In the Wrong Hands: Complementarities, Resource Allocation, and TFP

By Simeon D. Alder*

I explore mismatch between firms and their managers as a source of variation in aggregate output and total factor productivity (TFP). The model is calibrated to match observations on the size distribution of U.S. manufacturing firms, managerial compensation, and aggregate moments in the national accounts. Quantitatively, small deviations from assortative matching can have sizeable effects on output and TFP. "Cronyism", where managerial positions are allocated by status rather than talent, imposes a substantial burden on economic welfare. Moreover, the model can reconcile the seemingly contradictory evidence from numerous case studies with results from recent contributions to the assignment literature.
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One of the most striking facts in macroeconomics is the variation of income per capita across countries. Recent work suggests that the lion's share in these differences can be explained by cross-country variations in total factor productivity (TFP) rather than variations in human and physical capital accumulation.[1] Building on work by Restuccia and Rogerson (2008) and Hsieh and Klenow (2009), a recent literature emphasizes misallocation as a potential explanation for low TFP. The aim of this paper is to quantitatively explore one particular form of misallocation, namely mismatch between the attributes of projects and the people who run them, as a source of aggregate productivity losses.

To explore this question I embed the assignment problem of Terviö (2008) into an otherwise standard Lucas span-of-control model. The economy is populated by heterogeneous managers and projects, and a firm consists of a matched manager-project pair. They match in a frictionless market and the joint characteristics of the pair determine the firm's span of control. Decreasing returns to scale in capital and labor ensure that the competitive equilibrium exhibits a non-degenerate

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[1]Recent contributions to this literature include, among many others: Klenow and Rodriguez-Clare (1997); Prescott (1998); Hall and Jones (1999); Howitt (2000); Restuccia, Yang and Zhu (2008); Jones (2011).

distribution of firm sizes. The equilibrium features perfect sorting between the managers' and the projects' quality and I consider the quantitative effect of assignment frictions on aggregate output and TFP.

I calibrate the model to match salient features of U.S. manufacturing data on the size distribution of firms, the allocation of rents to managers and the owners of the non-reproducible project attributes, labor's income share, and the capital-output ratio. I then use the parameterized model to examine the role of mismatch on aggregate TFP. In a first set of counterfactual experiments, I quantify the effects of departures from perfect sorting in a way that allows for matching frictions to be correlated with project qualities. I then introduce idiosyncratic output distortions alongside non-assortative matching in order to explore how the two sources of misallocation interact with one another. Lastly, I connect the model to a sizeable case study literature using a simple microeconomic experiment of executive shake-up.

I find that matching frictions alone can lower TFP by almost 20 percent. Moreover, when these frictions are positively correlated with the quality of projects, even a fraction of randomly assigned managers as low as two percent can generate a productivity loss in excess of 10 percent. When, in addition to managerial matching frictions, the allocation of labor and capital inputs across firms is distorted by idiosyncratic output taxes and subsidies, I find that TFP falls by as much as 40 percent. The extent to which idiosyncratic distortions and managerial misallocation are correlated with project quality is quantitatively important. When the most productive 0.1 percent of all projects are run by incompetent "cronies" whose firms benefit from output subsidies, aggregate TFP can drop by as much as 20 percent.

The paper is motivated by a large literature on the importance of managers and management practices.[2] While the main focus of this case study literature is on the expertise of individual managers and their practices, this paper emphasizes the importance of assigning managers of different qualities to the "right" projects. Interestingly, recent structural models of CEO compensation that explore the effects of managerial misallocation find that it plays a limited role. Gabaix and Landier (2008) and Terviö (2008), for instance, find that top managers in publicly traded U.S. corporations are of very similar ability. Therefore, replacing CEOs of large U.S. corporations by an arbitrarily chosen peer hardly affects the market value of those firms and mismatch is an unlikely source of significant efficiency losses. However, two key assumptions bring about their results. First, the elasticity of substitution between the project's and the CEO's quality is set to unity. Second, since both models feature a partial equilibrium where the occupational choice is switched off, the participation constraint is an exogenous object. While I use the assignment mechanism in Terviö (2008) as a starting point, this paper

[2]Among the many contributions to the case study literature are: McKinsey Global Institute (1998), La Porta and de Silanes (1999), Garcia, Knights and Tilton (2001), Bebchuk and Cohen (2005), Cole et al. (2005), Bloom and Van Reenen (2007), Bennedsen, Pérez-González and Wolfenzon (2008), and Bloom et al. (2013).

advances the literature in two directions. To begin with, I endogenize the agents' occupational choice by embedding the differential rents model in a Lucas (1978) economy. Moreover, I admit non-multiplicative technologies in the calibration exercise. These modeling choices turn out to be important for both the calibration and the counterfactual experiments discussed earlier. Matching frictions entail large output and productivity losses and in this respect, the paper revisits the role of complementarities that was first explored in Kremer's *O-ring* paper (Kremer, 1993) and, more recently, in a supply chain model by Jones (2011). While the idea of "weak links" is at the heart of their models, I emphasize the effect of congestion externalities associated with matching frictions and they can be large when projects and managers are complements rather than substitutes.

The work of Restuccia and Rogerson (2008) and Hsieh and Klenow (2009) explores the aggregate effect of capital and labor misallocation by way of idiosyncratic factor or output price distortions. Here, I explore the interaction of idiosyncratic distortions with the misallocation of managerial talent through a formal mechanism that is ubiquitous in many countries: cronyism. Imagine, for instance, that output subsidies are used to compensate project owners for being run by a relatively incompetent "insider", then the effect on productivity is twofold. First, mismatch itself lowers the effective supply of project and managerial qualities. In addition, the idiosyncratic distortions associated with such a compensation scheme lead to a misallocation of capital and labor inputs across (already poorly matched) project-manager pairs and that, in turn, further depresses TFP. The results of the counterfactual experiments suggest that this may be an important mechanism. Admittedly, whether idiosyncratic distortions and matching frictions coincide in the data is still an open question and may be a promising area for future research.

In sum, this paper reconciles the seemingly contradictory results from the case study literature with those in Gabaix and Landier (2008) and Terviö (2008). In addition, the counterfactual experiments suggest the possibility of a close relationship between exogenous matching frictions and the idiosyncratic distortions emphasized by Restuccia and Rogerson (2008) and Hsieh and Klenow (2009) as sources of aggregate inefficiency.

The rest of this paper proceeds as follows. Section I sets up the benchmark model. I describe how latent project and manager attributes are inferred from observables and I define the competitive equilibrium. In section II, I describe the aggregate and firm-level data used in the empirics and estimate relevant model parameters. The model is calibrated to fit U.S. manufacturing data in section III. Matching frictions and the associated allocative distortions are discussed in section IV. Section V summarizes and concludes.

I. Economic Environment

This model combines a Lucas (1978) span-of-control economy with a differential rents problem as in Sattinger (1979, 1993), Gabaix and Landier (2008), and, most closely, Terviö (2008). Importantly, an individual's occupational choice between managing and working is endogenous, as is a prospective firm's choice to hire capital and labor inputs or to sit idle. The model generates predictions for the split of (competitive) rents between project owners and the CEOs that run them as well as for the entire distribution of firm sizes. While Gabaix and Landier (2008) and Terviö (2008) successfully characterize the former, they are silent on the latter. Conversely, the seminal span-of-control model in Lucas (1978) is a theory of the size distribution of firms, but assumes that the entrepreneur *is* the firm and hence appropriates all of its rents. Here, I take the view that the combination of a CEO's ability as well as the quality of the project she runs determines *(a)* the scale of the enterprise and *(b)* how the surplus is split. Unlike earlier contributions, I characterize the general equilibrium effects of assignment frictions and these will turn out to be quantitatively important.

A. *Population and Projects*

The model is populated by $\frac{1}{N}$ identical (family) households. Each household has a measure N of members and is endowed with NK units of physical capital. Summing across households, the economy has aggregate capital endowment of size K. Furthermore, each member is endowed with a single unit of labor and a, her quality as a manager. The distribution of a is denoted by F_a. In addition, each household owns N projects with quality q, the distribution of which is denoted by F_q. All households are endowed with a full support of managerial and project qualities. Members of each household choose between working and managing. If they choose the latter, their a is paired with a project of quality q. The manager and project characteristics are aggregated and jointly determine the span of control over capital and labor inputs, similar to Lucas (1978). All four inputs are traded in factor markets. As is standard in the literature, no transactions are carried out *within* a single household and a firm's factor inputs are supplied by distinct households.

B. *Preferences*

Households do not value leisure and order their preferences over consumption $\{C_t\}$ by $NU\left(\frac{C}{N}\right)$, where $U(\cdot)$ satisfies $U' > 0$, $U'' \leq 0$ and the Inada conditions $U'(0) = \infty$ and $U'(\infty) = 0$. Henceforth, let $c = \frac{C}{N}$. At every point in time, each of the N members of the household consumes an equal share of the household's aggregate consumption bundle C. The household sums the individual valuations of all its members.

C. *Technology and the Firm's Problem*

Firms produce the final good by combining managerial and project quality, a and q, respectively, with capital (k) and labor (ℓ) inputs. The production technology exhibits decreasing returns to scale with respect to ℓ and k, both individually and jointly. Output of an (a, q) firm is given by

$$y = f\big(a, q\big)^{1-\gamma}\big(k^{\alpha}\ell^{1-\alpha}\big)^{\gamma}, \tag{1}$$

where α and γ lie in $(0, 1)$. The project's owner and manager are joint residual claimants.

The owners of the project make two distinct production decisions: First, they select a manager who is compensated according to a wage profile $\omega(a, q)$, which, in general, depends on her own type a as well as the quality q of the project. An owner-manager team then hires capital and labor inputs at the competitive factor prices r and w, respectively. Given the common span-of-control γ, the idiosyncratic productivity $f(a, q)$ determines each firm's scale of operation in terms of factor inputs.

Before I proceed to the firm's problems, I need some additional notation. Given the nature of the assignment problem it turns out to be useful to assign labels to each project and manager. I assign a unique name i in $[0, 1]$ to each manager and j in $[0, 1]$ to each project. Then, $a[i]$ identifies the ability a associated with a particular name i and $q[j]$ does the same for projects.

I will first characterize the capital and labor demands of an arbitrary pair $(a[i], q[j])$. Once I have these factor demands, I describe the mechanism that assigns managers to projects.

Capital and Labor. — I ignore the possibility of incentive problems between managers and project owners. Instead, they maximize the joint surplus denoted by Φ. Each firm hires capital and labor to maximize:

$$\Phi\big(a[i], q[j]\big) = \max_{k\geq 0, \ell\geq 0} f(a[i], q[j])^{1-\gamma}\big(k^{\alpha}\ell^{1-\alpha}\big)^{\gamma} - rk - w\ell,$$

The first-order conditions of this concave problem are:

$$f(a[i], q[j])^{1-\gamma}\gamma\big(k^{\alpha}\ell^{1-\alpha}\big)^{\gamma-1}\alpha\big(\tfrac{k}{\ell}\big)^{\alpha-1} = r \tag{2}$$

$$f(a[i], q[j])^{1-\gamma}\gamma\big(k^{\alpha}\ell^{1-\alpha}\big)^{\gamma-1}(1-\alpha)\big(\tfrac{k}{\ell}\big)^{\alpha} = w \tag{3}$$

Dividing (2) by (3), one obtains the standard result of equalized capital-labor ratios across firms. Different (a, q) pairs simply differ in their *scale* of operation, but not their factor intensities. Moreover, it is straightforward to show that the factor demands are proportional to the firm's idiosyncratic productivity:

$$\ell(a[i], q[j]) \propto f(a[i], q[j]) \quad \text{and} \quad k(a[i], q[j]) \propto f(a[i], q[j]) \tag{4}$$

Assignment Problem. — The assignment problem in this model follows Terviö (2008) closely. While his characterization of stable (optimal) assignments is more general in many, if not most, respects I embed the problem in a Lucas (1978) occupational choice model, which enables me to characterize the effects of various matching frictions in general equilibrium. The general equilibrium channel will turn out to be qualitatively and quantitatively important.

Given that a project owner selects a manager endowed with a particular a rather than marginal units, this problem does not have standard first-order necessary conditions. Stability requires that the payoffs instead satisfy resource, sorting, and participation constraints as in Terviö (2008). It will be convenient to identify $(a[i], q[j])$ pairs and the corresponding payoffs by their names rather than their respective attributes. Accordingly, $\Phi[i,j]$, $\omega[i,j]$, and $\pi[i,j]$ denote the surplus, managerial pay, and compensation for project quality, respectively, associated with a particular $(a[i], q[j])$ pair. The firm's productivity, capital, and labor demands will be denoted analogously. For the remainder of the paper, I will refer to $\pi[i,j]$ as profits and, more generally, arguments in square brackets refer to labels while arguments in parentheses refer to the manager's and project's types.

Feasibility requires that:

$$\Phi[i,j] \geq \pi[i,j] + \omega[i,j] \tag{5}$$

The assignment problem's sorting constraints are:

$$\begin{aligned} \Phi[i,j] - \omega[i,j] &\geq \Phi[i',j] - \omega[i',j] && (6)\\ \Phi[i,j] - \pi[i,j] &\geq \Phi[i,j'] - \pi[i,j'] && (7) \end{aligned}$$

Equation (6) requires that project j prefers manager i over any alternative i' and, similarly, manager i prefers project j over any alternative j'. Finally, the participation constraints are:

$$\begin{aligned} \omega[i,j] &\geq w \\ \pi[i,j] &\geq v \end{aligned} \tag{8}$$

where w and v are the managers' and projects' outside options, respectively. While w is a general equilibrium object, v is an exogenous parameter of the model.

If $\Phi(a,q)$ is supermodular, then the stable matching of a and q is *positive assortative* (see Becker, 1973, among others). For the projects and managers who participate in this assignment market the stable match is a one-to-one correspondence from i to j. The following proposition provides a sufficient condition for the supermodularity and differentiability of Φ.

PROPOSITION 1: *Let Ω be the subset of $[0,1] \times [0,1]$ whose (i,j) elements satisfy the sorting constraints* (6) *and* (7) *and the participation constraints in* (8).

Then, for all (i, j) in Ω, $\Phi\big(a[i], q[j]\big)$ is supermodular and continuously differentiable if $f\big(a[i], q[j]\big)$ is supermodular and continuously differentiable.

The proof is in the appendix. For the remainder of the paper, I assume that f is both supermodular and continuously differentiable. The former implies that the matching of managers to projects is positive assortative and there exists a one-to-one relabeling from (i, j) to $\imath$ such that $\Phi[\imath, \imath]$ is increasing in $\imath$. Let $\bar{\imath}$ be the label for the marginal match that satisfies (8) with equality. Differentiability is a useful assumption when the attributes follow continuous distributions.

LEMMA 1: *For given outside options w and v, supermodular and continuously differentiable $\Phi(a, q)$, and continuously distributed a and q types, the profiles of the managers' compensation $\omega[\imath, \imath]$ and the projects' profits $\pi[\imath, \imath]$ are characterized by the following system of differential equations:*

$$\omega'[\imath, \imath] = \frac{\partial \Phi(a[\imath], q[\imath])}{\partial a} a'[\imath] \tag{9}$$

$$\pi'[\imath, \imath] = \frac{\partial \Phi(a[\imath], q[\imath])}{\partial q} q'[\imath] \tag{10}$$

with initial value condition:

$$\Phi(a[\bar{\imath}], q[\bar{\imath}]) = w + v \tag{11}$$

Since the conditions for assortative matching are satisfied, $a[\imath] \equiv F_a^{-1}(\imath)$ and $q[\imath] \equiv F_q^{-1}(\imath)$ are the inverse CDFs of the managers' and the projects' types. The corresponding partial derivatives are denoted by $a'[\imath]$ and $q'[\imath]$, respectively. Equations (9) and (10) are derived from the sorting constraints (6) and (7). Substitute $\imath'$ by $\imath - \epsilon$ in equation (6), rearrange, and divide both sides of the inequality by ϵ to obtain:

$$\frac{\Phi\big(a[\imath], q[\imath]\big) - \Phi\big(a[\imath - \epsilon], q[\imath]\big)}{\epsilon} \geq \frac{\omega[\imath, \imath] - \omega[\imath - \epsilon, \imath]}{\epsilon}$$

In the limit where ϵ approaches zero, the expression yields equation (9). Proceeding analogously for the manager's sorting constraint yields equation (10). The slope of the wage profile at $\imath$ is determined by the product of manager $a[\imath]$'s marginal contribution to the joint surplus $\left(\frac{\partial \Phi(a[\imath], q[\imath])}{\partial a}\right)$ and the slope of the ability distribution $(a'[\imath])$ at "location" $\imath$. The interpretation of manager $\imath$'s marginal product is standard. All else equal, managers who make a bigger marginal contribution to the surplus Φ can command a steeper wage increase. The slope of the ability distribution, on the other hand, reveals how close potential substitutes are in terms of their ability. When $a'[\imath]$ is positive, an alternative manager ranked just below, say at $\imath - \epsilon$, is less competent than $a[\imath]$. In contrast, when $a'[\imath] = 0$, then $a[\imath - \epsilon]$ is of the same type and hence a perfect substitute. The availability (or dearth) of close substitutes determines a manager's "bargaining power": managers who are unique can command a higher wage compared to managers surrounded by similar types. In the extreme $a'[\imath] = 0$ case, she commands no premium at all since local alternatives are of the same type and that implies $\omega'[\imath, \imath] = 0$. In sum, the slope of the wage profile is determined by two factors: the manager's marginal product and the abundance of comparable CEOs. Since the assignment problem is symmetric, the exact same intuition applies to equation (10) that governs the slope of the projects' profit profile.[3]

Output of the marginal firm $\bar{\imath}$ is such that, in general equilibrium, the marginal manager and project are paid their outside options w and v. Put differently, the competitive wage and the exogenous v satisfy the indifference condition:

$$w + v = \max_{k,\ell} f(a[\bar{\imath}], q[\bar{\imath}])^{1-\gamma} (k^{\alpha} \ell^{1-\alpha})^{\gamma} - rk - w\ell. \tag{12}$$

The payment profiles for managers and projects are, respectively,

$$\omega[\imath, \imath] = w + \int_{\bar{\imath}}^{\imath} \omega'[\jmath, \jmath] \mathrm{d}\jmath, \tag{13}$$

$$\pi[\imath, \imath] = v + \int_{\bar{\imath}}^{\imath} \pi'[\jmath, \jmath] \mathrm{d}\jmath. \tag{14}$$

The assumption of continuously distributed managerial abilities and project qualities simplifies the analysis. Differential rent problems of this type satisfy the no-surplus condition in Ostroy (1980, 1984) and hence meet the requirements of a competitive equilibrium. This eliminates the need for a bargaining protocol between projects and managers.[4]

Taking the payment profiles $\pi[\imath, \imath]$ and $\omega[\imath, \imath]$ as given (i.e., observable), equations (9) and (10) form a system of two ordinary differential equations in $a[\imath]$ and $q[\imath]$. The system's initial value condition (11) is a general equilibrium object in that $w + v$ must satisfy the indifference condition in equation (12). The assignment of CEOs to projects is static and permanent. Since projects who are not paired with a manager never produce any surplus, the option value of being unmatched is zero and setting $v = 0$ is a natural choice.[5]

D. Household's Problem

Recall that Ω is the subset of $[0, 1] \times [0, 1]$ that contains all matched managers and projects. Let $\mathbf{1}_{\Omega}(i, j)$ be a function that indicates whether (i, j) is in Ω. Then the household maximizes:

$$\max_{c} NU(c),$$

[3] Figure 1 in Terviö (2008) illustrates the intuition graphically.

[4] See a similar discussion in Terviö (2008). The standard references are Sattinger (1979, 1993).

[5] When $f(\cdot, \cdot)$ is Cobb-Douglas, this system of differential equations has an analytical solution. See Appendix B.

subject to the budget constraint:

$$(15)\quad c \leq rk + \int_0^1 \int_0^1 \big(\omega[i,j] + \pi[i,j]\big)\mathbf{1}_\Omega(i,j)\mathrm{d}i\mathrm{d}j + w\int_0^1 \int_0^1 \big(1 - \mathbf{1}_\Omega(i,j)\big)\mathrm{d}i\mathrm{d}j,$$

where $k = \frac{K}{N}$ is the capital stock per household member. Since the household has a family structure, there is no need to keep track of individual capital stocks; k is a sufficient summary statistic. The measure $\int_0^1 \int_0^1 \big(1 - \mathbf{1}_\Omega(i,j)\big)\mathrm{d}i\mathrm{d}j$ of household members who are not matched with a project in the assignment market supply their single unit of labor and are paid their outside option, the competitive wage rate w.

Note that in an equilibrium that features positive sorting, the household's budget constraint (15) effectively simplifies to:

$$c \leq rk + \int_{\bar{\imath}}^1 \big(\omega[\imath,\imath] + \pi[\imath,\imath]\big)\mathrm{d}\imath + w\bar{\imath}$$

E. Equilibrium

DEFINITION 1: *A positive assortative equilibrium consists of prices $\omega[\imath,\imath]$, $\pi[\imath,\imath]$, r, w, v, an occupational rank cutoff $\bar{\imath}$ (and hence a measure of active firms $1-\bar{\imath}$), per capita income c, factor inputs $k[\imath,\imath]$ and $\ell[\imath,\imath]$ such that, for given prices:*

1) *The households maximize utility subject to the budget constraint.*
2) *Each firm $\imath$ maximizes $\Phi[\imath,\imath]$.*
3) *The labor market clears:*
$$\bar{\imath} = \int_{\bar{\imath}}^1 \ell[j,j]\,dj$$
4) *The capital market clears:*
$$K = \int_{\bar{\imath}}^1 k[j,j]\,dj,$$
where K denotes the aggregate capital stock.
5) *Manager $\imath$ prefers project $\imath$ over any other assignment, given $\omega[\imath,\cdot]$.*
6) *The marginal manager (of rank $\bar{\imath}$) is indifferent between being hired as a manager and a worker:*
$$\omega[\bar{\imath},\bar{\imath}] = f(a[\bar{\imath}],q[\bar{\imath}])^{1-\gamma}\big(k[\bar{\imath},\bar{\imath}]^\alpha, \ell[\bar{\imath},\bar{\imath}]^{1-\alpha}\big)^\gamma - rk[\bar{\imath},\bar{\imath}] - w\ell[\bar{\imath},\bar{\imath}] - \pi[\bar{\imath},\bar{\imath}] = w$$
7) *Project $\imath$ prefers manager $\imath$ over any alternative, given $\pi[\cdot,\imath]$.*

One particularly useful property of the model is that neither the agents' occupational choice nor the firms' labor demands depend on the aggregate capital stock K.

LEMMA 2 (Occupational Choice): *The occupational cutoff $\bar{\imath}$ does not depend on the aggregate capital stock. In particular, $\bar{\imath}$ is characterized (implicitly) by*

$$\bar{\imath} = (1-\alpha)\frac{\gamma}{1-\gamma}\frac{\int_{\bar{\imath}}^{1} f\big(a[\imath], q[\imath]\big)\, d\imath}{f\big(a[\bar{\imath}], q[\bar{\imath}]\big)} \tag{16}$$

For a given distribution of types, a non-negative and strictly increasing $f(\cdot,\cdot)$, α, and γ, a unique $\bar{\imath}$ in $(0,1)$ satisfies (16). Trivially, the left hand side of the equation is increasing in $\bar{\imath}$. Since $f\big(a[\imath], q[\imath]\big)$ is non-negative and increasing in $\bar{\imath}$ and $\int_{\bar{\imath}}^{1} f\big(a[\imath], q[\imath]\big)\mathrm{d}\imath$ is non-negative and decreasing in $\bar{\imath}$, the right hand side is decreasing in $\bar{\imath}$. When $\bar{\imath} = 1$, the right hand side equals zero and that cannot satisfy (16). If, in addition, $\frac{\int_0^1 f\left(a[\imath],q[\imath]\right)\mathrm{d}\imath}{f\left(a[0],q[0]\right)} > 0$, $\bar{\imath} = 0$ cannot satisfy (16) and the solution must lie in $(0,1)$. Moreover, Lemma 2 implies that employment decisions do not depend on the capital stock either.

COROLLARY 1 (Labor Demand): *The firms' labor demands do not depend on K.*

The proof is in the appendix. Since the assignment of managers to projects and labor demands do not depend on the economy's capital endowment, we can easily embed this static model into an otherwise standard neoclassical economy.

F. *Endogenous Capital Accumulation*

In this environment, members of a household are infinitely lived and maximize their lifetime utility subject to a budget constraint and a law of motion for capital:

$$\max_{c_t, k_{t+1}} \sum_{t=0}^{\infty} \beta^t N U(c_t) \tag{17}$$

such that

$$\begin{aligned} c_t + x_t \leq & rk_t + \int_0^1 \int_0^1 \big(\omega[i,j] + \pi[i,j]\big)\mathbf{1}_{\Omega}(i,j)\mathrm{d}i\mathrm{d}j \\ & + w \int_0^1 \int_0^1 \big(1 - \mathbf{1}_{\Omega}(i,j)\big)\mathrm{d}i\mathrm{d}j \end{aligned} \tag{18}$$

$$x_t = k_{t+1} - (1-\delta)k_t, \tag{19}$$

where $x_t = \frac{X_t}{N}$ denotes investment per capita. The equilibrium of this dynamic economy consists of sequences of prices and quantities that satisfy *2.* through *8.* in the definition of the static equilibrium for each period t as well as the Euler equation associated with the household's problem in equations (17) - (19). Since capital accumulation is not particularly salient in the present context, I focus on the steady state of this economy and for the remainder of the paper the capital stock K is at the endogenous steady state level, unless noted otherwise.

II. Data

The distributions of managerial and project qualities are the fundamental building blocks of my model. Unfortunately, these are not directly observable and I instead rely on equations (4), (13), and (14) to parameterize the model. These equations characterize firm-level employment, occupational choices, as well as payments to managers and project owners, and I can use their empirical counterparts to back out the underlying distributions of a and q that are consistent with the data. In this section I describe the enterprise-level data from which I construct the empirical payment profiles corresponding to $\omega[\cdot,\cdot]$ and $\pi[\cdot,\cdot]$ in the model and the Census' Statistics of U.S. Businesses (SUSB) from which I compute the salient statistics of the distribution of firm sizes in U.S. manufacturing.

Companies that file with the U.S. Securities and Exchange Commission (SEC) are required to disclose information on executive compensation and submit audited financial statements in their annual proxy statements and on Form 10-K. *Compustat* aggregates these statements in two databases (*Compustat North America* and *ExecuComp*) and I select all firms with two-digit NAICS codes 31-33, that is, corporations whose main line of business is manufacturing.

The information on executive compensation goes as far back as 1992. Since coverage is scant in the first two years of the sample I only use data from 1994 to 2010. While I keep track of all senior executives, I pay particular attention to the level and composition of the CEOs' pay.[6] Salaries, bonuses, and items like 401(k) matching contributions or private use of corporate vehicles are all part of an executive's *current* compensation. They receive *deferred* compensation in the form of stock options or grants, typically with vesting restrictions.[7] They are deferred in the sense that firms do not incur the full resource cost at award time. In my calculations of the CEO's total *flow* compensation, I annuitize the value of her stock grants and options. Using an asset's fair value would be at odds with the model's mapping from the manager's contribution to current output (a) to her pay (ω). In the *Compustat* financial data, I keep track of dividends and capital gains to compute the flow value of the firm's payments (π) to the owners

[6] Firms report details for as many as 13 senior managers. 89 percent have information for at least five and virtually all of them have details for the top-four executives.

[7] Long-term incentive pay (LTIP) is current since the firm incurs the full resource cost immediately. Compensation, however, is for *past* contributions and, for that reason, I exclude it from my calculations. Since LTIP accounts for a small share of total pay, on average, the results are robust to including them.

of q.[8] Analogously to my treatment of stock options and grants, I annuitize the capital gains.

The *Compustat* sample of firms does not, of course, describe the entire size distribution of U.S. manufacturing firms. The sector consists of a large number of smaller firms that are not required to file with the SEC and I use a custom tabulation from the Census Bureau to characterize the entire employment size distribution as the empirical counterpart to equation (4). According to the Census, there were 306,303 manufacturing enterprises (and approximately 354,000 establishments) in 2000 and the average manufacturing firm employed 53 workers.[9] Figure 1 shows the distribution of manufacturing firm sizes in the U.S. in 2000 in log-log space. The plus signs plot the empirical distribution of employees in 44 size bins (the highest available "resolution" from the Census Bureau) and the solid line plots the Generalized Pareto distribution with the best fit. The maximum likelihood estimates for the shape and scale parameters are $\xi = 1.16$ and $\sigma = 6.39$, respectively.[10] My *Compustat* sample includes as many as 700 firms (in each year), corresponding to slightly less than 0.3 percent of the census universe. Accordingly, manager-project pairs above the 99.7$^{\text{th}}$ percentile of the size distribution are the model counterparts to the *Compustat* firms.

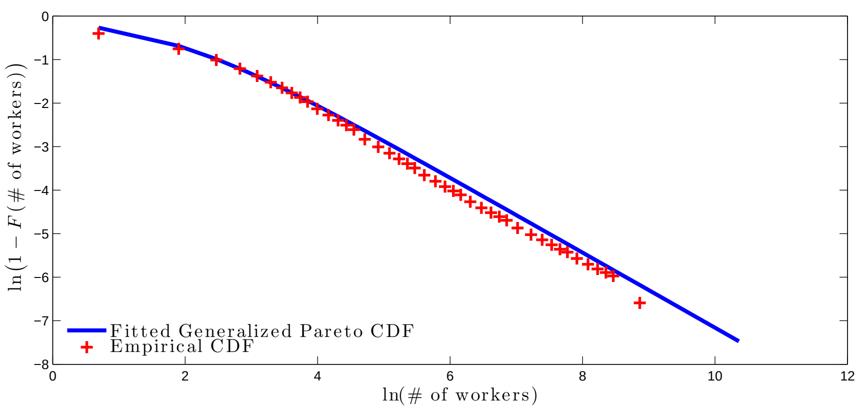

Figure 1. Distribution of U.S. Manufacturing Firms (2000)

[8]Terviö (2008) approaches the problem from a different angle and capitalizes all flow payments. My problem is essentially static and focuses on the manager's contemporaneous contribution to output.

[9]Mark Wright generously agreed to share the data. Firms and establishments are reported as having no employees if they have no one on payroll during the mid-March pay period, but with employees on payroll for at least one other pay period during the entire 2000 calendar year. According to the SUSB, total manufacturing employment was 16.4 million in 2000.

[10]The estimates are precise with standard errors of 0.004 and 0.026, respectively. Other processes have been used to describe U.S. firm sizes, among them the log-normal and standard Pareto distributions. Compared to Generalized Pareto distributions, the former tend to be too thin-tailed while the latter have tails that are too fat.

A. Flow Payments in the Data

Let $\Xi[i]_t$ denote firm i's profit after all factors of production have received their flow compensation, but prior to the distribution of financial assets.

$$\Xi[i]_t = y[i]_t - r_t k[i]_t - w_t \ell[i]_t - \omega[i]_{F,t} - \pi[i]_{F,t} \tag{20}$$

where it is understood that in equilibrium project i is of quality $q[i]$ and run by a manager of ability $a[i]$. $y[i]$ is firm i's output net of investment. $\omega[i]_{F,t}$ and $\pi[i]_{F,t}$ denote *current* payments – F for "flow" – to managers and owners of project quality, respectively.

Let $V[.]$ denote the firm's *ex dividend* market value net of fixed assets:

$$V[i]_t = \sum_{s=t}^{T} (1+r_s)^{-(s-t)} \Xi[i]_s \tag{21}$$

Note that the firm's market value is net of the physical capital stock. In the data, I compute $V[i]_t$ by subtracting tangible assets (at book value) from the firm's market capitalization. Owners and managers claim shares of $V[i]_t$ and receive them as stock options, grants, or in the form of capital gains.[11] Clearly, the asset value of these claims, denoted by $\omega[i]_{V,t}$ and $\pi[i]_{V,t}$ must satisfy:

$$V[i]_t = \omega[i]_{V,t} + \pi[i]_{V,t}.$$

The flow cost to the firm of compensating manager a and owner $q[i]$ with such claims at the end of the period is the amortization payment $\omega[i]_{A,t}$ and $\pi[i]_{A,t}$ of an annuity with present value $\omega[i]_{V,t}$ and $\pi[i]_{V,t}$, respectively:

$$\omega[i]_{V,t} = \sum_{s=t}^{T} (1+r_s)^{-(s-t)} \omega[i]_{A,s} \tag{22}$$

$$\pi[i]_{V,t} = V[i]_t - \omega[i]_{V,t} = \sum_{s=t}^{T} (1+r_s)^{-(s-t)} \pi[i]_{A,s} \tag{23}$$

The present value of options is the Black-Scholes value at grant date. In its Black-Scholes calculations, *ExecuComp* recognizes that options have vesting restrictions and assumes that they have an average maturity of seven years. I therefore set $T = 7$ in equation (22). In equation (23), on the other hand, I assume $T \to \infty$ since firms are infinitely-lived.

Let r_t^T be the annual yield to maturity of risk-free Treasury securities issued at

[11] Current profits are distributed to owners by means of dividends and to managers through bonuses. Since these are current payments, we need not worry about them here.

time t with maturity $T-t$. Then the corresponding annuity values are:[12]

$$\begin{aligned}\omega[i]_{A,t} &= r_t^T\Big(1-(1+r_t^T)^{-(T-t)}\Big)^{-1}\omega[i]_{V,t}\\ \pi[i]_{A,t} &= r_t^T\Big(1-(1+r_t^T)^{-(T-t)}\Big)^{-1}\pi[i]_{V,t}\end{aligned}$$

The sum of these annuity values and the flow payments $\omega[i]_{F,t}$ and $\pi[i]_{F,t}$ from equation (20) are the empirical counterparts to (13) and (14):

$$\begin{aligned}\omega[i]_t &= \omega[i]_{F,t}+\omega[i]_{A,t}\\ \pi[i]_t &= \pi[i]_{F,t}+\pi[i]_{A,t}\end{aligned}$$

Figure 2 plots the cross-sectional mean and median for $\omega[i]_t$ in million USD in the *Compustat* sample between 1994 and 2010. Mean CEO compensation grows at 2.3 percent per year, on average, while median pay increases at a rate of almost 2.5 percent. Executive compensation grows at a particularly healthy clip between 1994 and 2005 (8.7 percent and 6.0 percent for mean and median compensation, on average). There is a noticeable level adjustment around 2006 and the growth rates are more muted thereafter. Our measure of firm profits ($\pi[i]_t$) broadly follows the executive trends over time. Throughout the sample, the cross-sectional variation of CEO pay and profits follows the same pattern: a one dollar increase in profits is associated with a 25 cent increase in executive compensation. Table 1 summarizes the cross-sectional dispersion of CEO pay for select years during the time period. The dispersion is high throughout the sample period, particularly so around 2000, and executive pay is strongly right-skewed ("superstar" effect).

B. Estimation of Wage and Profit Profiles

Under the assumption that firms produce a single homogeneous final good and provided that $f(a,q)$ is supermodular, the decentralized equilibrium features positive sorting. The one-to-one mapping from types to rents in equations (9), (10), (13), and (14) implies unit rank correlation of payments between managers (ω) and firms (π). In the data, however, the year-by-year Spearman rank correlation between total flow payments to the CEO and payments to the firm fluctuates between 0.61 (in 1999) and 0.75 (in 2007).[13] There are, of course, several can-

[12] The U.S. government never issued consols. I therefore discount the perpetuities by the annual yield of 30-year bonds or by the longest available maturity when the U.S. government suspended issuance of long bonds between 2002 and 2006.

[13] Firms with negative book value (per share) – `bkvlps` in *Compustat* – are excluded from the sample. If I sum the compensation of the five most senior executives, the rank correlation rises to between 0.67 (1999) and 0.80 (2007). Both the CEO and Top Five correlations are higher than those reported in Terviö (2008). He capitalizes payments to the firm and payments to the CEO are taken directly from the *ExecuComp* database (variable `tdc1`). This variable lumps together current (cash or in kind)

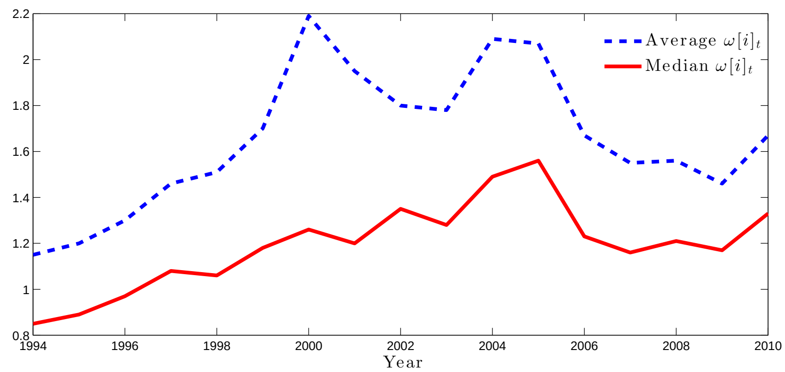

Figure 2. CEO Compensation in U.S. Manufacturing

Table 1—Cross-Sectional Dispersion of CEO Compensation

Year	Standard Deviation	Coefficient Variation	$90^{th}/10^{th}$ Percentile	$95^{th}/5^{th}$ Percentile	$99^{th}/1^{st}$ Percentile
1994	0.99	0.86	6.42	9.75	28.09
1996	1.07	0.83	6.92	11.51	30.62
1998	1.97	1.31	6.52	11.18	36.83
2000	4.79	2.19	9.63	16.98	56.62
2002	1.69	0.94	7.81	13.86	36.37
2004	1.82	0.87	7.75	14.35	40.07
2006	1.41	0.84	6.48	11.06	29.30
2008	1.40	0.89	5.28	9.21	66.43
2010	1.25	0.75	5.39	8.96	25.41
Average 1994-2010	1.76	1.03	6.90	11.90	38.41

didate explanations for deviations from perfect sorting. Search and matching frictions come to mind immediately. In addition, the heterogeneity across managers and firms is one-dimensional in the model and this may not fully capture the many factors on which top management's impact on firm productivity depends. Bertrand and Schoar (2003), for instance, describe the effects of different management styles on firm performance. Similarly, Bloom and Van Reenen (2007) and Bloom et al. (2007) document multiple dimensions of management ability that affect profitability, productivity, and survival rates.[14]

While the rank correlation is a sufficient statistic for sorting, the data suggests

compensation and asset transfers at fair value. Here, I construct a consistent flow measure of executive compensation.

[14]Theoretical models with multidimensional types typically do not exhibit sorting. Eisfeldt and Kuhnen (2013), for instance, generate imperfect assortative matching equilibria in a model with time-varying high-dimensional managerial skills. Lindenlaub (2014), Postel-Vinay and Lise (2014) and Alder, Meyer-ter Vehn and Ohanian (2014) are recent contributions to the literature on multidimensional sorting.

that there is more structure to the relationship between executive pay and firm profits and Figure 3 shows that it is approximately log-linear. According to Gabaix and Landier (2008), one of the "best documented empirical regularit[ies] regarding levels of executive compensation" is that CEO pay is proportional to firm size exponentiated by τ.[15] In honor of the original prediction the relationship is commonly called "Roberts' Law" (Roberts, 1956) and in Table 2, I report the coefficient estimates of the equation

$$\log(\omega[i]) = \alpha + \tau \log(\text{firm size}) + \beta X + \epsilon, \tag{24}$$

where X is a vector of industry fixed effects. I run the regression with two different proxies for firm size: (1) payments to q – which are denoted by π – and (2) the number of employees. For brevity, I only report the estimates for τ with (columns I and III) and without industry fixed effects (columns II and IV). The coefficients are robust to including industry dummies.

Table 2—Panel Regression of CEO Compensation on Firm Size

Independent Variable: log(CEO Compensation)				
Dependent Variable:	(I)	(II)	(III)	(IV)
$\log(\pi)$	.257	.253		
	(.009)	(.010)		
	(.006)	(.006)		
log(# of employees)			.304	.318
			(.005)	(.005)
			(.010)	(.009)
Industry Fixed Effects	No	Yes	No	Yes
N	8853	8853	8818	8818
R^2	.41	.43	.40	.44

Explanation: The panel covers manufacturing firms in *Compustat* between 1994 and 2010. Only firms with sufficient data on executive compensation are included. The first row of standard errors (in parentheses) is clustered by year. The second row of standard errors (in parentheses) is clustered at the firm level. The industry fixed effects are based on 3-digit NAICS codes.

III. Calibration

The aim of the calibration is to parameterize the economy to match aggregate and micro moments that characterize the U.S. manufacturing sector to their model counterparts.

Several parameters have analogues in a conventional neoclassical growth model and I choose their values using standard procedures. The depreciation rate δ is

[15] The original quote is from Baker, Jensen and Murphy (1988).

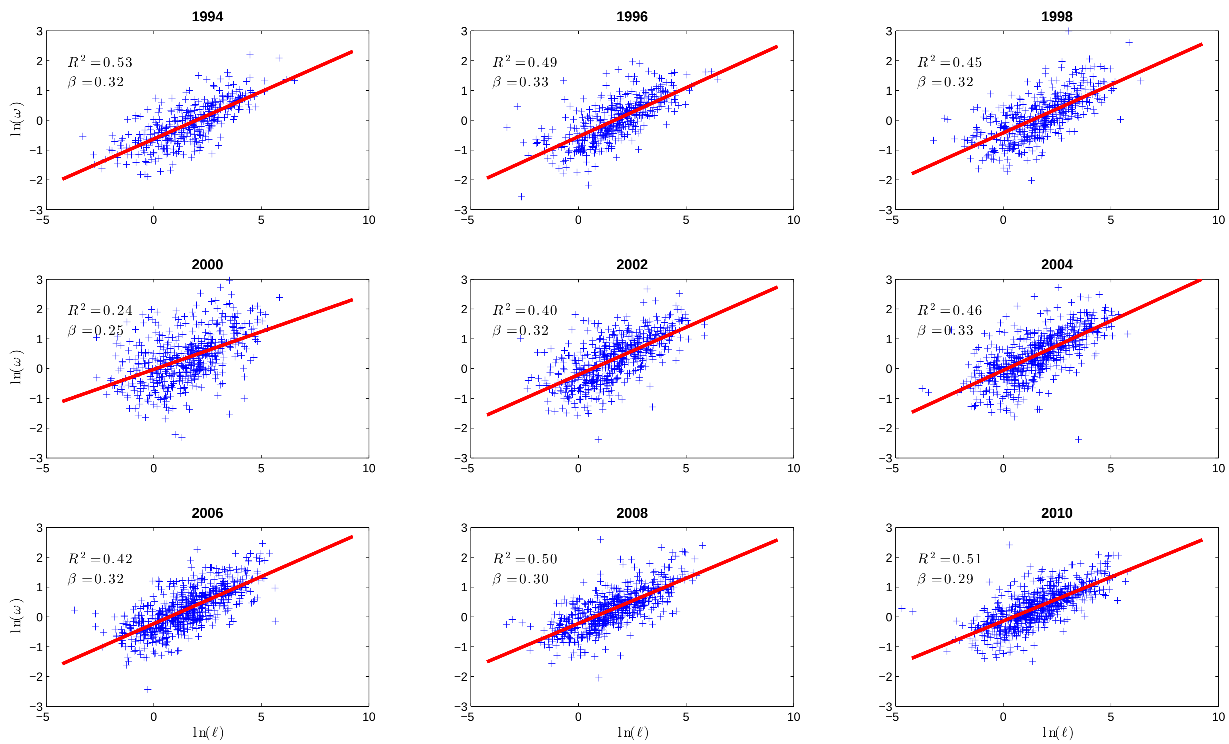

Figure 3. Roberts' Law for U.S. Manufacturing Firms

calibrated to match the average capital-output ratio for 1998-2005 in U.S. manufacturing. Based on the National Income and Product Accounts (NIPA), the average ratio for the period is 1.29. Gordon (1971) argues that the national accounts underestimate the price of capital and hence the capital-output ratio. I therefore follow Atkeson and Kehoe (2005) and assume a capital-output ratio of 1.46. The depreciation rate that matches the capital-output target is 12 percent.[16] I assume that a model period corresponds to one year in the data and the value of 0.96 for β is standard. Together, β and δ determine the real rental rate in the steady state. The extent of decreasing returns is an important parameter in the model and in line with the previous literature I set γ to 0.85 for the benchmark calibration (see Atkeson and Kehoe, 2005, for instance). Given γ, a value of 0.28 for α matches labor's average income share (including managerial compensation) in manufacturing between 1998 and 2005 (63 percent).

To match the cross-sectional micro moments of the firm size distribution, managerial compensation, and firm profits, I need to parameterize the technology $f(\cdot,\cdot)$ and the distributions of managerial talent and project productivities.

I assume that $f(a, q)$ is a standard CES function with parameters ν and ρ:

$$f(a,q) = \left(\nu a^{\rho} + (1-\nu)q^{\rho}\right)^{\frac{1}{\rho}} \tag{25}$$

The elasticity of substitution between a and q is given by $\frac{1}{1-\rho}$ and ν governs the relative importance of the manager's contribution to the firm's idiosyncratic productivity and hence to the surplus denoted by Φ.

Managerial abilities and productivities are drawn from unit-scale Generalized Pareto distributions with cumulative distribution functions:

$$i = G(x;\xi_x) = 1 - (1+\xi_x x)^{-\frac{1}{\xi_x}}, \text{ for } x \in \{a, q\}, \tag{26}$$

where ξ_x denotes the distributions' shape parameter. Note that the scale parameter for both distributions is set to unity and I will have more to say about this normalization shortly. The corresponding inverse CDFs are:

$$x[i] = \left((1-i)^{-\xi_x} - 1\right)\xi_x^{-1} \tag{27}$$

Numerical experiments suggest that when a and q follow Generalized Pareto distributions, the distribution of $f(a, q)$ will be closely approximated by a Generalized Pareto too. Since I know from section II that employment across U.S. manufacturing enterprises follows a Generalized Pareto, this parameterization is natural.[17]

[16]The depreciation rate is higher compared to the values used elsewhere, e.g. Restuccia and Rogerson (2008). However, I am targeting the capital-output ratio net of residential structures rather than the aggregate ratio, which is typically in the 2 to 2.5 range.

[17]See Rossi-Hansberg and Wright (2007) and Axtell (2001) for a detailed discussion.

As is standard in the literature, I use a calibration strategy that does not depend on the choice of the numeraire and is therefore invariant with respect to the scale of output, capital, and the factor price ratios $\frac{w}{r}$, $\frac{\omega[\cdot,\cdot]}{r}$, and $\frac{\pi[\cdot,\cdot]}{r}$.

The functional form restrictions in (25) and (26) may seem overly restrictive. In particular, what if Generalized Pareto distributions with non-unit scale parameters σ_a and σ_q and a more flexible specification of the firm's productivity as a function of a and q, namely $g(a,q) = \left(\nu_a a^{\rho_a} + \nu_q q^{\rho_q}\right)^{\frac{1}{\rho}}$, generated a better fit with the data? This concern turns out to be unwarranted. Thanks to two properties of the model, *scale* and *shape* invariance, the functional form restrictions in (25) and (26) are inconsequential.[18]

DEFINITION 2 (Scale Invariance): *An economy is scale invariant if for any K and parameterization $(\sigma_a, \sigma_q, \nu_a, \nu_q)$ there exists an alternative $(\widehat{\sigma}_a, \widehat{\sigma}_q, \widehat{\nu}, 1-\widehat{\nu})$ such that:*

1) *the occupational cutoff $\bar{\imath}$ and the firms' labor demands $\ell[i,i]$ are identical for $i \geq \bar{\imath}$; and*
2) *the prices for labor, capital, managerial abilities and project qualities are rescaled by a common factor $\Lambda\left(\frac{\widehat{\sigma}_a}{\sigma_a}, \frac{\widehat{\sigma}_q}{\sigma_q}\right)$.*

PROPOSITION 2 (Scale Invariance): *An economy characterized by the functional forms in* (25) *and* (26) *is scale invariant.*

The proof is in the appendix. The intuition is quite simple. The change in the scale of a and q, denoted by $\frac{\widehat{\sigma}_a}{\sigma_a}$ and $\frac{\widehat{\sigma}_q}{\sigma_q}$ respectively, can always be offset by a change of the share parameters to $\tilde{\nu}_a = \nu_a\left(\frac{\widehat{\sigma}_a}{\sigma_a}\right)^{-\rho}$ and $\tilde{\nu}_q = \nu_q\left(\frac{\widehat{\sigma}_q}{\sigma_q}\right)^{-\rho}$. In addition, there always exists a normalization $\widehat{\nu}_a = \frac{\tilde{\nu}_a}{\tilde{\nu}_a + \tilde{\nu}_q}$ and $\widehat{\nu}_q = 1 - \widehat{\nu}_a$, which is isomorphic to the original economy rescaled by $\Lambda\left(\frac{\widehat{\sigma}_a}{\sigma_a}, \frac{\widehat{\sigma}_q}{\sigma_q}\right) = \left(\widehat{\nu}_a + \widehat{\nu}_q\right)^{-\frac{1-\gamma}{\rho(1-\alpha\gamma)}}$. In particular, there exists a parameterization $(1, 1, \widehat{\nu}, 1-\widehat{\nu})$ that satisfies Definition 2.

The second property of interest is (asymptotic) shape invariance. For simplicity, I only consider the case with unit-scale Generalized Pareto distributions for abilities and productivities. According to Proposition 2 this restriction is without loss of generality.

DEFINITION 3 (Shape Invariance): *An economy is shape invariant if for any parameterization $(\xi_a, \xi_q, \rho_a, \rho_q)$ with $\xi_a > 0$ and $\xi_q > 0$ there exists an alternative $(\widehat{\xi}_a, \widehat{\xi}_q, \widehat{\rho}_a, \widehat{\rho}_q)$ such that the the occupational cutoff $\bar{\imath}$ and the firms' labor demands*

[18] The CDF of a random variable x that follows a GP distribution with scale parameter σ_x is $i = G(x;\xi_x) = 1 - (1 + \frac{\xi_x}{\sigma_x} x)^{-\frac{1}{\xi_x}}$.

$\ell[i,i]$ are identical for $i \geq \bar{\imath}$ across the two economies. An economy is asymptotically *shape invariant if the labor demand functions of firms indexed by $i \to 1$ are shape invariant.*

PROPOSITION 3 (Asymptotic Shape Invariance): *An economy characterized by the functional forms in* (25) *and* (26) *is asymptotically shape invariant.*

The proof is in the appendix. With functional forms as in (25) and (26), variations in labor demand by firm i that stem from a change in the tail indices (ξ_a and/or ξ_q) can be largely offset by adjustments of the curvature parameters ρ_a and/or ρ_q, provided i is sufficiently large. In the limit as the firms' indices i approach 1, labor demands are *exactly* invariant to changes in ξ_a and ξ_q since they can be fully offset by changes in ρ_a and ρ_q.[19] It follows immediately that the restriction to functions f with constant elasticity of substitution and returns to scale (i.e., $\rho_a = \rho_q = \rho$) is without loss of generality in the limit as i approaches 1. Since my calibration strategy is built around moments that are generated by the far right tail of the size distribution – roughly the top 0.3 percent of all manufacturing firms – the functional form restriction in (25) is mild. I am now in a position to parameterize the model.[20]

A. *Heuristic Calibration with* $\boldsymbol{\rho = 1}$

Even though it is somewhat uninteresting with respect to the sorting of managers and projects, it is still insightful to consider a specification of the model where the firm's idiosyncratic productivity is linear in a and q.[21] Since marginal products only depend on *own* types, the mapping from parameters to model moments is particularly transparent and I use this heuristic case to build some intuition for my calibration strategy. Two of my target moments involve payments to firms and CEOs in the far right tail of the firm size distribution and thus depend on properties of the underlying type distributions that prevail when i approaches unity. In particular, the tails of Generalized Pareto distributions have power law properties and this allows me to derive some intuitive limiting results for the special linear case.

With $\rho = 1$, *Roberts' Law*, the elasticity of executive compensation with respect to firm size, only depends on the ratio of shape parameters ξ_a and ξ_q. The intuition

[19] When ability and productivity follow lognormal or power law distributions, the economy is *exactly* rather than asymptotically shape invariant. I nonetheless prefer the functional form in (26) since it fits the empirical size distribution of firms more closely, especially in the upper tail.

[20] The interested reader may refer to León-Ledesma, McAdam and Willman (2010) for a similar discussion of identification problems in models with non-unitary CES functions and directed technological change. Similarly, Cantore and Levine (2012) discuss the importance of *normalization* or *reparameterization* in the context of an RBC model with a general CES production function.

[21] When $\rho = 1$, CEOs and projects whose marginal products satisfy the participation constraint are indifferent between all possible assignments. Since an arbitrarily small amount of curvature restores positive sorting, one can think of this exercise as an attempt to highlight the model properties as the substitution elasticity $\frac{1}{1-\rho}$ approaches infinity.

is that in log-log space, the slopes of the ability and productivity distributions asymptote constants as $i \to 1$ and the elasticity of pay with respect to firm size is given by the ratio of these two slopes. Formally, $\frac{\partial \ln \omega[i]}{\partial \ln \phi[i]} \xrightarrow{i \to 1} \frac{\xi_a}{\xi_q}$ and my calibration target is an elasticity of 0.31.

The two shape parameters also characterize the occupational cutoff $\bar{\imath}$ and hence the average firm size. Since $\frac{\partial^2 \left(\nu a[i]+(1-\nu)q[i]\right)}{\partial \xi_a \partial i}$ and $\frac{\partial^2 \left(\nu a[i]+(1-\nu)q[i]\right)}{\partial \xi_q \partial i}$ are strictly positive for any i in the open interval $(0,1)$, equation (16) implies that $\bar{\imath}$ is increasing in both ξ_a and ξ_q. Since only a particular ${}^{\xi_a}/_{\xi_q}$ satisfies *Roberts' Law*, the first two targets identify the two shape parameters, for a given ν.

Finally, given some ξ_a and ξ_q, the evolution of CEO's surplus share across the firm size distribution is governed by ν. Recall that the projects' outside option v is assumed to be zero and the marginal manager appropriates the entire surplus, that is, ${}^{\omega[\bar{\imath}]}/_{\phi[\bar{\imath}]} = 1$. The share parameter ν governs the rate at which the CEO's compensation share declines from unity: the lower ν, the sharper the decrease. More formally, one can show that $\partial \frac{\omega[i]}{\phi[i]} / \partial i \Big|_{\nu=1} = 0$ and $\frac{\partial \frac{\omega[i]}{\phi[i]} / \partial i \Big|_{i=\bar{\imath}}}{\partial \nu} > 0$. I select a value $1 > \nu \geq 0$ that matches the 7 percent target share for the subset of CEO-project pairs whose pay and profit profiles also satisfy *Roberts' Law*.

To summarize, I select a ratio of shape parameters ${}^{\xi_a}/_{\xi_q}$ that matches the 0.31 elasticity of pay with respect to firm size in the data. Conveniently, this model moment does not depend on ν. Next, for a given ν, the occupational cutoff condition (16) pins down the ξ_a (or ξ_q) that hits the average firm size target of 53 employees. Lastly, for a given ξ_a and ${}^{\xi_a}/_{\xi_q}$, I can identify the share parameter ν that matches the CEO's average surplus share.

In contrast, when $\rho < 1$, the CEOs' and projects' marginal products, the occupational threshold, and the CEOs' average share of the surplus are functions of all three parameters simultaneously. Due to the supermodularity of $f(\cdot,\cdot)$ and hence $\phi(\cdot,\cdot)$, the slope of the CEOs' pay profile is not just increasing in the tail shape of the ability distribution (ξ_a) but also in the shape of the productivity distribution (ξ_q). By the same token, the slope of the profit profile is increasing in both ξ_a and ξ_q. In these more general cases I jointly calibrate ξ_a, ξ_q, and ν to generate model moments that match the empirical counterparts.

B. Benchmark Calibration

In the benchmark calibration I set ρ to -1, which corresponds to a substitution elasticity of $\frac{1}{2}$. The parameter has not been pinned down securely elsewhere in the literature and I will show later that the effect of assignment frictions is sensitive to variations in ρ. Terviö (2008) and Gabaix and Landier (2008), for instance, use a unit-elasticity technology in the quantitative exploration of their assignment models and find that matching frictions hardly matter. Jones (2011) sets the substitution elasticity between intermediate goods to $\frac{1}{2}$ and reports robustness results for alternative values. Similarly, I will report results for several substitution elasticities on the Leontief and linear sides of Cobb-Douglas.

Table 3 lists the parameter values that match the three calibration targets in the benchmark case with $\rho = -1$ and in the illustrative linear case ($\rho = 1$). In the latter, the ratio ξ_a/ξ_q is indeed close to the target elasticity of 0.31 and the value of ν close to unity ensures that the CEOs' share of the match surplus drops gradually enough to match the 7 percent target for firms at the 99.7^{th} percentile and above. When ability and productivity are gross complements, the ratio ξ_a/ξ_q exceeds one, which is somewhat counterintuitive given that I am targeting a profit profile whose slope is steeper than the wage profile's. In contrast to the linear case, however, ν not only governs the CEO's share of the surplus but also affects the elasticity of her wage with respect to firm size and the drop in ν from almost 1 to 0.946 counteracts the relative rise in ξ_a.

Table 3—Benchmark Calibrations

Panel *A*: Standard Parameters

Parameter	Calibrated Value	Target
β	0.96	Real rate of return
δ	0.12	Capital-output ratio
γ	0.85	Atkeson and Kehoe (2005)
α	0.28	Income share of capital

Panel *B*: Additional Parameters

Parameter	Calibrated Value		Target
ρ	-1	1	
ξ_a	1.593	0.438	Average firm size
ξ_q	0.858	1.469	"Roberts' Law"
ν	0.946	0.999	Average CEO share

By construction, the parameter values in Table 3 generate model moments that match their targeted empirical counterparts. Those that I do not target, however, can vary across different values of ρ and Table 4 lists several higher moments of the distribution of wages and profits as well as firm size quantiles. In general, the model does not generate enough dispersion and skewness in CEO pay and profits compared to the data.[22] Moreover, the benchmark calibration generates a distribution of firm sizes with a right tail that is slightly too thin and with a censored left tail. While the marginal firm in the data has a single employee, the smallest firms in the model have four of them and, accordingly, the size percentiles for firms with just a few employees are slightly off. Figure 4 plots the distribution using all 44 size bins from the U.S. Census and illustrates the gap between the model (circles) and empirical (solid line) distributions. While the linear parameterization performs better with respect to the dispersion of profits and wages, it generates excess skewness and the size distribution of firms (crosses)

[22]It is worth noting, however, that the profit ratios are volatile in the data. This is particularly true for the $99^{\text{th}}/50^{\text{th}}$ ratio that ranges from a low of 68 in 1994 to a high of 216 in 2000 (from 58 to 157, respectively, when the the top and bottom 1 percent observations are eliminated).

is quite counterfactual.

Table 4—Additional Moments

	Model		Data	
	$\rho = -1$	$\rho = 1$	(all)	(trimmed)
CEO compensation:				
$90^{\text{th}}/10^{\text{th}}$ percentile	1.76	2.46	6.90	6.43
$99^{\text{th}}/1^{\text{st}}$ percentile	2.49	4.74	38.41	22.79
Coefficient of variation	0.25	0.47	1.03	0.75
Skewness	1.55	2.32	4.42	1.70
Profits:				
$90^{\text{th}}/50^{\text{th}}$ percentile	3.72	8.65	14.60	13.32
$99^{\text{th}}/50^{\text{th}}$ percentile	12.62	67.18	138.42	98.43
Coefficient of variation	1.24	2.70	3.33	2.69
Skewness	3.69	5.77	6.98	5.0
Fraction of firms with fewer than:				
5 employees	14.3	31.3	38.0	–
10 employees	50.2	80.9	56.5	–
25 employees	77.0	95.5	76.6	–
100 employees	93.8	99.0	93.2	–
500 employees	98.9	99.7	98.5	–
5,000 employees	99.9	99.9	99.8	–

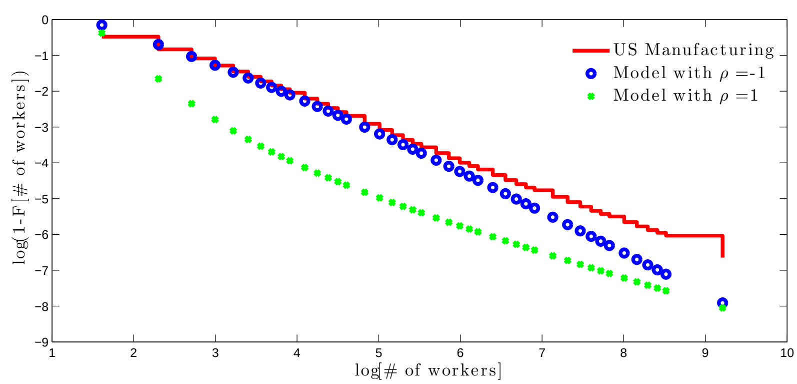

Figure 4. Distribution of Firm Sizes

C. Alternative Values for $\boldsymbol{\rho}$

While the *ExecuComp* data is not rich enough to identify the degree of complementarity between a and q, it is still worth exploring parameterizations for values of ρ that are different from the benchmark $\rho = -1$. Figure 5 plots the share parameter ν and the tail indices ξ_a and ξ_q that match the three empirical targets over a range of substitution elasticities. What stands out immediately is the discontinuity at $\frac{1}{1-\rho} = 1$. As the substitution elasticity approaches unity from below, the shape parameter of the *ability* distribution (ξ_a) rises while ν moves in the opposite direction. In contrast, as $\frac{1}{1-\rho}$ approaches one from above, the right tail of the *productivity* distribution becomes heavier whereas ξ_a drops below zero (suggesting that the support of the ability distribution has a finite upper bound). In the Cobb-Douglas limit, no model parameterization can match all three calibration targets since the power law properties in the far right tail of Generalized Pareto Distributions imply $\frac{\partial \ln \omega[i]}{\partial \ln \phi[i]} \xrightarrow{i \to 1} 1$ as long as $\nu\xi_a > 0$ or $(1-\nu)\xi_q > 0$.[23]

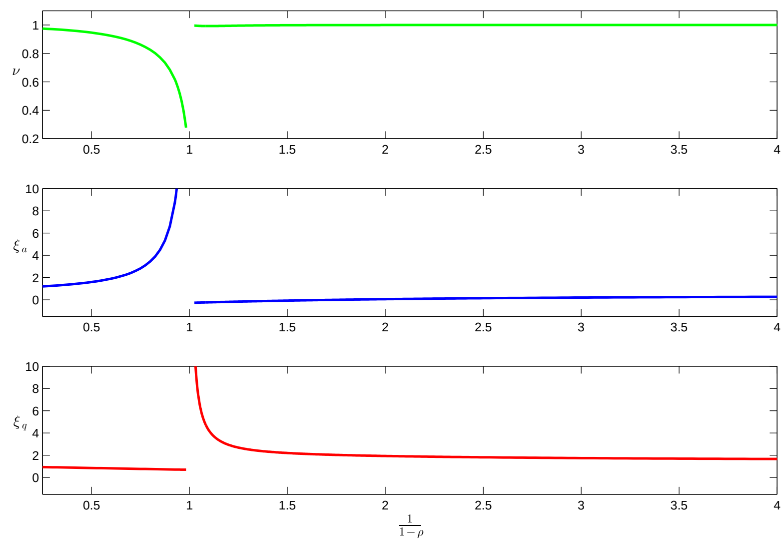

Figure 5. Targeted Parameters

Figure 6 plots the dispersion and skewness statistics for CEO compensation and profits that are also in Table 4 for the benchmark calibration. When $\frac{1}{1-\rho} > 1$, the model generates more dispersion and skewness in the wage and profit profiles, compared to parameterizations where ability and productivity are gross complements. While the measures of wage and profit dispersion on the linear side of

[23]The high value of ξ_a or ξ_q in the neighborhood of $\rho = 0$ poses a computational challenge. In particular, integrands involving the inverse CDF of fat-tailed distributions tend to be singular when the upper bound of the numerical integration approaches 1 and these singularities are ubiquitous as ρ approaches zero from above or below. Since, in addition, I cannot parameterize the model to match Roberts' Law when $\rho = 0$, Figure 5 only plots calibration results for ρ contained in $[0.25, 0.98]$ and $[1.02, 4]$. One could parameterize the model to match the two remaining targets. In this case, the shape invariance property discussed earlier implies that ν is not identified separately from ξ_a and that $1-\nu$ is not identified separately from ξ_q. Clearly, ν is a free parameter and the calibration would require a normalization.

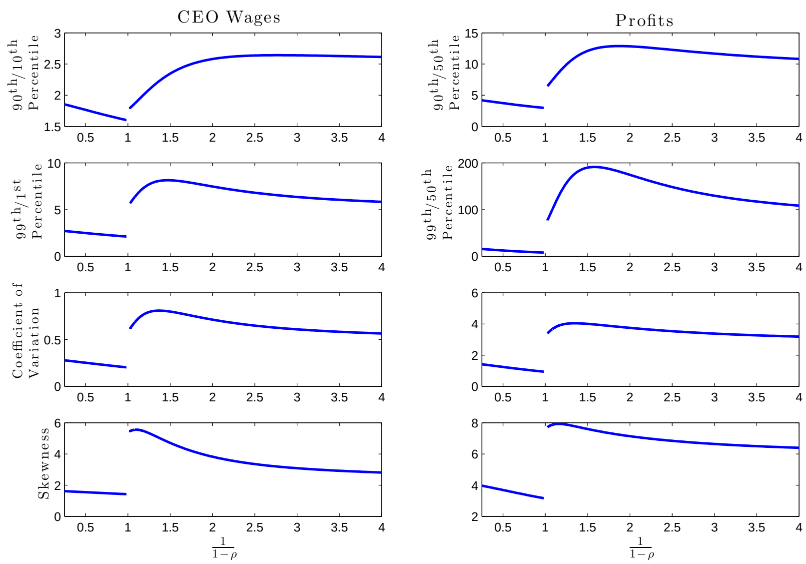

FIGURE 6. NON-TARGETED MOMENTS

Cobb-Douglas are in better agreement with the data, the skewness measures and firm size percentiles (not plotted) on the Leontief side are closer to their empirical counterparts. The non-targeted moments alone offer hardly any guidance in terms of pinpointing the empirically relevant substitution elasticity. This lack of identification merits further attention since it is closely related to a modeling choice I have not discussed thus far.

The model abstracts from executive turnover entirely and this choice reflects limitations imposed by the *ExecuComp* data. In principle, instances of CEOs that run different firms over the course of their careers and cases where firms hire chief executives from or loose them to other firms give rise to variations in managerial pay and profits that can be informative about the degree of complementarity between attributes of managers and those of projects. In practice, however, all except for a handful of turnover events in *ExecuComp* involve CEOs that enter or exit the sample and, for that reason, do not generate variations in pay or profits that identify the substitution elasticity between a and q.

The intuition for the link between wages and profits on one hand and the substitution elasticity on the other is straightforward. Models with equilibria characterized by perfect sorting imply a one-to-one correspondence between project and CEO types. By construction, this rules out the identification of the distribution of types separately from the technology that determines the match surplus. In contrast, imagine a model with an equilibrium where a particular CEO type is assigned to different projects (and vice versa) both in the cross section and over time. Then, given a particular CEO's history of assignments, variations in her marginal product are driven by the variation in project qualities in that assignment history as well as the functional form of $f(\cdot,\cdot)$. To see why, consider a CEO once again in the special case of (25) with $\rho = 1$. Since the CEO's marginal product only depends on her own type, this implies that her compensation does not vary with the quality of the projects she is assigned to. In contrast, when managers and projects are complements, marginal products – and hence wages and profits – vary systematically across the CEOs' and projects' assignment histories. Holding the distribution of CEO and project qualities as well as the assignment histories fixed, the extent of this variation in wages or profits is increasing in the degree of complementarity and that is a moment that we can, in principle, take to the data. Now, to identify the empirically relevant degree of complementarity we need micro data with "comparable" employment spells as well as wage and profit histories. Since the *ExecuComp* sample has virtually no instances of CEOs with more than one employment spell, I cannot identify ρ and I instead use a calibration strategy based on cross-sectional moments where the substitution elasticity is an exogenous parameter rather than a calibration object.[24]

IV. Quantitative Effects of Non-Assortative Matching

The undistorted equilibrium of the model is a meritocracy where high-quality projects are run by competent managers. While the claim that non-meritocratic practices are widespread is fairly uncontroversial, it is an open question to what extent they contribute to differences in measured TFP between rich and poor countries.[25] In this section, I will show that the misallocation of managerial abilities can be a significant source inefficiency.

To quantify the aggregate effects of managerial misallocation, I consider two types of matching frictions. The first counterfactual experiment follows Fernandez and Rogerson (2001), where a known fraction θ of agents is matched randomly *ex post*. The parameter θ parsimoniously captures aspects of assignment markets that may prevent assortative matching but are not modeled explicitly in this paper.[26] In the second experiment, the matching frictions are correlated with the project productivities: projects above the $\left(100\times(1-\theta)\right)^{\text{th}}$ percentile are assigned to a random CEO, the remaining projects sort positively with the unmatched managers. In both cases, I first consider the effects in partial equilibrium where

[24] In ongoing work, we develop an assignment model where managers learn their true abilities from a sequence of signals. Following each update of their belief, managers may opt to be assigned to a different employer and this generates endogenous employment spells and compensation histories. To parameterize the model, we use a rich employer-employee dataset covering the entire Danish labor force over a decade and the panel structure of the data is rich enough to identify the empirically relevant degree of complementarity (Alder and Groes, 2014).

[25] See, for instance, the discussion of the incidence of dynastic management in Caselli and Gennaioli (2013).

[26] Among them are matching or separation costs, information frictions, matching between agents that differ in more than one dimension, or the accumulation of match-specific capital.

the occupational cutoff is identical to the benchmark calibration. I then solve for the general equilibrium where the cutoff – denoted by $\hat{\imath}$ – responds to the change in firm-level productivities associated with matching frictions. Across all counterfactual experiments I find that these matching frictions have sizeable effects on aggregate productivity. While the drop in the occupational cutoff ($\hat{\imath} < \bar{\imath}$) is *by itself* quantitatively unimportant, it is symptomatic for the large inframarginal effect of assignment frictions.[27]

In section IV.B, I examine how matching frictions interact with idiosyncratic distortions in the tradition of Hsieh and Klenow (2009) or Restuccia and Rogerson (2008). I find that both channels can be quantitatively important, particularly so when mismatch and distortions are correlated, for instance, when high-quality projects run by incompetent managers face particularly high distortions.[28]

A. *Partial Random Matching*

In this experiment I consider an environment where agents who participate in the assignment market are matched assortatively with probability $1-\theta$. With the complementary probability θ they are randomly assigned to a participating project, similar to Fernandez and Rogerson (2001). By construction, any random match generates a joint rent that is at least as large as $\Phi[\hat{\imath},\hat{\imath}]$, where $\hat{\imath}$ indexes the (perfectly matched) marginal types. Since there is always a split of the rent such that a randomly matched project and CEO receive at least their outside options, no one wishes to exit the market once the uncertainty has been resolved.[29] The presence of random matches has general equilibrium effects and the marginal type $\hat{\imath}$ is generally different from the marginal type $\bar{\imath}$ in the efficient benchmark.

In the perfectly assortative economy, the "aggregate supply" of talent and quality is given by $F[\bar{\imath},\bar{\imath}] = \int_{\bar{\imath}}^{1} f[\imath,\imath]\mathrm{d}\imath$. With partial random matching, the effective supply is a mixture of perfect and random assignments:

$$F^U_\theta[\hat{\imath},\hat{\imath}] = (1-\theta)\int_{\hat{\imath}}^{1} f[\imath,\imath]\mathrm{d}\imath + \tfrac{\theta}{1-\hat{\imath}}\int_{\hat{\imath}}^{1}\int_{\hat{\imath}}^{1} f[\imath,\jmath]\mathrm{d}\imath\mathrm{d}\jmath$$

In the partial equilibrium, I set the average firm size to 53 employees exogenously

[27]In this class of models, counterfactual experiments that *only* affect the marginal project or CEO typically have inconsequential aggregate effects. In contrast, the effect of experiments that alter the tail shape of firm productivities – such as the mismatch of managers and projects – can be sizeable.

[28]There is also a large literature on the correlation of talent and wealth. Buera, Kaboski and Shin (2011), Caselli and Gennaioli (2013), Jeong and Townsend (2007), and Paulson and Townsend (2004), among others, argue that financial or contractual frictions can be a quantitatively important source of aggregate inefficiency. The observation that low productivity is relatively common in family firms is discussed by both Caselli and Gennaioli (2013) (contractual constraints) and Bloom and Van Reenen (2007) (primogeniture). While the implications of financial frictions are clearly interesting and important, questions of family ownership, managerial control, and succession are beyond the scope of this paper.

[29]Most, if not all, standard bargaining protocols will assign at least the respective outside options to each match partner. The details of the split are not important as long as no one regrets participating in the assignment market *ex post*. To characterize the aggregate effects of these random assignments I do not need a fully specified bargaining protocol.

(i.e. $\hat{\imath} = \bar{\imath}$). In the general equilibrium, on the other hand, the perfectly matched marginal type $\hat{\imath}$ is indifferent between being a CEO and being a worker, just as in the efficient benchmark economy:

$$\omega_\theta^U[\hat{\imath}] = f[\hat{\imath},\hat{\imath}]^{1-\gamma}\big(k[\hat{\imath}]^\alpha, \ell[\hat{\imath}]^{1-\alpha}\big)^\gamma - r_\theta^U k[\hat{\imath}] - w_\theta^U \ell[\hat{\imath}] = w_\theta^U,$$

where r_θ^U and w_θ^U denote the market clearing factor prices in the presence of matching frictions.

Figures 7 – 9 illustrate the effect of uncorrelated matching frictions on aggregate TFP (in partial and general equilibrium) and the average firm size (which is given by $\frac{\hat{\imath}}{1-\hat{\imath}}$) for select values of $\frac{1}{1-\rho}$. The general equilibrium effect lowers the occupational cutoff $\hat{\imath}$ and hence the average firm size. Note that the effect is decreasing in the substitution elasticity $\frac{1}{1-\rho}$ and rather insignificant on the linear side of Cobb-Douglas (right panel of Figure 9). Despite the magnitude of the drop in $\hat{\imath}$ for low elasticities of substitution, the general equilibrium effect with respect to TFP is modest. The difference between the left panels of Figures 7 and 8 is at most 2.5 percentage points (or 15 percent of the total decline in productivity) and productivity barely drops when the substitution elasticity between managers and projects is higher than unity (right panels of Figures 7 and 8). To understand why this is the case, it is useful to take another look at Figure 5. Parameterizations on the linear side of Cobb-Douglas are characterized by a relatively homogeneous pool of CEOs. Since, in addition, high substitutability implies that a CEO's marginal product does not vary a whole lot across assignments, mismatch is a quantitatively negligible source of efficiency losses.[30] In contrast, in parameterizations with $\frac{1}{1-\rho} < 1$ managers are quite a bit more heterogeneous (clearly, $\xi_a > \xi_q$ in Figure 5) and mismatch entails a significant reallocation of talent across firms. Moreover, complementarity implies that a CEO's marginal product varies considerably across different assignments. Together, heterogeneity and complementarity imply that matching frictions are costly in the aggregate.

In the second counterfactual experiment, matching frictions are correlated with the project productivities. Projects from the $\big(1-\theta(1-\hat{\imath})\big)^{\text{th}}$ percentile to the top of the size distribution are matched randomly with CEOs. Since the randomization occurs after the individuals' occupational choices have already been made – just like in the previous experiment – these projects are assigned with equal probability to any manager between the $\hat{\imath}^{\text{th}}$ and 100^{th} percentile of the ability distribution. The projects between the $\hat{\imath}^{\text{th}}$ and $\big(1-\theta(1-\hat{\imath})\big)^{\text{th}}$ percentile of the productivity distribution are matched assortatively to the remaining managers, of which there is a measure $(1-\hat{\imath})(1-\theta)$ with abilities ranked between $\hat{\imath}$ and 1.

With correlated random matching, the effective supply of talent and quality is again a mixture of perfect and random assignments, but the mismatch is "con-

[30]When projects and managers are *perfect* substitutes, their marginal products do *not* depend on assignments at all and random matching has no effect on aggregate TFP.

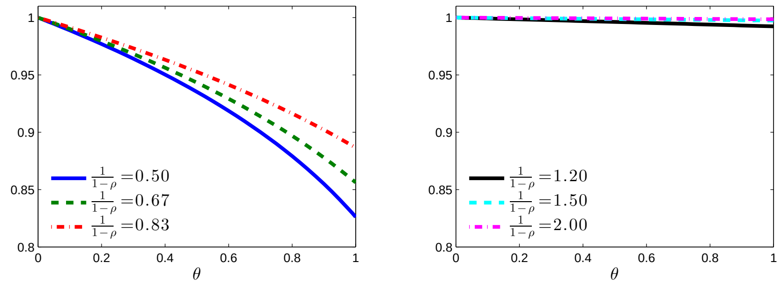

FIGURE 7. TFP (PARTIAL EQUILIBRIUM)

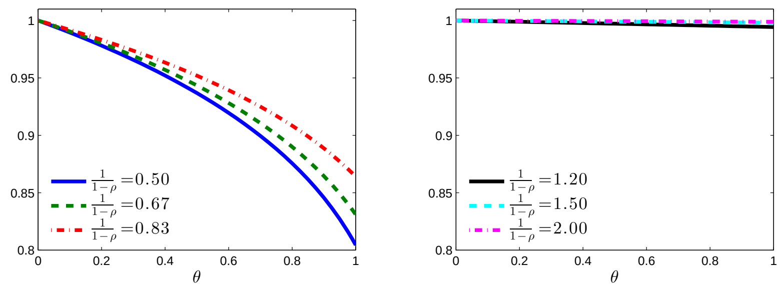

FIGURE 8. TFP (GENERAL EQUILIBRIUM)

centrated" in the right tail, above the $\big(1-\theta(1-\hat{\imath})\big)^{\text{th}}$ percentile to be precise, of the size distribution:

$$F^C_\theta[\hat{\imath},\hat{\imath}] = \int_{\hat{\imath}}^{1-\theta(1-\hat{\imath})} f\left[\tfrac{\imath-\hat{\imath}\theta}{1-\theta},\imath\right]\mathrm{d}\imath + \tfrac{1}{1-\hat{\imath}}\int_{1-\theta(1-\hat{\imath})}^{1}\int_{\hat{\imath}}^{1} f[\imath,\jmath]\mathrm{d}\imath\mathrm{d}\jmath$$

In the extreme case with $\theta = 1$, no manager is in the "sorting pool" of the assignment problem and the first term drops out completely. Like before, the average firm size is set to 53 employees in the partial equilibrium exercise. In general equilibrium, the marginal manager in the sorting pool (with index $\hat{\imath}$) must again be indifferent between being a CEO and a worker:

$$\omega^C_\theta[\hat{\imath}] = f[\hat{\imath},\hat{\imath}]^{1-\gamma}\big(k[\hat{\imath}]^\alpha,\ell[\hat{\imath}]^{1-\alpha}\big)^\gamma - r^C_\theta k[\hat{\imath}] - w^C_\theta \ell[\hat{\imath}] = w^C_\theta,$$

where r^C_θ and w^C_θ denote the market clearing factor prices.

Figures 10 – 12 show the effect of correlated matching frictions on TFP and the average firm size. In the polar cases with $\theta = 0$ and $\theta = 1$, the effects are identical to the uncorrelated case. In contrast to the previous exercise, however, even small

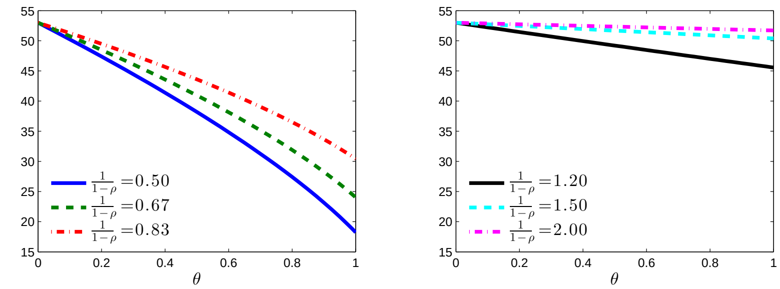

Figure 9. Average # of Workers per Firm (General Equilibrium)

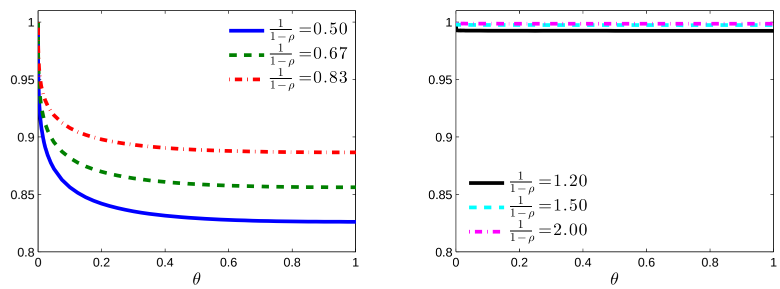

Figure 10. TFP (Partial Equilibrium)

fractions of mismatched projects can generate sizable aggregate effects when the matching frictions are correlated with project qualities (or size). To compare the effects of correlated and uncorrelated matching frictions, let me consider the fraction θ of random assignments that achieves 50 percent of the productivity loss generated by $\theta = 1$. On the Leontief side of Cobb-Douglas (left panel of Figures 10 and 11), a fraction as low as 0.018 pushes aggregate TFP to this halfway point when the misallocation of managerial talent is correlated with the productivities. In contrast, when the misallocation of talent is uncorrelated with size, roughly two thirds of all assignments must be random to generate a productivity drop of this magnitude. Figure 13 illustrates the contrast between the correlated and uncorrelated misallocation of managerial abilities for the benchmark parameterization with $\frac{1}{1-\rho} = \frac{1}{2}$. Clearly, the extent to which matching frictions are correlated with project attributes plays a quantitatively important role in these counterfactual experiments.

On the linear side of Cobb-Douglas (right panel of Figures 10 and 11), the bulk of the productivity drop can again be generated by a small fraction of random assignments. In sharp contrast to the cases with $\frac{1}{1-\rho} < 1$, however, the aggregate productivity losses are smaller by at least one order of magnitude and these results

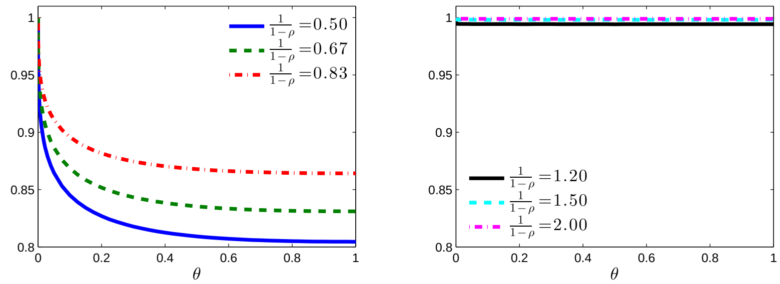

FIGURE 11. TFP (GENERAL EQUILIBRIUM)

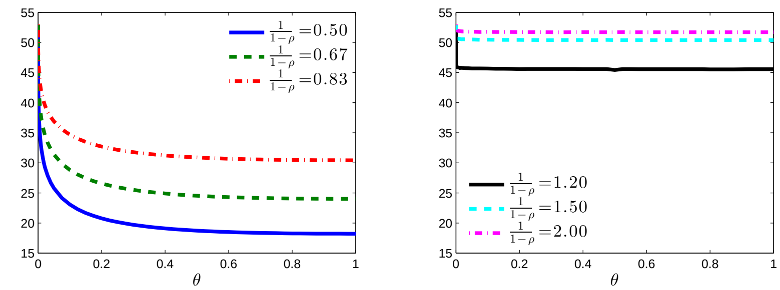

FIGURE 12. AVERAGE # OF WORKERS PER FIRM (GENERAL EQUILIBRIUM)

are driven by the combination of *(a)* the substitutability between projects and managers and *(b)* a fairly homogeneous pool of managerial talent (see the plot for ξ_a in Figure 5).

Lastly, Figure 14 plots relative TFP for substitution elasticities ranging from $\frac{1}{2}$ to 2 when the random and sorting pools are of equal measure ($\theta = 0.5$). Clearly, the aggregate costs associated with the misallocation of managerial talent are particularly high when the assignment frictions are correlated with project qualities and when managers and projects are gross complements. In contrast, when they are gross substitutes, mismatch is not particularly costly in both the correlated and uncorrelated cases.[31]

[31] The aggregate effects of these counterfactual experiments are qualitatively robust to different values of the span of control parameter γ. With $\gamma = 0.9$, the drop in aggregate TFP associated with matching frictions is approximately half as big while it can be more than 50 percent larger when γ is set to 0.8. Regardless of the value of γ, the aggregate effects are always decreasing in ρ and small for $\frac{1}{1-\rho} > 1$.

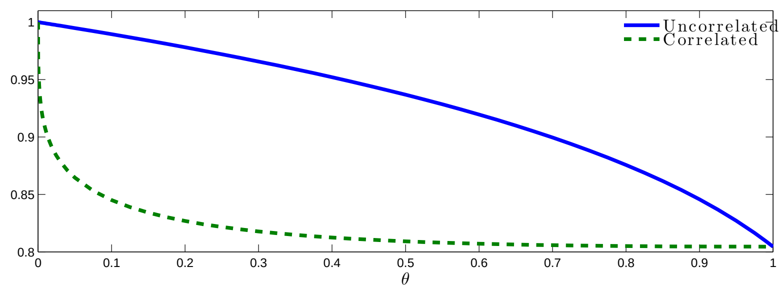

Figure 13. Relative TFP with $\frac{1}{1-\rho} = 0.5$

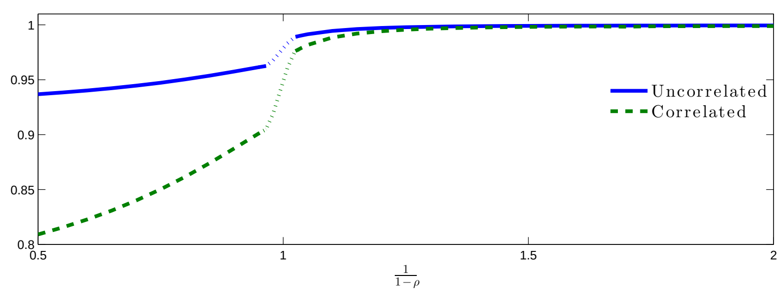

Figure 14. Relative TFP with $\theta = 0.5$

B. Mismatch and Distortions

One natural question to ask is how matching frictions interact with idiosyncratic distortions à la Restuccia and Rogerson (2008) and Hsieh and Klenow (2009). Just as in the counterfactuals in section IV.A, the correlation between project attributes, mismatch, and distortions is quantitatively important. To illustrate the key interaction channel, I use a stylized example where output distortions are uncorrelated with matching frictions. I then consider a "crony" economy where both mismatch and distortions are correlated with project attributes.

Uncorrelated Distortions. — In order to disentangle the direct and indirect effects of matching frictions in an environment where firms are facing idiosyncratic distortions, let me first consider the case where all managers and projects are sorting perfectly. A fraction $1-\theta$ of these firms are facing no idiosyncratic output distortions, $\frac{\theta}{2}$ of all firms are taxed at rate τ, and an equal measure is taxed at rate $-\tau$ (i.e. these firms are subsidizes). For simplicity, I assume that the distortions are uncorrelated with the firms attributes and I ignore general equilibrium effects by setting the rank of the marginal types to $\bar{\imath}$ (from the benchmark calibration).

TFP in the distorted economy with sorting is:

$$TFP_1 = (1-\bar{\tau}_1)^{-\gamma}\Big[(1-\theta)F + \theta F\Big\{\tfrac{1}{2}(1-\tau)^{\frac{\gamma}{1-\gamma}} + \tfrac{1}{2}(1+\tau)^{\frac{\gamma}{1-\gamma}}\Big\}\Big]^{1-\gamma},$$

where $F = \int_{\bar{i}}^{1} f[i,i]di$ and

$$\bar{\tau}_1 = 1 - \frac{(1-\theta)F + \theta F\Big\{\tfrac{1}{2}(1-\tau)^{\frac{1}{1-\gamma}} + \tfrac{1}{2}(1+\tau)^{\frac{1}{1-\gamma}}\Big\}}{(1-\theta)F + \theta F\Big\{\tfrac{1}{2}(1-\tau)^{\frac{\gamma}{1-\gamma}} + \tfrac{1}{2}(1+\tau)^{\frac{\gamma}{1-\gamma}}\Big\}}. \tag{28}$$

Let me now consider what happens when the projects facing distortions are randomly assigned to rather than assortatively matched with managers. Productivity in this economy is:

$$TFP_2 = (1-\bar{\tau}_2)^{-\gamma}\Big[(1-\theta)F + \theta \widehat{F}\Big\{\tfrac{1}{2}(1-\tau)^{\frac{\gamma}{1-\gamma}} + \tfrac{1}{2}(1+\tau)^{\frac{\gamma}{1-\gamma}}\Big\}\Big]^{1-\gamma},$$

where $\widehat{F} = \frac{1}{1-\bar{i}}\int_{\bar{i}}^{1}\int_{\bar{i}}^{1} f[i,j]didj$ and

$$\bar{\tau}_2 = 1 - \frac{(1-\theta)F + \theta \widehat{F}\Big\{\tfrac{1}{2}(1-\tau)^{\frac{1}{1-\gamma}} + \tfrac{1}{2}(1+\tau)^{\frac{1}{1-\gamma}}\Big\}}{(1-\theta)F + \theta \widehat{F}\Big\{\tfrac{1}{2}(1-\tau)^{\frac{\gamma}{1-\gamma}} + \tfrac{1}{2}(1+\tau)^{\frac{\gamma}{1-\gamma}}\Big\}}. \tag{29}$$

The difference between F and $\widehat{F}$ captures the direct effect of mismatch. Since $f[\cdot,\cdot]$ is supermodular, random matching implies that the integral over idiosyncratic productivities drops, i.e. $\widehat{F} < F$. In addition, matching frictions have an indirect effect that is captured by the difference between the "average" distortion rates $\bar{\tau}_1$ and $\bar{\tau}_2$ in equations (28) and (29), respectively. Since $\frac{1}{2}(1-\tau)^{\frac{1}{1-\gamma}} + \frac{1}{2}(1+\tau)^{\frac{1}{1-\gamma}} > \frac{1}{2}(1-\tau)^{\frac{\gamma}{1-\gamma}} + \frac{1}{2}(1+\tau)^{\frac{\gamma}{1-\gamma}}$ for any $\tau > 0$, $\bar{\tau}_2$ is negative and decreasing in $\widehat{F}$. Put differently, the decrease in the "sum" of idiosyncratic productivities of firms facing distortions also has a countervailing effect on the economy's average distortion rate. The left panel of Figure 15 plots relative TFP for $\tau = 0.2$, θ between 0 and 1, and $\rho = -1$ from the benchmark calibration. The solid line shows the effect of introducing idiosyncratic distortions. The dashed line adds the *direct* effect of matching frictions and the dotted line combines distortions with the direct *and* indirect effects of mismatch. For a given θ, the vertical drop from the solid to the dashed line captures the productivity drop associated with matching friction alone. The distance between the dashed and dotted lines captures the difference between $\bar{\tau}_1$ and $\bar{\tau}_2$ (shown in the right panel). While the direct and indirect effects move in opposite directions in this example, this need not be the case when distortions are correlated with the attributes of projects or managers or both. In the following section, I turn my attention to a case where mismatch and distortions are no longer orthogonal events. In particular, I consider a set of output distortions that allocates managers to large firms based on exogenous *status* rather than managerial quality.

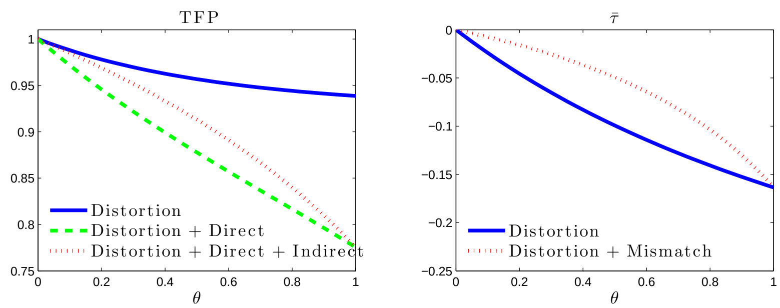

Figure 15. Mismatch and Uncorrelated Distortions

Correlated Distortions: "Cronyism". — In this counterfactual experiment I assume that once all agents have decided whether or not to participate in the assignment market, a fraction θ of all managers are designated "insiders". In addition to their ability a, insiders are characterized by their uniformly distributed status $s \sim U[0,1]$, which is uncorrelated with a. They enter the matching market in the order of their status: $s = 1$ goes first, $s = 0$ enters last. Status is economically valuable since it is paired with access to resources in the form of output distortions (think taxes and subsidies). Insiders approach the owner of a project with the promise of rents large enough to make him indifferent between hiring her and the assortative match in the undistorted assignment. This steers insiders toward the highest-quality projects that have not been assigned yet. Let $\tilde{\imath}$ denote the (endogenous) rank of the marginal outsider. Then projects between size rank $\tilde{\imath} + (1-\theta)(1-\tilde{\imath})$ and 1 are run by insiders. The remaining $(1-\theta)(1-\tilde{\imath})$ projects, which have not previously been assigned to an insider CEO, are matched assortatively with outsiders.

Let $j(s)$ be a CEO of rank j in the ability distribution with status s. Similarly, $i(s)$ is a project with quality rank i matched with a CEO of status s. Since insiders aim for the highest-quality project that is unmatched, the sorting is perfect between insider status and project type:

$$i(s) = 1 - (1-s)(1-\theta)(1-\tilde{\imath})$$

To make the owner of project i indifferent between insider $j(s)$ and the perfect match of identical rank i, an insider offers a firm-specific output subsidy τ such that:

$$\tau[j(s), i(s)] = 1 - \left(\frac{f\left[i(s), i(s)\right]}{f\left[j(s), i(s)\right]}\right)^{1-\gamma}, \tag{30}$$

where the numerator is the idiosyncratic productivity of a perfect manager-project match and the denominator is the productivity of a status-based assignment as in (IV.B). "Cronies" have special ties to politicians or bureaucrats who can generate rents by way of subsidies, exclusive licenses, preferential access to financial markets, selective anti-trust enforcement, and myriad other forms of anti-competitive policies.[32]

Projects below quality rank $\tilde{\imath}+(1-\theta)(1-\tilde{\imath})$ are assigned to an outsider (since the economy has run out of insiders). These firms are taxed to make them indifferent between hiring the best available outsider and the perfect match (of equal rank):

$$\tau\left[\tfrac{i-\tilde{\imath}\theta}{1-\theta}, i\right] = 1 - \left(\frac{f[i,i]}{f\left[\frac{i-\tilde{\imath}\theta}{1-\theta}, i\right]}\right)^{1-\gamma}, \text{ for } \tilde{\imath} \leq i < \tilde{\imath} + (1-\theta)(1-\tilde{\imath}) \tag{31}$$

Since the denominator is greater than the numerator, $\tau\left[\frac{i-\tilde{\imath}\theta}{1-\theta}, i\right]$ is positive for all projects managed by outsiders. While there is perfect sorting between outsiders and projects, the assignment is "shifted" by the presence of insiders. Just like insider-run firms, outsider-project pairs hire the same capital and labor inputs as an untaxed $f[i,i]$ firm. The general equilibrium effects associated with these output distortions imply that $\tilde{\imath} < \bar{\imath}$.[33]

The average distortion $\bar{\tau}$ can be written as a function of the assignment of insiders and outsiders to firms and the corresponding distortions from equations (30) and (31):

$$\bar{\tau} = 1 - \frac{\int_{\tilde{\imath}}^{1} f[j,j]\mathrm{d}j}{\underbrace{\int_{j=1-\theta(1-\tilde{\imath})}^{1}\int_{i=0}^{1} f[i,j]^{1-\gamma} f[j,j]^{\gamma}\mathrm{d}i\mathrm{d}j}_{\text{firms matched with insiders}} + \underbrace{\int_{j=\tilde{\imath}}^{1-\theta(1-\tilde{\imath})} f\left[\tfrac{j-\tilde{\imath}\theta}{1-\theta}, j\right]^{1-\gamma} f[j,j]^{\gamma}\mathrm{d}j}_{\text{firms matched with outsiders}}}$$

The effective supply of ability and quality in the crony economy, denoted by F_τ, is a function of the average distortion $\bar{\tau}$, the idiosyncratic distortions in (30) and

[32]The Indian "License Raj", anti-competitive business activities by individuals in Soeharto's inner circle in Indonesia, and systematic preferential lending in South Korea (see Kang, 2002*a*,*b*, for instance), are examples how political leaders allocate rents to insiders.

[33]The marginal CEO has rank $\tilde{\imath}$ regardless of status. While it is straightforward to consider distinct cutoffs for insiders and outsiders, nothing in the data disciplines the exercise and I omit it in the interest of brevity.

(31), and the idiosyncratic productivities, $f[i,j]$:

$$F_\tau = \Bigg(\int_{j=1-\theta(1-\tilde{\imath})}^{1} \int_{i=0}^{1} f[i,j]^{1-\gamma} f[j,j]^{\gamma} \mathrm{d}i\mathrm{d}j + \int_{j=\tilde{\imath}}^{1-\theta(1-\tilde{\imath})} f\big[\tfrac{j-\tilde{\imath}\theta}{1-\theta}, j\big]^{1-\gamma} f[j,j]^{\gamma} \mathrm{d}j \Bigg) (1-\bar{\tau})^{-\frac{\gamma}{1-\gamma}} \tag{32}$$

Aggregate output and TFP are functions of K, the size of the workforce $\tilde{\imath}$, and the effective supply of ability and quality, F_τ:

$$\begin{aligned} Y_\tau &= K^{\alpha\gamma} \tilde{\imath}^{(1-\alpha)\gamma} F_\tau^{1-\gamma} \\ \mathrm{TFP}_\tau &= \frac{Y_\tau}{K^\alpha \tilde{\imath}^{1-\alpha}} = K^{\alpha(\gamma-1)} \tilde{\imath}^{(1-\alpha)(\gamma-1)} F_\tau^{1-\gamma} \end{aligned}$$

Since the economy has an exogenous capital endowment, relative TFPs are given by the simple expression $\big(\frac{F_\tau}{F}\big)^{1-\gamma}\big(\frac{\tilde{\imath}}{\bar{\imath}}\big)^{(1-\alpha)(\gamma-1)}$. The factor prices w_τ and r_τ are rescaled by the changes in the effective supply of ability and talent and the size of the labor force:

$$w_\tau = w\big(\tfrac{F_\tau}{F}\big)^{1-\gamma}\big(\tfrac{\tilde{\imath}}{\bar{\imath}}\big)^{(1-\alpha)\gamma} \quad \text{and} \quad r_\tau = r\big(\tfrac{F_\tau}{F}\big)^{1-\gamma}\big(\tfrac{\tilde{\imath}}{\bar{\imath}}\big)^{(1-\alpha)\gamma-1}$$

The idiosyncratic taxes and subsidies are not revenue neutral. However, the government's budget can be balanced by lump-sum taxes or transfers to the representative household. This is without loss of generality since none of the relevant margins (labor supply, occupational choice, and factor demands) are affected. The inclusion of lump-sum taxes and transfers in the households' budget constraint is a trivial change of equation (15) and the government's budget balance constraint is similarly straightforward. In the interest of brevity, I omit them here.

Figure 16 plots the aggregate effects of mismatch (dashed lines) and mismatch combined with idiosyncratic distortions (solid lines) for $\frac{1}{1-\rho} = \frac{1}{2}$ and $\frac{1}{1-\rho} = 2$. In the benchmark parameterization with $\frac{1}{1-\rho} = \frac{1}{2}$ even small fractions of insiders have sizable effects on aggregate productivity. When just the top 0.1 percent of all projects are assigned to insiders, aggregate productivity drops by more than 19 percent. Approximately one quarter (five percentage points) of the decline are generated by matching frictions while idiosyncratic distortions account for the remainder (about thirteen percentage points). When the top 1 percent are run by insiders, mismatch accounts for one third and distortions for two thirds of the 29 percent drop in aggregate productivity. In contrast, when projects and CEOs are gross substitutes, the effects are negligible. Moreover, distortions play a less prominent role (right panels of Figures 16 and 17).

The experiments in IV.A and IV.B differ from one another in one important respect. In the former, the firm size distribution varies systematically with the

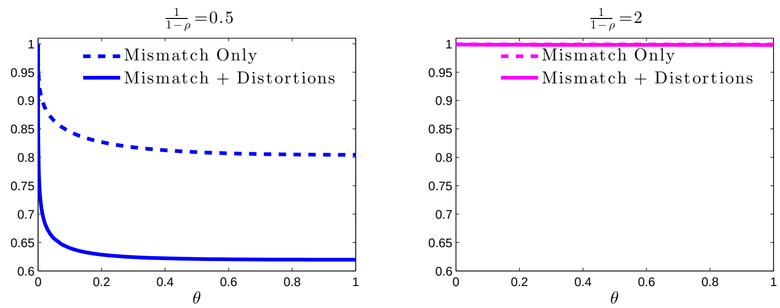

FIGURE 16. MISMATCH AND CORRELATED DISTORTIONS

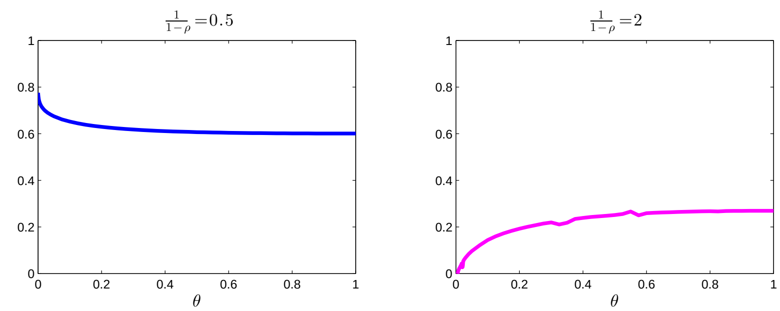

FIGURE 17. CONTRIBUTION OF IDIOSYNCRATIC DISTORTIONS

fraction of randomly assigned project-manager pairs (the average firm size is decreasing in the fraction of random matches) while the marginal products of capital and labor are equalized across firms. In the insider-outsider economy, on the other hand, the idiosyncratic distortions offset (up to a general equilibrium effect) the changes in the size distribution caused by mismatch while the dispersion of marginal products is increasing in the insider share.[34] The evidence suggests that both facts are salient. Poschke (2014), for instance, shows that the shape of the firm size distribution varies systematically with GDP per capita (and hence aggregate productivity) while Hsieh and Klenow (2009) find that the dispersion of marginal products is systematically related to aggregate TFP.

More generally, the misallocation of managerial talent may be part of a nexus linking competition to productivity. To the extent that $f[\cdot,\cdot]$ is a proxy for a firm's organization capital – and that is, in fact, my favorite interpretation – the predictions of the insider model with idiosyncratic distortions are in line with the evidence in Bloom and Van Reenen (2007) and Bloom et al. (2013), where poor management practices (low organization capital) are more common

[34]The strength of this countervailing effect on labor demands depends, of course, on the specification of $\tau[i, j]$.

in firms that are not exposed to product market competition. Moreover, the insider-outsider model predicts that CEO compensation is not correlated with $f[\cdot,\cdot]$ after controlling for firm size, which, again, is what Bloom and Van Reenen find in the data.[35]

Schmitz (2005), Holmes and Schmitz (2010) and Syverson (2011), among others, emphasize the importance of an alternative link between competition and productivity. They find that in response to a rise in foreign competition, iron ore producers in the Great Lakes region of the United States achieved sizable productivity gains by overhauling restrictive *work* rules. Interestingly, they argue that improved labor-management relations played a key role in the process and I view the role of *work* rules and the *management* practices emphasized by Bloom and his coauthors as complementary rather than competing links between competition and productivity.[36]

C. Microeconomics of Executive Turnaround

The counterfactual experiments in IV.A and IV.B show that, depending on ρ, matching frictions *can* have sizable macroeconomic effects. While the discussion at the end of section III.C highlighted the challenges associated with identifying the empirically relevant degree of complementarity in this class of models, several case studies provide some evidence for complementarities on the Leontief side of Cobb-Douglas. Cole et al. (2005) report that the exodus of skilled foreign managers and experts associated with the nationalization of the Venezuelan oil industry led to a sharp decline in output and labor productivity. Using data from Garcia, Knights and Tilton (2001) they also find that the entry of more efficient private copper mining companies in Chile in the 1990s led to an acceleration of labor productivity growth at the state-owned Codelco mines by using superior technology and/or better expertise. Importantly, the technologies and expertise were available prior to the reversal of the 1971 nationalization, but senior management was too incompetent or unwilling to introduce either. In La Porta and de Silanes (1999), survey respondents report that improved profitability in formerly state-owned Mexican enterprises was chiefly the result of new production processes and the firing of old managers and directors (in that order). They find that one half to two thirds of a 24-percentage-point increase in the mean ratio of operating income to sales is accounted for by productivity gains.[37] Similarly, according to the McKinsey Global Institute (1998), management's inability to implement lean production and overly complex manufacturing processes can explain a sizable portion of the 50 percent labor and total factor productivity gap in the Korean auto industry in the mid 1990s. Bennedsen, Pérez-González and Wolfen-

[35]See Bloom and Van Reenen (2007) on management practices and competition (p. 1358-60) and on CEO compensation and management scores (p. 1385-86), respectively.

[36]Bloom, Draca and Van Reenen (2011) and Pavcnik (2002) find that foreign trade exposure affects productivity growth mostly through R&D and labor reallocation toward more productive firms.

[37]The remainder is split between price increases and transfers from worker to shareholders.

zon (2008) also find that exogenous variations in managerial input are associated with sizeable differences in firm performance among Danish limited liability firms.

I can use my model to quantify the micro effects of a senior management shake-up and verify whether it is empirically plausible in light of these case studies. Consider, for instance, the case where a relatively incompetent incumbent is replaced by the optimal (assortatively matched) CEO. I only consider managers and projects whose rank exceed the cutoff $\bar{\imath}$. I index them by i (managers) and j (projects), respectively, such that 0 labels the marginal type and 1 the most able (productive). For a given substitution elasticity and firm rank i, the model predicts the productivity gain associated with a switch from executive $0 \leq j < i$ to the perfect match i. Figure 18 plots the results for a median firm and the usual values of $\frac{1}{1-\rho}$. If a marginal manager is replaced by the "correct" median type, productivity increases by 11 percent when the substitution elasticity is less than unity and by about two percent when managers and projects are gross substitutes. In addition, the gains are linear in the manager's rank (up to $i = 0.5$) to a first approximation. Figure 19 plots the productivity gains for a large firm (99.7^{th} percentile). Given the longer rank distance between the marginal and "correct" manager, the gains are, of course, much larger. In addition, the fat tails of the quality distribution are reflected in the concavity of the gains profile: in the benchmark with $\rho = \frac{1}{2}$, productivity doubles when a median type is replaced. The jump from a marginal incumbent to the perfect match yields "only" an additional thirty percentage points for a total gain of 130 percent. While the gains are sizable for other values of the substitution elasticity on the Leontief side of Cobb-Douglas, they drop by an order of magnitude or more when managers and projects are substitutes (right panel).[38] While this exercise yields no conclusive evidence on the empirically relevant elasticity of substitution, it suggests that only sufficiently strong complementarities can reconcile the model with the case study findings.

Let me conclude the mismatch discussion with a final observation on managerial abilities. While the distribution is exogenous here, the effects of matching frictions are magnified if individuals with heterogeneous abilities make investments in managerial skills as in Bhattacharya, Guner and Ventura (2013). Frictions dull the incentives to invest across the board. Insiders clearly have no incentive to accumulate skills since their payoff is determined by status alone. Outsiders, on the other hand, acquire fewer skills compared to the frictionless benchmark since the presence of insiders lowers the return to skill by depressing the wage rate and hence the entire profile of managerial compensation. Unambiguously, the effective supply of abilities and qualities drops below F_τ in equation (32) since, in addition to the effects stemming from distortions, the economy's endogenous distribution

[38] Keep in mind that this is a "micro" exercise where the project's quality is held fixed. The gains depend on the distribution of managerial quality and the value of ρ, but not on the distribution of project qualities. The 99.7^{th} percentile corresponds to the cutoff for "large" firms in the model. Firms above this cutoff are the model counterpart to corporations that file audited financial statements with the SEC, for which I compute the CEOs' compensation share and Roberts' Law.

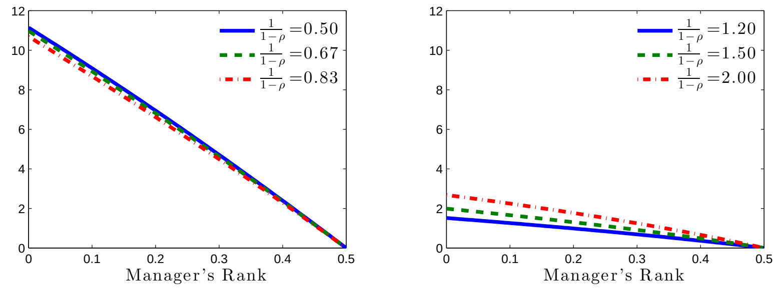

FIGURE 18. EXECUTIVE TURNAROUND AT MEDIAN FIRM

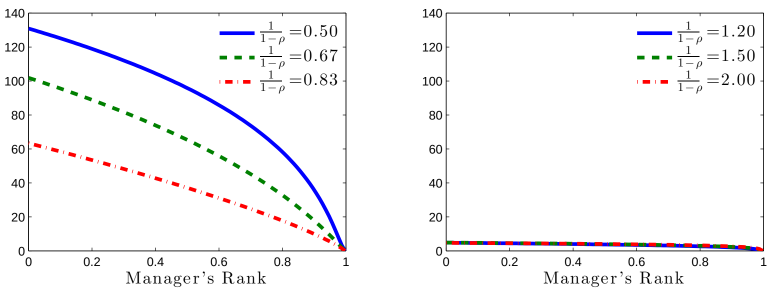

FIGURE 19. EXECUTIVE TURNAROUND AT LARGE FIRM (99.7^{th} PERCENTILE)

of skill is first-order dominated by $a[i]$. Clearly, the effect of matching frictions is further amplified by an endogenous drop in the accumulation of managerial skills.

V. Conclusion

This paper reconciles the results from the recent literature on misallocation in monopolistically competitive or Lucas span-of-control environments (Hsieh and Klenow, 2009; Restuccia and Rogerson, 2008), structural models of CEO-to-project assignments (Gabaix and Landier, 2008; Terviö, 2008), and the empirical literature on management practices (Bloom and Van Reenen, 2007, among others). An important insight from the calibration in section III and the counterfactuals in section IV is that the misallocation of managerial talent *can* have sizable aggregate effects. The exact magnitude depends on *(a)* the degree of complementarity between the attributes of projects and their managers and *(b)* the extent to which mismatch is correlated with project quality. Moreover, the effects of matching frictions are distinct from those generated by idiosyncratic distortions in the Restuccia and Rogerson (2008) and Hsieh and Klenow (2009) tradition. The former affect TFP by reshaping the distribution of firm productivities. The latter, on the other hand, lower productivity by dispersing the marginal products of capital and labor. While systematic variations in the distribution of firm sizes and the dispersion of marginal products are salient stylized facts, their interaction and relative contribution to measured TFP are open empirical questions.

Which degree of complementarity truly governs the interaction of projects and CEOs remains an important and, as of yet, unanswered question. To identify the empirically relevant elasticity one needs a model where at least some CEOs choose to manage different projects over the course of their careers. This generates endogenous variations in the CEOs' and the projects' marginal products and differences in executive pay and corporate profits across spells are informative about the underlying substitution elasticity. The dynamic assignment model with endogenous separations we develop in Alder and Groes (2014) lays the theoretical foundation for future empirical research in this direction.

Proofs

PROOF OF PROPOSITION 1:

Let $\mathbf{1}_{\Omega}(i,j)$ be an indicator function takes a unit value when i is assigned to j and zero otherwise. Then $\bar{\imath}(\Omega) = 1 - \int_0^1 \int_0^1 \mathbf{1}_{\Omega}(i,j)\mathrm{d}i\mathrm{d}j$ denotes the measure of workers in the economy and $F(\Omega) = \int_0^1 \int_0^1 f\big(a[i], q[j]\big)\mathbf{1}_{\Omega}(i,j)\mathrm{d}i\mathrm{d}j$ characterizes the aggregate supply of managerial abilities and project qualities.

It is straightforward to show that

$$\begin{aligned} w &= (1-\alpha)\gamma\left(\frac{K}{\bar{\imath}(\Omega)}\right)^{\alpha\gamma}\left(\frac{F(\Omega)}{\bar{\imath}(\Omega)}\right)^{1-\gamma} \\ r &= \alpha\gamma\left(\frac{K}{\bar{\imath}(\Omega)}\right)^{\alpha\gamma}\left(\frac{F(\Omega)}{\bar{\imath}(\Omega)}\right)^{-\gamma}\frac{F(\Omega)}{K} \end{aligned}$$

For all (i,j) in Ω,

$$\begin{aligned} \ell[i,j] &= f[i,j]\frac{\bar{\imath}(\Omega)}{F(\Omega)} \\ k[i,j] &= f[i,j]\frac{K}{F(\Omega)} \end{aligned}$$

Note that for a given $\bar{\imath}(\Omega)$ the factor prices do not depend on $f[i,j]$. Substituting the expressions for factor inputs into the profit function to obtain:

$$\begin{aligned} \Phi[i,j] =& f[i,j]^{1-\gamma}\big(f[i,j]\tfrac{K}{F(\Omega)}\big)^{\alpha\gamma}\big(f[i,j]\tfrac{\bar{\imath}(\Omega)}{F(\Omega)}\big)^{(1-\alpha)\gamma} - wf[i,j]\tfrac{\bar{\imath}(\Omega)}{F(\Omega)} - rf[i,j]\tfrac{K}{F(\Omega)} \\ =& f[i,j]\Big[\big(\tfrac{K}{F(\Omega)}\big)^{\alpha\gamma}\big(\tfrac{\bar{\imath}(\Omega)}{F(\Omega)}\big)^{(1-\alpha)\gamma} - w\tfrac{\bar{\imath}(\Omega)}{F(\Omega)} - r\tfrac{K}{F(\Omega)}\Big] \end{aligned}$$

Clearly, if $f[i,j]$ is supermodular, then so is $\Phi[i,j]$. ■

PROOF OF LEMMA 2:

Substitute the expressions for factor prices and inputs in the proof of Proposition 1 into equation (12). After setting $\bar{\imath} = \bar{\imath}(\Omega)$ the equation defines the cutoff $\bar{\imath}(\Omega)$ implicitly:

$$\bar{\imath}(\Omega) = (1-\alpha)\frac{\gamma}{1-\gamma}\frac{F(\Omega)}{f\big(a[\bar{\imath}(\Omega)], q[\bar{\imath}(\Omega)]\big)}$$

Clearly, the cutoff does not depend on K. ■

PROOF OF COROLLARY 1:

Since $\bar{\imath}(\Omega)$ does not depend on the capital stock, neither does $\ell[i,j] = f[i,j]\frac{\bar{\imath}(\Omega)}{F(\Omega)}$. ■

PROOF OF PROPOSITION 2:

Let

$$g[i,i] = \Big[\nu_a\big\{[(1-i)^{-\xi_a}-1]\tfrac{\sigma_a}{\xi_a}\big\}^\rho + \nu_q\big\{[(1-i)^{-\xi_q}-1]\tfrac{\sigma_q}{\xi_q}\big\}^\rho\Big]^{\frac{1}{\rho}}$$

and define

$$\widehat{\sigma}_a \equiv \lambda_a\sigma_a \quad \text{and} \quad \widehat{\sigma}_q \equiv \lambda_q\sigma_q.$$

Then, for any $\lambda_a > 0$ and $\lambda_q > 0$, it is straightforward to show that:

$$g[i,i] = \Lambda(\lambda_a,\lambda_q)\Big[\widehat{\nu}\big\{[(1-i)^{-\xi_a}-1]\tfrac{\widehat{\sigma}_a}{\xi_a}\big\}^\rho + (1-\widehat{\nu})\big\{[(1-i)^{-\xi_q}-1]\tfrac{\widehat{\sigma}_q}{\xi_q}\big\}^\rho\Big]^{\frac{1}{\rho}},$$

where $\Lambda(\lambda_a,\lambda_q) = (\nu_a\lambda_a^{-\rho}+\nu_q\lambda_a^{-\rho})^{\frac{1}{\rho}}$ and $\widehat{\nu} = \frac{\nu_a}{\nu_a\lambda_a^{-\rho}+\nu_q\lambda_a^{-\rho}}$. In particular, there exist $\lambda_a = \sigma_a^{-1}$ and $\lambda_a = \sigma_a^{-1}$ such that $f[i,i] = \frac{g[i,i]}{\Lambda(\lambda_a,\lambda_q)}$ satisfies (25) and (26). Using Lemma 2 one can show that $\bar{\imath}$ does not depend on the scale of $f[i,i]$. Lastly, Corollary 1 implies that output, factor prices, the surplus $\Phi[i,i]$, and the wage and profit profiles in the $(\widehat{\sigma}_a,\widehat{\sigma}_q,\widehat{\nu},1-\widehat{\nu})$ economy are equal to those of the original parameterization rescaled by the factor $\Lambda(\lambda_a,\lambda_q)^{-(1-\gamma)}$. Lastly, for $\lambda_a = \sigma_a^{-1}$ and $\lambda_q = \sigma_q^{-1}$, the distributions of managers and projects have unit scale as in (26).

■

PROOF OF PROPOSITION 3:

For simplicity, consider only the first term in the function $g(\cdot,\cdot)$ from page 19:

$$\nu_a\big\{[(1-i)^{-\xi_a}-1]\tfrac{\sigma_a}{\xi_a}\big\}^{\rho_a},$$

which implies that $a \sim \text{GPD}(\sigma_a,\xi_a)$. Now assume that $\widehat{\xi}_a = \lambda_a\xi_a$ and we need to show that there exists a $\widehat{\rho}_a$, $\widehat{\sigma}_a$, and $\widehat{\nu}_a$ such that

$$\widehat{\nu}_a\big\{[(1-i)^{-\widehat{\xi}_a}-1]\tfrac{\widehat{\sigma}_a}{\widehat{\xi}_a}\big\}^{\widehat{\rho}_a} \xrightarrow{i\to 1} \nu_a\big\{[(1-i)^{-\xi_a}-1]\tfrac{\sigma_a}{\xi_a}\big\}^{\rho_a}$$

For $\widehat{\xi}_a > 0$, one can easily show that $\widehat{\nu}_a\big\{[(1-i)^{-\widehat{\xi}_a}-1]\tfrac{\widehat{\sigma}_a}{\widehat{\xi}_a}\big\}^{\widehat{\rho}_a} \xrightarrow{i\to 1} \widehat{\nu}_a(1-i)^{\widehat{\rho}_a\widehat{\xi}_a}\Big(\frac{\widehat{\sigma}_a}{\widehat{\xi}_a}\Big)^{\widehat{\rho}_a}$ and similarly for the right hand side of equation (A). What remains to be shown is that

$$\nu_a(1-i)^{\widehat{\rho}_a\widehat{\xi}_a}\Big(\frac{\widehat{\sigma}_a}{\widehat{\xi}_a}\Big)^{\widehat{\rho}_a} = \nu_a(1-i)^{\rho_a\xi_a}\big(\tfrac{\sigma_a}{\xi_a}\big)^{\rho_a}.$$

It is easily verified that $\widehat{\rho}_a = \frac{\rho_a}{\lambda_a}$ and $\widehat{\sigma}_a = \sigma_a^\lambda\lambda\xi_a^{1-\lambda}$ solve the equation. Put differently, a manager i drawn from a Generalized Pareto Distribution with scale parameter $\sigma_a^\lambda\lambda\xi_a^{1-\lambda}$ and tail index $\lambda_a\xi_a$ and whose contribution to the match surplus is governed by the curvature parameter $\frac{\rho_a}{\lambda_a}$ is asymptotically (i.e. as $i \to 1$) isomorphic to a manager i drawn from a GPD(σ_a, ξ_a) distribution subject to the curvature parameter ρ_a. An analogous argument applies to the distribution and curvature parameters for projects. We can then use Proposition 2 to normalize $\sigma_a = \sigma_q = 1$ and set the share parameters in the CES function for managers and projects to $\widehat{\nu}$ and $1 - \widehat{\nu}$, respectively. Finally, for any $(\xi_a, \xi_q, \rho_a, \rho_q)$ there always exist λ_a and λ_q such that $\widehat{\rho}_a = \widehat{\rho}_a = \rho$.

Let $\bar{\imath}$ be the occupational cutoff in the $(\xi_a, \xi_q, \rho_a, \rho_q)$ economy and $\hat{\bar{\imath}}$ the corresponding rank in $(\widehat{\xi}_a, \widehat{\xi}_q, \widehat{\rho}_a, \widehat{\rho}_q)$. Then Lemma 2 implies $\bar{\imath} - \hat{\bar{\imath}} \to 0$ as $\min\{\bar{\imath}, \hat{\bar{\imath}}\} \to 1$. Finally, let $\ell[i, i]$ and $\hat{\ell}[i, i]$ denote labor demands in the two economies. Corollary 1 then implies that for $\max\{\bar{\imath}, \hat{\bar{\imath}}\} \leq i < 1$, the labor demands $\ell[i, i] - \hat{\ell}[i, i] \to 0$ as $\min\{\bar{\imath}, \hat{\bar{\imath}}\} \to 1$.

■

Differential Equations when $f(a[i], q[j]) = a[i]^\nu q[j]^{1-\nu}$

Under the assumption that $a[i]$ and $q[j]$ are combined by a Cobb-Douglas technology, the system of differential equations formed by (9) and (10) has a closed form solution.[39] To begin with, recall from Appendix A that $\Phi[i, j] = \phi f[i, j]$, for some $\phi \in \boldsymbol{R}^{++}$. Then the two equations can be written as:

$$\begin{aligned} \frac{a'[i]}{a[i]} &= \omega'[i] \frac{a[i]^{-\nu} q[j]^{\nu-1}}{\phi \nu} \\ \frac{q'[j]}{q[j]} &= \pi'[j] \frac{a[i]^{-\nu} q[j]^{\nu-1}}{\phi(1-\nu)} \end{aligned}$$

Since payments exhaust the available surplus,

$$\Phi[i, j] = \phi a[i]^\nu q[j]^{1-\nu} = \omega[i] + \pi[j].$$

Moreover, let $\tilde{\Phi}[i, j] \equiv \ln \Phi[i, j] = \ln(\omega[i] + \pi[j])$.

$$\begin{aligned} \tilde{\Phi}'[i, j] &= \frac{1}{\Phi[i,j]}(\omega'[i] + \pi'[j]), \\ \tilde{a}'[i] &\equiv \frac{\mathrm{d} \ln a[i]}{\mathrm{d} i} = \frac{\omega'[i]}{\nu \Phi[i,j]}, \\ \tilde{q}'[j] &\equiv \frac{\mathrm{d} \ln q[j]}{\mathrm{d} j} = \frac{\pi'[j]}{(1-\nu)\Phi[i,j]}. \end{aligned}$$

I can then solve for:

$$\ln\left(\frac{a[\imath]}{a[\bar{\imath}]}\right) = \tilde{a}[\imath] - \tilde{a}[\bar{\imath}] = \int_{\bar{\imath}}^{\imath} \tilde{a}'[j] \mathrm{d}j = \int_{\bar{\imath}}^{\imath} \omega'[j] \big(\nu \Phi[j, j]\big)^{-1} \mathrm{d}j = \int_{\bar{\imath}}^{\imath} \frac{\omega'[j]}{\nu(\omega[j] + \pi[j])} \mathrm{d}j$$

[39] I thank Andrew Hollenhorst for pointing out the existence of such a solution.

Exponentiating both sides of the equation:

$$\frac{a[\imath]}{a[\bar{\imath}]} = \exp\left(\int_{\bar{\imath}}^{\imath} \frac{\omega'[j]}{\nu(\omega[j]+\pi[j])}\mathrm{d}j\right)$$

Similarly, I can characterize the exceedences of $q[\imath]$ over a threshold as:

$$\frac{q[\imath]}{q[\bar{\imath}]} = \exp\left(\int_{\bar{\imath}}^{\imath} \frac{\pi'[j]}{(1-\nu)(\omega[j]+\pi[j])}\mathrm{d}j\right)$$

*

REFERENCES

Alder, Simeon D., and Fane N. Groes. 2014. "A Tale of Two C(. . .)s: Competence and Complementarity." Working Paper.

Alder, Simeon D., Moritz Meyer-ter Vehn, and Lee E. Ohanian. 2014. "Dynamic Sorting." Working Paper.

Atkeson, Andrew, and Patrick J. Kehoe. 2005. "Modeling and Measuring Organization Capital." *Journal of Political Economy*, 113(5): 1026 – 1053.

Axtell, Robert L. 2001. "Zipf Distribution of U.S. Firm Sizes." *Science*, 293: 1818–1820.

Baker, George P., Michael C. Jensen, and Kevin J. Murphy. 1988. "Compensation and Incentives: Practice vs. Theory." *Journal of Finance*, 43(3): 593–616.

Bebchuk, Lucian A., and Alma Cohen. 2005. "The Costs of Entrenched Boards." *Journal of Financial Economics*, 78(2): 409 – 433.

Becker, Gary S. 1973. "A Theory of Marriage: Part I." *Journal of Political Economy*, 81(4): 813–846.

Bennedsen, Morten, Francisco Pérez-González, and Daniel Wolfenzon. 2008. "Do CEOs Matter?" Working Paper.

Bertrand, Marianne, and Antoinette Schoar. 2003. "Managing with Style: The Effect of Managers on Firm Policies." *Quarterly Journal of Economics*, 118(4): 1169–1208.

Bhattacharya, Dhritiman, Nezih Guner, and Gustavo Ventura. 2013. "Distortions, Endogenous Managerial Skills and Productivity Differences." *Review of Economic Dynamics*, 16(1): 11–25.

Bloom, Nicholas, and John Van Reenen. 2007. "Measuring and Explaining Management Practices Across Firms and Countries." *Quarterly Journal of Economics*, CXXII(4).

Bloom, Nicholas, Benn Eifert, Aprajit Mahajan, David McKenzie, and John Roberts. 2013. "Does Management Matter? Evidence from India." *Quarterly Journal of Economics*, 128(1): 1–51.

Bloom, Nicholas, Mirko Draca, and John Van Reenen. 2011. "Trade Induced Technical Change? The Impact of Chinese Imports on Innovation, IT and Productivity." NBER Working Paper 16717.

Bloom, Nicholas, Stephen Dorgan, John Dowdy, and John Van Reenen. 2007. *Management Practice and Productivity: Why They Matter.* McKinsey & Company.

Buera, Francisco J., Joseph P. Kaboski, and Yongseok Shin. 2011. "Finance and Development: A Tale of Two Sectors." *American Economic Review*, 101(5): 1964–2002.

Cantore, Christiano, and Paul Leslie Levine. 2012. "Getting Normalization Right: Dealing with 'Dimensional Constants' in Macroeconomics." *Journal of Economic Dynamics and Control*, 36(12): 1931–1949.

Caselli, Francesco, and Nicola Gennaioli. 2013. "Dynastic Management." *Economic Inquiry*, 51(1): 971–996. Working Paper.

Cole, Harold L., Lee E. Ohanian, Alvaro Riascos, and James A. Schmitz. 2005. "Latin America in the Rearview Mirror." *Journal of Monetary Economics*, 52(1): 69–107.

Eisfeldt, Andrea L., and Camelia M. Kuhnen. 2013. "CEO Turnover in a Competitive Assignment Framework." *Journal of Financial Economics*, 109(2): 351–372.

Fernandez, Raquel, and Richard Rogerson. 2001. "Sorting and Long-Run Inequality." *Quarterly Journal of Economics*, 116(4): 1305–1341.

Gabaix, Xavier, and Augustin Landier. 2008. "Why Has CEO Pay Increased So Much?" *Quarterly Journal of Economics*, 123(1): 49–100.

Garcia, Patricio, Peter F. Knights, and John E. Tilton. 2001. "Labor Productivity and Comparative Advantage in Mining: the Copper Industry in Chile." *Resources Policy*, 27(2): 97–105.

Gordon, Robert J. 1971. "Measurement Bias in Price Indexes for Capital Goods." *Review of Income and Wealth*, 17(2): 121–174.

Hall, Robert E., and Charles I. Jones. 1999. "Why Do Some Countries Produce So Much More Output Per Worker Than Others?" *Quarterly Journal of Economics*, 114(1): 83–116.

Holmes, Thomas J., and James A. Schmitz. 2010. "Competition and Productivity: A Review of Evidence." *Annual Review of Economics*, 2: 619–642.

Howitt, Peter. 2000. "Endogenous Growth and Cross-Country Income Differences." *American Economic Review*, 90(4): 829–846.

Hsieh, Chang-Tai, and Peter J. Klenow. 2009. "Misallocation and Manufacturing TFP in China and India." *Quarterly Journal of Economics*, 124(4): 1403–1448.

Jeong, Hyeok, and Robert M. Townsend. 2007. "Sources of TFP Growth: Occupational Choice and Financial Deepening." *Economic Theory*, 32(1): 179–221.

Jones, Charles I. 2011. "Intermediate Goods and Weak Links in the Theory of Economic Development." *American Economic Journal: Macroeconomics*, 3(2): 1–28.

Kang, David C. 2002*a*. "Bad Loans to Good Friends: Money Politics and the Developmental State in South Korea." *International Organization*, 56(1): 177–207.

Kang, David C. 2002*b*. *Crony Capitalism: Corruption and Development in South Korea and the Philippines. Cambridge Studies in Comparative Politics*, Cambridge University Press.

Klenow, Peter J., and Andres Rodriguez-Clare. 1997. "The Neoclassical Revival in Growth Economics: Has It Gone Too Far?" *NBER Macroeconomics Annual*, 12: 73–103.

Kremer, Michael. 1993. "The O-Ring Theory of Economic Development." *Quarterly Journal of Economics*, 108(3): 551–575.

La Porta, Rafael, and Florencio López de Silanes. 1999. "The Benefits of Privatization: Evidence from Mexico." *Quarterly Journal of Economics*, 114(4): 1193–1242.

León-Ledesma, Miguel A., Peter McAdam, and Alpo Willman. 2010. "Identifying the Elasticity of Substitution with Biased Technical Change." *American Economic Review*, 100(4): 1330–1357.

Lindenlaub, Ilse. 2014. "Sorting Multidimensional Types: Theory and Application." Working Paper.

Lucas, Robert E. 1978. "On the Size Distribution of Business Firms." *Bell Journal of Economics*, 9(2): 508–523.

McKinsey Global Institute. 1998. McKinsey Global Institute.

Ostroy, Joseph M. 1980. "The No-Surplus Condition as a Characterization of Perfectly Competitive Equilibrium." *Journal of Economic Theory*, 22(2): 183–207.

Ostroy, Joseph M. 1984. "A Reformulation of the Marginal Productivity Theory of Distribution." *Econometrica*, 52(3): 599–630.

Paulson, Anna L., and Robert M. Townsend. 2004. "Entrepreneurship and Financial Constraints in Thailand." *Journal of Corporate Finance*, 10(2): 229–262.

Pavcnik, Nina. 2002. "Trade Liberalization, Exit and Productivity Improvement: Evidence from Chilean Plants." *Review of Economic Studies*, 69(1): 245–276.

Poschke, Markus. 2014. "The Firm Size Distribution Across Countries and Skill-Biased Change in Entrepreneurial Technology." Working Paper.

Postel-Vinay, Fabien, and Jeremy Lise. 2014. "Multidimensional Skills, Sorting, and Human Capital Accumulation." Working Paper.

Prescott, Edward C. 1998. "Lawrence R. Klein Lecture 1997: Needed: A Theory of Total Factor Productivity." *International Economic Review*, 39(3): 525–551.

Restuccia, Diego, and Richard Rogerson. 2008. "Policy Distortions and Aggregate Productivity with Heterogeneous Establishments." *Review of Economic Dynamics*, 11: 707–720.

Restuccia, Diego, Dennis Tao Yang, and Xiaodong Zhu. 2008. "Agriculture and Aggregate Productivity: A Quantitative Cross-Country Analysis." *Journal of Monetary Economics*, 55(2): 234 – 250.

Roberts, David R. 1956. "A General Theory of Executive Compensation Based on Statistically Tested Propositions." *Quarterly Journal of Economics*, 70(2): 270–294.

Rossi-Hansberg, Esteban, and Mark L.J. Wright. 2007. "Establishment Size Dynamics in the Aggregate Economy." *American Economic Review.*

Sattinger, Michael. 1979. "Differential Rents and the Distribution of Earnings." *Oxford Economic Papers*, 31(1): 60–71.

Sattinger, Michael. 1993. "Assignment Models of the Distribution of Earnings." *Journal of Economic Literature*, 31(2): 831–80.

Schmitz, James A. 2005. "What Determines Productivity? Lessons from the Dramatic Recovery of the U.S. and Canadian Iron Ore Industries Following Their Early 1980s Crisis." *Journal of Political Economy*, 113(3): 582–625.

Syverson, Chad. 2011. "What Determines Productivity?" *Journal of Economic Literature*, 49(2): 326–365.

Terviö, Marko. 2008. "The Difference That CEOs Make: An Assignment Model Approach." *American Economic Review*, 98(3): 642–68.