period which govern the probability of the firm initiating a layoff. Workers face two types of uninsurbale idiosyncratic risk. The first is idiosyncratic income risk as each period the worker may involuntarily enter unemployment depending on the match productivity shock. The second is risk in the workers stochastic human capital evolution.

2.1 Model environment

Time is discrete and runs forever. There is a unit measure of risk averse workers and a continuum of potential risk neutral entrant firms. Both firms and workers discount the future by $\beta \in (0, 1)$. There are two types of workers - unemployed and employed. Employed workers provide one unit of indivisible labor that the firm converts into output. Jobs are destroyed exogenously with probability δ and endogenously due firm specific layoff decisions. Each period, workers stochastically die with probability ψ and measure ψ are born into unemployment. In addition to firms and workers, there is also a government who provides two transfers in the labor market - Unemployment Insurance and Firm Transfers by collecting taxes on workers earnings. The original version of the model in Menzio and Shi (2011) without a government is *Block Recursive* meaning that the model can be solved without keeping track of the aggregate distribution of agents across states. The model presented in this paper is *Conditionally Block Recursive* as agents take the tax rate that balances the government budget as given when they make search decisions.

Worker Heterogeneity. Workers are heterogeneous in three total dimensions. First, workers can either be unemployed or employed and transition between the two states. If unemployed, the worker consumes unemployment benefit b_u, which expire with probability ζ each period they are unemployed. An unemployed worker with expired benefits consumes

[8] Recent papers that incorporate Ljungqvist and Sargent (1998) style of human capital are Huckfeldt (2022) and Kaiser (2024).

$\nu < 1$ fraction of previous benefits. The consumption for an unemployed worker with expired benefits is νb_u, which represents the non - UI component of UI benefits. Unemployed workers search over a menu of piece-rates, $\omega \in [0,1] \equiv \mathcal{W}$. Piece - rate ω is the fraction of output the worker consumes once matched with a firm. When a worker successfully matches with a firm, they produce output $f(\cdot)$ together, the worker earns wage ω and consumes $\omega f(\cdot)$. In addition to searching for their piece-rate, ω, the worker is also heterogeneous in their human capital level, $h \in [\underline{h}, \overline{h}] \equiv \mathcal{H} \subseteq \mathbb{R}_+$, which stochastically evolves according to Ljungqvist and Sargent (1998). Employed workers and unemployed workers have different processes for their evolution of human capital. Unemployed workers skills' are likely to depreciate at the onset of job loss. With probability π_u, an unemployed worker loses Δ_h amount of human capital. The stochastic human capital process for an unemployed worker is

$$h' = \begin{cases} h & \text{w/ probability } (1 - \pi_u) \\ h - \Delta_h & \text{w/ probability } \pi_u \end{cases}$$

With probability π_w, an employed worker gains Δ_h amount of human capital and with probability $(1 - \pi_w)$, the employed worker level of human capital does not change. The evolution of human capital for an employed worker is:

$$h' = \begin{cases} h & \text{w/ probability } (1 - \pi_w) \\ h + \Delta_h & \text{w/ probability } \pi_w \end{cases}$$

Define H_e as the markov transition matrix of employed worker human capital such that the law of motion of human capital for an employed worker is $h' \sim H_e(h'|h)$. Similarly for an unemployed worker, define H_u as the markov transition matrix of unemployed worker human capital such that $h' \sim H_u(h'|h)$.

All worker types are subject to an exogenous death shock each period with probability ψ. Measure ψ of workers enter the labor market each period unemployed and draw from the initial human capital distribution $h \sim H_u(h)$. Once matched with a firm, all firm-worker matches become hetergenous in both human capital, h, and specific match productivity, z.

Firms. Each period, firms are subject to match productivity shocks z. The conditional cumulative density function over the productivity distribution is $F(z|z_{-1})$. Incumbent firms draw their new productivity value from the conditional distribution, $F(z|z_{-1})$ and newly formed firms draw productivity from the ergodic distribution of z, denoted as $F(z)$.

Once matched with a worker, the firm produces according to their production function, $f(z,h) = zh$. Recall that the worker earns a constant piece rate, ω, of production over the duration of the match. The firm keeps $(1-\omega)$ fraction of the total output. In the spirit of Hopenhayn (1992), each period, existing firms must also pay a fixed cost of operating, c_f, which is critical to generate endogenous separations in the model. This is a modeling technique also used in Schaal (2017), who uses a directed search model to understand time varying idiosyncratic uncertainty and implications for the labor market. This fixed cost can be thought of as fixed resources the firm needs to use each period to produce (i.e. rent or machinery). The firms total flow profit each period is $(1-\omega)f(z,h) - c_f$ where the amount that the flow revenue adjusts to productivity shocks, $(1-\omega)$, is fixed for the duration of the match.

Due to the nature of directed search of the model, firms and workers search and match in segmented submarkets. Each submarket is defined by a piece rate ω and level of human capital of worker, h. Firms enter competitively in each submarket and all entering firms pay the same cost κ to post a vacancy in a specific submarket. In accordance with free entry, firms will enter a specific submarket until their expected profits from matching with a worker are zero. Free entry determines the endogenous measure of firms entering each submarket. For example, the mass of firms entering a specific (ω, h) submarket and amount of vacancies depends on the worker's level of human capital. Given a piece-rate ω, a low human capital worker does not provide as much value to the firm as a high human capital worker, thus less firms will enter a submarket for a low human capital worker. Lastly, firms treat each job independently and thus a match consists of only one worker and one firm.

Firms do not know their match productivity when posting vacancies and learn this upon matching with a worker and enter submarkets in expectation of their idiosyncratic productivity level. This captures the uncertainty that firms face when they make big decisions over hiring and investment. In the model, firms cannot commit to paying their worker if the realized value of the idiosyncratic shock z makes the net present value of the firm today negative. The layoff rule that firms follow is an endogenous object, denoted as $\underline{z}(\omega, h)$. The layoff rule is an indicator function that represents for a given combination of (ω, z, h), the discounted present value of the match to the firm is negative. A more formal definition of the layoff rule is defined in the firm value function section.

Layoffs and Separations. Employed workers exogenously separate from their current employer with probability $\delta \in (0, 1)$. In equilibrium, fraction δ of matches dissolve. In this case, the firm gets 0 and the worker enters unemployment. The second way that firm and workers separate is endogenous. Given the realized productivity level, firms make layoff decisions. If the idiosyncratic productivity makes the present discounted value of employing the worker to the firm fall below 0, then the firm initiates a layoff. This occurs when the newly drawn match productivity $z < \underline{z}(\omega, h)$. In this case, the firm exits and receives 0 and the worker enters unemployment. I discuss how some layoff decisions can be inefficient and involuntary in the eyes of the worker in Section 2.4.

Labor Market. Search by entering firms and unemployed workers is directed across segmented submarkets that are specific to the worker and firm characteristics. Given their human capital level, h, unemployed workers direct their search across submarkets for a piece-rate, ω, that is fixed for the duration of the firm-worker match. Thus the initial wage contract is defined by (ω, h) and together they define a submarket for all $\mathcal{H}x\mathcal{W}$ combinations. Let $u(\omega, h)$ denote the number of unemployed workers searching in a specific submarket and similarly $v(\omega, h)$ denote the number of vacancies posted by firms. The market tightness, in each submarket is the ratio of the number of vacancies that firms will post to the number of workers searching in that submarket, $\theta(\omega, h) = \frac{v(\omega,h)}{u(\omega,h)}$. A constant returns to scale matching function, $M(u(\omega, h), v(\omega, h))$, governs how many matches will be formed in each (ω, h) submarket. The probability a worker matches with a firm in a given submarket is $p(\theta(\omega, h)) = \frac{M(u(\omega,h),v(\omega,h))}{u(\omega,h)}$ and the probability a firm matches with a worker is given by $q(\theta(\omega, h)) = \frac{M(u(\omega,h),v(\omega,h))}{v(\omega,h)}$. In equilibrium, the market tightness $\theta(\omega, h)$ determines the endogenous entry of firms into each submarket making $p(\theta(\omega, h))$ and $q(\theta(\omega, h))$ exogenous functions of $\theta(\omega, h)$. Thus, the exogenously given matching function, $M(u(\omega, h), v(\omega, h))$, gives the number of firm-worker matches that will occur in that submarket in equilibrium. Furthermore, as proved formally in Appendix 7.6.1, because the tightness function, $\theta(\omega, h)$, is independent of the distribution of workers across states, the equilibrium of this model is *Conditionally Block Recursive* as in Menzio and Shi (2011).

All matches are initially formed in expectation over the match productivity draw. Thus, submarkets and market tightness are not conditional on the idiosyncratic productivity of the match z. This is due to a timing assumption, that firms and workers only realize their match productivity after the match has been formed. This assumption can be easily relaxed.

Government Policy and Transfers in the Labor Market. The government provides transfers to unemployed workers and firms by levying taxes on workers earnings. More formally, a government policy consists of three objects. The first object is proportional tax rate, τ, that government levies on employed workers earnings to fund the transfers in the labor market. The second object is the amount of transfers given to unemployed workers, b_u. This benefit expires with probability ζ to represent the non UI component of unemployed transfers.

The third object that formally defines a government policy are Firm Transfers, denoted as $b_f(\omega, z, h; \alpha, z_g)$. The Firm Transfer is a function of the current total earnings of the worker, which are a function of the piece-rate, ω, the human capital level of the worker, h, and the idiosyncratic productivity of firm, z. This transfer is multidimensional object and can be written as a piecewise function:

$$b_f(\omega, z, h; \alpha, z_g) = \begin{cases} \alpha \underbrace{\omega z h}_{\text{worker earnings}} & \text{if} \quad \underline{z}(\omega, h; \alpha, z_g) \leq z \leq z_g \\ 0 & o.w. \end{cases} \tag{1}$$

The government can vary the firm transfer on two dimensions i) $\alpha \in [0, 1]$ governs the amount of payroll subsidized ii) $z_g \in [\underline{z}, \overline{z}]$, governs which types of firms are eligible to receive the transfer. When $\alpha = 1$ the government is subsidizing 100% of the current payroll costs of the firm. When $z_g = \overline{z}$, the government is subsidizing all firms in the economy.

Firm Transfers are contingent on not exiting and thus not laying off the worker. While some firms may be eligible to receive $b_f(\omega, z, h; \alpha, z_g)$, the amount provided may insufficient to make the firm value positive and thus not enough to incentivize the firm to maintain current worker. Thus, the firm will exit the economy and not accept the transfer, which is not counted in the government budget. This is represented by the lower bound on the types of firms who receive the transfer in Equation 1. This new layoff rule is weakly less than the layoff rule without this policy, $\underline{z}(\omega, h; \alpha, z_g) \leq \underline{z}(\omega, h; 0, \emptyset)$[9] implying that less firm-worker matches will be endogenously destroyed.

For simplicity, I denote $g_{b_f} = (\alpha, z_g)$ to represent the combination of the government tools used to vary the firm transfer. Denote $g_{b_f} = (\emptyset)$ when there is no government intervention to firms. Define a government policy:

$$\{b_u, g_{b_f}, \tau\}$$

The government balances the budget each period so that the total amount of revenue collected from taxes equals the outlays of the two types of transfers in the labor market. The formal government budget is given in Section 2.5.

[9] The $\emptyset$ notation represents the world without Firm Transfers. Government does not have access to z_g or α.

Timing. The timing of one period is divided into multiple sub-periods.

1. Measure ψ of workers die and are replaced by new unemployed workers
2. Unemployed workers realize their stochastic benefit expiration with probability ζ
3. Workers draw their new human capital level, h
4. Firms and workers face the exogenous separation shock, δ
5. Entering firms post vacancies in submbarkets (ω, h)
6. Unemployed workers and entering firms search and match given tightness $\theta(\omega, h)$
7. Firms draw their productivity level
   - If worker is coming from unemployment, draw from the unconditional distribution of productivity, $\tilde{z} \sim F(\tilde{z})$
   - If already matched with worker, draw from conditional distribution of productivity, $z \sim F(z|z_{-1})$
8. Eligible firms - $z \leq z_g$ receive Firm Transfer, $b_f(\omega, z, h; \alpha, z_g)$
   - Lower bound on firms who receive Firm Transfers defined, $\underline{z}(\omega, h; g_{b_f})$:
     - If $\tilde{z} < \underline{z}(\omega, h; g_{b_f})$ newly matched firm makes layoff decision and exit - firm receives 0 and worker enters unemployment
     - If $z < \underline{z}(\omega, h; g_{b_f})$ exiting matched firm makes layoff decision and exit - firm receives 0 and worker enters unemployment
   - If $\tilde{z} \geq \underline{z}(\omega, h; g_{b_f})$ or $z \geq \underline{z}(\omega, h; g_{b_f})$, firms and workers remain together
9. Production and consumption

2.2 Firm and Worker Problems

This section explains the Bellman Equations of the workers and firms. The value functions below are described at the end of one period, when production and consumption is taking place in subperiod 9. Unemployed workers have realized their benefit expiration and employed workers and firms have realized their separation shocks.

2.2.1 Firm Problem

The firm pays fixed cost of production, c_f and produces $f(z,h)$ with their worker today, keeping $(1-\omega)$ fraction of output. The firm discounts their future by $(1-\psi)\beta$ taking into account the probability their current employee will experience the stochastic death shock tomorrow. Tomorrow with probability δ, the firm and the worker separate exogenously. With probability $(1-\delta)$ the firm and worker remain together. Next the firm draws a new match productivity, $z' \sim F(z'|z)$, which can lead to a layoff decision for the firm if $z' < \underline{z}(\omega,h)$. In this event, the productivity shock causes the present discounted value to the firm to fall below 0. The firm decides to exit the economy and receive 0. If the firm draws $z' \geq \underline{z}(\omega,h)$, they remain with the worker and produce.

$$J_{\text{stay}}(\omega,z,h;g_{b_f}) = (1-\omega)f(z,h) - c_f + b_f(\omega,z,h;g_{b_f}) + (1-\psi)\beta(1-\delta)\mathbb{E}_{h'|h,z'|z}V(\omega,z',h';g_{b_f}) \tag{2}$$

$$V(\omega,z',h';g_{b_f}) = \max\{J_{\text{stay}}(\omega,z',h';g_{b_f}), J_{\text{exit}}\}$$
$$J_{\text{exit}} = 0$$

$$\underline{z}(\omega,h';g_{b_f}) \text{ solves } V(\omega,\underline{z}(\omega,h';g_{b_f}),h';g_{b_f}) = 0 \tag{3}$$

The first part of 2 represents the flow revenue to the firm. The second part of the firm value function represents the expected continuation value. The firm keeps track of the employed workers human capital level which evolves next period according to $h' \sim H_e(h'|h)$. The firm then takes a draw from the productivity distribution, $z' \sim F(z'|z)$. There are four total cases with what happens next. The first is that firm draws $z' > z_g$ and $z' < \underline{z}(\omega,h';g_{b_f})$, making them ineligible to receive a Firm Transfer and they exit the economy and lay off their worker. The second case is that firm draws $z' > z_g$ and $z' \geq \underline{z}(\omega,h';g_{b_f})$. In this case, the firm is not eligible to receive a Firm Transfer and since they draw high enough productivity above the layoff threshold, they remain with their worker. The third case is the firm realizes $z' \leq z_g$ and $z' < \underline{z}(\omega,h;g_{b_f})$, making the firm eligible to receive a Firm Transfer but not enough to make the firm value positive. In this case, the firm does not accept the transfer, exits the economy, and lays off their worker. In the last case, the firm draws $z' \leq z_g$ and $z' \geq \underline{z}(\omega,h;g_{b_f})$. In this case, the firm is eligible to receive a Firm Transfer and the amount is enough to make the firm value positive, thus incentivizing the firm to remain with their current worker. The endogenous layoff threshold in 3 changes depending on the generosity of the Firm Transfer policy. A high amount of transfers to firms prevents firm exits in the economy. The equilibrium layoff rule in 3 is increasing in the piece-rate of the worker, holding the human capital level fixed. A worker with high a piece-rate leaves the firm with a low flow production value, subjecting the firm to an increase in endogenous layoff risk. This is represented by the max operator in the firm's continuation value.

2.2.2 Unemployed Worker Problem

In this section, I show the bellman equations for unemployed workers with benefits and with expired benefits. The unemployed workers wage search depends on their benefit expiration status. Both value functions are shown below.

Benefits An unemployed worker with benefits has flow consumption b_u. At the beginning of next period, they face a stochastic death shock with probability ψ. Next, they draw a new level of human capital according to the law of motion, $h' \sim H_u(h'|h)$ before entering the search stage. As shown in the second part of Equation 4 unemployed workers enter the labor market and direct their search over piece-rates, ω', which represents a fixed per-period fraction of output they will produce if matched with a firm. With probability $(1-p(\theta(\omega', h'; g_{b_f})))$ they do not match with a firm and remain unemployed with their new human capital level, h'. The unemployed worker whose benefits have not expired now faces the stochastic benefit expiration shock. With probability ζ the unemployed worker's benefit expire and consume expired benefits for the remainder of their unemployment spell.

$$U(h, b_u) = u(b_u) + (1-\psi)\beta\mathbb{E}_{h'|h}\left[\max_{\omega'}(1-p(\theta(\omega', h'; g_{b_f})))\hat{U}(h', b_u) + p(\theta(\omega', h'; g_{b_f}))\tilde{W}(\omega', \tilde{z}, h')\right] \tag{4}$$

$$\hat{U}(h', b_u) = \zeta U(h', \nu b_u) + (1-\zeta)U(h', b_u) \tag{5}$$

$$\tilde{W}(\omega', \tilde{z}, h') = \underbrace{\int_{\underline{z}(\omega', h'; g_{b_f})}^{\infty} W(\omega', \tilde{z}, h')dF(\tilde{z})}_{\text{Keep Job}} + \underbrace{\int_{-\infty}^{\underline{z}(\omega', h'; g_{b_f})} \hat{U}(h', b_u)dF(\tilde{z})}_{\text{Enter Unemployment}} \tag{6}$$

With probability $p(\theta(\omega, h; g_{b_f}))$ the unemployed worker successfully matches with a firm and now faces the endogenous separation shock due to the initial draw of the match productivity shock. Newly formed firms draw from the ergodic distribution of the productivity distribution, $\tilde{z} \sim F(\tilde{z})$. The expected value of the unemployed worker remaining with this firm is given by the expected value of working, $\tilde{W}(\omega', \tilde{z}, h')$. The expected value of working is show in Equation 6 which is divided into two parts. The first part represents the total expected value of the employed worker if the firm draws a high enough match productivity value, $\tilde{z} \geq \underline{z}(\omega', h'; g_{b_f})$. Survive the initial productivity shock, they consume earnings $\omega f(z, h)$ and have match value $W(\omega', \tilde{z}, h')$. The second part represents the total expected value of leaving the firm which occurs if the firm draws an initial match productivity value such that $\tilde{z} < \underline{z}(\omega', h'; g_{b_f})$. The worker goes back into unemployment and faces the stochastic benefit expiration shock.

The policy decision resulting from solving 4 for an unemployed worker with benefits with human capital level, h is denoted as $\omega_u^{'}(h, 1; g_{b_f})$. The human capital level of the unemployed worker influences their probability of matching. For example for a given piece-rate, consider a high human capital worker. Since high human capital workers are more attractive to the firm, firms will post many vacancies for these high human capital jobs. This increases the probability the worker will match with a high human capital firm.

Expired Benefits Unemployed workers realize stochastic benefit expiration with probability ζ each period over the duration of their unemployment spell. An unemployed worker with expired benefits has flow consumption, νb_u. This represents the non-UI component of UI benefits. The value function for an unemployed worker with expired benefits is given in 7. Similar to an unemployed worker with benefits, they face a stochastic death shock next period with probability ψ and draw new human capital level according to the law of motion, $h' \sim H_u(h'|h)$.

$$U(h, \nu b_u) = u(\nu b_u) + (1-\psi)\beta \mathbb{E}_{h'|h}\left[\max_{\omega'} (1 - p(\theta(\omega', h'; g_{b_f})))U(h', \nu b_u) + p(\theta(\omega', h'; g_{b_f}))\tilde{W}(\omega', \tilde{z}, h') \right] \tag{7}$$

$$\tilde{W}(\omega', \tilde{z}, h') = \underbrace{\int_{\underline{z}(\omega', h'; g_{b_f})}^{\infty} W(\omega', \tilde{z}, h') dF(\tilde{z})}_{\text{Keep Job}} + \underbrace{\int_{-\infty}^{\underline{z}(\omega', h'; g_{b_f})} U(h', \nu b_u) dF(\tilde{z})}_{\text{Enter Unemployment}} \tag{8}$$

With probability $(1 - p(\theta(\omega, h; g_{b_f})))$ the unemployed worker does not match with a firm and remained unemployed with expired benefits. With probability $p(\theta(\omega, h; g_{b_f}))$ the unemployed worker matches with a firm and face the endogenous layoff decision based on the firms initial draw of productivity. Their continuation value is similar to the unemployed worker with benefits, except they are no longer subject to the stochastic benefit expiration shock, which is shown in the second part of Equation 8.

Unemployed workers with benefits and with expired benefits have different wage search behaviors. For a given level of human capital, an unemployed worker with expired benefits will apply to lower wage jobs with higher probability of matching. This is due to the fact that consumption for unemployed workers with expired benefits is lower than with benefits. They prefer to become employed faster. Define the unemployed workers with expired benefit optimal search decision as $\omega_u^{'}(h, 0; g_{b_f})$.

2.2.3 Employed Worker Problem

This section describes the employed worker bellman equation. The value for an employed worker is given in Equation 9. An employed worker with current human capital level h earns $\omega f(z, h)$ when they produce with a firm. Employed workers pay proportional tax τ on their earnings to fund the two types of transfers in the labor market. Thus, an employed worker has flow consumption $(1-\tau)\omega f(z, h)$.

$$\begin{aligned} W(\omega, z, h) &= u((1-\tau)\omega f(z, h)) \\ &+ (1-\psi)\beta \mathbb{E}_{h'|h}\left[\delta \hat{U}(h', b_u) + (1-\delta)\int_{\underline{z}(\omega, h'; g_{b_f})}^{\infty} W(\omega, z', h')dF(z'|z) + \int_{-\infty}^{\underline{z}(\omega, h'; g_{b_f})} \hat{U}(h', b_u)dF(z'|z)\right] \end{aligned} \tag{9}$$

At the beginning of the next period, employed workers die with probability ψ. If the worker survives the stochastic death shock, then they draw their new human capital level $h' \sim H_e(h'|h)$. With probability δ the worker experiences an exogenous separation shock and enters unemployment where they immediately face a risk of their benefits expiring as show in Equation 5. With probability $(1-\delta)$ they do not separate from their current firm and now face endogenous layoff risk. The firm draws new match productivity, $z' \sim F(z'|z)$. If the firm draws productivity $z' \geq \underline{z}(\omega, h'; g_{b_f})$, then the worker keeps their job today and continues on with value $W(\omega, z', h')$. If the firm draws a productivity value that is strictly below the layoff threshold, $z' < \underline{z}(\omega, h'; g_{b_f})$, then the firm initiates the separation and the employed worker enters unemployment. The worker then immediately faces the stochastic benefit expiration shock, which is given in Equation 5.

2.3 Free entry and market tightness

Submarkets specify a piece-rate, ω, and human capital level of worker h. Given the tightness in submarket (ω, h), the probability the firm matches with the worker is given by $q(\theta(\omega, h; g_{b_f}))$. The equilibrium mass of firms in each submarket (ω, h) is endogenously determined through competitive free entry. All entering firms pay a fixed cost κ to enter into a submarket. The firm does not know its idiosyncratic value when posting the vacancies, so the free entry condition is taken over expectation of the stationary distribution of productivity, $\tilde{z} \sim F(\tilde{z})$. Formally define the value of a vacancy to a firm, $V(\omega, h; g_{b_f})$

$$V(\omega, h; g_{b_f}) = -\kappa + q(\theta(\omega, h; g_{b_f}))\mathbb{E}_{\tilde{z}}[J(\omega, \tilde{z}, h; g_{b_f})]$$

Firms maximize profits in expectation around their future idiosyncratic match productivity. They decide how many vacancies to post in each submarket by facing a tradeoff: posting a high ω job means firms face lower market tightness and have a higher probability of meeting a worker and thus filling the vacancy. However, this means they will be extracting less of the surplus and make less expected profits.

With free entry, the expected value of a vacancy $V(\omega, h; g_{b_f}) = 0$ as the expected profits from the vacancy are enough to cover the cost of posting a vacancy. Then, if expected value net of vacancy cost of entering a submarket yields profits, $\mathbb{E}_{\tilde{z}}[J(\omega, \tilde{z}, h; g_{b_f})q(\theta(\omega, h; g_{b_f}))] - \kappa \geq 0$, then a firm will post a vacancy in that submarket. When this condition is met, $\theta(\omega, h; g_{b_f}) > 0$. Otherwise if $\mathbb{E}_{\tilde{z}}[J(w, \tilde{z}, h; g_{b_f})q(\theta(\omega, h; g_{b_f}))] - \kappa < 0$ then no new firms will enter and that submarket will be closed in equilibrium, implying that the tightness will be 0. Thus in equilibrium, it must hold that:

$$q(\theta(\omega, h; g_{b_f}))\mathbb{E}_{\tilde{z}}[J(\omega, \tilde{z}, h; g_{b_f})] - \kappa \leq 0, \quad \theta(\omega, h; g_{b_f}) \geq 0, \forall(\omega, h) \tag{10}$$

The two above inequalities hold with complementary slackness. Then a formal definition of market tightness is

$$\theta(\omega, h; g_{b_f}) = \begin{cases} 0 & otherwise \\ q^{-1}\left(\frac{\kappa}{\beta\mathbb{E}_{\tilde{z}}[J(\omega, \tilde{z}, h; g_{b_f})]}\right) & \beta \cdot \mathbb{E}_{\tilde{z}}[J(\omega, \tilde{z}, h; g_{b_f})] - \kappa \geq 0 \end{cases} \tag{11}$$

To understand how $\theta(\cdot), q(\cdot), p(\cdot)$ work together, consider a submarket with a high job finding rate, $p(\theta(\omega, h; g_{b_f}))$. Then this submarket must also have a relatively high market tightness, meaning there are a lot of vacancies posted by firms relative to number of unemployed individuals searching with human capital level, h. There would be a lot of vacancies because the expected value of the job filled to the firm yields a high expected firm value, and hence must have a low piece-rate offer. Futhermore, because there are relatively fewer applicants for one job posting, the probability of the firm matching to the worker, $q(\theta(\omega, h; g_{b_f}))$, is low, compared to the high probability of the applicant matching for the low piece-rate job, $p(\theta(\omega, h; g_{b_f}))$.

The value of posting a vacancy also depends on the probability of separation given the piece-rate ω and human capital level h. Holding the piece-rate fixed, higher human capital jobs are less likely to be destroyed due to the low productivity shock. This increases the value of the match to the firm and thus an entering firm will post many vacancies for this (ω, h) worker, leading to a high match rate for high human capital workers. Now holding human capital fixed, higher piece-rate matches are subjected more to the endogenous layoff shock compared to lower piece-rate matches. This is because a high piece-rate worker earns a lot of the flow profits from the firm, thus leaving the firm with lower profits. Relatively, this increase the value of posting vacancies for lower piece-rate matches.

2.4 Inefficient Layoffs

To illustrate the firm's layoff decisions in relation to the value of the workers, I plot the values of the firm, employed and unemployed worker in Figure 1. Some layoff decisions made by firms are inefficient and involuntary in the eyes of the worker. This can be seen by the "Inefficient Layoff" region depicted in Figure 1. This happens due rigidity in the wage contract. In Menzio and Shi (2011), firms internalize the value of the worker and update the wage contract each period according to the idiosyncratic productivity shock. In this case, wages are fully flexible and the surplus is unchanged as a result. Thus, all layoff or separation decisions are socially efficient in Menzio and Shi (2011). The key deviation I take from Menzio and Shi (2011) is to incorporate rigidity in the wage contract. The total earnings of the worker can adjust to the idiosyncratic productivity shock, but it is not fully flexible as the amount that earnings adjust is fixed per period by ω. Thus, the endogenous layoff rule firms follow, $\underline{z}(\omega, h; g_{b_f})$, is not socially efficient, meaning that firms and workers optimally want to leave the match at different times. Furthermore, the rigidity causes the firm to destroy potentially positive surplus matches due to the high value of the employed worker.

The figure represents how some layoff decisions are inefficient. To the left of the first horizontal dotted line is the productivity region where both the firm and worker would like to separate. The firm wants to leave because their net present value of the match is negative. The employed worker wants to leave because the value of unemployment, as show as the gray line, is weakly higher than being employed, the blue line. The worker would be better

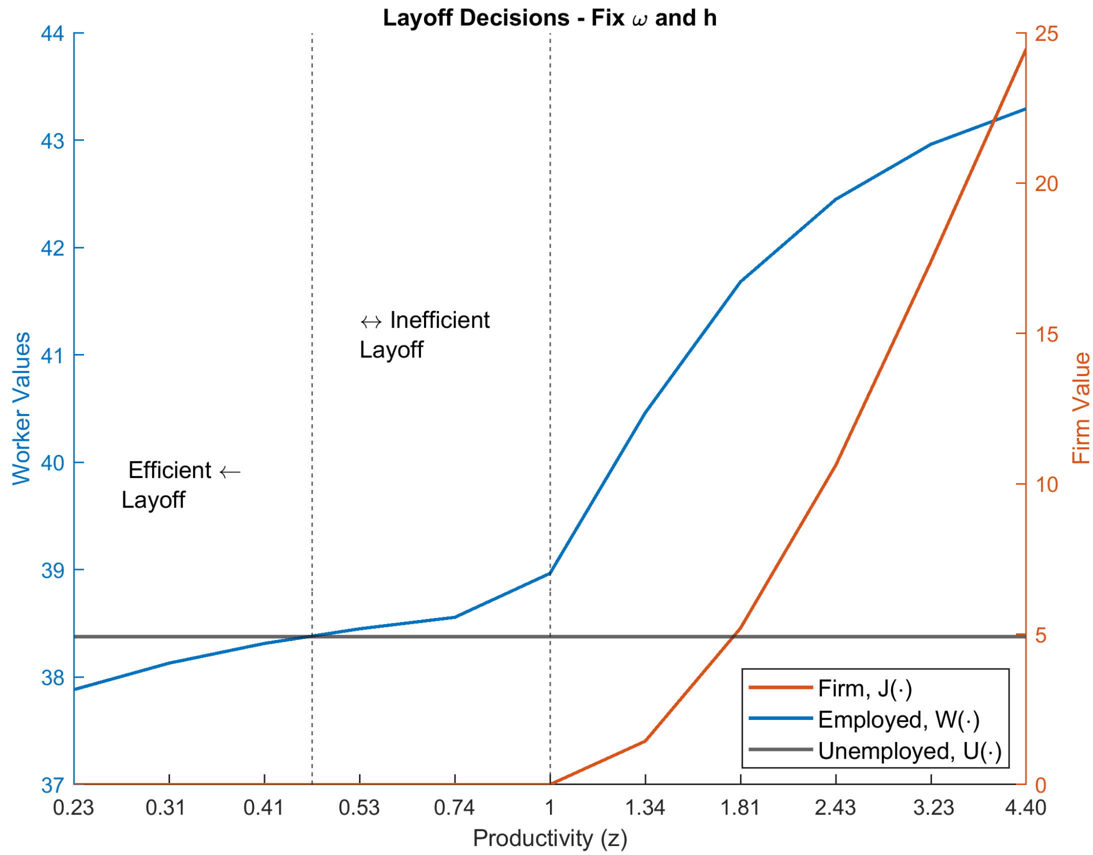


Figure 1: Firm Layoff Decisions

off entering unemployment and getting a chance to draw again from the match productivity distribution. This highlights the value of searching for a new job. This type of layoff decisions is characterized as an efficient layoff.

The right hand side of the figure shows the inefficient layoff region. To the right of the first horizontal line is the region where the firm is strictly better off laying off the worker and exiting the economy but the worker is better off remaining in the match. This can be seen from the value of remaining employed is weakly greater than the value of unemployment. This is characterized as an inefficient layoff. This layoff decisions induces the endogenous layoff rule, $\underline{z}(\omega, h; g_{b_f})$ that is influenced by the firm transfer policy, $b_f(\omega, z, h; g_{b_f})$. Lastly, to the right of the second horizontal dotted line is the productivity region where both the firm and worker want to stay. The value of the job filled to the firm is strictly positive and the value of employment is higher than entering unemployment. In this region of the state space, neither the firm nor worker want to leave.

2.5 Government Budget

The government must balance the budget each period to fund the two types of labor market transfers, one given to unemployed workers, b_u and νb_u, and one given to firms, $b_f(\omega, z, h; g_{b_f})$. Equation 12 shows that the goverment must balance the budget by equalizing taxes collected on workers earnings to the amount spent on the two types of labor market insurance programs. Let $\Phi(\omega, z, h)$ denote the resulting equilibrium distribution of jobs or firm-worker matches for each (ω, z, h) pair. Let u_b denote the measure of unemployed workers that have benefits and u_{nb} represent the measure of unemployed workers that have expired benefits.[10]

$$\underbrace{\int b_f(\omega, z, h; \alpha, z_g) d\Phi(\omega, z, h)}_{\text{transfer to firm}} + \underbrace{b_u u_b + \nu b_u u_{nb}}_{\text{transfers to unemployed}} = \underbrace{\tau \int \omega z h d\Phi(\omega, z, h)}_{\text{tax revenue}} \tag{12}$$

Then right hand side of the government budget in Equation 12 shows that total amount of revenue that the government collects from taxes on individuals earnings. The left hand side of 12 is split into two parts. The first part shows the total amount spent on the firm transfer policy. Recall that the government only finances transfers to firm if the benefit is amount to make the firm want to stay in the match. The second part of the right hand side represents the total amount spent on transfers to unemployed workers. The two transfers given to unemployed workers, b_u and νb_u are weighted by the measures of unemployed workers in each state, u_b and u_{nb}. In total, the right hand side of Equation 12 is the total amount spent on the two types of labor market insurance programs.

The left hand side of the government budget represents the total amount of taxes to balance the budget given the government policy. The government levies proportional tax, τ on all workers earnings, which is represented by the integral across different firm-worker matches. Each period, the government must balance the budget.

2.6 Dynamics of Distributions for Firms and Unemployed Workers

In this section, I show the law of motion for the distributions for agents across states. There are three laws of motion to keep track of - the distribution for firm-worker matches, $\Phi(\cdot)$, the distribution of unemployed workers with benefits, $\Upsilon(h, 1)$, and the distribution of unemployed worked with expired benefits, $\Upsilon(h, 0)$. Recall that the cumulative conditional productivity distribution is denoted as $F(z|z_{-1})$ and the initial productivity distribution is $F(z)$ and has a

[10] $u_b + u_{nb} = u$, where u_{nb} denotes the measure of unemployed workers with benefits and u is the total measure of unemployed workers.

pdf, $f(z)$. Recall that the human capital distribution for an unemployed worker is given by $H_u(h|h_{-1})$ and newly born workers draw from the initial human capital distribution, $H_u(h)$. Employed workers draw human capital from $H_e(h|h_{-1})$. The laws of motions are shown at the end of one period, where production and consumption takes place and all layoffs have been realized.

Equation 13 shows the law of motion for the distribution of firm-worker matches.

$$\begin{aligned}
\Phi'(\hat{\omega},\hat{z},\hat{h}) &= T_1^*(\Phi,\Upsilon,\underline{z}(\cdot),\omega'(\cdot))(\hat{\omega},\hat{z},\hat{h}) \\
&= \underbrace{(1-\delta)(1-\psi)\int\int \Phi(\hat{\omega},z,h)F(\hat{z}|z)H_e(\hat{h}|h)\mathbb{1}\{\hat{z}\geq \underline{z}(\hat{\omega},\hat{h};g_{b_f})\}dzdh}_{\text{Employed workers}} \\
&+ \underbrace{(1-\psi)f(\hat{z})\int \Upsilon(h,1)H_u(\hat{h}|h)\mathbb{1}\{\omega'(\hat{h},1;g_{b_f})=\hat{\omega}\}p(\theta(\hat{\omega},\hat{h};g_{b_f}))dh}_{\text{Hired unemployed workers with unexpired benefits}} \\
&+ \underbrace{(1-\psi)f(\hat{z})\int \Upsilon(h,0)H_u(\hat{h}|h)\mathbb{1}\{\omega'(\hat{h},0;g_{b_f})=\hat{\omega}\}p(\theta(\hat{\omega},\hat{h};g_{b_f}))dh}_{\text{Hired unemployed workers with expired benefits}} \qquad (13)
\end{aligned}$$

Equation 14 gives the law of motion for an unemployed worker with benefits.

$$\begin{aligned}
\Upsilon'(\hat{h},1) &= T_2^*(\Phi,\Upsilon,\underline{z}(\cdot),\omega'(\cdot))(\hat{h},1) \\
&= \underbrace{(1-\zeta)\delta\int\int\int \Phi(\omega,z,h)H_e(\hat{h}|h)d\omega dzdh}_{\text{Exogenously fired employed workers}} \\
&+ \underbrace{(1-\zeta)(1-\delta)(1-\psi)\int\int\int \Phi(\omega,z,h)F(\hat{z}|z)H_e(\hat{h}|h)\mathbb{1}\{\hat{z}<\underline{z}(\omega,\hat{h};g_{b_f})\}d\omega dzdh}_{\text{Endogenously laid off employed workers}} \\
&+ \underbrace{(1-\zeta)(1-\psi)\int \Upsilon(h,1)H_u(\hat{h}|h)[1-p(\theta(\omega'(\hat{h},1;g_{b_f}),\hat{h}))]dh}_{\text{Unemployed workers who searched and didn't find a job}} \\
&+ \underbrace{(1-\zeta)(1-\psi)\int\int \Upsilon(h,1)H_u(\hat{h}|h)p(\theta(\omega'(\hat{h},1;g_{b_f}),\hat{h}))F(\hat{z})\mathbb{1}\{\hat{z}<\underline{z}(\omega',\hat{h};g_{b_f})\}dhdz}_{\text{Unemployed workers searched and matched but endogenously laid off}} \\
&+ \underbrace{(1-\zeta)\psi H_u(\hat{h})}_{\text{New borns with start life with benefits}} \qquad (14)
\end{aligned}$$

Equation 15 gives the law motion for an unemployed worker with expired benefits.

$$
\begin{aligned}
\Upsilon'(\hat{h},0) &= T_3^*(\Phi,\Upsilon,\underline{z}(\cdot),\omega'(\cdot))(\hat{h},1) \\
&= \underbrace{\zeta\delta(1-\psi)\int\int\int \Phi(\omega,z,h)H_e(\hat{h}|h)d\omega dz dh}_{\text{Exogenously fired employed workers}} \\
&+ \underbrace{\zeta(1-\delta)(1-\psi)\int\int\int \Phi(\omega,z,h)F(\hat{z}|z)H_e(\hat{h}|h)\mathbb{1}\{\hat{z}<\underline{z}(\omega,\hat{h};g_{b_f})\}d\omega dz dh}_{\text{Endogenously fired employed workers}} \\
&+ \underbrace{\zeta(1-\psi)\int \Upsilon(h,1)H_u(\hat{h}|h)[1-p(\theta(\omega'(\hat{h},1;g_{b_f}),\hat{h}))]dh}_{\text{Unemployed workers with benefits who searched and didn't find a job and lost their benefits}} \\
&+ \underbrace{(1-\psi)\int \Upsilon(h,0)H_u(\hat{h}|h)[1-p(\theta(\omega'(\hat{h},1;g_{b_f}),\hat{h}))]dh}_{\text{Unemployed workers without benefits who searched and didn't find a job}} \\
&+ \underbrace{(1-\psi)\int\int \Upsilon(h,0)H_u(\hat{h}|h)p(\theta(\omega'(\hat{h},0;g_{b_f}),\hat{h}))F(\hat{z})\mathbb{1}\{\hat{z}<\underline{z}(\omega',\hat{h};g_{b_f})\}dhdz}_{\text{Unemployed workers searched and matched but endogenously laid off}} \\
&+ \underbrace{\zeta\psi H_u(\hat{h})}_{\text{New borns with start life without benefits}} \qquad (15)
\end{aligned}
$$

2.7 Equilibrium Definition and Block Recursivity

Denote $\mu : \{e, z, w, h\} \rightarrow [0, 1]$ as the distribution of agents across $\mathcal{E}x\mathcal{Z}x\mathcal{W}x\mathcal{H}$ space where e denotes employment status: $e = \{U, E\}$. Denote $\Phi(\omega, z, h)$ the equilibrium distribution of firm-worker matches. Denote $\mathcal{T}^* = \{\mathcal{T}^*{}_1, \mathcal{T}^*{}_2, \mathcal{T}^*{}_3\}$ as the law of motion for μ.

A *Recursive Competitive Equilibrium* in this economy is a set of individual policy functions for unemployed wage search with and without benefits, $\{\omega_u'(h, 1), \omega_u'(h, 0)\}$, layoff rule $\{\underline{z}(\omega, h; g_{b_f})\}$, market tightness function $\{\theta(\omega, h; g_{b_f})\}$, value functions, $\{W(\cdot), U(\cdot), J(\cdot)\}$, government policy, $\{b_u, g_{b_f}, \tau\}$ and $\mathcal{T}^*$ the transition of the aggregate state μ that satisfy:

1. Unemployed and employed wage search are optimal and yield policy functions $\{\omega_u'(h, 1), \omega_u'(h, 0)\}$
2. Market tightness function satisfies the free entry condition in all submarkets and satisfies Equation 11
3. The tax rate τ balance the government budget in Equation 12.

4. $\mathcal{T}^*$ is consistent with policy functions $\{\omega_u^{'}(h,1), \omega_u^{'}(h,0)\}$

5. μ is fixed point of $\mathcal{T}^*$ such that $\mu = \mathcal{T}^*(\mu)$

The equilibrium of this model is *Conditionally Block Recursive* for an exogenous τ. A *Block Recursive Equilibrium* defined by Menzio and Shi (2011) implies that the individual agents policy functions are independent of the aggregate distribution of other agents across states. This greatly simplifies computation. A *Conditionally Block Recursive* equilibrium means that for an exogenous guess of the tax rate, τ, the aggregate distribution μ do not affect the prices in the model. A formal proof is provided in Appendix 7.6.1.

3 Calibration

In this section, I describe how I take the model to the data. I use a combination of both external and internal calibration. All parameters used in the model are described in Table 1. The parameters in Table 2 are jointly estimated via Simulated Methods of Moments. The model can match moments in the macroeconomy and the aggregate labor market. Appendix 7.7 gives detailed instructions about solving the model with the government budget.

Preferences and demographics: Time period is set to one quarter. Workers stochastically die each period with probability ψ. I calibrate $\psi = 0.00833$ so that workers have a 30 year working life on average.[11] Newly born individuals enter the labor force as an unemployed worker. Their initial human capital is draw from an exponential distribution with parameter λ_h. I calibrate this parameter to match the dispersion in residualized earnings. Using data from CPS ASEC between 2010 - 2019, I find the 90th-10th percentile of residualized earnings is 2.09.[12] Setting $\lambda_h = 0.5$, I measure this dispersion in residualized earnings as 2.08 running a similar regression in the model simulated data. The grid for human capital is linearly spaced $h \in [1, 25]$ with step size $\Delta_h = 0.352$.[13] The lower and upper bounds on the human capital grid are chosen so that in equilibrium there is not a large mass of agents at either end of the distribution.

Firms are risk neutral and workers are risk averse. Preferences over consumption are given as:

$$u(c) = \frac{c^{(1-\sigma)} - 1}{1 - \sigma}$$

[11] The expected working life is 120 quarters. Then to solve for $\psi = \frac{1}{120}$.

[12] In the data, I run the follow regression on yearly log earnings: $w_t = \alpha + \text{age}_t + \epsilon_t$

[13] $\Delta_h = \frac{h_{ub} - h_{lb}}{nh-1} = 0.352$

I set the risk aversion parameter, $\sigma = 2$, which is standard in the literature. Workers and firms share the same discount factor, β. I set $\beta = 0.99$ to equal a quarterly discounted annualized risk free rate of 4%. Firms and workers produce according to $f(z, h) = zh$.

Labor Market The probability that a worker matches with a firm, $p(\theta)$ and the probability a firm matches with a worker, $q(\theta)$, are determined by the constant returns to scale matching function which takes the functional form:

$$M(u, v) = \frac{u \cdot v}{(u^{\gamma} + v^{\gamma})^{\frac{1}{\gamma}}} \in [0, 1)$$

The matching elasticity, γ, governs the job finding rate of searching workers to market tightness. I set $\gamma = 1.6$ as measured in Schaal (2017).

Employed workers' human capital increases with probability π_w, which is calibrated calibrate to match the yearly returns on earnings from tenure. Running a regression of log yearly earnings on tenure using the CPS ASEC supplement from 2010 - 2020, I find that this return is 2.35% annualized. Setting $\pi_w = 0.2$ in the model gives annual return to tenure of 2.61%. This is line with estimates from Topel (1991), Topel and Ward (1992), and Haltiwanger et al. (2024). Unemployed workers' human capital decreases with probability, π_u, which is calibrated to match the earnings around job loss. Setting $\pi_u = 0.8$, earnings one year after job loss drop by 10.7% relative to pre-displacement earnings. Using earnings from the Displaced Workers Supplement between 2010 - 2019, I find on average earnings around job loss are 10.4% lower after one year of job loss. Firms match productivity follows an AR(1) process

$$\ln(z') = \rho_z \ln(z) + \eta_z', \quad \eta_z \sim \mathcal{N}(0, \sigma_z^2)$$

I discretize the continuous variable according to Tauchen (1986). The two parameters that govern this process are the persistence of the productivity shock, ρ_z, and the shock to the innovation each period, σ_z. I set $\rho_z = .95, \sigma_z = 0.034$ as in Khan and Thomas (2008), Schaal (2017), and Bloom et al. (2018). Discretizing productivity yields a truncated log normal distribution with 11 possible states. Firms pay a fixed cost of production each period, c_f, which is essential for the endogenous separations in the model. Setting $c_f = 1.8$, the fixed cost targets a steady state unemployment rate of 6.23%.

The cost of posting a job vacancy, $\kappa = 1.5$, is set to target the quarterly labor costs of posting a vacancy as measured in Silva and Toledo (2009) and used in Hagedorn and Manovskii (2008). The exogenous job destruction rate separation rate $\delta = 0.02$ targets a quarterly E-U transition rate of 5.75% as measured in the BLS 2010 - 2019. The productivity process,

Table 1: Model Parameters

Variable	Description	Value
Nonestimated		
β	Discount factor (quarterly)	0.99
σ	Risk aversion	2
γ	Matching elasticity	1.6
ρ_z	Persistence in match productivity	0.95
σ_z	Shock to Productivity	0.038
ν	Fraction of unemployed transfer after expiration	0.250
ψ	Stochastic death	0.00833
ζ	Probability UI benefit expiration	0.33
Jointly Estimated		
π_u	Probability of human capital decrease	0.8
π_w	Probability of human capital increase	0.2
c_f	Fixed cost of production	1.8
b_u	Labor market transfer to unemployed	2.2
κ	Labor cost of posting a vacancy	1.5
δ	Probability of exogenous job destruction	0.02
λ_h	Exponential parameter for initial human capital	0.5

the exogenous separation rate and the fixed cost of production all jointly determine the E-U transition rates.

Transfers in the Labor Market. In the baseline version of the model, only unemployed workers are eligible to receive a public transfer, b_u. This transfer is estimated to match the 41.2% replacement rates of unemployed workers - change in unemployed transfers over the change in annual pre-displacement income - as measured in Braxton et al. (2023). As in Mitman and Rabinovich (2015) unemployment benefits expire stochastically with probability ζ. I set $\zeta = 0.33$ to approximate benefits expiring after 26 weeks following Braxton et al. (2023). At the onset of benefit expiration, unemployed workers consume ν fraction of their previous benefit amount. I set $\nu = 0.250$ to approximate the drop in non-durable spending at the onset of UI benefit expiration as measured in Braxton et al. (2023) and Ganong and Noel (2019).

Table 1 shows all parameters in the model and their values.

Table 2 compares the simulated moments from the model to the data.

Table 2: Jointly Estimated Parameters

Parameter	Value	Moment	Data	Model	Source
π_u	0.8	Earnings loss 4Q after layoff	-10.4%	-10.7%	DWS 2010 - 2019
π_w	0.2	Annual return of tenure on earnings	2.35%	2.61%	CPS ASEC 2010 - 2019
b_u	2.2	Unemployment transfer replacement rates	41.2%	41.7%	Braxton, Herkenhoff, Philips (2024)
c_f	1.8	Quarterly Unemployment Rate	6.43%	6.59%	BLS 2010 - 2019
κ	1.5	Labor cost of posting vacancy	4.5%	9.03%	Hagedorn and Manovskii (2008)
δ	0.02	Quarterly E-U Transition Rate	5.75%	3.74%	BLS 2010 - 2019, Shimer (2005)
λ_h	0.5	P90/P10 log wage residuals	2.09	2.08	CPS ASEC 2010 - 2019

4 Model Validation

In this section of the paper, I test the model's prediction on employment. To validate the model mechanism, I simulate the Paycheck Protection Program. The baseline calibration of the model presented in Section 2.2 does not include Firm Transfers, $b_f(\cdot; g_{b_f})$. The employment gains found by simulating the Paycheck Protection Program are untargeted and serve as a model validation exercise.

4.1 Background on Paycheck Protection Program

The Paycheck Protection Program (PPP) was a payroll subsidy program initiated at the onset of the COVID-19 induced recession under the CARES Act.[14] This policy was aimed at helping smaller business, who were adversely affected by COVID-19 due to lockdowns, to maintain payroll and avoid bankruptcy. It was the first time that the U.S. government provided forgivable loans to small business. The program exhausted $500 million in the first four months of eligibility, making it one of the most expensive firm-based fiscal policies in U.S. history.

To be eligible to apply for a PPP loan, the business had to have less than 500 employees. The loan was capped at $10,000,000 and could be no more than 2.5 times the average monthly payroll from the prior year. A loan could also be used for covering workers benefits, paid leave, and healthcare costs. The loans size was capped at $100,000 per employee. The loan was forgivable if the firm spent more than 60% of the loan on payroll costs. As of October 6th, 2024, 91% of loans were forgiven.[15] As policymakers at the time thought COVID was a short term aggregate shock, the loan could only cover up to 10 weeks of a business's payroll costs.

In the United States, almost 99.9% of businesses have less than 500 employees. However their labor share is much smaller. Using U.S. Census data, I compute that the number of

[14] https://home.treasury.gov/policy-issues/coronavirus/about-the-cares-act

[15] https://www.pandemicoversight.gov/data-interactive-tools/data-stories/how-many-paycheck-protection-program-loans-have-been-forgiven

workers employed at small businesses accounted for 46.4% of the labor force in 2019.[16] Additionally using data from the Small Business Administration (SBA), I find that the average amount of money spent at each firm that received a loan was \$8,093. While this number is representing up to 2.5 times the monthly average payroll cost, I approximate this to be a quarterly amount to match the frequency of the model. According to the Bureau of Labor Statistics Occupational Employment and Wage Statistics, the average annual wage in 2019 was \$53,490, making the average quarterly wage \$13,372. This implies that on average, each worker was eligible to receive a payment equal to 54.8% of the quarterly wage pre-policy.

4.2 Simulating PPP In The Model

To mimic the transfers given to small businesses during the PPP and in my model, I replicate two things in the model: the eligibility threshold and the amount. In the model, the government has access to two policy parameters: α, which governs the amount given to firms and z_g, which governs the eligibility to receive a transfer. In the model, I find z_g^{PPP}[17] that is equivalent to 37% of the labor force receiving a transfer, which is a lower bound on the 46.4% I find in the data. Setting $\alpha = 1.1$ implies that the amount on average each firms receives is 56.7% of the average quarterly wage in the economy, which matches the amount given to firms under the PPP. To define more formally the firm transfer used in this experiment:

$$b_f(\omega, z, h; 1.1, z_g) = \begin{cases} 1.1\omega z h & \text{if} \quad z \leq z_g^{\text{PPP}} \\ 0 & o.w. \end{cases}$$

Given that the model can align with the the amount and eligibility around the PPP, the next paragraph discusses the timing of the experiment.

The empirical literature studying the employment effects of the PPP have generally found positive employment gains at firms who were eligible to receive a PPP loan. Chetty et al. (2023) uses a difference-in-difference and find a statistically significant 2.48% increase in employment at firms who are eligible to receive PPP ($<$ 500 employees) compared to those firms who were not ($\geq$ 500 employees). Using similar approach, Autor et al. (2022a) finds between 2 - 12% increase in employment depending on firm size and Hubbard and Strain (2020) find between 1.3 - 1.8% increase in employment at firms eligible to receive a PPP loan.

A key difference between the payroll subsidy program considered in this paper and the Paycheck Protection Program is the policy was enacted at the onset of an large and unex-

[16] http://www.census.gov/programs-surveys/susb/

[17] In the productivity distribution, this is equal to one value lower from the median productivity level.

pected aggregate shock. Because of the nature of the shock and the policy, the empirical literature was easily able to exploit timing and size variation in eligibility requirements of the PPP. In order to speak to the findings from this literature, I consider the employment gains along the unexpected transition path on my model. To the left of the horizontal line of Figure 2 is the pre - policy level of steady state employment.

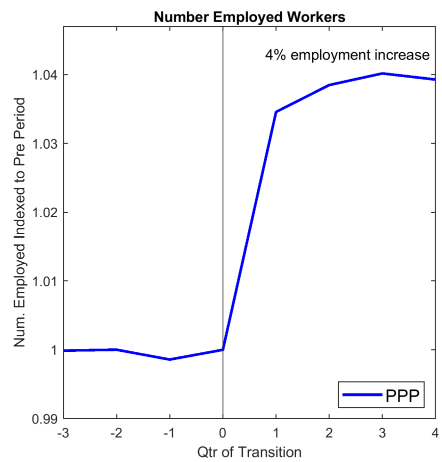


Figure 2: PPP Model Validation - Employment

I find that employment increases by 4% along the transition path, which is line with the empirical estimates. Figure 3 shows that the amount of endogenous layoffs decrease dramatically to almost zero. The amount of employment gains appears relatively small to the large amount of the payment that the firms are receiving. There is one main reason for this - that the average layoff rates in steady states for z_g^{PPP} are less than 2%. In Section 5, I show the layoff rates in the baseline version of the model and the PPP version of the model. The probability of layoff for a low productivity firm is at most 0.2%. It makes sense that the model does not produce very large employment gains given that the layoff rates at the firms receiving the transfer is very small.

The transition path abstracts away from balancing the government budget. The tax rate used along the transition path is the one from the baseline version of the world without Firm Transfers. This scenario to synonymous to the PPP as taxes were not increased immediately in the economy to finance transfers. As this program was quite costly, in Section 6.2, I discuss the general equilibrium effects and welfare implications of the PPP. I find that while this had significant employment gains, the policy lead to high distortionary taxes on earnings and large welfare loss.

The results of this section show that the estimated model can qualitatively capture the employment gains measured in the data at the onset of the PPP. In the next section, I discuss

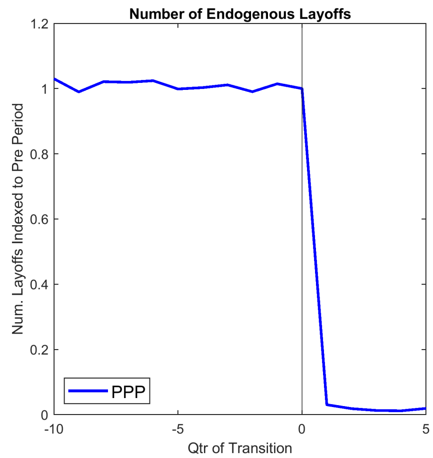


Figure 3: PPP Model Validation - Layoffs

more broadly the macroeconomic implications of Firm Transfers.

5 Macroeconomic Implications of Firm Transfers

In this section, I show how introducing Firm Transfers to the economy with only UI changes the macroeconomy. Using the same policy experiment from the model validation exercise, Figure 4 I show how the layoff decisions change in the world with the Firm Transfers. On the right hand side of Figure 4, I show the layoff rates by the productivity type of the firm conditional that a firm and worker has remained together for at least one period and not separated at the initial meeting. The layoff rates are very low for the lowest productivity firm because there a very low probability that the lowest productivity firm will produce with a worker when realized this productivity. Due to the nature of the policy, that only lower productivity firms are able to get the transfer, when firms are eligible to receive the Firm Transfer, I see layoff rate decrease for the lowest productivity firms. This is a key tradeoff in the model as the lowest productivity firms are now less likely to layoff workers, which can be bad for overall productivity and output but less workers are entering unemployment and face risk of losing their skills.

There is hetereogenity in the way that Firm Transfer impact the layoffs of workers. Figure 5 shows the layoff rates for given levels of worker human capital. The left hand side graph shows the separation rates for all levels of human capital in the baseline version of the model. The layoff rates are highest for the lower human capital workers as firms are more likely to become liquidity constrained as lower human capital workers contribute less to output than higher human capital workers. With the introduction of Firm Transfers as show in the right

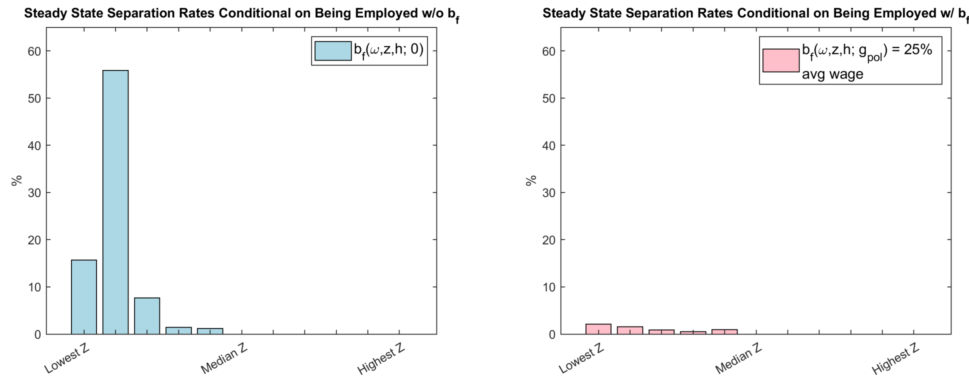


Figure 4: Layoff Rates by Firm Productivity Level

hand side of Figure 5 shows the decrease in layoff rates for higher to medium levels of human capital workers. Firms matched with lower human capital workers face larger revenue loss

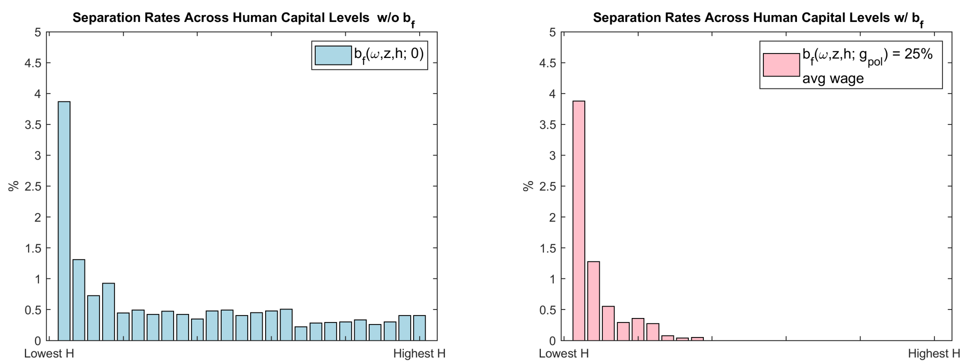


Figure 5: Layoff Rates by Worker Human Capital Level

compared to higher humcan capital workers, implying that Firm Transfers are not enough to prevent a layoff for some of these matches. In this case, it is optimal for the firm and worker to leave the match and enter unemployment, where the lower human capital worker may be able to match with a higher productivity firm.

Introducing Firm Transfers to the economy with only UI changes the layoff decisions of firms given their productivity level and human capital of worker. This also changes firms' incentives to enter submarkets and post job vacancies. One way that the increase in job vacancies can be seen is through the increase in job finding rates. Figure 6 plots the job finding rates in the baseline world and in the world with firm transfers.

Higher human capital worker face higher job finding rates. This is because a worker with higher skills is more valuable to the firm, so there will be a large number of vacancies in submarkets for high human capital workers. Similarly, lower human capital workers face lower job finding rates, contributing to costly unemployment. With the introduction of Firm

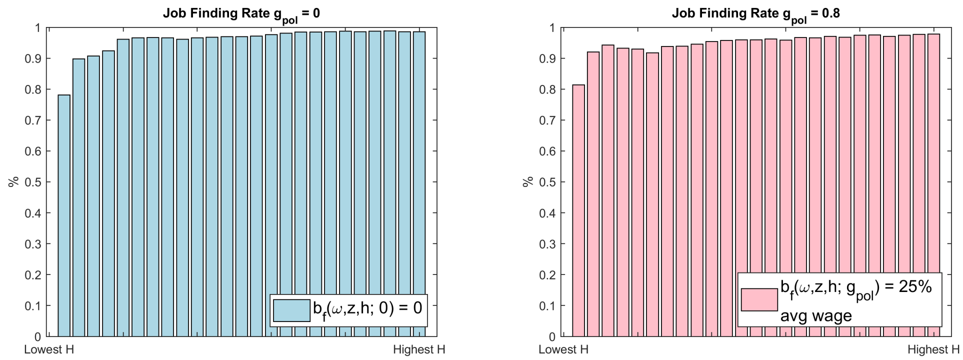


Figure 6: Job Finding Rates

Transfers, lower human capital vacancies are now more attractive, increasing the incentive for firms to enter that submarket and post more vacancies.

6 Welfare Implications of Firm Transfers

This section explores the welfare implications of Firm Transfers. Welfare is measured in steady state across economies with different level of labor market insurance policies. The first part of this section I discuss the welfare measure and define the utilitarian welfare function.

6.1 Welfare Measure and Objective Function

Welfare is computed using a newborn agent "behind the veil of ignorance" before they have realized their benefit expiration state and has not yet realized their human capital draw. Recall that newborn agents draw their human capital from the initial distribution $H_u(h)$. Then social welfare is given as:

$$W = \int H_u(h)(\zeta U(h,0) + (1-\zeta)U(h,1))dh \tag{16}$$

I define the optimal transfers in the labor market to be the levels of $\{b_u, g_{b_f}, \tau\}$ that maximizes social welfare, W when all parameters are fixed from the baseline model besides $\{b_u, g_{b_f}, \tau\}$. Welfare gains or losses from moving to the baseline model to an alternative policy are measured by consumption equivalence. The consumption equivalence is the amount of consumption that an agents would be willing to pay (if positive) or have to be paid (if negative) in all future periods and states to achieve the allocation in the alternative policy model environment. Define $W^{Baseline}$ as the welfare from the baseline model and W^{Policy} as the welfare from the model with alternative policy, $\{b_u, g_{b_f}, \tau\}$. Then define consumption equivalent, λ

$$\lambda = \left(\frac{W^{policy} + \frac{1}{(1-\sigma)(1-\beta)}}{W^{Baseline} + \frac{1}{(1-\sigma)(1-\beta)}} \right)^{\frac{1}{1-\sigma}} - 1$$

The formal derivation of λ is show in Appendix 7.7.

A government policy is $\{b_u, g_{b_f}, \tau\}$. The utilitarian government weights all individuals in the economy equally. The optimal policy is one that maximizes the utilitarian welfare effect under W^{Policy}

$$\{b_u^*, g_{b_f}^*, \tau^*\} = \underset{\{b_u, g_{b_f}, \tau\}}{\arg\max}\, W^{Policy} \tag{17}$$

6.2 Welfare Analysis of PPP

This section measures the general equilibrium and welfare effects of the Paycheck Protection Protection. The results in Table 3 shows the steady state and general equilibrium implications of implementing PPP level transfers while keeping all parameters fixed from the baseline model without Firm Transfers. Recall from Section 4, to simulate the PPP, setting $\alpha = 1.1$ equates on average firms receiving a transfer that is 57% of the average wage in the economy. To target the eligibility requirements of the policy, setting $z_g = z_g^{\text{PPP}}$ equates 37% of the labor force receiving a firm transfer.

Table 3: Welfare Effects of PPP

		Baseline	PPP
	b_u as % of mean wage	35.8%	32.8%
	$b_f(\omega, z, h; \alpha, z_g)$ as % of mean wage	0%	57%
	α % of payroll	0%	110%
	z_g - eligibility of LF	0%	37%
Macro	Mean Match Productivity as %Δ	-	-14.8%
	Output as %Δ	-	-6.2%
	Unemployment Rate	6.46%	3.12%
	Consumption 1 YR Post Layoff	93.7%	80.4%
	Taxes	2.52%	23.6%
Labor	Layoff Rates	2.62%	0.04%
	Mean Earnings	6.13	6.69
	Mean Job Finding Rate	88.1%	90.8
Welfare	Consumption Equivalence	-%	-8.91%

PPP level of transfers were very generous in terms of amount - equating to a transfer that is 1.1 times more than the average quarterly payroll amount. Along with the large eligibility, the transfers are funded by very high taxes on workers earnings - 8.36 times more than in the economy without PPP level of transfers. As seen be the endogenous layoff rates dropping to 0, almost all lower productivity firms are subsidized. This level of transfers prevents workers from entering unemployment again and matching with a higher productivity firm. Thus workers applying for new jobs are more likely to match with a lower productivity firm, which can seen by the lowered consumption after job loss. The decrease in output shows that this high level of transfers can lead to a missallocation of resources. However, this policy helps to keep workers out of unemployment and allows their human capital to grow, which can be seen the difference in the percent change from the changes in firm level productivity and output. The gains in human capital do not offset the high amount of taxes to fund PPP level of transfers, thus welfare decreases by 9%. Agents much rather prefer the baseline economy.

The results presented in Table 3 show that the PPP was untargetted, distortionary and led to misallocation of resources. In the next section, I solve for the optimal level of combination of Firm Transfers and Unemployment Insurance.

6.3 Optimal Transfers in Labor Market

In this section, I solve for the combination of transfers to firms that prevent layoff and transfers to unemployed worker that maximize steady state welfare. In the baseline economy, there are no transfers to firms, but unemployed workers receive an amount of unemployment insurance that matches the level in the U.S. To solve for the optimal amount of transfers in the labor, I use the utilitarian welfare criterion as show in Equation 17.

I compute optimal policy in the labor market for two types of government policies. The first experiment shows the welfare effects of moving the current U.S. labor market insurance policies - only UI and no FT - to a world with optimal FT. In the second experiment I show the welfare effects of moving from the baseline economy to an economy with the optimal level of UI. The third experiment measures the welfare from moving to the baseline economy to the world with optimal combination of transfers to both the firm and unemployed workers.

Table 4 presents the results of the welfare experiments. Column (1) of this table represents the baseline estimation where unemployed workers receive a transfer equal to 36% of the average economy wide wage and firms receive zero transfers. Column (2) reports the economy for optimal level of FT where I maximize over the two parameters that govern FT, the amount α and the eligibility, z_g and hold all other parameters fixed from Column (1). Under this policy, about 44% of firm-worker matches are receiving a FT that equals 5.7% of the average wage. The steady state level of layoffs decrease as well as the unemployment rate. To fund the transfer to the firms, workers must pay higher tax rate. The policy expensive, which is why the average transfers to firms are only 5.7% of the average wage compared to unemployed workers receiving a transfer that equals 36% of the average wage in the baseline economy. While the amount given to unemployed workers has not varied from Column (1), the replacement rates have gone down because the introduction of FT increases the average earnings in the economy. So workers receive relatively less in unemployment, but the amount of time they spend in unemployment decreases as the job finding rates have increased. Furthermore, Column (2) shows that while welfare increases, the introduction of firm transfers can decrease the aggregate productivity of the economy. Even though the firm transfers are expensive and productivity declines, risk averse workers are willing to to give up 0.10% of their baseline consumption to be in a world with optimal level of FT.

I next consider the optimal level of UI. Column (3) reports the steady state economy for an optimal level of UI. I find that the optimal level of UI equals a transfer of 41% of the average wage. The increased transfer at layoff increases the consumption smoothing ability of the unemployed worker. This relatively lowers the outside option of becoming employed and thus the unemployed worker applies to higher wage jobs. However, higher wage jobs also have lower matching probabilities, which ultimately leads to an increase in the unemployment duration and unemployment rate of the macroeconomy. Higher wage jobs have a higher risk of layoff, so the layoff rates increase under this policy. However, due to more generous transfers, the tax rate on labor income must increase to balance the government budget.

There are two main reasons why the welfare gain from optimal UI in Column (3) is higher than optimal FT in Column (2). The first is because layoff rates increase, there are fewer lower productivity firms operating in the economy, which is why match productivity increases. Entering unemployment more frequently gives unemployed workers a chance to match with higher productivity firm and thus increase earnings. The second reason is that the overall cost of UI is much lower than FT as there are less workers receiving UI than firm-worker matches receiving FT. Increasing UI is a cheaper way to provide insurance in the labor market and ultimately, the benefits of increased UI outweigh the moral hazard costs. Given the benefits of increased consumption upon layoff, workers are willing to give up 0.93% of lifetime baseline consumption to be in the world with an unemployment transfer equal to 41% of the average wage.

I next consider the optimal level of transfers to unemployed workers jointly with the amount of transfers given to firms, where the utilitarian government optimizes over both the amount given to firms and the eligibility requirement of FT. This table represents how UI and FT work optimally together and are complementary policies. Column (4) of Table 4 shows that optimal policy given to firms is one that subsidizes 22% of firm-worker matches. Firms on average receive 7.5% of the average wage in the economy. Unemployed workers

Table 4: Optimal Transfers in Labor Market

		Baseline	Optimal FT	Optimal UI	Optimal FT & UI
	b_u as % of mean wage	35.8%	35.3%	40.6%	40.9%
	$b_f(\omega, z, h; \alpha, z_g)$ as % of mean wage	0%	5.7%	0%	7.5%
	α - fraction of payroll	0%	5%	0%	10%
	z_g - % of LF eligible for FT	0%	44%	0%	22%
Macro	Mean Match Productivity as %Δ	-	-0.47%	3.5%	1.5%
	Output as %Δ	-	-0.31%	1.7%	-0.48%
	Unemployment Rate	6.46%	6.16%	7.51%	6.95%
	Taxes	2.52%	3.83%	3.47%	4.59%
Labor	Layoff Rates	2.62%	2.55%	3.56%	3.23%
	Mean Qrtly Unemployment Duration	1.81	1.74	2.1	2.0
	Mean Earnings as %Δ	-	1.4%	3.8%	3.1%
	Mean Job Finding Rate	88.1%	89.1%	85.7%	86.8%
Welfare	Consumption Equivalence	-	0.10%	0.93%	1.02%

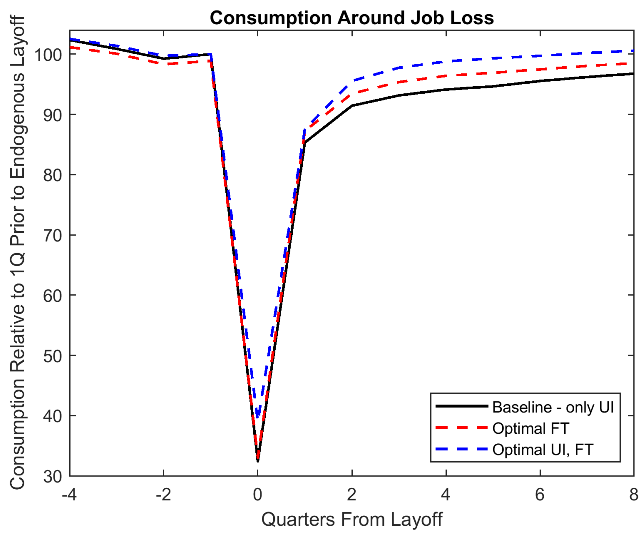
receive a transfer that equals 41% of the average wage in the economy, higher compared to the baseline economy in Column (1). In comparison to Column (1) the unemployment rate increases, driven by the moral hazard effect of UI - unemployed workers apply for higher wage jobs that 1) have lower matching probabilities 2) have higher separation rates. In comparison to Column (3), the layoff rates decrease, due to Firm Transfers. The average unemployment duration decreases due to the increase in job finding rates. The moral hazard effect of UI is partially offset by the increase in job finding rates. As workers avoid entering unemployment where they face risk of human capital erosion, workers earnings' on average are increasing. Column (4) of Table 4 highlights the general equilibrium effects of both UI and FT jointly. FT relatively increases the option value of working, workers spend less time in unemployment on average due to increased job finding rates. Since UI is cheaper to finance than FT, unemployed workers want more insurance upon job loss as they are more frequently entering unemployment due to increased layoff rates from applying to higher wage jobs. The introduction of FT decreases the layoffs rate at higher wages but since layoffs are still higher than in Column (1), unemployed workers want more insurance at job loss. Workers are willing to give up 1% of their consumption in the baseline economy in order to live in the economy with optimal UI and FT.

The main finding of Table 4 is that both of these policies are complementary, similar to Birinci, Karahan, Mercan, and See (2021) who study payroll subsidies and unemployment insurance in the context of a pandemic. Furthermore, contrary to the empirical finding in

Cahuc, Kramarz, Nevoux (2021), who find that subsiding payroll costs only helps firms who are severely hurt, my quantitative results show that only subsidizing firms who face small losses is welfare enhancing.

Figure 7 shows the path of consumption around job loss for workers who were endogenously laid off in the baseline model. The black line shows the consumption for a worker who is laid off in the baseline version of the model with only UI. In this economy, 2 years following job loss, workers have recovered 96% of their pre displacement consumption. The red dashed line shows that path of consumption for that same individual in the economy with optimal levels of Firm Transfers as shown in Column (2) of Table 4. Under this policy regime, workers recover 98% of the pre-displacement consumption after 2 years of job loss. This is because workers can transition faster to employment from unemployment with the increased job finding rates. This benefits the worker as they face less skill depreciation while in unemployment and can apply to higher human capital vacancies than compared to the baseline economy as represented by the black line of Figure 7.

Figure 7: Consumption Path of Displaced Workers



The blue dashed line in Figure 7 represents the path of consumption for a displaced worker under the optimal policy regime. After 2 years of job loss, workers make a fully recover in their consumption as they recover 101% of their pre-displacement consumption. As show in Column (4) of Table 4, this recovery from job loss increases due to the increase in job finding rates from Firm Transfers and the increase in Unemployment Insurance at the onset of job loss. Furthermore, due to increased Unemployment Insurance, workers apply for higher wage jobs, which increases the endogenous layoff rates. While this could be bad for workers, as shown by the increased in match productivity in Column (4) of Table 4, unemployed workers are more likely to match with a higher productivity firm, which helps them recover from job loss.

6.3.1 Welfare Decomposition

In this section, I consider two welfare decomposition exercises to help understand which one of the two channels is the largest contributor in welfare gains of Firm Transfers. Recall that Firm Transfers provide insurance through two channels - lowered unemployment risk and increase job creation. The welfare decomposition exercise is done by solving the baseline model without Firm Transfers where firms make endogenous layoff decisions according to $\underline{z}(\omega, h; \emptyset)$. To compute the gains in welfare from the lowered unemployment risk channel, I re-solve the baseline model but using the firm's layoff decisions from the optimal policy, $\underline{z}(\omega, h; g_{b_f}^{\text{OPT}})$ and then compute welfare gains from the baseline version of the model to the model with the optimal layoff threshold.

Table 5: Welfare Decomposition

	Baseline to Optimal FT
Total Welfare	0.104
Welfare from $\underline{z}(\omega, h; g_{b_f}^{\text{OPT}})$	0.0139
Unemployment Risk Channel	13.4%
Job Finding Rate Channel	86.6%

The first Column 1 of Table 5 represents the contribution of each insurance channel to the overall welfare gain of optimal Firm Transfers. When I re-solve the baseline model using the firms' layoff threshold from the optimal FT policy, $\underline{z}(\omega, h; g_{b_f}^{\text{OPT}})$, I find that the welfare gain from the baseline is 0.0139% of lifetime consumption. This implies that the remaining welfare gains are coming from the job creation channel, which accounts for almost 90% of the welfare gains. In Section 6.3 I discuss the implications of adding Firm Transfers to an economy with only Unemployment Insurance. The welfare gains are not as large as the gains from optimal UI because FT is a very costly policy. Furthermore, only 34% of layoffs are inefficient in the baseline model - meaning that 66% of layoffs are efficient. If Firm Transfers are very large, then many low productivity firms will be subsidized which can lead to workers having lowered earnings and consumption. Ultimately, large transfers and mitigating all separations is not optimal because of the returns to unemployment - workers can potentially search and match with a higher productivity firm.

7 Conclusion

In this paper, I introduce an alternative way to provide insurance to workers against the high cost of job loss through a policy called Firm Transfers. I then ask the question: which policy standard Unemployment Insurance or Firm Transfers or a combination of both maximizes social welfare? I find that social welfare is maximized when both Unemployment Insurance and Firm Transfers are used jointly together as I show they are complementary policies. The biggest contribution this paper makes is developing a framework that can measure the macroeconomic benefits and costs of Firm Transfers. Many countries around the world use policies similar to Firm Transfers, however there is very little know about their overall general equilibrium and welfare effects. The model is able to replicate dynamics of the aggregate U.S. labor market as well as the individual level of the cost of job loss. I show that Firm Transfers prevent inefficient layoffs and provide insurance through two channels - lowered unemployment risk and increased job creation. Second, I use the model to solve for the optimal combination of Unemployment Insurance and Firm Transfers. I find that there are welfare gains from increasing the generosity of Unemployment Insurance and Firm Transfers compared to the current levels in the U.S. The job creation channel of Firm Transfers accounts for the largest contributor to welfare gains under the optimal policy. Lastly, I show that this optimal policy largely mitigates the scarring effect of job loss allowing workers to fully recover from job loss after two years of displacement.

Beyond the quantitative contribution of this paper, understanding the general equilibrium effects of Unemployment Insurance and Firm Transfers jointly together is important for policy makers to understand given the amount of resources that go into public insurance programs for governments across the world. Extensions of this paper are to consider asymmetric information around the firm's productivity process to speak to the well known adverse selection problem of policies similar to Firm Transfers.

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Appendix A

7.1 Simple Model of Firm & Unemployment Insurance

To motivate the intuition of introducing firm insurance into a directed search environment, consider a simple static model. The simple model makes predictions that I will test empirically over the summer. In Section 2.1, I consider an extended version of this simple model into a quantitative model of directed search.

Consider a one period model with three three separate agents: firms, workers, and government. All firms are ex-ante homogeneous and subject to an idiosyncratic productivity shock, $z_s \in \{z_H, z_L\}$, which can either be high (z_H) or low (z_L). The high productivity shock occurs with probability π and the low productivity shock occurs with probability $(1-\pi)$. Firms face a simple production function: $y_s = z_s$ so their production is perfectly correlated with their idiosyncratic shock. Firms offer one fixed wage contract, w. With probability $(1-\pi)$, firms receive the bad productivity shock. In the counterfactual, firms are eligible to receive a transfer from the government, b_f, which is funded through taxes on earnings.

There is a unit measure of risk neutral workers who consume wage w. Unemployed workers receive unemployment benefit b_u. The government taxes working agents at rate τ to fund both types of public transfers, b_u and b_f.

For simplicity, assume that workers' probability of matching to the job, $p(\theta(w)) = 1$ so that any worker who searches for a job gets matched with certainty. Additionally, assume that $z_H > w > z_L$. This crucial assumption means that if struck with high productivity, firms production is strictly higher than their fixed wage cost and if struck with low productivity, the firm cannot pay their fixed wage to the worker and the worker goes into unemployment, motivating transferring money to firms. The timing of the static model is as follows:

i) Firm posts vacancies according to free entry condition: $\kappa \geq q(\theta(w))\mathbb{E}[J(z,w)]$

ii) Search and match occurs

iii) z_s realized

iv) Firm transfer, b_f, is enacted

v) Firm separation decision

vi) Ex-post value of match to firm: $J(z,w)$

vii) Workers consume either w or b_u.

Now I consider two environments of this simple static model. The first is the baseline economy presented in Section 7.2, that has endogenous separations between firm and worker in the event of low productivity shock. The second is the counterfactual in Section 7.3, where the firm and worker do not have to separate in the presence of low productivity shock as the firm receives a transfer b_f from the government. Then I discuss the intuition and key model predictions in Section 7.4.

7.2 Only Unemployment Insurance

In the environment with no firm transfers, if firms realize z_L, then the match value is negative and the worker will be forced to separate from the firm. Then with probability $(1-\pi)$, the firm and worker will produce nothing. On the other hand, with probability π, the firm-worker match realize a positive productivity shock and the worker remains employed and firm pays wage w. The value of the match to the firm can be written as:

$$J(z,w) = \begin{cases} z_H - w, & \text{with probability } \pi \\ 0, & \text{with probability } (1-\pi) \end{cases}$$

Now, denote the expected aggregate consumption of both unemployed and employed workers as $E(C)$. If the firm realizes the high productivity state, then the worker consumes wage w and pays tax rate τ on her wage to fund public transfer b_u. With probability $(1-\pi)$ a worker is thrown into unemployment and she consumes b_u. Thus, the expected aggregate consumption of workers in this environment is:

$$\mathbb{E}(C) = \pi w(1-\tau) + (1-\pi)b_u$$

Fraction π of the workers receive a positive productivity match and consume $w(1-\tau)$ and fraction $(1-\pi)$ must separate from the firm, as the firm cannot pay them wage w, and consume unemployment benefit b_u, which is transferred through the government. The government budget must balance. The government collects money from the fraction of individuals working and that must equate the value of unemployment insurance given out. Then the government budget constraint is:

$$(1-\pi)b_u = \tau w \pi$$

The right hand side of the government budget constraint is the total amount of unemployment insurance they must provide. The left hand side of the government budget constraint is the total amount of revenue collected from the fraction π of workers who remain employed. By doing some simple algebra with the government budget constraint, I can rewrite expected aggregate consumption of a worker as:

$$\mathbb{E}(C) = \pi w$$

In the version of the simple static model without firm transfers, the value of aggregate consumption is the fraction of individuals who matched with a high productivity firm. In the next section, I run a similar exercise but with the introduction of firm insurance, the expected consumption is greater. The intuition for this is explained in Section 7.4.

7.3 Both Unemployment + Firm insurance

In the environment with both firm and unemployment transfers from the government, the probability the firm-worker match separates is zero. With probability $(1-\pi)$, the firm realizes z_L, then the firm gets a firm transfer $b_f = w - z_L$. This value is equal to the exact difference between the match output and zero, thus if the firm receives a that is enough for the firm to pay wage w to the worker. In this scenario, the firm is indifferent between separating and producing. On the other hand, with probability π, the firm-worker match realize a positive productivity shock and does not receive transfer b_f as the worker remains employed and firm pays wage w. Then write the value of the match to the firm as:

$$J(z, w) = \begin{cases} z_H - w, & \text{with probability } \pi \\ z_L - w + b_f = 0, & \text{with probability } (1 - \pi) \end{cases}$$

The firm transfer, b_f, is funded by taxes (τ) on wages that the employed worker will bear. Note that the firm is indifferent between a world with and without firm transfers. Denote the value of aggregate expected consumption with firm transfers, C_{ft}. In this simple model, the value of the firm transfer b_f is just enough to insure that the firm and worker will not separate. Thus, with the introduction of the firm transfer, b_f, no worker will become unemployed, so the aggregate consumption will be denoted as:

$$\mathbb{E}(C_{ft}) = \pi w(1 - \tau) + (1 - \pi)w(1 - \tau) = w \cdot (1 - \tau)$$

With probability π, the firm realizes high state of the world, z_H, and thus the worker gets to keep her job and consumes after tax wage $w(1-\tau)$ in this state of the world. With probability $(1 - \pi)$ the firm realizes the bad state of the world. With the introduction of $b_f = w - z_L$, the firm can pay worker w and the worker can remain employed at her current job, but must pay taxes on her earnings in order to fund this. In this version of the static model with both firm and unemployment insurance, all workers consume $w(1-\tau)$. The government budget for the two public transfers also has to balance:

$$(1-\pi)b_f = w \cdot \tau[\pi + (1-\pi)]$$

The right hand side of the government budget constraint is income from the taxes collected from working individuals in both productivity states (since those with a low match still get a wage due to b_f transfer). The left hand side is the amount the government will spend on the firm transfer program. Thus, it is the total amount spent on the firm level transfers, b_f, for the fraction of firms that are hit with a low productivity shock.

If we rearrange the government budget constraint, we get $\tau = \frac{(1-\pi)b_f}{w}$. We can re-write the expected consumption in the environment with firm transfers as:

$$E(C_{ft}) = \pi w + z_L(1-\pi)$$

The aggregate expected consumption of workers in a world with both unemployment and firm transfers is equal to the expected consumption with only unemployment insurance plus the fraction of low productivity output, $z_L(1-\pi)$.

7.4 Predictions From Simple Model

To determine model predictions, we first determine if firm transfers are welfare enhancing. Consider comparing expected consumption in the world with firm transfers and the world without firm transfers.

$$\begin{aligned} E(C_{ft}) &> E(C) \\ \pi w + z_L(1-\pi) &> \pi w \\ z_L(1-\pi) &> 0 \\ z_L &> 0 \end{aligned}$$

The value of expected aggregate consumption is strictly greater in the world with firm transfers.

- **Key Model Prediction 1**: Firm transfers are welfare enhancing if match productivity is positive.

- **Key Model Prediction 2**: Firm transfers are welfare enhancing regardless of the value of unemployment insurance

The simple one period model predicts that firm insurance is a welfare enhancing policy if the match productivity is positive. As the wage is not endogenous in this simple model, optimal policy is to only include firm insurance in a low productivity state. In tandem with key model prediction 2, workers are willing to do this regardless of the value of unemployment insurance. This is a feature of the static model that changes in the dynamic model presented in Section 2. There, agents face the costs of searching for a new job versus staying at their current job which is feasible via b_f. In general, there has not been a time in history when the U.S. relied only on firm insurance and not unemployment insurance as a stabilization policy. Thus, by calibrating the value of the firm transfer to match the failure and success rates of firms after the PPP loans, I can use my quantitative model presented in Section 2 as a laboratory to analyze model prediction 2 and determine if firm insurance is a strictly better policy in the absence of unemployment insurance. Due to the nature of *Block Recursivity*, my model can estimate the macroeconomic impacts for both aggregate shocks and idiosyncratic shocks. Based on my preliminary results from running a similar policy experiment in my full blown quantitative section, I predict that only relying on firm transfers will not be welfare enhancing and that when a wage ladder is introduced, a combination of both is optimal for social insurance. Some quantitative models (Birinci, Karahan, Mercan, See (2021)) study both subsiding firm costs and unemployment insurance and find that both policies are complementary to each other. Additionally, this paper studies both quantitatively and empirically each policy separately and jointly together.

Appendix B

7.5 Quantitative Model of Firm Transfers with Self-Insurance

7.5.1 Model environment

Time is discrete and runs forever. There are $T \geq 2$ overlapping generations of households and each individial lives for T periods.[18] There is a unit measure of risk averse workers and a continuum of potential entrant firms. Both firms and workers discount their future value by $\beta \in (0, 1)$. In addition to firms and workers, there is also a government who collects taxes on earnings to fund the two types of labor market insurance.

Worker Heterogeneity. Workers are either unemployed or employed. If unemployed,

[18] This is for tractability reasons.

the worker consumes unemployment benefit b_u, which never expire over the duration of their unemployment. If employed, a worker earns wage $w \in [\underline{w}, \overline{w}] \equiv W \subseteq \mathbb{R}_+$. In addition to either being unemployed or employed, workers are heterogeneous along three total dimensions. In addition to searching for their wage, w, the worker can choose how much to save in a risk free asset, $a \in [\underline{a}, \overline{a}] \equiv A \subseteq \mathbb{R}$. Agents cannot borrow as $\underline{a} = 0$ and the upper bound is not binding.

Additionally, employed workers have firm-match productivity value, $z \in [\underline{z}, \overline{z}] \equiv Z \subseteq \mathbb{R}_+$. The value of z evolves according to a discretized $AR(1)$ process, which I denote as $G(\cdot)$. The employed workers match productivity today is draw from the conditional markov probability distribution, $G(z|z_{-1})$ and explained in more detail in Section **??**. Note that only unemployed workers can search for jobs. In Section **??** I show the model solved with on-the-job search where both unemployed and employed workers can search for jobs.[19] Households are ex-ante identical but as they start searching for wages, become heterogeneous in their wealth, wage, and productivity level.

Firms. Firms face idiosyncratic level shocks, z, each period. They post vacancies according to the free entry condition in expectation of their idiosyncratic productivity level. Because of this, firms cannot commit to paying their worker if the realized value of the idiosyncratic shock z makes the net present value of the firm today negative. I denote this value as $\underline{z}(w, a)$ which indicates that the discounted present value of the match to the firm has dropped below zero. This cutoff is formally defined in the firm value function section. As a counterfactual, I denote the transfer to the firm as b_f when they realize this low productivity draw. Firms treat each jobs independently, so there is one worker and one firm in a match. When a firm is matched with a worker, they produce $f \cdot z$ together where f is normalized to one for all firms. The measure of firms is determined by competitive entry and they pay vacancy cost of κ each period to post vacancies in each submarket.

Separations. Each period, δ fraction of firm-worker matches are destroyed. In this situation, the employed worker gets thrown into unemployment and then the firm gets 0. The second type of separation is the separation due to a low idiosyncratic match productivity draw. With probability $G(\underline{z}(a, w)|z_{-1})$, the firm draws a $(z < \underline{z}(w, a))$. In this situation, the firm gets 0 and the employed workers goes into unemployment. Section **??** of this paper examines the macroeconomic and welfare effects of subsidizing firm level transfers so that the cutoff of when a firm will fire a worker is lower.

[19]The interaction of firm transfers and on the job is not well understood.

Labor Market. Unemployed workers direct their search across submarket for a wage, w, that is fixed for the duration of the firm-worker match. Let u denote the number of unemployed workers searching in a specific submarket and similarly v denote the number of vacancies. Then a constant returns to scale matching function, $M(u, v)$, determines how many unemployed workers will search in a specific submarket and how many vacancies the firms will post in that submarket. The submarkets are indexed by the state variables available to the firm at the time of posting. These are the age of the worker (t), wealth of worker (a), and wage contract (w). Note that the firm does not post new vacancies according to their match productivity. The match productivity z is draw after the match has been formed. Let θ denote the market tightness in each submarket, which is the ratio of the number of vacancies v to the number of unemployed workers u. Then the probability a worker matches with a firm in a given submarket is $p_t(\theta(a, w)) = \frac{M_t(u_t(a,w), v_t(a,w))}{u_t(a,w)}$ and the probability a firm will match with a worker is given by $q_t(\theta(a, w)) = \frac{M_t(u_t(a,w), v_t(a,w))}{v_t(a,w)}$. Once matched with a firm, the firm and worker produce $f \cdot z$ and the worker earns wage w for the duration of the match. The tightness function is independent of the distribution of workers across states, ultimately making the equilibrium of this model *Block Recursive* as in Menzio and Shi (2011).

Transfers: Unemployed workers receive b_u from the government. In the event that a firm draws $z < \underline{z}(a, w)$, then they are eligible to receive b_f, which is a lump sum payment to the firm. The firm has to spend this money on keeping their worker. The transfer to the firm, b_f, will not be enough to save every match that experiences $z < \underline{z}(a, w)$. The firm still accepts this payment even though it will not save the match. Both types of transfers b_u and b_f are funded through taxes on earnings, τ. The government balances the budget each period so that the total amount of revenue collected from taxes equals the outlays of the two types of labor market insurance.

Timing. The timing of each period is divided into multiple sub-periods. In the first stage, firms pay κ to post vacancies in each submarket indexed by (a, w) around their expectation of their idiosyncratic productivity. In the second stage, unemployed search in submarkets with tightness, $\theta(a, w)$. In the third stage, exogenous separations occur with probability δ. In the next stage, firms realize their idiosyncratic productivity value. In the fifth sub-period, a firm can receive transfer b_f from the government to avoid separation from current worker match. In the sixth stage, firms make a separation decision: if the discounted Present Value of the match falls below zero, the firm and worker endogenously split and the worker goes into unemployment. Lastly if the worker and firm remain together, they produce $f \cdot z$. Regardless at the end of every period, unemployed and employed workers make their savings and consumption decisions. production and consumption/savings decisions occur.

7.5.2 Bellman Equations

This section explains the Bellman Equations of the workers and firms. The value functions below are described in the middle of the period, workers have realized their separation shocks and going to make their consumption/savings decisions.

Decisions on consumption and savings. Given an employed worker with wealth (a), age (t), match productivity (z), and consumes wage (w) has value when making consumption and savings decisions is denoted as $V_{t,e}(w,a,z)$. After making savings choice a' for tomorrow, their expected continuation value can be denoted as $\beta \cdot \mathbb{E}_{z'|z}\tilde{V}_{t+1,e}(w,a',z')$. They take an expectation over the conditional markov distribution, $G(z'|z)$ to measure the expected probability they will remain employed tomorrow at their job. An employed worker optimizes:

$$V_{t,e}(w,a,z) = \max_{a'} u(c) + \beta \cdot \mathbb{E}_{z'|z}\tilde{V}_{t+1,e}(w,a',z') \quad \forall t \leq T \tag{18}$$

$$V_{T+1,e}(w,a,z) = 0 \tag{19}$$

$$\begin{aligned} &s.t. \\ c + a' &= w(1-\tau) + a(1+r) \\ z' &\sim G(z'|z) \end{aligned} \tag{20}$$

Employed agents maximize their expected discounted utility subject to their budget constraint. Workers must pay taxes, denotes as τ, on their earnings to fund both the unemployment and firm insurance. Let $c_{t,e}(w,a,z), a'_{t,e}(w,a,z)$ denote policy functions for optimal consumption and savings for employed individual.

Similarly, we can generally define the consumption and savings problem for an unemployed individual. Given an unemployed individual with wealth (a) and age (t) their value function when making consumption savings decision is denoted as $V_{t,u}(a)$. The unemployed worker takes an expectation over two things. The first is going to be matching with their desired firm. Conditional on matching with the new firm, the must also survive the endogenous separation shock, ($\tilde{z} \geq \underline{z}(a,w)$). The unemployed worker takes an expectation over the probability of staying with that firm using the unconditional ergodic distribution of the markov process, $G(\tilde{z})$. After making their savings choice a' for tomorrow, the continuation value for an unemployed worker is defined as $\beta \cdot \mathbb{E}_{\tilde{z}}\tilde{V}_{t+1,u}(a')$. Then we can define an unemployed agents optimal consumption and savings decisions that solve:

$$V_{t,u}(a) = \max_{a'} u(c) + \beta \cdot \mathbb{E}_{\tilde{z}} \tilde{V}_{t+1,u}(a') \quad \forall t \leq T \tag{21}$$

$$V_{T+1,u}(a) = 0 \tag{22}$$

$$s.t.$$

$$c + a' = b_u + a(1 + r)$$

$$\tilde{z} \sim G(\tilde{z}) \tag{23}$$

Unemployed agent maximizes their discounted expected future consumption subject to their budget constraint. Denote $c_{t,u}(a), a'_{t,u}(a)$ consumption and saving policy functions for an unemployed individual.

Unemployed wage search. An unemployed agent at age t receives unemployment benefit equal to b_u and chooses a' savings for tomorrow. Then the unemployed agent directs their search to submarket with tightness $\theta(a', w')$ and gets matched to a firm with probability $p_{t+1}(\theta(a', w'))$. The first part of the unemployed continuation value represents her value if she does not get matched, she remains unemployed at value $V_{t,u}(a')$. With probability $p_{t+1}(\theta(a', w'))$, the worker gets matches and draws $\tilde{z}$ from the stationary ergodic markov distribution. With some probability $G(\underline{z}(a', w')$, the newly employed worker matches with a firm that draws a low productivity and cannot pay the worker. In this scenario, the worker go back into unemployment, $V_{t+1,u}(a')$. This part of the unemployed worker's continuation value is with an indicator function denoted as $\mathbb{1}_{\tilde{z}<\underline{z}(a',w)}$. If the unemployed worker matches with a firm who draws a high enough productivity, then the worker continues with the firm at value $V_{t+1,e}(w', a', \tilde{z})$. Thus denote the unemployed expected continuation value $\mathbb{E}_{\tilde{z}} \tilde{V}_{t+1,u}(a')$:

$$\mathbb{E}_{\tilde{z}} \tilde{V}_{t+1,u}(a') = \max_{w'} (1 - p_{t+1}(\theta(a', w')) \cdot V_{t+1,u}(a') \tag{24}$$

$$+ p_{t+1}(\theta(a', w')) \mathbb{E}_{\tilde{z}} \left[\mathbb{1}_{\tilde{z} \geq \underline{z}(a',w)} \{V_{t+1,e}(a' w', \tilde{z})\} + \mathbb{1}_{\tilde{z}<\underline{z}(a',w)} \{V_{t+1,u}(a')\} \right] \tag{25}$$

Let the policy function for the optimal search target be defined as $w_u^{'}(b_u, a')$, which is a function of end of period wealth. Define submarket tightness searched by the unemployed worker as $\theta(a', w') \equiv \theta(a', w'(b_u))$. This formulation of the unemployed consumption, savings, and wage search decisions shows that directed search serves as self-insurance. Workers are risk averse and thus would like to accumulate precautionary savings. An unemployed worker knows her wealth and can direct her search to best smooth her consumption. Consider a low wealth worker, she would like to build up precautionary savings and targets low wage jobs that have a relatively high job finding probability. If not, she will dis-save in her wealth until her consumption is zero which would not be optimal. To avoid this, she targets low paying jobs with high matching probability. Thus a lower level of unemployment benefit, b_u, means that the unemployed agent will target lower wages. On the other hand, for higher levels of unemployment benefit, the unemployed worker will target higher wages as her outside option has increased.

Employed wage search. An employed agent receives wage from fixed wage contract w, chooses a' for tomorrow. With probability δ, the worker goes into unemployment. With probability $1-\delta$, the worker survives the exogenous probability shock. Then the worker undergoes the endogenous separation shock. With probability $G(z' = (z < \underline{z}(a,w)|z)$ they draw a match productivity with their firm that is below the cutoff and the worker is forced into unemployment with value $V_{t+1,u}(a')$. This is denoted with an indicator function $\mathbb{1}_{z'<\underline{z}(a',w)}$. If they survive the low productivity shock, they continue on with their current firm with value $V_{t+1,e}(w,a',z')$. Denote the workers expected continuation value $\mathbb{E}_{z'|z}\tilde{V}_{t+1,e}(w,a',z')$:

$$\mathbb{E}_{z'|z}V_{t+1,e}(w,a',z') = \delta V_{t+1,u}(a') + (1-\delta)\mathbb{E}_{z'|z}\left[\mathbb{1}_{z'\geq\underline{z}(a',w)}\{V_{t+1,e}(w,a',z')\} + \mathbb{1}_{z'<\underline{z}(a',w)}\{V_{t+1,u}(a')\}\right]$$

Define employed workers optimal wage search function, $w_e^{'}(w,a')$. A workers wealth influences their probability of matching. For example, consider a high wage and asset worker. Since they have built up precautionary savings, they can afford to apply to high wage jobs that have low matching probabilities. The chance of them leaving their current firm-worker match is smaller thus there is a higher chance they will not leave their job. The value of this job filled to the firm and worker will be higher than in the case of a low asset, low age worker in the absence of idiosyncratic shocks. However, the probability $G(z' = \underline{z}(a',w)|z)$ is increasing in the wage of the worker. For higher wage contracts, the total value of the firm is lower and thus is more susceptible to realizing a negative discounted present value from a negative idiosyncratic realization. In the world without social insurance given to the firm, the worker internalizes this and applies to lower wage jobs as they prefer to be employed than unemployed.

7.5.3 Firms and market tightness

Consider a filled job with fixed wage contract w, then the current profit of the job is $f \cdot z - w$ where f is homogenous across matches and z is the idiosyncratic productivity value. Tomorrow, the firm will exogenously separate from its worker with probability δ. In this state, the firm gets zero. With probability $1 - \delta$, the firm and worker survive the exogenous employment shock, then they continue onto the productivity shock. Occurring with probability $G(z' = \underline{z}(a', w)|z)$, the firm and worker will endogenously separate as the realized value of z today makes the present value of the firm today negative.

In the firms value function below, the state of the world in which they separate is denoted as $\mathbb{1}_{z' < \underline{z}_{a',w}}$ where the future value of this job filled is zero as the firm and worker separated. In either case, if the firm loses its worker due to on the job search or due to low idiosyncratic productivity draw, the future value of the job to the firm will be zero. If no separation occurs, then the value to the firm of the future job will be $J_{t+1}(w, a', z')$, where z' is their idiosyncratic shock tomorrow and a' is the wealth choice of the worker who remains at the same job tomorrow with fixed wage contract w. The total value to the firm without a firm transfer is denoted as $\tilde{J}(a, w, z)$. With the introduction of the government financed firm transfer b_f, the firm's bellman becomes $J(a, w, z)$. Thus the firms bellman can be denotes below as:

$$\tilde{J}_t(a, w, z) = zF - w + \beta \cdot (1 - \delta)\mathbb{E}_{z'}\left[\mathbb{1}_{z' \geq \underline{z}(a',w)}\{J_{t+1}(a', w, z') + \mathbb{1}_{z' < \underline{z}_{(a',w)}} 0\}\right]$$

$$J_t(a, w, z) = \begin{cases} \tilde{J}_t(a, w, z) + b_f & \tilde{J}_t(a, w, z) < 0 \\ \tilde{J}_t(a, w, z) & \tilde{J}_t(a, w, z) \geq 0 \end{cases} \tag{26}$$

$$\underline{z}(w) \quad \text{solves} \quad \tilde{J}_t(a, w, \underline{z}(a, w)) = 0$$

Given a submarket with tightness $\theta((a', w))$, the probability the firm matches with the worker is given by $q(\theta(a', w))$. Each submarket tightness is determine by competitive free entry condition where firms pay κ for posting each vacancy. The firm does not know its idiosyncratic value when posting the vacancies, so the free entry condition is taken over expectation of the stationary distribution of z, denoted as $\tilde{z}$. Then expected value net of vacancy cost of entering a submarket is $\mathbb{E}_{\tilde{z}}[J_{t+1}(a', w, \tilde{z})q_{t+1}(\theta(a, w))] - \kappa \geq 0$. Firms will post in each submarket if this condition is true, otherwise if $\mathbb{E}_{\tilde{z}}[J_{t+1}(a', w, \tilde{z})q_{t+1}(\theta(a, w))] - \kappa < 0$ then no recruiting firm will enter that submarket. Thus in equilibrium, it must hold that:

$$\mathbb{E}_{\tilde{z}}[J_{t+1}(a', w, \tilde{z})q - t + 1(\theta(w))] - \kappa \leq 0, \qquad \theta(a', w) \geq 0, \forall (a', w) \tag{27}$$

The two above inequalities hold with complementary slackness and define market tightness

$$\theta(a', w) = \begin{cases} 0 & otherwise \\ q^{-1}\left(\frac{\kappa}{\beta \mathbb{E}_{\tilde{z}}[J_{t+1}(a', w, \tilde{z})]}\right) & \beta \cdot \mathbb{E}_{\tilde{z}}[J_{t+1}(a', w, \tilde{z})] - \kappa \geq 0 \end{cases} \tag{28}$$

To understand how θ, q, p work together, consider a submarket with a high job matching probability, $p(\theta(a', w))$, for an unemployed worker. Then that submarket must also have a relatively high market tightness, meaning there are a lot of vacancies relative to number of individuals searching. There would be a lot of vacancies because the expected value of the job filled to the firm yields a high expected firm value, and hence has a low wage offer. Futhermore, because there are relatively fewer applicants for one job posting, the probability of the firm matching to the worker, $q(\theta(a', w))$, is low, compared to the high probability of the applicant matching for the low wage job. To understand the implication of on the job search in the firms value function, consider a high wage and high wealth worker. For example, consider a high wage and asset worker. The firm has full knowledge of the workers wealth. Since they have built up precautionary savings, they can afford to apply to high wage jobs that have low matching probabilities. The chance of them leaving their current firm-worker match is smaller thus there is a higher chance they will not leave their job. This increases the value of the match to the firm and thus the firm will not be posting as many vacancies for this (a', w) combination, leading to a low match rate for a high wealth, high wage job posting.

Workers are risk averse and have the following utility form.

$$u(c) = \frac{c^{1-\sigma} - 1}{1 - \sigma}$$

The probabilities $q(\theta)$ and $p(\theta)$ are determined by the constant returns to scale matching function and take functional form:

$$q(\theta) = \frac{\frac{1}{\theta}}{[(\frac{1}{\theta})^{\gamma} + 1]^{\frac{1}{\gamma}}}, \quad p(\theta) = q(\theta) \cdot \theta, \quad \theta = \frac{v}{u}$$

where the labor market tightness, θ, is defined as the vacancies posted by firms v, divided by the number of unemployed people u. The above matching probabilities are found by using the constant returns to scale matching function

$$M(u, v) = \frac{u \cdot v}{(u^{\gamma} + v^{\gamma})^{\frac{1}{\gamma}}}$$

External	Description		Value	Data	Source
σ	Utility Curvature		2	-	Standard
γ	Matching Elasticity		1.6	-	Schaal (2017)
ρ_z	Productivity Persistence		.95	-	Khan and Thomas (2008)
η_z	Shock to Productivity		.038	-	Khan and Thomas (2008)
Internal		Value	Model	Data	
κ	Unemployment Rate	0.35	6.32%	6.8%	BLS 2010 - 2019
λ	E-E Rate	0.43	2.7%	9.1%	-
b_u	Replacement Rate	0.325	42.8%	41.2%	PSID
δ	Separation Rates	0.025	5.31%	5.744%	BLS 2010 - 2019
β	P70 Net Liquid Asset/Income	0.974	27.01%	29.36%	Survey of Consumer Finance 2010, 2013

Table 6: Model Parameters

where γ governs the elasticity between the probability that an unemployed worker will get matched with a firm to the market tightness. The model is calibrated to a quarterly frequency. The markov process on the firms productivity is a discretized $AR(1)$ process using Tauchen (1986) [20].

$$z' = \rho_z z + \epsilon_z$$
$$\epsilon_z \sim \mathcal{N}(0, \sigma_z^2)$$

The parameter ρ_z is currently set to match firms' persistence in their investment measured in Khan and Thomas (2008). Schaal (2017) who studies a directed search model and firms uncertainty uses a similar parameterization. In Section **??**, I discuss how I will estimate ρ_z using the U.S. Census data.

The top portion of Table 7.5.3 shows the externally calibrated parameters. The utility curvature value is set to 2 so that agents value life as in Hall and Jones (2007). A discount factor of 0.996 corresponds to a 4% annual interest rate. The quarterly interest rate 0.367% compounded quarterly which corresponds to a 4% annual interest rate. The total exogenous and endogenous separations rates is calibrated 10% to match the total endogenous and exogenous separations as documented in Shimer (2005). The matching elasticity parameter is set as in Schaal (2017) who also studies a directed search model with time varying firm level idiosyncratic risk.

Table 7.5.3 show the remaining parameters that are calibrated internally. The cost of posting a vacancy κ is set very low so that I can match the unemployment rate in the data. The transfer to the unemployed individual b_u is set to match the average replacement rate of previous earnings in the data. The graphs below show the main mechanisms in the model.

Figure **??** shows how the job finding rates increase when b_f is turned on. The worker faces

[20] The results in the paper include 5 different idiosyncratic states

high job finding rates for each contract she considers. The figure is taken as an expectation across all $\tilde{z}$ draws that a firm can take after they form a match with their worker. The firm value increases, implying that it is more profitable to post a vacancy and leading more firms to enter that submarket. This results in a low probability the firm will meet the worker, but leads to a higher probability that the worker will meet the firm, which is show in Figure **??**.

Figure **??** shows how with the introduction of transfers to firm leads to less unemployment risk. The transfer allows the worker to stay with its current firm match. In the next section, I validate my model by running a counterfactual experiment where I set b_f equal the Paycheck Protection Program received at firms normalized by the average wage in the economy. The methodology and results are in Section **??**. In the next section, Table 7 defines the value of the firm transfer as a percent of wage.

7.5.4 Quantitative Results

In the first column of Table 7, I show aggregate moments from the baseline version of the model. While the current calibration is a work in progress, the table gets at the basic tradeoffs of giving money to firms before they lay off their workers versus giving money to unemployed workers.

In the second column of Table 7, I solve for the optimal social insurance policies to both workers and firms. The value of the firm transfer in this optimal policy about 30% of the average wage in the economy and unemployment insurance is increased to about 55% of the pre-job loss replacement wage. With the introduction of the firm transfer, we see a drop by about $\frac{1}{2}$ in the aggregate unemployment rate and an increase in the average equilibrium wage in the economy. Two forces are at play here. First, the increase in unemployment insurance induces workers to apply for higher wage jobs than in the baseline model and second, with firm level subsidy, these high jobs survive the low idiosyncratic productivity draw. Furthermore, the consumption upon job loss is higher as firm transfers decrease unemployment risk and lead to higher equilibrium wage. Even with a higher tax rate, consumption on average is higher in this state of the world, so agents are willing to give up a fraction of their consumption in the baseline version of the model to transfer to the optimal policy with two types of social insurance.

In Column 3 of Table 7 highlights the optimal firm transfer given the baseline level of unemployment insurance. The firm transfer remains the same even in the absence of an increase in the unemployment insurance. Similar to Column 2, there is an increase in the wage and decrease in unemployment rate. However, the consumption drop upon job loss is not as high as in Column 2. This is coming from the fact that unemployment insurance is not increased. However, it is higher than in the baseline version of the model suggesting that transfer to firms insure consumption at longer horizons than unemployment insurance.

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Table 7: Welfare Results

	Baseline	FT, UI	Optimal FT	UI	PPP FT
b_f % of Wage	0%	46.5%	47.1%	0%	56.7%
b_u % of pre-job loss Wage	40.18%	47.01%	36.5%	51.3%	42.7%
Average Wage	0.88	0.835	0.89	0.828	0.881
Unemployment Rate	6.34%	4.19%	4.18%	6.29%	3.73%
Consumption upon job loss	74.76%	79.07%	79.45%	73.73%	92.1%
Consumption 4Q post job loss	89.44%	94.7%	92.0%	91.57%	92.1%
Mean Job Finding Rate	83.7%	84.9%	84.9%	83.8%	85.4%
Separation Rates	4.56%	3.4%	3.5%	5.30%	1.01%
Aggregate Prod. as % from baseline	-	-11.82%	-11.32%	0%	-11.9%
Taxes	2.67%	13.6%	13.2%	10.8%	16.9%
SS Welfare	-	3.33%	1.65%	3.18%	-11.5%

The main finding of Table 7 is that both of these policies are complementary, similar to Birinci, Karahan, Mercan, and See (2021) who study payroll subsidies and unemployment insurance in the context of a pandemic. Furthermore, contrary to the empirical finding in Cahuc, Kramarz, Nevoux (2021), who find that subsiding payroll costs only helps firms who are severely hurt, my quantitative results show that only subsidizing firms who face small losses is welfare enhancing.

In the second column of Table 7, I solve for the optimal social insurance policies to both workers and firms. The value of the firm transfer in this optimal policy about 30% of the average wage in the economy and unemployment insurance is increased to about 55% of the pre-job loss replacement wage. With the introduction of the firm transfer, we see a drop by about $\frac{1}{2}$ in the aggregate unemployment rate and an increase in the average equilibrium wage in the economy. Two forces are at play here. First, the increase in unemployment insurance induces workers to apply for higher wage jobs than in the baseline model and second, with firm level subsidy, these high jobs survive the low idiosyncratic productivity draw. Furthermore, the consumption upon job loss is higher as firm transfers decrease unemployment risk and lead to higher equilibrium wage. Even with a higher tax rate, consumption on average is higher in this state of the world, so agents are willing to give up a fraction of their consumption in the baseline version of the model to transfer to the optimal policy with two types of social insurance.

The results in Table 7 highlight two things. The first being that removing the endogenous separation risk for a higher wage increases welfare and second that in order for these two policies to work, they need to be enacted together, increasing UI alone or enacting a firm benefit alone increases welfare, but not as much when as when they are used together. Furthermore, my results differ from Guipponi and Landais (2022) as I find these policies work well together in the presence of persistent shocks. While they do not examine idiosyncratic shocks, my model can easily incorporate both idiosyncratic and aggregate persistent and/or temporary shocks due to the nature of *Block Recursivity*. In Section **??** I discuss the parameter values that can govern an aggregate uncertainty process.

Other than compare and contract welfare gains and losses from each of these policy experiments, it is important to understand how these policies affect workers' wealth. Blundell, Pistaferri, and Preston (2008) were one of the first to show the existence of individuals self-insurance. Chaumont and Shi (2022) use a model of directed search to highlight the importance of workers savings and borrowings in setting optimal unemployment insurance. In Figure (8), the horizontal axis is the workers age and the vertical axis is their current wealth

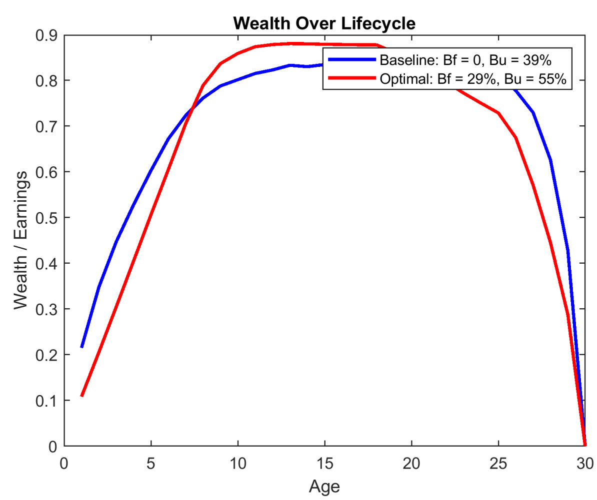

Figure 8: Wealth Distribution Over Lifecycle

divided by their earnings. The workers in general are saving a lot, with even the lowest percentile of the distribution have a wealth almost two times her earnings in the baseline economy. In the baseline and optimal wealth distributions, workers build up precautionary savings early in life and deplete their wealth towards the end of their life. Workers build up their precautionary savings faster in the optimal equilibrium and deplete their savings faster. This is because while firm transfers do not completely eliminate endogenous separations, they do allow workers to maintain their high wage jobs and ultimately allow them to save more. While I realize that workers are saving almost all of their wealth in their prime age of working, this is all subject to future calibration.

In Figure (9), equilibrium earnings over the lifecycle is shown. Both of the earnings are normalized to earnings at Age 1 in the baseline version of the model. Total earnings are

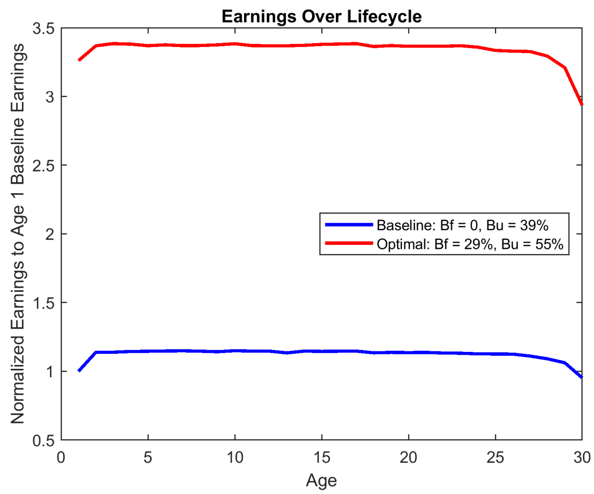

Figure 9: Earnings Distribution

about 3x higher in the optimal policy version of the model. This is coming from the fact that transfers to firms allow high wage jobs to survive a low idiosyncratic productivity shock. Before the firm transfer is turned on, According to the consumption equivalence calculated in Columns 1 & 2 of Table 7, the risk averse workers would rather pay a higher tax rate to eliminate unemployment risk and earn a higher wage.

In Figure (10), I plot the consumption around job loss. In the baseline version of the model, consumption drops by about 7.5% upon job loss and never fully recovers even 2 years after job loss. Ganong and Noel (2019) estimate that spending on non-durables drops by about 6.4% upon job loss. This serves as one way to validate my model, as it can almost match the drop in consumption around job loss in the data. The main intuition for my model is found in the optimal policy regime here. Consumption upon job loss is higher with the firm transfer and increased unemployment insurance. This is because workers are applying for higher wage jobs and due to the firm transfer, they keep those jobs. While consumption does not fully recover after 2 years of job loss, I find that firms transfers can insure consumption at longer horizons as the change in consumption prior to job loss and 2 years out is much

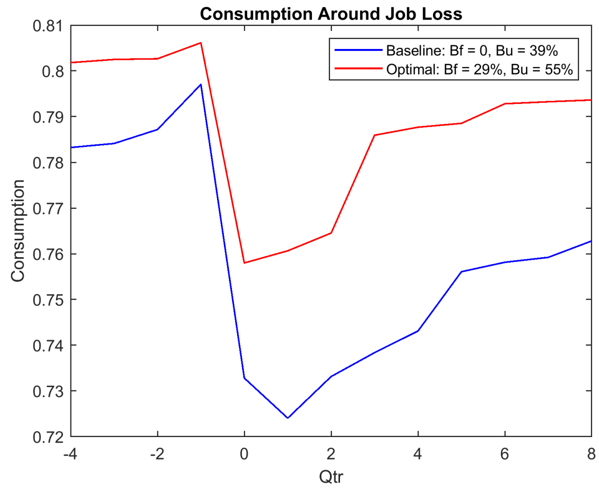

Figure 10: Consumption Around Job Loss

smaller in this economy regime.

7.6 Quantitative Extension: Fully Rigid Wage Contracts

7.6.1 Bellman Equations

This section explains the Bellman Equations of the workers and firms. The value functions below are described in the middle of the period, workers have realized their separation shocks and going to make their consumption decisions.

Value of Match to Firms - no b_f

The firm produces $f(z,h)$ with their worker today and pays them a rigid wage contract, w. Tomorrow, they take a new productivity draw, z' that determines if they will stay together with their worker. If they draw a productivity such that the present value of the match is positive, then they remain together and pay their worker, fraction of output $\alpha f(z,h)$ and have match productivity z'. If the present value of the match is negative tomorrow, then the firm makes a layoff decision. In this scenario, the firm exits and gets 0.

The value of the match to the firm with a worker paying them fixed wage contract w is given as:

$$J(w,z,h) = f(z,h) - w + \beta \cdot \mathbb{E}_{z'|z}\left[\mathbb{1}_{z' \geq \underline{z}(w,h)}\{J(w,z',h)\} + \mathbb{1}_{z' < \underline{z}(w,h)}\{0\}\right]$$

$$\underline{z}(w,h) \text{ solves } J(w,\underline{z}(w,h),h) = 0$$
$$\mathbb{1}_{z' \geq \underline{z}(w,h)} = \mathbb{1}\{J(w,z) \geq 0\}, \quad \mathbb{1}_{z' < \underline{z}(w,h)} = \mathbb{1}\{J(w,z,h) < 0\}$$

Value of Match to Firms - with b_f

Below gives the value of the match to the firm with the introduction of b_f. This section describes how a firm can become eligible to receive the transfer and what is the value of the firm transfer. The firm transfer is perfectly targetted, meaning that the value of b_f is how much the firm needs to exactly break even and this it is a function of all the current state variables to the firm:

$$b_f(w,z,h) = \begin{cases} |J(w,z,h)| & \text{if } J(w,z,h) < 0 \\ \\ 0 & \text{if } J(w,z,h) \geq 0 \end{cases}$$
$$J_{b_f}(w,z,h) = \begin{cases} J(w,z,h) + b_f(w,z,h) & \text{if } J(w,z,h) < 0 \\ \\ J(w,z,h) & \text{if } J(w,z,h) \geq 0 \end{cases}$$

$$\underline{z_{b_f}}(w,h) \text{ solves } J_{b_f}(w,\underline{z_{b_f}}(w,h),h) = 0$$

According to the value functions above, every match remains and there are no separations. However, in Section 6.3, I solve for the optimal quality of firm-worker matches that should be implemented to enhance welfare.

Decisions for Unemployed Workers

An unemployed worker wakes up today with human capital level h and consumes public benefit, b_u. Tomorrow, they wake up and draw their new level of human capital, $h' \sim H_u(h'|h)$. Then, knowing their new level of human capital, they direct their search for a fixed wage contract, w'. They match with the firm with probability $p(\theta(w',h'))$. With probability $1-p(\theta(w',h'))$ they do not match with a firm and remain unemployed with value $U(h')$. Once matched with a firm, the are subject to the match productivity shock that the firm draws from the ergodic distribution, $\tilde{z} \sim G(z)$. If they survive the productivity shock, they consume wage w' and have match value $W(w',\tilde{z},h')$. If they do not survive the productivity shock, then they got back into unemployment with value $U(h')$ and consume the unemployment benefit b_u. The value of searching for an unemployed worker when they wake up today is:

$$U(h) = b_u + \beta \cdot \mathbb{E}_{h'|h} \max_{w'} \underbrace{(1 - p(\theta(w', h')))U(h')}_{\text{do not match}} + \underbrace{p(\theta(w', h'))\mathbb{E}_{\tilde{z}}\tilde{W}(w', \tilde{z}, h')}_{\text{match and face productivity shock}}$$

$$\tilde{W}(w', \tilde{z}, h') = \mathbb{1}_{\tilde{z} \geq \underline{z}(w', h')} W(w', \tilde{z}, h') + \mathbb{1}_{\tilde{z} < \underline{z}(w', h')} U(h')$$

and the law of motion for an unemployed worker human capital,

$$h' = \begin{cases} h & \text{w/ probability } (1 - \pi_w) \\ \\ h + \Delta_h & \text{w/ probability } \pi_w \end{cases}$$

On average, an unemployed worker will lose human capital.

Decisions for Employed Workers

An employed worker wakes up today with their fixed human capital level, h and consumes their constant wage w and pay taxes τ to the government to fund the two types of labor market insurance b_f, b_u. While employed, workers human capital on average increases and at the beginning of every period, they draw their new human capital level, h'. With probability π_w, they gain Δ_h amount of human capital. Tomorrow they draw a new human capital level, $h' \sim H_w(h'|h)$ and realize the opportunity λ_e to search on the job. With total probability $\lambda_e p(\theta(w, h))$ they match with a new firm and expect be paid new wage w'. With probability $(1 - \lambda_e p(\theta(w, h)))$ they do not realize the opportunity to search on the job and they remain at current firm. Now the employed worker face layoff risk from either a new firm or a current firm. The employed worker faces layoff risk at their current firm and their new firm The separation shocks are similar to that of an unemployed worker, however, the productivity distribution is conditional. The current firm draws new match productivity from the conditional distribution, $z' \sim G(z'|z)$. A newly formed firm draws productivity from the ergodic distribution $\tilde{z} \sim G(\tilde{z})$. The bellman for an employed worker is given as

$$W(w, z, h) = w(1 - \tau) + \beta \cdot \mathbb{E}_{h'|h} \max_{w'} \mathbb{E}_{z'|z} \underbrace{(1 - \lambda_e p(\theta(w', h')))\hat{W}(w, z', h')}_{\text{do not match w/ new firm}} + \mathbb{E}_{\tilde{z}} \underbrace{\lambda_e p(\theta(w', h'))\hat{W}(w', \tilde{z}, h')}_{\text{match w/ new firm}}$$

$$\hat{W}(w, z', h') = \mathbb{1}_{z' \geq \underline{z}(w,h')}\{W(w, z', h')\} + \mathbb{1}_{z' < \underline{z}(w,h')}\{U(h')\}$$
$$\hat{W}(w', \tilde{z}, h') = \mathbb{1}_{\tilde{z} \geq \underline{z}(w',h')}\{W(w', \tilde{z}, h')\} + \mathbb{1}_{\tilde{z} < \underline{z}(w',h')}\{U(h')\}$$

and the law of motion for the employed workers human capital

$$h' = \begin{cases} h & \text{w/ probability } \ (1 - \pi_w) \\ \\ h + \Delta_h & \text{w/ probability } \ \pi_w \end{cases}$$

On average, an employed worker will gain human capital. With some probability the employed worker who did not search on the job remains with their current firm and survives the low productivity shock. In this case, the worker continues on with wage w, match productivity z', and human capital level, h'. The worker gets unlucky and enters unemployment with value $U(h')$ if the firms draws $z < \underline{z}(w, h)$.

Define employed workers optimal wage search function, $w_e^{'}(w, z, h')$. A workers human capital influences their probability of matching. For example, consider a high wage and high human capital worker. Since high human capital workers are more attractive to the firm, firms will post many vacancies for these high human capital jobs. This increases the probability the worker will match with a high human capital firm. However, the probability of separation, which occurs when $(z' < \underline{z}(w, h))$ is increasing in the wage of the worker. For higher wage contracts, the total value of the firm is lower and thus is more susceptible to realizing a negative discounted present value from a negative idiosyncratic realization. In the world without firm transfers, $b_f(w, h, z)$, given to the firm, the worker internalizes this and applies to lower wage jobs as they prefer to be employed than unemployed.

Appendix C

7.7 Proof of Block Recursive Equilibrium

In this Appendix, I formally prove that the equilibrium admitted by this model is *Block Recursive* as in Menzio and Shi (2011). This means that the equilibrium of distribution of agents across states does not impact prices in the model. With an endogenous government tax rate, τ, the property of *Block Recursive* may not hold. This is because the tax rate need to balance the budget depends the amount of revenues collected from employed workers and how much of those revenues are spent on transfers to firms and unemployed workers. Workers search behavior changes for a given the level of τ.

For the purpose of the proof, suppose that τ is exogenously given and the government budget need not balance. In this case, the workers and firms treat τ as a parameter given to them in the model just like any other parameter value. Then the optimal decisions of workers and firms are independent of the aggregate distribution of agents across states, μ. To solve for the balanced budget, then iterate over different value of τ until the budget is balanced.

Proposition 1: *Suppose τ is exogenously given and the government budget need not balance. Assume that the utility function meets standard conditions ($u' > 0, u'' < 0, \lim_{c\to\infty} u'(c) = 0$ and u is invertible), the matching function is invertible and constant returns to scale, and there is a bounded support for the choice of piece-rates $\omega \in [0,1]$, then a Block Recursive Equilibrium Exists.*

Proof. The following proof is a direct proof done by construction.

1. First hypothesize that the value of a job filled, $J(\omega, z, h)$ is independent of μ.

2. Then the expected value of a job filled is computed by applying an expectation operator over $J(\omega, z, h)$. Hypothesize that expected value of a job filled $\mathbb{E}_{\tilde{z}}(J(\omega, \tilde{z}, h))$ is also independent of μ as the expectation of the ergodic distribution of productivity is exogenously given and independent of μ.

3. Then the competitive entry of firms into submarkets (ω, h) is independently determined from μ, making the market tightness in each submarket independent from μ as shown in Equation 11

4. Then as the matching probabilities $p(\theta(\omega, h)), q(\theta(\omega, h))$ are exogenous functions of θ, they are also independent of μ

5. Workers and firms can calculate their value functions given $p(\theta(\omega,h)), q(\theta(\omega,h))$ independently from μ

6. Optimal choices of wages and layoffs are independent of μ. As the piece-rate ω is on a bounded interval, the extreme value theorem guarantees at least one solution to the unemployed optimal search in 4 and the firms optimal layoff rule in 2. These optimal choices are solved independent of μ

7. Then the expected value of a job filled $\mathbb{E}_{\tilde{z}}(J(\omega,\tilde{z},h)))$ remains independent from μ

8. As the ergodic distribution of productivity is independent of μ, then it holds from the initial hypothesis in step (1) that $J(\omega,z,h)$ independent from μ. QED.

For a simpler version, consider the following mini proof. The free entry condition in each submarket that firms face states that the expected value of the job filled must be equal to the cost they pay to post in that submarket. Using the free entry condition, we can back out the market tightness, $\theta(\cdot)$ in each of these submarkets. Note that $\theta(\cdot)$ is not dependent on the aggregate distribution μ. Thus, the vacancy filling rates, $q(\theta(\cdot))$ and the job finding rates that workers face $p(\theta(\cdot))$ are also independently determined from μ. Then it must be that the workers policy functions are independent of μ.

Appendix D

The model is solved using value function iteration on discrete grids for piece-rates. Possible piece rates lie on grid [0.01, 0.99] with 100 grid points. Productivity lies on a grid [0.22, 4.39] where I use Tauchen (1986) to discretize the productivity process and there are 11 possible productivity states. Human capital lies on a grid [1, 10] which are evenly spaced. There are 25 possible human capital grid points. In the simulation to check if government budget is balanced, I simulate 10,00 individuals for 360 periods burning the first 120 iterations. Solving the model with balanced government budget satisfies the following steps:

1. **Taxes:** Guess τ.

2. **Firms Bellman:** Guess the value to a firm of job being filled (Equation 2)

3. **Endogenous Layoff Threshold**: Based off the guess for the firm bellman function, solve for the guess of the endogenous layoff threshold (Equation 3).

4. **Free entry**: Using the guess of the firm value, take expectation over productivity using the ergodic distribution for a given human capital level, h. Invert the free entry condition to obtain the market tightness $\theta(\omega,h)$ (Equation 11)

5. **Unemployed Wage Search:** Given the estimate of market tightness, $\theta(\omega, h)$ (Equation 11), solve the unemployed optimal wage search.

6. **Iterate Until Convergence to Fixed Point** Set update guesses equal to old guesses and run until convergence

   (a) Set updated guess for firm value equation equal to old guess (Equation 2)

   (b) Set updated guess for endogenous layoff threshold equal to old guess (Equation 3)

   (c) Set updated guess for value of unemployed worker equal to old guess (Equation 4)

   (d) Set updated guess for value function for employed worker equal to old guess (Equation 9)

7. **Budget Balance:** Check to see if budget balances (Equation 12) by simulating a large mass of individuals, if not update guess of τ using bisection. Run steps 1 - 7 until government spending = government revenue as shown in Equation 12.

Appendix E

Consumption equivalence: What fraction of consumption would a person in steady state of the baseline model be willing to pay (if positive) or have to be paid (if negative) in all future periods/states to achieve the allocation in the alternative policy model environment?

Define this fraction of consumption as λ. Denote the welfare from the alternative policy environment as W^{policy} and $W^{Baseline}$ as the welfare measure from the baseline version of the economy. The risk aversion of the agents is σ. Then define λ as one that solves:

$$W^{Policy} = \mathbb{E}\left(\sum_{t=0}^{\infty} \beta^t \frac{[(1+\lambda)W^{Baseline}]^{1-\sigma} - 1}{1-\sigma}\right)$$

Then solving the above equation for λ yields:

$$\lambda = \left(\frac{W^{Policy} + \frac{1}{(1-\sigma)(1-\beta)}}{W^{Baseline} + \frac{1}{(1-\sigma)(1-\beta)}}\right)^{\frac{1}{1-\sigma}} - 1$$

The formal derivation is given below.

$$
\begin{aligned}
W^{Policy} &= (1+\lambda)^{1-\sigma}\mathbb{E}\left(\sum_{t=0}^{\infty}\beta^t\frac{[W^{Baseline}]^{1-\sigma}-1}{1-\sigma}\right)\\
&= (1+\lambda)^{1-\sigma}\mathbb{E}\left(\sum_{t=0}^{\infty}\beta^t\frac{(W^{Baseline})^{1-\sigma}}{1-\sigma}-\sum_{t=0}^{\infty}\frac{\beta^t}{1-\sigma}\right)\\
&= (1+\lambda)^{1-\sigma}\mathbb{E}\left(\sum_{t=0}^{\infty}\beta^t\frac{(W^{Baseline})^{1-\sigma}}{1-\sigma}\right)-\frac{1}{(1-\sigma)(1-\beta)}\\
&= (1+\lambda)^{1-\sigma}\mathbb{E}\left(\sum_{t=0}^{\infty}\beta^t\frac{(W^{Baseline})^{1-\sigma}}{1-\sigma}+\left(-\frac{1}{1-\alpha}+\frac{1}{1-\alpha}\right)\right)-\frac{1}{(1-\sigma)(1-\beta)}\\
&= (1+\lambda)^{1-\sigma}\mathbb{E}\left(\sum_{t=0}^{\infty}\beta^t\frac{[W^{Baseline}]^{1-\sigma}-1}{1-\sigma}+\left(\frac{1}{1-\sigma}\right)\right)-\frac{1}{(1-\sigma)(1-\beta)}\\
&= (1+\lambda)^{1-\sigma}\mathbb{E}\left(\sum_{t=0}^{\infty}\beta^t\frac{[W^{Baseline}]^{1-\sigma}-1}{1-\sigma}+\left(\sum_{t=0}^{\infty}\beta^t\frac{1}{1-\sigma}\right)\right)-\frac{1}{(1-\sigma)(1-\beta)}\\
&= (1+\lambda)^{1-\sigma}\left(W^{Baseline}+\frac{1}{(1-\sigma)(1-\beta)}\right)-\frac{1}{(1-\sigma)(1-\beta)}\\
W^{Policy}+\frac{1}{(1-\sigma)(1-\beta)} &= (1+\lambda)^{1-\sigma}\left(W^{Baseline}+\frac{1}{(1-\sigma)(1-\beta)}\right)\\
\left(W^{Policy}+\frac{1}{(1-\sigma)(1-\beta)}\right)^{\frac{1}{1-\sigma}} &= (1+\lambda)\left(W^{Baseline}+\frac{1}{(1-\sigma)(1-\beta)}\right)^{\frac{1}{1-\sigma}}\\
\left(\frac{W^{Policy}+\frac{1}{(1-\sigma)(1-\beta)}}{W^{Baseline}+\frac{1}{(1-\sigma)(1-\beta)}}\right)^{\frac{1}{1-\sigma}} &= (1+\lambda)\\
\implies \lambda &= \left(\frac{W^{Policy}+\frac{1}{(1-\sigma)(1-\beta)}}{W^{Baseline}+\frac{1}{(1-\sigma)(1-\beta)}}\right)^{\frac{1}{1-\sigma}}-1
\end{aligned}
$$

Where W^{Policy} is the welfare of the agents in the baseline model and $W^{Baseline}$ is the welfare measure of agents in the alternative policy world.