Deceptive Redistribution: Online Appendix*

Simeon D. Alder[†] Guillermo L. Ordoñez[‡]

August 23, 2016

*This document contains the online appendix associated with the paper "Deceptive Redistribution" by the same authors.

[†]University of Notre Dame. E-mail: salder@nd.edu
[‡]University of Pennsylvania and NBER. E-mail: ordonez@econ.upenn.edu

A Many Informed Agents and Free Riding

Is the government's type revealed when the number of informed sectors is large enough? In this section we argue that free riding prevents this from happening, and hence our analysis is robust to relaxing the assumption of two sectors and $\eta > \frac{1}{2}$.

Consider the case with $N \in \mathbb{Z}^{++}$ sectors of equal size, but with the same production and endowment structures as in section 3. Assume also there are two distinct groups of sectors: $1 \leq N_I < N$ sectors are targeted by a policy (i.e., they are informed) while $N_U = N - N_I$ are not targeted by any policy, uninformed sectors that constitute the majority. As in the text, the median voter is uninformed, which constitutes the interesting case of analysis. Analogously to section 2, aggregate output is a function of all β's and α's. The optimal policy for all governments in state β^L is α^L and in state β^H is α^H, but bad governments have private incentives to implement an inefficient policy α^L when the true fundamental is β^H.

We are interested in the behavior of informed sectors, who observe both the fundamental β and the policy α. Symmetry implies that the expected gains from getting rid of a bad government are the same for all distorted sectors. When $\bar{\theta}$ is in the *intermediate* region, a *single* informed agent with evidence is sufficient to inform the clueless voters in uninformed sectors about the incumbent's type and trigger her replacement. Finding evidence, however, is costly and this is where the free-riding problem arises.

In what follows we develop the analysis for state (H, L), but a parallel one exists for state H, H, just replacing $\bar{\theta}$ with 0. Let $\hat{\pi}_i$ denote sector i's net expected gain from getting rid of the incumbent, i.e., obtaining $(1-\eta)Y^H$ instead of loosing $(1-\eta)\bar{\theta}$.

$$\widehat{\pi} \equiv F(\widehat{z}_{HL})(1-\eta)Y^H - \big(1 - F(\widehat{z}_{HL})\big)(1-\eta)\bar{\theta}.$$

We are looking for a symmetric mixed strategy equilibrium, where p denotes the probability an individual informed sector exerts effort to find credible evidence. If it does, the expected net gain is $\widehat{\pi} - \widehat{z}^*_{HL}$. If it does not, it realizes a net gain of 0 with probability $(1-p)^{N_I - 1}$ (no other informed sector finds evidence) or $\widehat{\pi}$ with probability $1 - (1-p)^{N_I - 1}$ (at least one other informed sector finds evidence).

Any individual sector is indifferent between looking for evidence and free riding whenever

$$\widehat{\pi} - \widehat{z}^*_{HL} = \widehat{\pi}\big(1 - (1-p)^{N_I - 1}\big).$$

where $\widehat{z}^*_{HL}$ is determined by the maximization of

$$\max_{\widehat{z}_{HL}} \widehat{\pi}(1-p)^{N_I - 1} - \widehat{z}_{HL}$$

and then

$$F'(\widehat{z}^*_{HL})(1-\eta)(Y^H + \bar{\theta})(1-p)^{N_I - 1} = 1,$$

The (symmetric) probability of exerting $\widehat{z}^*_{HL}$ is

$$p = 1 - \left(\frac{\widehat{z}^*_{HL}}{\hat{\pi}}\right)^{\frac{1}{N_I - 1}}.$$

The probability that at least one informed sector exerts effort is

$$1 - (1-p)^{N_I} = 1 - \left(\frac{\widehat{z}^*_{HL}}{\hat{\pi}}\right)^{\frac{N_I}{N_I - 1}}.$$

In the limit, with $N_I \to \infty$ the corresponding probabilities are

$$\begin{aligned} p &\to 0 \\ &\text{and} \\ 1 - (1-p)^{N_I} &\to 1 - \frac{\widehat{z}^*_{HL}}{\hat{\pi}} > 0, \\ &\quad \text{since } \frac{N_I}{N_I - 1} \to 1 \text{ as } N_I \to \infty. \end{aligned}$$

That is, the probability that at least one informed sector finds evidence in the presence of free-riding does *not* converge to one as the number of informed sectors grows. The reason is that the probability of a single informed sector exerting effort $\widehat{z}^*_{HL}$ goes to zero at a faster rate than the number of informed agents goes to infinity. Put differently, in the model with a continuum of informed agents and free riding, the uncertainty about the government's type is *not* completely resolved in the presence of free-riding. Bad governments can maintain their reputation in expectation even when they are distorting a large number of sectors.

B Endogenous Value of Reputation

In the main text we assume a single period model and an exogenous continuation value, $\Pi(\phi)$, which was increasing in ϕ. Reputation, however, is an intrinsically dynamic concept, and its continuation value arises endogenously in a dynamic game. In this section we show how to determine the value $\Pi(\phi)$ endogenously and we discuss why the solution for a *single period* as we characterize in the main text is effectively the solution for *any period* that is sufficiently far from a terminal period T.

Where does the value of reputation come from? As long as retaining power has some value and reputation increases the likelihood of staying in office, reputation has a positive value. In what follows we describe how continuation values are determined in a model with an arbitrarily large number of periods and what the conditions are for these values to be increasing.

To introduce dynamics we assume an overlapping generation structure, in which three cohorts coexist in every period; young, old and retired. When young, agents are uninformed about the real effects of government's policies, hence they play the role of sector $\mathbb{2}$ in the simple model. When old, they are informed and assume the role of sector $\mathbb{1}$. The retired simply consume a fixed amount of endowment and die before the end of the period. The timing within each period is the same as in the main text, with one exception: at the end of each period the current young and old elect the government with updated posterior reputation ϕ for one more period, or replace the incumbent with a successor who, by assumption, has reputation ϕ_0.

To avoid deterministic outcomes and to ensure that continuation values are smooth functions of reputation, we assume that there is a random utility gain associated with reelecting the incumbent, for all cohorts and in every period, denoted by $\nu_t \sim F_\nu[-\underline{\nu}, \underline{\nu}]$.

Only young and old cohorts vote. Since the old retire at the end of the period, they cast their vote only based on ν, this is because their consumption does not depend on the government type. In contrast, the young are old next period and decide based on ν and the probability that the incumbent is good, ϕ. Under this assumption elections themselves do not reveal additional information about the government's type.

How are the government's continuation values $\Pi_t(\phi)$ determined at the end of each period t? Assume that the economy comes to an end in period T and $\Pi_T(\phi) = 0$ for all ϕ. To fix ideas we focus on the case where $\bar{\theta}$ is in the *large* region but the same line of reasoning extends to the other two regions. In this region, we still have $\tau_R(\phi) = 1$ for all ϕ, and since there are no penalties in terms of loosing reputation (the left hand side of equation (6) is zero), bad governments always distort in the final period, that is $\tau_{X,T}^{B,HL}(\phi) = \tau_{X,T}^{B,HH}(\phi) = 1$ for all ϕ. Intuitively, bad governments have no incentive to stay in office and prefer to extract the short-term rents by distorting while they can.

Since agents anticipate this behavior, they are more likely to reelect an incumbent at the end of period $T-1$ if he has a good reputation. The expected gains for the young from reelecting an incumbent with reputation ϕ at the end of $T-1$ is

$$E^y(U_T|\phi) = E(C_T|\phi)+\nu_T = (1-\eta)\left[\zeta(1-\gamma)\bar{\theta} + (1-\zeta)[\phi(Y^H + (1-\gamma)\bar{\theta}) + (1-\phi)(1-2\gamma)\bar{\theta}]\right]+\nu_T.$$

In contrast, the expected gains from replacing the incumbent, at the end of $T-1$, with a new type ϕ_0 is

$$E^y(U_T|\phi_0) = E(C_T|\phi_0) = (1-\eta)\left[\zeta(1-\gamma)\bar{\theta} + (1-\zeta)[\phi_0(Y^H + (1-\gamma)\bar{\theta}) + (1-\phi_0)(1-2\gamma)\bar{\theta}]\right].$$

This implies that young voters reelect the incumbent for the terminal period if

$$\nu_T > \bar{\nu}_T(\phi) \equiv E(C_T|\phi_0) - E(C_T|\phi) = (\phi_0 - \phi)(1-\eta)(1-\zeta)(Y^H + \gamma\bar{\theta}) > 0.$$

Trivially, old voters reelect the incumbent for the final period if $\nu_T > 0$. Intuitively, the incumbent is reelected if the utility gains from confirming the incumbent are larger than the expected gains from ousting her. For the young cohort this is given by the expected lower probability of distortions by her successor at time T (this is in terms of probabilities given by $\phi_0 - \phi$), which translates into higher output and consumption. Naturally, if $\phi > \phi_0$, young voters re-elect the incumbent even if it comes at some utility cost $\nu_T < 0$. An incumbent is re-elected at the end of period $T-1$ if

$$\xi_T(\phi) = \tfrac{1}{2}\left[1 - F_\nu(\bar{\nu}_T(\phi))\right] + \tfrac{1}{2}\left[1 - F_\nu(0)\right],$$

which is clearly increasing in ϕ (since $\bar{\nu}_T(\phi)$ is decreasing in ϕ). The better the incumbent's reputation, the more likely her re-election.

A higher reelection probability is important for bad types at $T-1$ since there are gains from distorting in the future and obtaining $(1-\zeta)\Delta$. If, in addition, fixed and exogenous rents $\bar{\Pi}$ are associated with being in office (power, prestige, etc), it is straightforward to see that the governments' continuation value at $T-1$ is increasing in reputation:

$$\Pi_{T-1}(\phi) = \xi_T(\phi)[(1-\zeta)\Delta + \bar{\Pi}]$$

Following the same logic, the expected gains to the young from reelecting an incumbent with reputation ϕ at the end of $T-2$ is

$$E^y(U_{T-1}|\phi) = (1-\eta)\left[\zeta(1-\gamma)\bar{\theta} + (1-\zeta)[\widehat{\phi}_{T-1}(Y^H + (1-\gamma)\bar{\theta}) + (1-\widehat{\phi}_{T-1})(1-2\gamma)\bar{\theta}]\right] + \nu_{T-1}$$

where

$$\widehat{\phi}_{T-1} = \phi + (1-\phi)[\gamma(1-\tau^{B,HL}_{X,T-1}(\phi)) + (1-\gamma)(1-\tau^{B,HH}_{X,T-1}(\phi))]$$

and both $\tau^{B,HL}_{X,T-1}(\phi)$ and $\tau^{B,HH}_{X,T-1}(\phi)$ are determined by equation (6).

Similarly, the expected gains from replacing the incumbent, at the end of $T-2$, with a new ϕ_0 type is

$$E^y(U_{T-1}|\phi_0) = (1-\eta)\left[\zeta(1-\gamma)\bar{\theta} + (1-\zeta)[\widehat{\phi}_{0,T-1}(Y^H + (1-\gamma)\bar{\theta}) + (1-\widehat{\phi}_{0,T-1})(1-2\gamma)\bar{\theta}]\right]$$

where

$$\widehat{\phi}_{0,T-1} = \phi_0 + (1-\phi_0)[\gamma(1-\tau^{B,HL}_{X,T-1}(\phi_0)) + (1-\gamma)(1-\tau^{B,HH}_{X,T-1}(\phi_0))]$$

This implies that young voters reelect the incumbent at the end of period $T-2$ if

$$\nu_{T-1} > \bar{\nu}_{T-1}(\phi) = (\widehat{\phi}_{0,T-1} - \widehat{\phi}_{T-1})(1-\eta)(1-\zeta)(Y^H + \gamma\bar{\theta}) > 0$$

Since old voters at $T-2$ reelect the incumbent if $\nu_{T-1} > 0$, the probability of reelection of an incumbent with reputation ϕ at the end of period $T-2$ is

$$\xi_{T-1}(\phi) = \frac{1}{2}\left[1 - F_\nu(\bar{\nu}_{T-1}(\phi))\right] + \frac{1}{2}\left[1 - F_\nu(0)\right],$$

which is also increasing in ϕ under the sufficient condition that $\bar{\nu}_{T-1}(\phi)$ is decreasing in ϕ.

As the government's continuation value take the possibility of staying in power until period T into account,

$$\Pi_{T-2}(\phi) = \xi_{T-1}(\phi)[(1-\zeta)[\gamma(1-\tau^{B,HL}_{X,T-1}(\phi))+(1-\gamma)(1-\tau^{B,HH}_{X,T-1}(\phi))]\Delta+\bar{\Pi}]+\delta\mathbb{E}_\phi\Pi_{T-1}(\phi).$$

Assuming that the incentives to distort are decreasing in ϕ (for example for relatively low reputation levels), there is a counter-force that tends to depress the government's continuation value. On the one hand, continuation values increase with reputation because the probability of reelection increases with reputation. On the other hand, the lower probability of distortion associated with a better reputation decreases the prospective gains from distortion: they don't behave opportunistically very often.

To summarize, the government's continuation values are increasing in reputation if and only if the gains from being in power, $\bar{\Pi}$, are large enough to compensate for the decrease in opportunities to extract rents. In other words, an equilibrium with continuation values increasing in ϕ and distortion probabilities decreasing in ϕ is sustained when the benefit associated with reelection, $\bar{\Pi}$, is large enough compared to the opportunistic incentives of rent-seeking.

Finally, following Ordoñez (2013), continuation values are a contraction mapping such that, for periods far away from the terminal period T, they converge to a stationary solution for all ϕ, which implies the distortion probabilities $\tau_{t,X}(\phi)$ are also stationary for all ϕ. In this sense, the solution characterized in the main text for a *single period* is effectively the solution for *any period* sufficiently far from the terminal period, and then it is the unique limit, as $T \to \infty$, to the finite horizon Markov perfect equilibria described in Proposition 1.

References

Ordoñez, Guillermo L., "Fragility of Reputation and Clustering of Risk-Taking," *Theoretical Economics*, 2013, *8* (3), 653–700.