## A Note on Financial Frictions and Mismatch

## Simeon D. Alder\*

There is a sizeable body of work that discusses the correlation of ability and wealth in the presence of market imperfection. Buera et al. (2011); Caselli and Gennaioli (2005); Jeong and Townsend (2007); Paulson and Townsend (2004), among others, argue that financial or contractual frictions can be a quantitatively important source of aggregate inefficiency. Borrowing constraints have an analogous effect in my companion paper Alder (2012).

The model is identical to the environment in section 2 of that paper except for the presence of an asset *b*. For simplicity, assume that each member owns the family household's per capita asset portfolio *b*. In addition, let there be a fraction  $\varphi \in [0, 1]$  of members who have unencumbered access to credit. The remainder,  $1 - \varphi$ , is financially constrained as in Buera et al. (2011).<sup>1</sup> They deposit their assets with a financial intermediary. The intermediary, in turn, uses these assets to collateralize loans to those managers who are credit-constrained. The severity of the constraint is summarized by a parameter  $\phi$ . The credit limit  $\overline{k}(a, q; \phi)$  of a CEO with ability *a* assigned to a project *q* is defined implicitly by

$$\max_{\ell} \left\{ f(a,q)^{1-\gamma} \Big( \overline{k}(a,q;\phi)^{\alpha} \ell^{1-\alpha} \Big) - w\ell \right\} - \tilde{\pi}(q) - (r+\delta)\overline{k}(a,q;\phi) + (1+r)b$$
$$= (1-\phi) \left[ \max_{\ell} \left\{ f(a,q)^{1-\gamma} \Big( \overline{k}(a,q;\phi)^{\alpha} \ell^{1-\alpha} \Big) - w\ell \right\} - \tilde{\pi}(q) + (1-\delta)\overline{k}(a,q;\phi) \right]$$
(1)

 $\phi$  has a natural interpretation. It is the share of output net of factor payments to labor and project rents plus undepreciated capital the intermediary can repossess should the

<sup>\*</sup>University of Notre Dame, Department of Economics, Email: simeon.alder.l@nd.edu

<sup>&</sup>lt;sup>1</sup>Alternatively, we can consider the effect of universal credit limits when managers are heterogeneous in wealth. This turns out to be onerous for technical reasons. Heterogeneity along two dimensions poses a challenging problem in assignment models. Moreover, it does not sync with our representative family household structure.

borrower default. Put differently, the credit limit implies that credit-constrained managers can only sign self-enforcing contracts. Without loss of generality I will focus my attention on sufficiently high values of  $\phi$  such that the credit limit is not binding for any low-ability managers, that is, those who run small firms since they are – in equilibrium – matched with low-quality projects.

Let  $i^*$  index the marginally unconstrained agent;  $\kappa$  indexes the occupational cutoff as in section 2.5.2. For any  $i \in [\kappa, i^*]$  equations (10), (11), and (12) in the companion paper characterize the non-linear prices  $\tilde{\pi}[i] = \pi[i]$  and  $\omega[i]$ . Project owners above  $i^*$  solve a more demanding problem: while some agents are hampered by credit limits, others are not. Just as before, an equilibrium assignment must satisfy sorting and participation constraints. Here, however, they take into account that constrained and unconstrained agents of equal ability are distinct types.

While the raw distribution of managerial talent is unchanged from the efficient benchmark, credit-constrained CEOs are effectively less "competent". Exactly how different they are depends on equation (1) and, critically, the (equilibrium) rent  $\tilde{\pi}(q)$  paid to the owner of the matched project. The constraint leads equally talented managers to choose different capital and labor inputs compared to unconstrained ones. "Effective" talent in the economy above *i*<sup>\*</sup> is thus an endogenous mixture distribution, the inverse CDF of which I denote by  $\tilde{a}[i]$ . Moreover, let  $\tilde{a}[i, u]$  and  $\tilde{a}[i, c]$  denote the underlying talent distributions associated with unconstrained and constrained managers, respectively. Finally, let g(a,q) be value-added net of factor payments to capital and labor. This is the surplus to be split between project owners and managers. More specifically, q(a[j, u], q[i]) is the surplus when a *j*-ranked, unconstrained CEO is assigned to the *i*-ranked project. Similarly, g(a[j, u], q[i]) is the surplus when an equally competent but constrained agent is matched with that same project. Any discrepancy in the surplus stems from differences in factor inputs between constrained and unconstrained managers.<sup>2</sup> Since the matching between *effective* talent and project quality remains as*sortative*, the profile of rents satisfies:

$$g_q(q[i], \tilde{a}[i])q'[i] = \tilde{\pi}'[i] \tag{2}$$

<sup>&</sup>lt;sup>2</sup>Note that in equilibrium, these two managerial types are *not* assigned to the same project, even though they are equally talented. The capital and labor inputs associated with a given match are pinned down by equation (1) and the embedded first order condition for labor.

The managers' compensation depends, of course, on their standing in financial markets:

$$g_a(q[i], \tilde{a}[i_u, u])\tilde{a}'[i_u, u] = \tilde{\omega}'[i_u, u]$$
(3)

$$g_a(q[i], \tilde{a}[i_c, c]) \tilde{a}'[i_c, c] = \tilde{\omega}'[i_c, c]$$
(4)

with initial condition  $g(q[i^*], a[i^*]) = \omega[i^*] + \pi[i^*] = \tilde{\omega}[i^*] + \tilde{\pi}[i^*]$ .<sup>3</sup> Note that  $i \neq i_u \neq i_c$ , in general.

Since  $\tilde{a}[i]$  is a mixture of distributions with endogenous and non-linear weights, I cannot explicitly solve for  $\tilde{\pi}[.]$ ,  $\tilde{\omega}[., u]$ , and  $\tilde{\omega}[., c]$ . I can, however, sketch the effects of financial frictions and distinguish them from those associated with the presence of insiders with preferential market access.

For all  $i > i^*$ , the factor-input-adjusted distribution of managers who face credit limits has a thinner tail than the distribution of unconstrained CEOs:

$$\tilde{a}'[j,c] < \tilde{a}'[j,u]$$
 and  $\tilde{a}[i^*,c] = \tilde{a}[i^*,u]$ 

Project owners do not care whether they are matched with a constrained or unconstrained CEO, provided the match yields the same rent. That rent depends on the distribution of effective talent and since that mixture is a convex combination of constrained and unconstrained managers, it must be that

$$\tilde{a}'[j] < a'[j]$$

Together with  $\tilde{a}[i^*] = a[i^*]$  this implies that the mixture is again thinner-tailed than the distribution of raw talent.

The effect of credit limits on efficiency is twofold:

1. In terms of raw, rather than input-adjusted, managerial talent, the equilibrium assignment to projects is not assortative (*mismatch effect*).

<sup>&</sup>lt;sup>3</sup>Recall,  $i^*$  is such that  $a[i^*] = \tilde{a}[i^*, u] = \tilde{a}[i^*, c]$ . It indexes the ability of the marginally constrained CEO.

2. For a given assignment, projects run by a constrained CEO are operated at inefficiently small scale and factor intensity (*factor input effect*).

What distinguishes financial frictions from cronyism are the patterns of distortion. Since cronies run the most productive projects, distortions are concentrated in the far right tail of the distribution. In theory, these firms are too large (in terms of employment or value added). With financial frictions, the distortions are concentrated elsewhere in the distribution. Competent, but constrained CEOs are pushed from the right tail to the left and the most talented among them are subject to the most severe distortions. The theory tells us that these firms are "too small."

Recall that managers and owners are treated symmetrically in the unconstrained version of the model in that they are joint residual claimants of output net of factor payments. While I impose collateral constraints on CEOs in this note, it is straightforward to apply borrowing constraints to owners instead.

## References

- Alder, Simeon D, "In the Wrong Hands: Complementarities, Resource Allocation, and TFP," 2012. Working Paper.
- **Buera, Francisco J, Joseph P Kaboski, and Yongseok Shin**, "Finance and Development: A Tale of Two Sectors," *American Economic Review*, 2011, *101* (5), 1964–2002.
- **Caselli, Francesco and Nicola Gennaioli**, "Dynastic Management," 2005. Working Paper.
- Jeong, Hyeok and Robert M Townsend, "Sources of TFP growth: Occupational Choice and Financial Deepening," *Economic Theory*, 01 2007, 32 (1), 179–221.
- **Paulson, Anna L and Robert M Townsend**, "Entrepreneurship and Financial Constraints in Thailand," *Journal of Corporate Finance*, 02 2004, *10* (2), 229–262.