

Maturity Walls

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Motivation

- **Maturity walls**: a **majority of debt** scheduled to mature within **short period** (< 2 yrs)
 - Large source of rollover risk
 - Failure to rollover \rightarrow cut investment, fire workers, and default
 - Common feature of non-financial firms' debt structure
- Dimension of debt structure **important** to rating agencies
 - Existing frameworks not well suited to consider impact of maturity walls
- Pose **understudied risks** to the aggregate economy
 - May amplify aggregate shocks if many firms refinance maturity walls during crisis

Research Questions

1. Why do firms concentrate debt payments, and how do they impact borrowing and default risk?
2. How much do maturity walls amplify transmission of a credit market freeze?

What I do in this paper

- Construct **novel** measure of **debt maturity dispersion**
 - 47% firms **choose** maturity walls (firms w/ 1 bond outstanding)
 - Maturity walls associated w/ higher credit risk (higher expected defaults and credit spread)
 - Why choose maturity wall? Large **fixed costs** to issue bonds (underwriter fee \downarrow issuance size)
- Develop dynamic heterogeneous firm **credit risk model** where:
 - Receive persistent income shocks
 - Pick **level** of long-term debt
 - Choose to **concentrate** or **disperse** debt payments
- Mechanism
 - Tax benefit of debt \rightarrow firms want to borrow
 - Trade-off: **Fixed debt** (**convex equity**) issuance costs \rightarrow **concentrate** (**disperse**) payments
 - Interaction btwn costs & benefits determines level and dispersion of debt payments
- Estimate model via SMM, externally validate, & quantify risks of maturity walls

Preview of results

1. How much do maturity walls matter for firm credit risk?

- In equilibrium: account for 8% of firm defaults
- Causal effect: \uparrow default rates by 36 bps (25%) & borrowing costs by 30 bps (21%)

2. Are firms less risky if it is cheaper to issue debt?

- Solve for counterfactual economy w/ lower debt issuance costs
- Higher eqm default (1 pp) & credit spreads (1.2 pp) b/c firms \uparrow borrowing compared to baseline

3. Do maturity walls amplify an aggregate credit shock to firm defaults?

- Firms w/ maturity walls due at shock are most likely to default
- Account for 16% of firm defaults

4. What do we get wrong by omitting maturity walls?

- Underestimate transmission of credit shock to default rates by 14% – 60%

Literature

Determinants of corporate debt structure:

- Stohs and Mauer (1996); Huang, Oehmke, Zhong (2019); Choi, Hackbarth, and Zechner (2018, 2021); Mota and Siani (2024)

Contribution: First to focus on maturity walls & impacts on firm default risk

Long-term debt and firm dynamics:

- Leland and Toft (1996); Chatterjee & Eyigungor (2012); Arellano & Ramanarayanan (2012); Diamond and He (2014); He and Milbradt (2016); Geelen (2019); DeMarzo and He (2021); Dangl and Zechner (2021); Jungherr and Schott (2021), Chaderina (2023)

Contribution: Literature assumes stylized schedule of debt payments, at odds with data
Incorporate maturity walls to fill gap

Financial heterogeneity and aggregate shocks:

- Crouzet (2017); Ottonello and Winberry (2020); Jungherr, Meier, Reinelt, Schott (2022); Crouzet and Tourre (2023)

Contribution: Aggregate implications of maturity walls

Roadmap

Empirical Facts

Model

Mapping Model to Data

Model Mechanics

Quantitative Exercises

Empirical Facts

Data

Mergent FISD: Universe of Corporate Bond Issuances

Bank Debt

- Focus: Non-financial corporate bonds
- Bonds excluded: foreign currency, asset-backed, convertible, and foreign issuer bonds
- Provides terms and history of bond issue
 - date of issuance, maturity at issuance, coupon payments, and repurchases
 - credit spreads, yield at issuance, underwriter fees
- Construct amount of debt outstanding by maturity for all (parent) firm-year pairs

Compustat: Balance sheet information

Sample Period: Annual, 1995 - 2019

Constructing a measure of debt payment dispersion

Share of debt due in m years:

$$s_{m,t} = \frac{\overbrace{b_{m,t}}^{\text{Debt due in } m \text{ yrs}}}{\sum_{m=1}^M b_{m,t}}$$

Standard deviation of debt maturity dates:

$$\sigma_{Mat,t} = \sqrt{\sum_{m=1}^M s_{m,t} (m - \underbrace{\mu_{Mat,t}}_{\text{Avg. Mat}})^2}$$

- Low $\sigma_{Mat,t}$ \rightarrow concentrated debt payments

When measuring payment dispersion, two features are desirable:

1. How large $s_{m,t}$
2. Size of neighboring debt payments (i.e: $s_{m-1,t}$ and $s_{m+1,t}$)
 - Firm A: pays 1/2 debt in $m = 1$ and $m = 2$ ($\sigma_{Mat} = 0.5$ years, $HI_{Mat} = 0.5$)
 - Firm B: pays 1/2 debt in $m = 1$ and $m = 20$ ($\sigma_{Mat} = 9.5$ years, $HI_{Mat} = 0.5$)
 - Reasonable to think rollover risk between firms is different

Summary of Empirical Facts

1. **47%** of firms have maturity walls

Primarily made up of firms with 1 bond outstanding

[More](#)

2. Maturity walls are **not** byproduct of firms financing over life cycle

[More](#)

3. Maturity walls are associated with higher credit risk

↑ 1 pp prob. default & 25 bps credit spread

[More](#)

4. Firms face **economies of scale** in bond issuance

Consistent with presence of fixed issuance costs

[More](#)

Next: Quantitative model informed by these facts

- Firm optimally choose:
 - i. How much to borrow
 - ii. How concentrated their debt payments are
- Key trade-off: convex equity costs (rollover risk) v. fixed debt issuance costs

Model

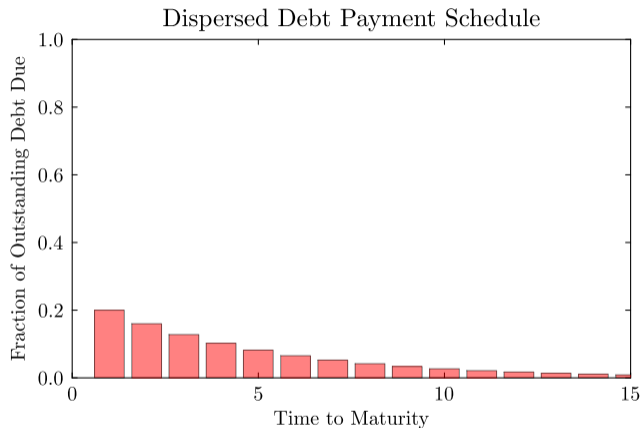
Firms

- Continuum of risk neutral firms that maximize dividend stream over infinite horizon
- Heterogeneous in states $\mathcal{S} \equiv (b_D, b_C, y, \eta, \varepsilon)$
 - b_D : debt w/ dispersed payments
 - b_C : debt w/ concentrated payment
 - y : firm revenue $y \sim G(y|y_{-1})$
 - η : iid repayment shock $Pr(\eta = 1) = \lambda$
 - $\varepsilon(b'_D, b'_C)$: iid manager pref. shocks over debt choices $\varepsilon \sim \text{Type 1 EV}(0, \sigma_\varepsilon)$
- Firm chooses:
 - $b'_D \in \mathcal{B}_D \equiv \{b_{1,D}, b_{2,D}, \dots, b_{n_D,D}\}$
 - $b'_C \in \mathcal{B}_C \equiv \{b_{1,C}, b_{2,C}, \dots, b_{n_C,C}\}$

Firms

- Debt Prices:
 - Priced by representative lender
 - Firm specific prices $\{q_D(b'_D, b'_C, y), q_C(b'_D, b'_C, y)\}$ that depends on debt choices
- Frictions:
 - Tax benefit of debt: deduct per-period coupon payment $(b_D + b_C)\tilde{c}$ off pre-tax income (y)
 - Convex equity issuance cost: α
 - Fixed debt issuance cost: c_I
 - Limited liability: firms can default on debt obligations
 - Liquidation costs: lender recovers fraction of firm's assets (χ) if firm defaults

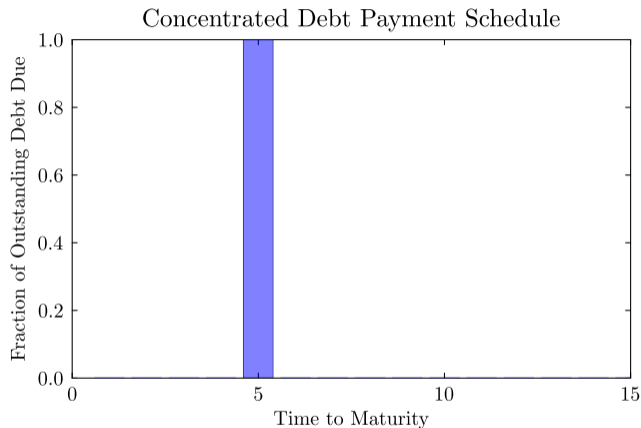
Modeling dispersed and concentrated debt payments



Dispersed Debt Payments (b_D):

- Exponentially maturing coupon bonds with constant amortization rate λ
- Each period: λb_D units of required principal repayments from maturing bonds

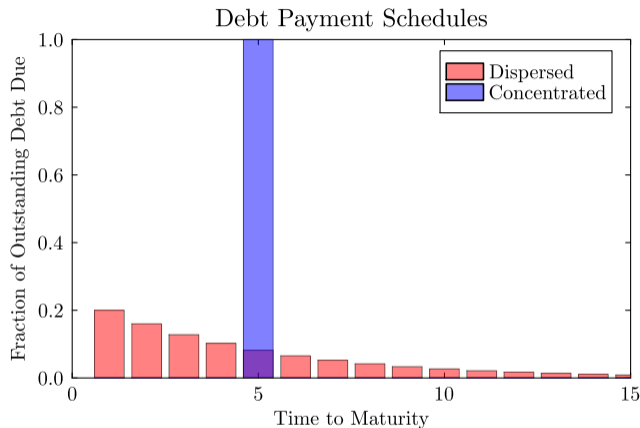
Modeling dispersed and concentrated debt payments



Concentrated Debt Payment (b_C):

- Bond pays a coupon until **random** expiration ($\eta = 1$) which arrives w/ probability λ
- When bond expires ($\eta = 1$), firm must fully repay b_C

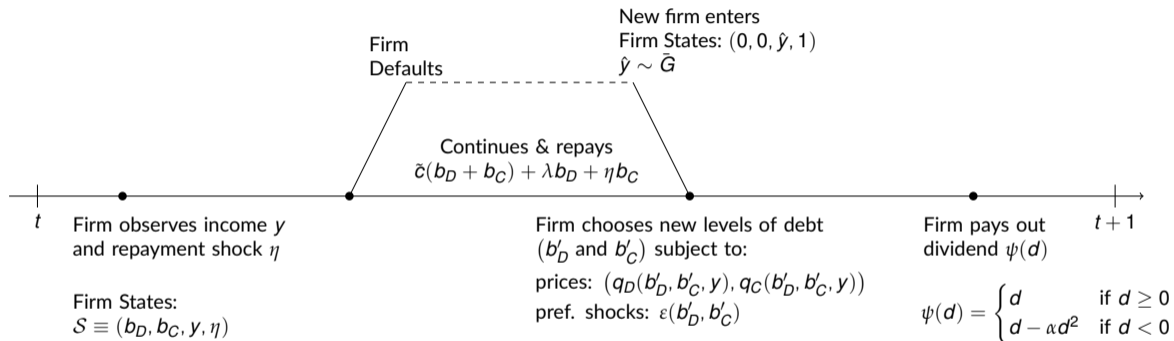
Modeling dispersed and concentrated debt payments



Remarks:

- Firm required to pay $\underbrace{\tilde{c}(b_D + b_C)}_{\text{Coupon}} + \underbrace{\lambda b_D + \eta b_C}_{\text{Principal}}$ to avoid default
- Bonds are identical in terms of (i) payment amount and (ii) average maturity ($1/\lambda$)
- Differ only in terms of schedule of payments

Timing


[Firm's Problem](#)
[Lender's Problem](#)
[Equilibrium Definition](#)

Mapping Model to Data

Mapping the model to the data

Model is estimated on annual FISC & Compustat data from 1995 - 2019

- Parameters divided into **externally calibrated** and **internally estimated**
- **Externally calibrated** parameters are chosen outside model
 - Estimate income process to capture underlying asset value fluctuations
 - Income is mapped to annual sales data
 - $\log(y') = \rho_y \log(y) + \sigma_y \varepsilon_y, \quad \varepsilon_y \sim N(0, 1)$
 - Average maturity $1/\lambda$ is matched to average maturity of corp. bonds
- **Internally estimated** parameters are jointly estimated via SMM
 - Match empirical moments important for debt issuance, rollover, and concentration
 - Construct model equivalent St. Dev of Debt Maturity Dates

Mapping b_D & b_C to σ_{Mat}

Model Estimation

Parameter	Description	Value	SE	Target/Reference	Data	Model
Externally Calibrated						
β	Discount factor	0.960	-	4% Annual Risk Free Rate	-	-
\bar{c}	Per-period coupon payment	$1/\beta - 1$	-	Eqm price of riskless debt is 1	-	-
τ	Corporate tax rate	0.300	-	Hennessy & Whited (2007)	-	-
ρ_y	Persistence: income shock	0.660	-	Auto-correlation of log sales	0.66	0.66
σ_y	St. dev: income shock	0.310	-	Log sales volatility	0.31	0.31
$1/\lambda$	Average Maturity of debt	8.300	-	Avg. debt maturity	8.30	8.30
Internally Estimated						
c_f	Fixed cost of production	0.967	0.244	Default rate (%)	1.13	1.20
α	Equity issuance cost	0.011	0.002	Avg. debt to income	2.22	2.22
σ_e	St. dev: pref. shock	0.001	0.000	St. dev debt to income	5.36	5.34
χ	Lender recovery fraction	0.093	0.040	Avg. credit spread	1.87	1.70
c_l	Fixed debt issuance cost	0.003	0.001	Avg. dispersion maturity dates	2.61	2.62
				Avg. underwriter fee (%)	0.79	0.75

Model fit: Leverage Distribution

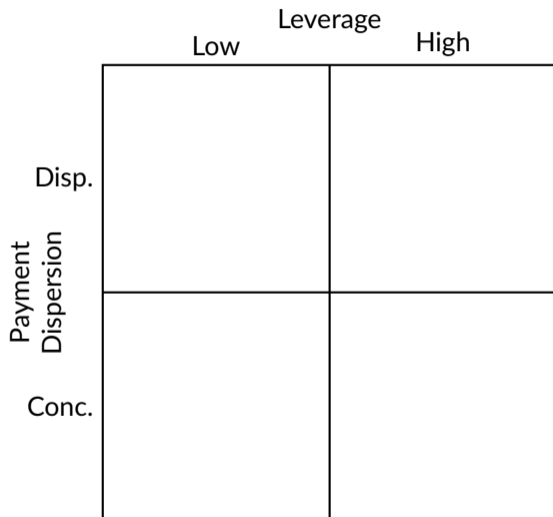
Model fit: σ_{Mat} Distribution

Model fit: Underwriter Fee Distribution

Model Mechanics

1. Joint choice of debt level and dispersion
2. Default / spreads depends on debt payment dispersion

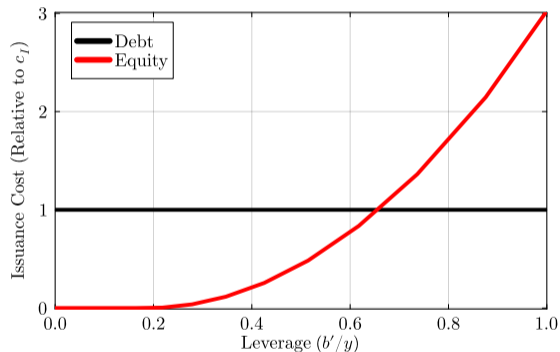
Choosing leverage & dispersion bundles



Choosing leverage & dispersion bundles: Dispersion choice

		Leverage	
		Low	High
Payment Dispersion	Disp.	X	
	Conc.		X

Cost difference between debt and equity issuance



Estimated model:
Avg debt cost / equity cost ≈ 3

[More](#)

Choosing leverage & dispersion bundles: Leverage choice

Firm chooses between two bundles:

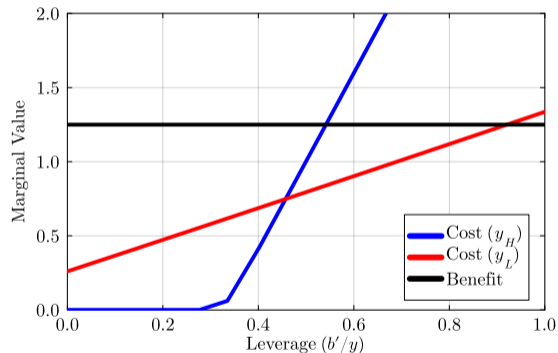
1. Low leverage, concentrated debt payment
2. High leverage, dispersed debt payments

Equate marginal benefit w/ marginal cost

Decrease in firm income (y)

- More likely to inject equity for **low** b/y
 \implies Increase in MC curve
- For given b/y , b lower in level
 \implies Flattening of MC curve

Marginal benefit & marginal cost of higher leverage



External validation of model mechanics

Model Predictions

- High leverage \implies dispersed debt payments
- High income \implies concentrated debt payments
(because lowly levered)

Untargeted Conditional Correlations

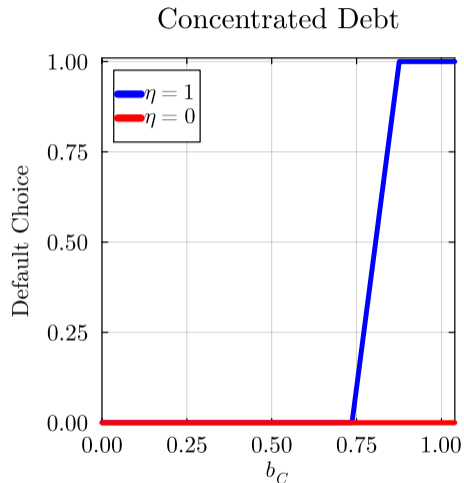
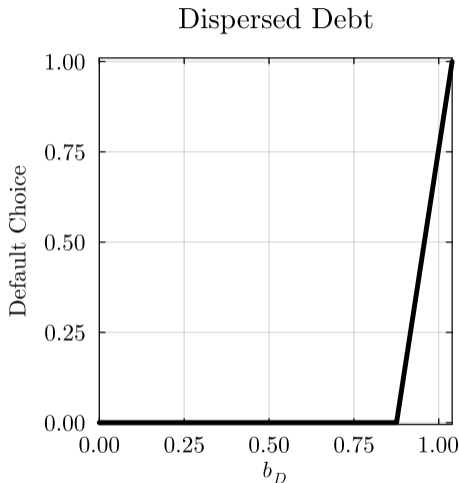
	St. Dev Maturity Dates (σ_{Mat})	
	Data	Model
Leverage	0.182	0.189
Income (Profit)	-0.071	-0.033
Additional Firm Controls	Yes	—

Note: Additional controls include: Size, Age, Average Maturity, Cash, Fraction of Bond Debt, IG Dummy

Model Mechanics

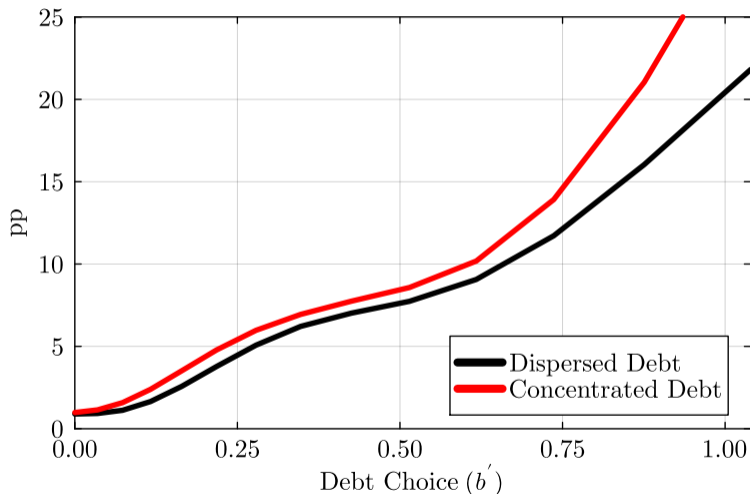
1. Joint choice of debt level and dispersion
2. Default / spreads depends on debt payment dispersion

Default behavior depends on debt payment concentration choice



Firm w/ b_C cannot sustain as high a level of debt as b_D

Dispersed debt payments \rightarrow lower interest rates



Takeaway: Interest rates price in firm def. risk \rightarrow borrowing cost $b_C >$ borrowing cost b_D

External validation of model mechanics II

Model Predictions

- High leverage \implies higher credit spread
- Dispersed debt payments \implies lower credit spread

Untargeted Conditional Correlations

	Credit Spread (bps)	
	Data	Model
Leverage	29.69	72.80
σ_{Mat}	-12.35	-19.79
Income (Sales)	-36.93	-46.55
Additional Firm Controls	Yes	—

Note: Additional controls include: Size, Age, Average Maturity, Cash, Fraction of Bond Debt, IG Dummy

Quantitative Exercises

1. How much do maturity walls contribute to credit risk?
2. Are firms less risky if issuing debt is cheaper?
3. Do maturity walls amplify transmission of credit market freeze?

How much do maturity walls contribute to credit risk?

In equilibrium:

- 8% of defaults are from firms failing to repay maturity walls

Causal effect of maturity walls on credit risk:

- Can't compare two firms w/ & w/o maturity wall, since it is endogenous choice
- Use structural model to generate exogenous variation in debt structure

Counterfactual economy:

- i firm's total leverage decision is **held constant** at baseline values
- ii firm's borrow all in b_C
- iii firm's optimally choose to default
- iv lender's optimally price debt to make zero profits

	bps	%	Baseline Value
Δ Default Rate	35.9	25.0	1.2%
Δ Credit Spread	30.0	21.1	1.7%

Quantitative Exercises

1. How much do maturity walls contribute to credit risk?
2. Are firms less risky if issuing debt is cheaper?
3. Do maturity walls amplify transmission of credit market freeze?

Are firms less risky if issuing debt is cheaper?

Underwriter fees (c_I) are important input in firm's choice to have maturity wall or not

Manconi, Neretina, Renneboog (2019)

- Corporate bond underwriters have market power
- Economically significant:
 - **Mean** market power: **12.2 bps** (16%) of underwriter fee
 - **Max** market power: **19.4 bps** (25%) of underwriter fee

How does eliminating underwriter market power affect firm's spreads, default, and market value?

- Counterfactual equilibrium: underwriter fee in perfectly competitive economy
- Reduce underwriter fee by mean/max percentage of underwriter market power estimates

How does underwriter market power impact default & spreads?

Underwriter fees (c_I):

- Pushes firms to concentrate debt issuance and payments
- Reducing c_I : firm can disperse debt payments at lower cost

Ex-ante **unclear** how decreasing issuance cost will impact default & spreads:

- **Composition** of debt changes:
borrow **more** b_D and less b_C : \downarrow default risk & credit spreads
- Total debt **level** increases:
maintain **higher total debt** with b_D : \uparrow default risk & credit spreads

In economy with competitive underwriter market

- \uparrow risk from borrowing more $>$ \downarrow reduced risk from debt composition changes
- Credit spreads & default rates are higher

Counterfactual equilibrium: competitive underwriter market

	Baseline	$0.84c_I$	$0.75c_I$
Share of debt held in b_D	18.25%	42.46%	50.00%
Debt Maturity Dispersion σ_{Mat}	2.62 years	4.85 years	5.62 years
Book leverage	20.72%	38.03%	42.48%
Market leverage	14.67%	24.26%	26.59%
Credit spread on b_D	1.44%	2.54%	2.84%
Credit spread on b_C	1.70%	2.65%	2.89%
Average credit spread	1.69%	2.64%	2.88%
Firm default rate	1.19%	1.96%	2.15%
Δ Market value	—	0.85%	1.01%

Takeaway: Firm's borrowing increases \rightarrow \uparrow credit spreads & default rates

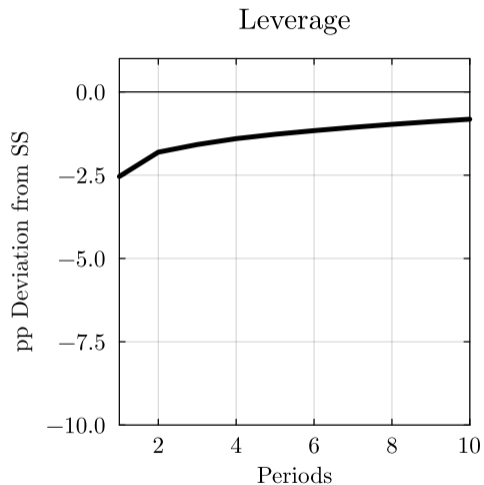
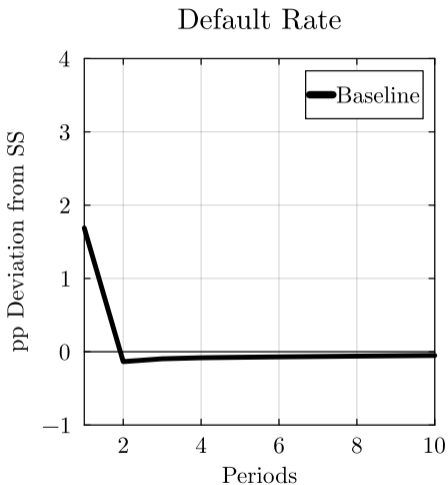
Quantitative Exercises

1. How much do maturity walls contribute to credit risk?
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Macroeconomic implications of maturity walls on market freezes

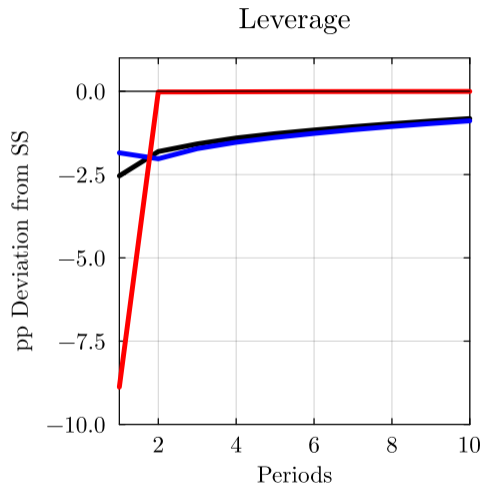
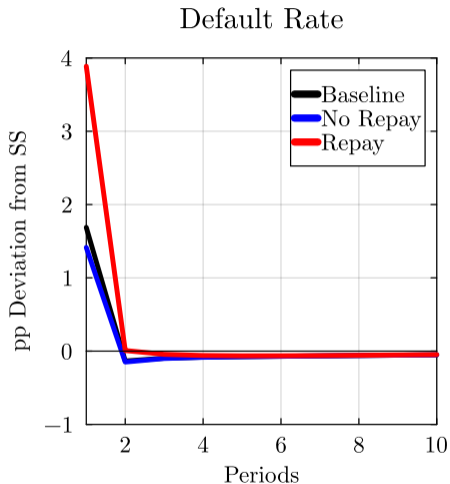
- Firms with maturity walls are more exposed to rollover risk
- ~50% of firms have maturity wall
- What are macro implications of maturity walls on credit market freeze?
 - Credit market freeze \rightarrow large decline in volume of transactions in primary market
- Credit Market freeze: unanticipated one period shock where
 - debt market shuts down (no new borrowing or early pre-payment)
 - equity issuance cost rises (calibrated to match \uparrow in default rate observed in GFC)
- Firms may be “unlucky” at having to repay maturity wall at time of market freeze
 - Unable to rollover
 - Amplifies default

Aggregate effects of credit market freezes



Default rate increases by 168 bps in market freeze

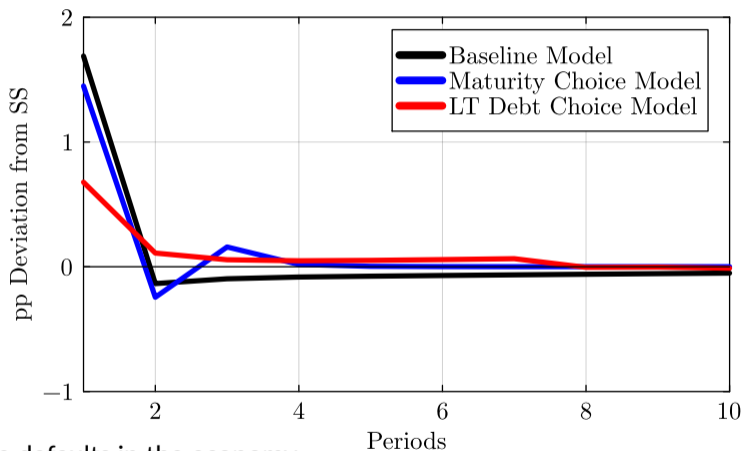
Heterogeneous effects of credit market freezes



16% of defaults are by firms who need to pay b_C at time of market freeze (2x from baseline)

What do we miss without concentrated debt payments?

Default Rate



We underestimate defaults in the economy

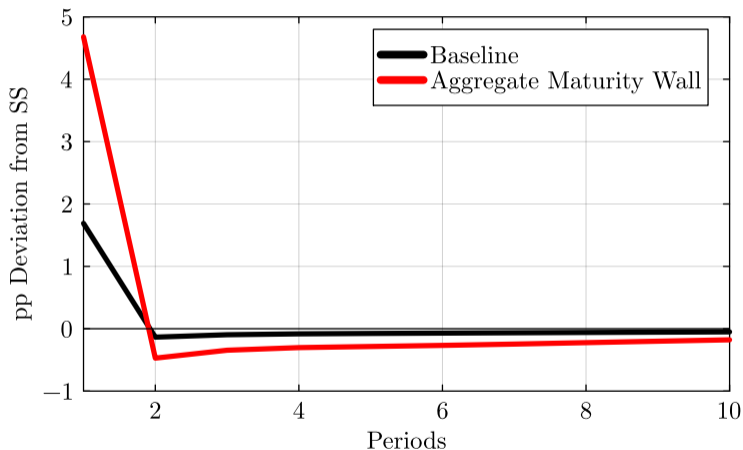
- 60% (100 bps) compared to **LT debt choice** model
- 14% (25 bps) compared to **maturity choice** model

(DeMarzo & He, 2021)

(He & Milbradt, 2016)

Aggregate maturity wall during credit market freeze

Default Rate



Aggregate maturity wall (η common across firms) \rightarrow 299bps (178%) higher default rate

Conclusion

Maturity walls matter for borrowing level, default, and borrowing costs

- Structural model of firm debt and maturity date concentration
- Key trade-off: issuance cost v. rollover risk
- Joint determination of leverage and maturity walls is important

Today I showed:

- Maturity walls \uparrow default rates by 36 bps (25%) & borrowing costs by 30 bps (21%)
- Removing market power in underwriter market \rightarrow higher eqm default compared to baseline
- Miss up to 60% of defaults during credit market freeze by omitting maturity walls
- Aggregate maturity walls amplify defaults during credit market freeze 299 bps

Appendix

What about bank debt?

Conditional on being bond issuer:

- Bond debt accounts for 87% of bank + bond debt
- Bank debt accounts for 60% of total debt (bank, bond, mortgages, credit lines, CP, etc.)

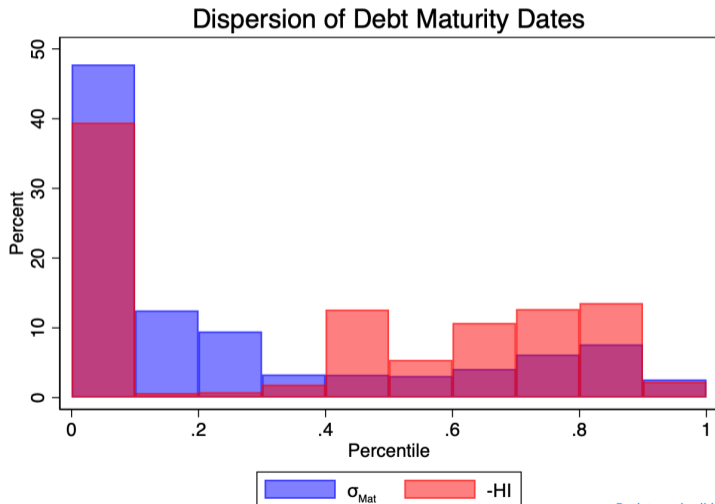
At aggregate level (Flow of Funds):

- Bond debt accounts for 55% of aggregate corporate borrowing
- Bank debt accounts for 10% of aggregate corporate borrowing

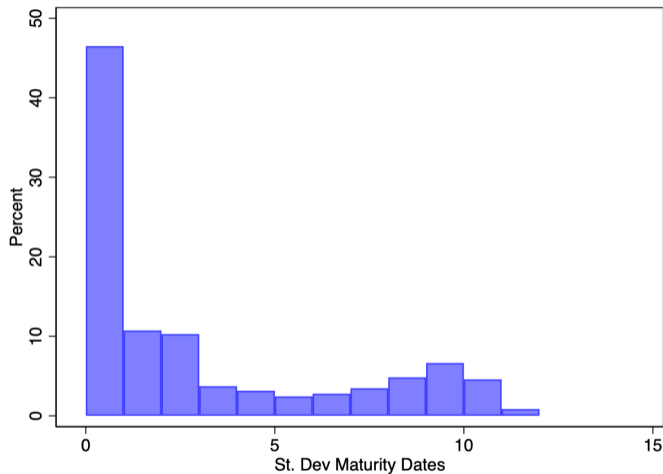
Constructing a measure of maturity walls

$$HI_{Mat,t} = \sum_{m=1}^M s_{m,t}^2$$

- High HI_{Mat} \rightarrow concentrated debt payments
- σ_{Mat} picks up in maturity walls missed by Mat



Fact 1: 47% of firms have maturity walls



Weighted by age

Weighted by size

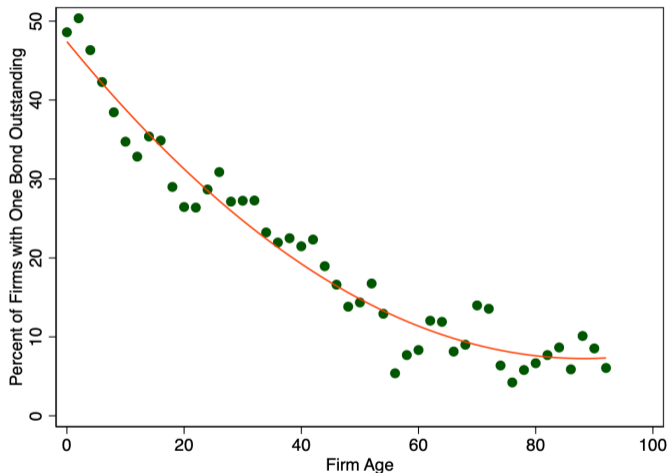
Weighted by leverage

Weighted by bond fraction

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- Avg σ_{Mat} : 2.6 years
Median σ_{Mat} : 1.5 years
- Maturity Wall: $\sigma_{Mat} \leq 1$
(Antero's σ_{Mat} at rating downgrade)
- Firms w/ maturity walls typically issue few bonds
 - Avg. # of bonds: 1.8
 - Median # of bonds: 1
 - P75 # of bonds: 2
- Maturity wall proxy going forward:
Firms w/ 1 bond outstanding

Fact 2: Maturity walls not driven by financing over firms' life cycle



Young firms (≤ 5)

- $\sim 50\%$ have one bond outstanding

Old firms (≥ 30)

- $\sim 20\%$ have one bond outstanding
- *Could have* chosen to have multiple bonds outstanding but didn't

Which firms are holding maturity walls?

	1{One Bond}	
	(1)	(2)
Leverage	-0.107*** (0.007)	-0.111*** (0.008)
Profit	0.082*** (0.006)	0.076*** (0.006)
Size	-0.277*** (0.012)	-0.281*** (0.013)
Age	-0.025*** (0.007)	-0.023*** (0.008)
No. Bonds Outstanding	-0.038*** (0.008)	-0.035*** (0.008)
Avg. Bond Maturity	-0.023*** (0.007)	-0.021*** (0.008)
Observations	12564	11852
R^2	0.282	0.295
Fixed Effects	Year	Ind & Year

Firms w/ maturity walls associated w/:

- ↑ leverage, concerned about rollover risk
→ disperse payments
- ↑ profit, less concern about rollover risk
→ concentrate payments

Fact 3: Firms with maturity walls appear more risky

	Prob. Default (pps)	Credit Spread (bps)
Leverage	4.690*** (0.402)	44.808*** (9.504)
Avg. Bond Maturity	-0.420** (0.203)	-3.688 (3.361)
1{One Bond}	0.955* (0.490)	24.478** (12.387)
Observations	6692	1269
R^2	0.407	0.690
Firm Controls	Yes	Yes
FEs	Yes	Yes

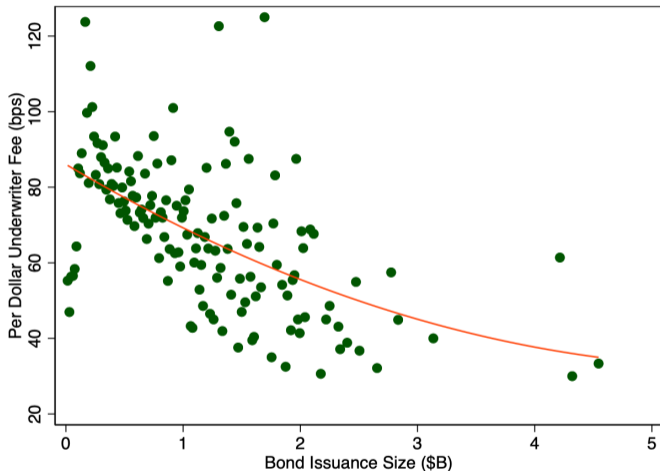
Takeaway: Firms w/ maturity walls associated w/ \uparrow prob. default & credit spreads

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Fact 4: Firms face economies of scale issue bonds

Underwriter fees:

- Cost to *issue* corporate bond (fixed + variable cost)
- Spread out fixed cost by issuing larger amounts
- Underwriter spread (Fee / Issue Size)
 - < \$1B issue: 80 bps
 - > \$3B issue: 40 bps

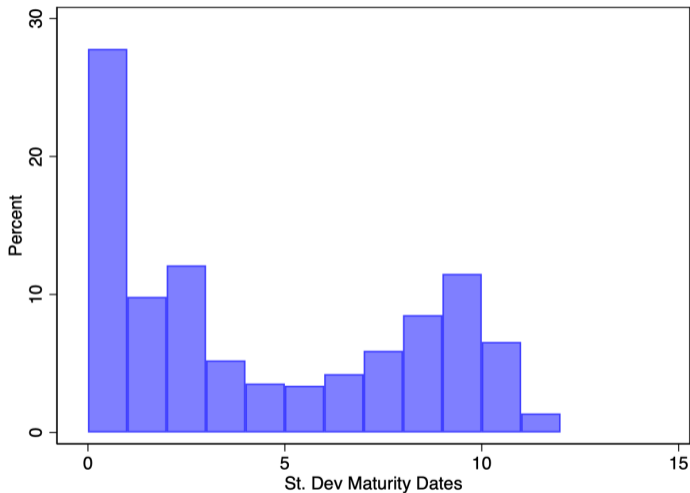


Takeaway: Economies of scale consistent with large fixed issuance costs

[Bond Issuance Freq](#) [Robustness](#)

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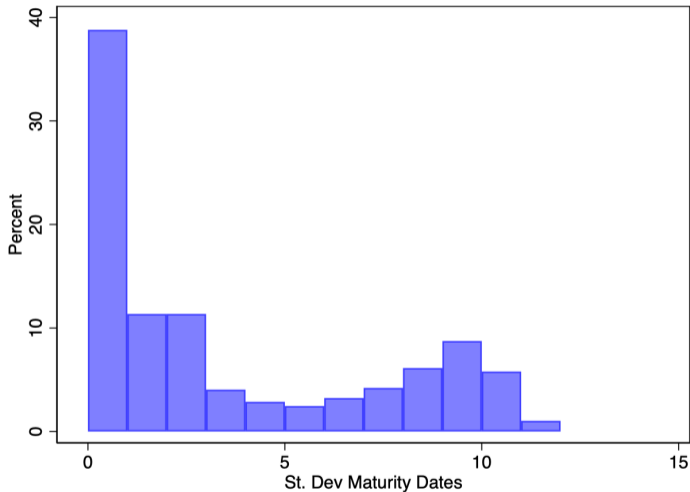
Firms concentrate debt maturity dates (weighted by firm age)



Average: 3.1 years

Median: 4.5 years

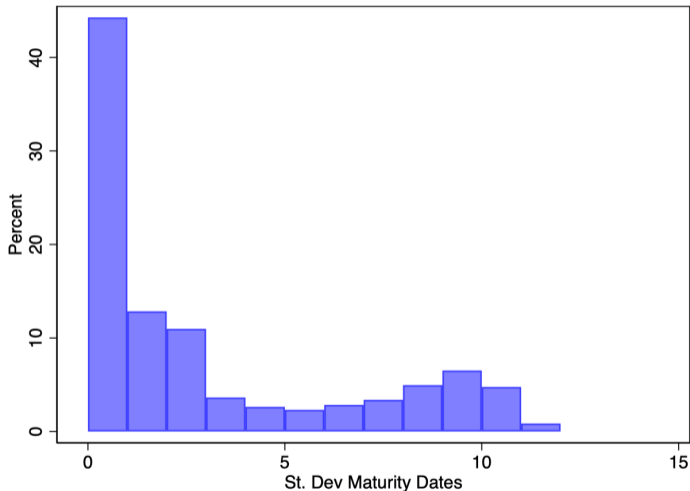
Firms concentrate debt maturity dates (weighted by firm size)



Average: 3.8 years

Median: 2.7 years

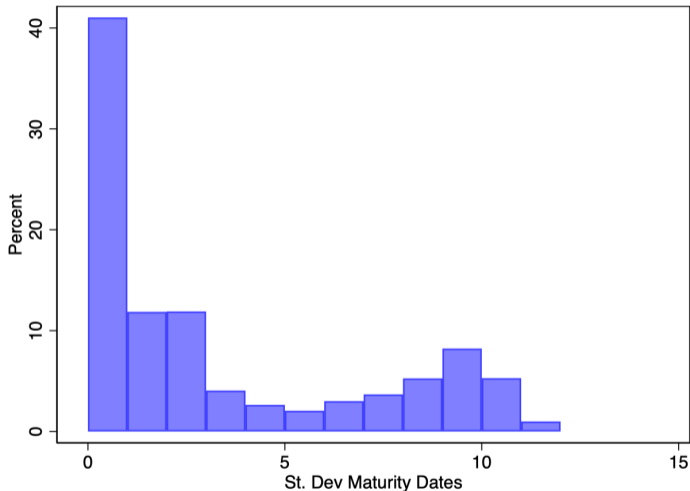
Firms concentrate debt maturity dates (weighted by firm leverage)



Average: 3.3 years

Median: 1.8 years

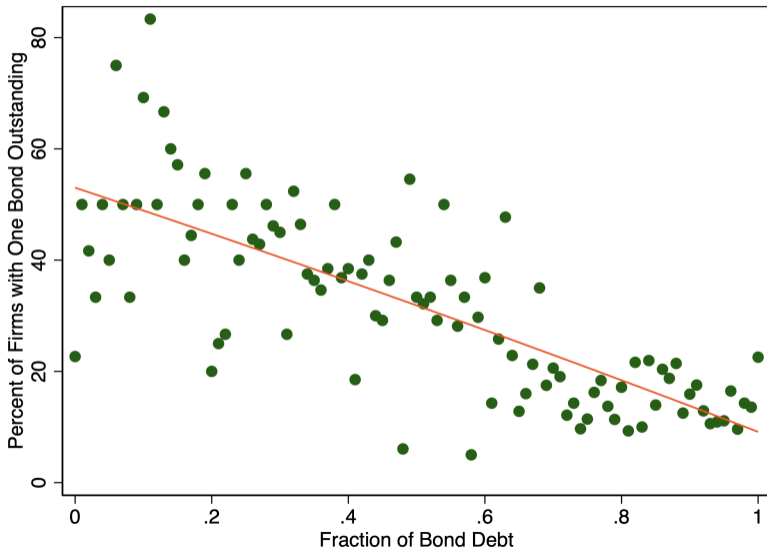
Firms concentrate debt maturity dates (weighted by bond debt pct)



Average: 3.4 years

Median: 1.9 years

Fact 2: Maturity walls not driven by financing over firms' life cycle



Firms infrequently issue corporate bonds

	Mean	Std. Dev
Bond Issuance Frequency	0.114	0.318
Number of Bonds Issued	1.874	1.360
Time Since Last Issue (Years)	3.359	2.263
Bond Issuance Size (\$M)	435.980	421.972
Bond Amount Outstanding (\$M)	1,681.264	3,658.709
Bond Issuance to Amount Outstanding	0.407	0.334
Number of Bonds Issued Last 5 Years	1.037	2.590
Number of Bonds Issued Last 10 Years	1.902	4.358
Number of Bonds Issued Last 20 Years	3.098	6.663

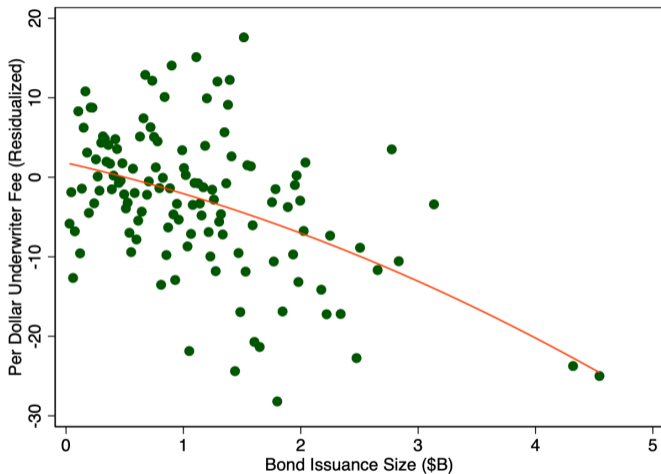
Firms issue bonds infrequently:

- 11.4% obs feature bond issuance
- Avg bonds issued in last 10 years is 1.9

Firms issue large amounts

- New bond issuance accounts for 40% of total amount outstanding

Economies of scale robust to firm and bond controls



Do firms behave differently before large repayment dates?

	Cash	Investment	$\mathbf{1}_{\text{Buyback}}$
s_1	-0.002 (0.007)	0.007 (0.008)	-0.241*** (0.052)
s_2	-0.003 (0.008)	0.002 (0.008)	-0.228*** (0.051)
s_3	-0.002 (0.007)	0.003 (0.008)	-0.217*** (0.049)
s_4	-0.006 (0.007)	0.002 (0.008)	-0.253*** (0.048)
s_5	-0.007 (0.006)	0.002 (0.007)	-0.222*** (0.048)
Fixed Effects	Firm & Year	Firm & Year	Firm & Year
Additional Controls	Yes	Yes	Yes

Note: s_m is share of long-term debt due in m years

How has the literature modeled long-term debt?

Modeling long-term debt with exponentially declining maturity structure:

- Hatchondo and Martinez (JIR, 2009); Arellano and Ramanarayanan (JPE, 2012); Aguiar et. al (ECMA, 2019)
- He and Xiong (JF, 2012); Dangi & Zechner (RFS, 2021); DeMarzo & He (JF, 2021); Jungherr and Schott (RED, 2021); Jungherr et. al (R&R ReSTUD, 2023)

Modeling long-term debt randomly maturing “lumpy” bond:

- Geelen (R&R JF, 2019); Gomes and Schmid (JF, 2021); Chen et al (JFE, 2021)

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(Continuing) Firm's Problem

$$V(S) = \max_{\{b'_D, b'_C\}} \left\{ \psi(d) + \overbrace{\varepsilon(b'_D, b'_C)}^{\text{Manager Pref. Shock}} + \beta \mathbb{E}_{\{y', \eta', \varepsilon'\}} \max \{ V(S'), 0 \} \right\}$$

Subject to:

$$d = \overbrace{(y - \underbrace{c_F}_{\text{Production Cost}} - \tilde{c}(b_D + b_C))}_{\text{After tax income}} (1 - \tau) - \overbrace{(\lambda b_D + \eta b_C)}^{\text{Debt repaid}}$$

$$+ \underbrace{q_D(b'_D, b'_C, y) I_D}_{\text{Dispersed debt issuance revenue}} + \underbrace{q_C(b'_D, b'_C, y) I_C}_{\text{Concentrated debt issuance revenue}} - \underbrace{c_I(\mathbb{1}_{I_D > 0} + \mathbb{1}_{I_C > 0})}_{\text{Debt issuance cost}}$$

$$I_D = b'_D - (1 - \lambda)b_D$$

$$I_C = b'_C - (1 - \eta)b_C$$

$$\psi(d) = \begin{cases} d & \text{if } d \geq 0 \\ d - \underbrace{\alpha d^2}_{\text{Eq. issuance Cost}} & \text{if } d < 0 \end{cases}$$

Lender's Problem

Debt is priced by rep lender making zero-profits in expectation

- $\delta(S')$: default decision in state $S' \equiv (b'_D, b'_C, y', \eta')$
- $\mathcal{R}(b_D, b_C, y)$: lender's recovery value in default

[Full Definition](#)

Price of unit of dispersed debt

$$q_D(b'_D, b'_C, y) = \beta \mathbb{E}_{\{y', \eta', \varepsilon'\}} \left\{ (1 - \delta(S')) \left(\underbrace{\tilde{c} + \lambda}_{\text{Payment tomorrow}} + \underbrace{(1 - \lambda)q_D(b''_D, b''_C, y')}_{\text{Expected future revenue to lender}} \right) + \delta(S')\mathcal{R}(b'_D, b'_C, y') \right\}$$

Price of unit of concentrated debt

$$q_C(b'_D, b'_C, y) = \beta \mathbb{E}_{\{y', \eta', \varepsilon'\}} \left\{ (1 - \delta(S')) \left(\underbrace{\tilde{c} + \eta}_{\text{Payment tomorrow}} + \underbrace{(1 - \eta)q_C(b''_D, b''_C, y')}_{\text{Expected future revenue to lender}} \right) + \delta(S')\mathcal{R}(b'_D, b'_C, y') \right\}$$

where $Pr(\eta = 1) = \lambda$

Equilibrium Definition

A recursive Markov equilibrium is a set of value and policy functions $\{V^*, b_D^*, b_C^*\}$ and debt prices $\{q_D^*, q_C^*\}$ such that:

1. Given prices q_D^* and q_C^* , firms optimize yielding V^* , b_D^* , and b_C^*
2. The default decision is consistent with firm decision rules
3. Debt prices q_D^* and q_C^* are such that the representative lender expects to earn zero profits
4. Stationary distribution of firms determined by firm decision rules and law of motion for y and η
 - Mass of defaulting firms are replaced with an equal mass of firms with $b_D = 0$, $b_C = 0$, $\eta = 1$ and $y \sim \bar{G}$

Debt is priced by rep lender making zero-profits in expectation

- $\delta(S')$: default decision in state $S' \equiv (b'_D, b'_C, y', \eta', \varepsilon')$
- $\tilde{V}(y) = \psi((y - c_F)(1 - \tau)) + \beta \mathbb{E}_{\{y'\}} \max \{ \tilde{V}(y'), 0 \}$
- $\mathcal{R}(b_D, b_C, y) = \min [1, \chi \tilde{V}(y) / b_D + b_C]$

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Price of unit of dispersed debt

$$q_D(b'_D, b'_C, y) = \beta \mathbb{E}_{\{y', \eta', \varepsilon'\}} \left\{ (1 - \delta(S')) \left(\overbrace{\tilde{c} + \lambda}^{\text{Payment tomorrow}} + \overbrace{(1 - \lambda)q_D(b''_D, b''_C, y')}^{\text{Expected future revenue to lender}} \right) + \delta(S')\mathcal{R}(b'_D, b'_C, y') \right\}$$

Price of unit of concentrated debt

$$q_C(b'_D, b'_C, y) = \beta \mathbb{E}_{\{y', \eta', \varepsilon'\}} \left\{ (1 - \delta(S')) \left(\overbrace{\tilde{c} + \eta}^{\text{Payment tomorrow}} + \overbrace{(1 - \eta)q_C(b''_D, b''_C, y')}^{\text{Expected future revenue to lender}} \right) + \delta(S')\mathcal{R}(b'_D, b'_C, y') \right\}$$

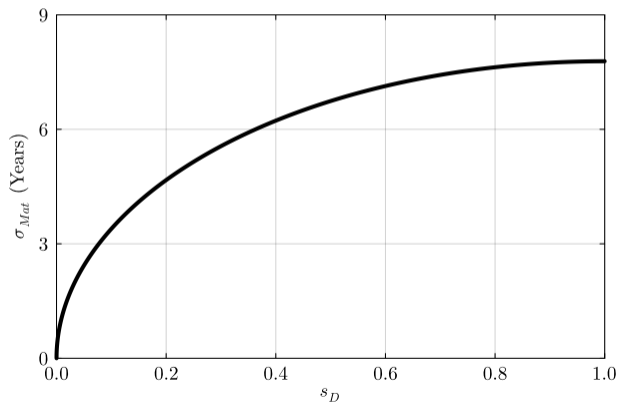
Constructing σ_{Mat} in the model

Let

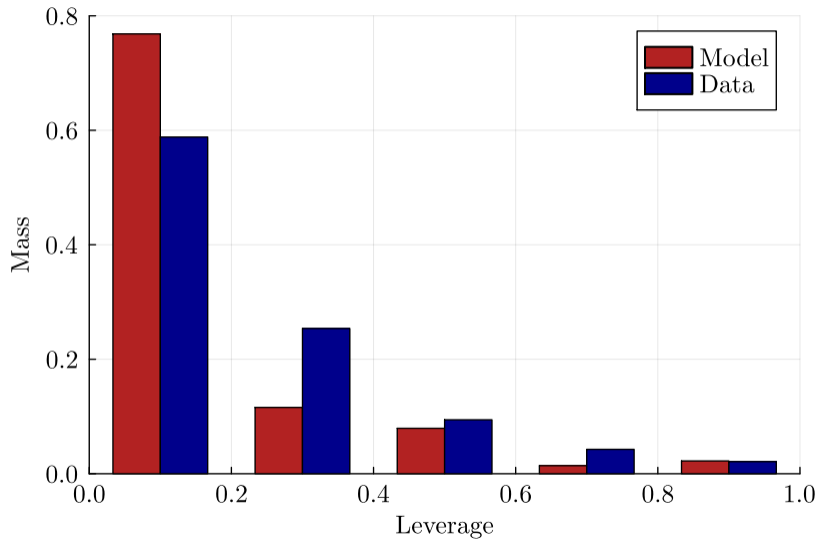
$$s_D = \frac{b_D}{b_D + b_C}$$

The mapping from s_D to σ_{Mat} is

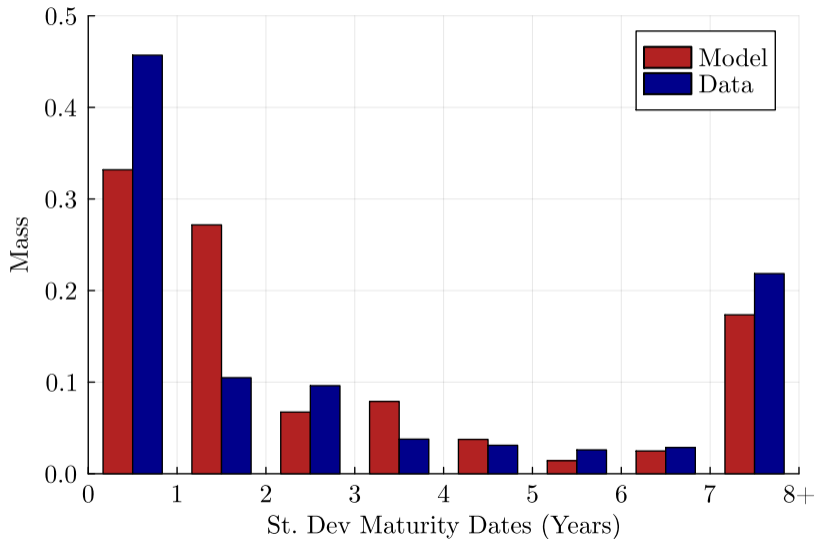
$$\sigma_{Mat} = \frac{\sqrt{(1 - \lambda)(2s_D - s_D^2)}}{\lambda}$$



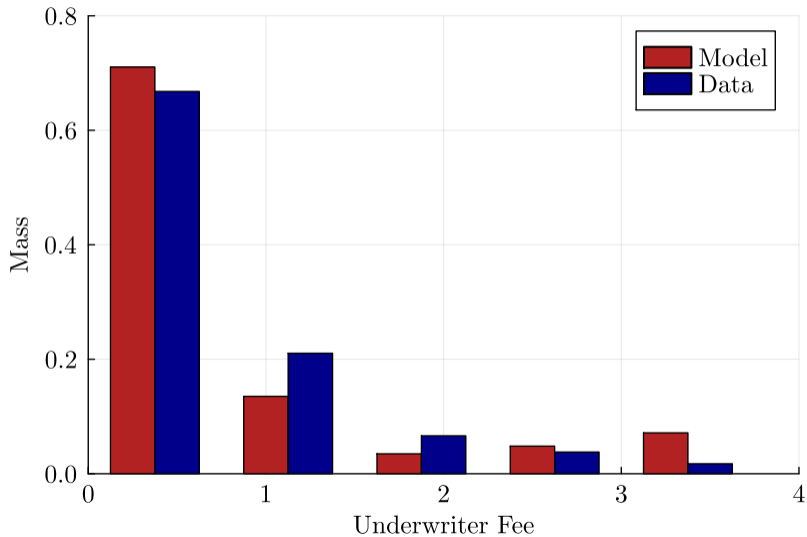
Model Fit: Market Leverage Distribution



Model Fit: St. Dev of Maturity Dates Distribution



Model Fit: Underwriter Fee Distribution



Relative cost difference between debt and equity issuance

- b_D smooths equity issuance costs [More](#)
- b_C less frequent debt issuances [More](#)

Low b : debt costs $>$ equity issuance costs

- Optimal to minimize number of debt issuances w/ b_C

High b : debt costs $<$ equity issuance costs

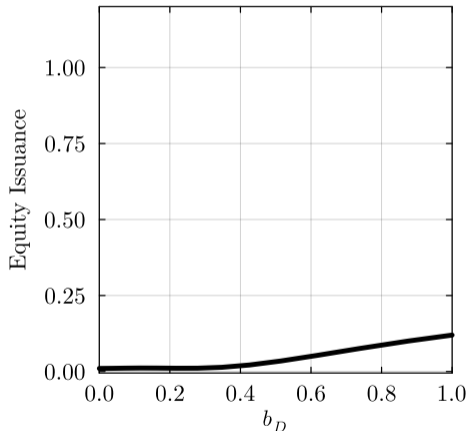
- Optimal for firm to substitute to b_D to smooth equity issuance costs

Takeaway: Relative costs of debt and equity issuance influence choice of b_D vs b_C

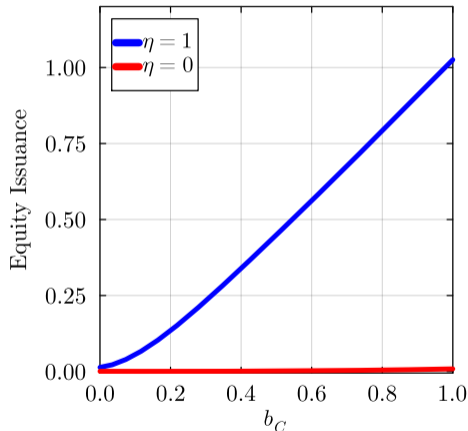
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Dispersed debt smooths equity issuances

Dispersed Debt

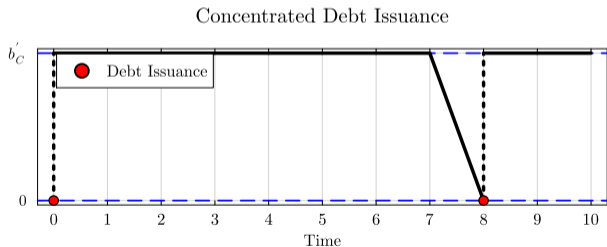
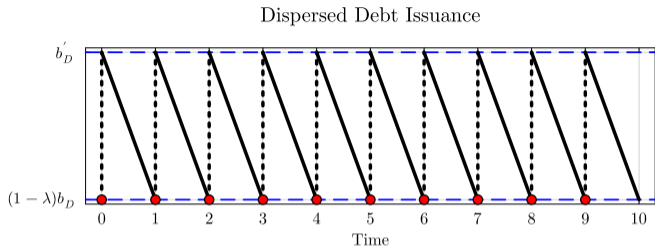


Concentrated Debt



Firm wants to smooth equity issuances (rollover risk) b/c of convex costs

Dispersed debt requires repeated issuance cost payment



Note: y is held constant at y_M

At stable leverage ratio, costly to repeatedly issue dispersed debt b/c frequent top-ups

Lender's Problem

Debt is priced by rep lender making zero-profits in expectation

- $\delta(S')$: default decision in state $S' \equiv (b'_D, b'_C, y', \eta')$
- $\mathcal{R}(b_D, b_C, y)$: lender's recovery value in default

Price of unit of dispersed debt

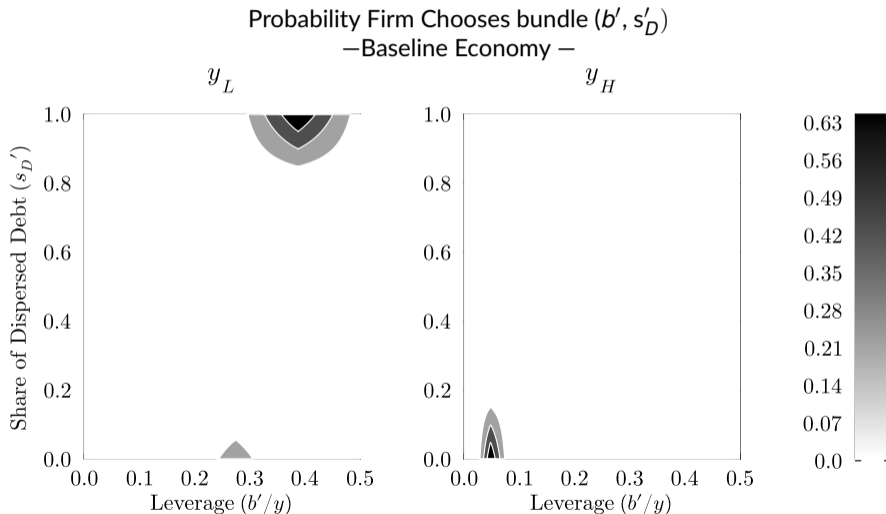
$$q_D(b'_D, b'_C, y) = \beta \mathbb{E}_{\{y', \eta', \varepsilon'\}} \left\{ (1 - \delta(S')) \left(\overbrace{\tilde{c} + \lambda}^{\text{Payment tomorrow}} + \overbrace{(1 - \lambda)q_D(b''_D, b''_C, y')}^{\text{Expected future revenue to lender}} \right) + \delta(S')\mathcal{R}(b'_D, b'_C, y) \right\}$$

Price of unit of concentrated debt

$$q_C(b'_D, b'_C, y) = \beta \mathbb{E}_{\{y', \eta', \varepsilon'\}} \left\{ (1 - \delta(S')) \left(\overbrace{\tilde{c} + \eta}^{\text{Payment tomorrow}} + \overbrace{(1 - \eta)q_C(b''_D, b''_C, y')}^{\text{Expected future revenue to lender}} \right) + \delta(S')\mathcal{R}(b'_D, b'_C, y) \right\}$$

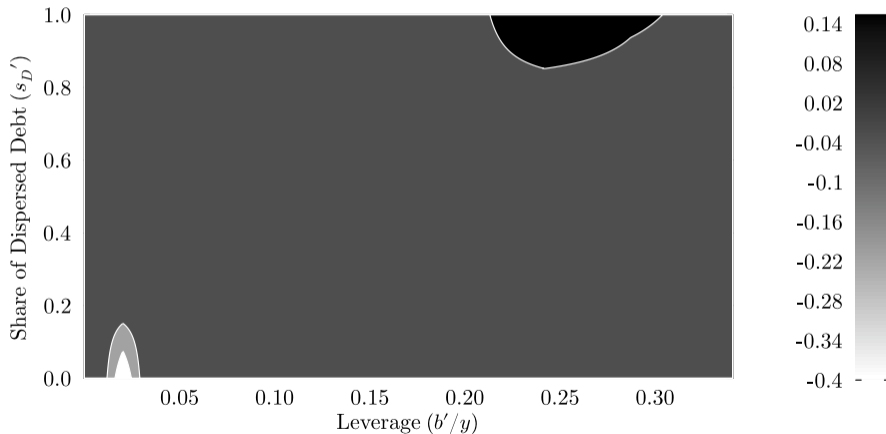
where $Pr(\eta = 1) = \lambda$

Counterfactual equilibrium: competitive underwriter market

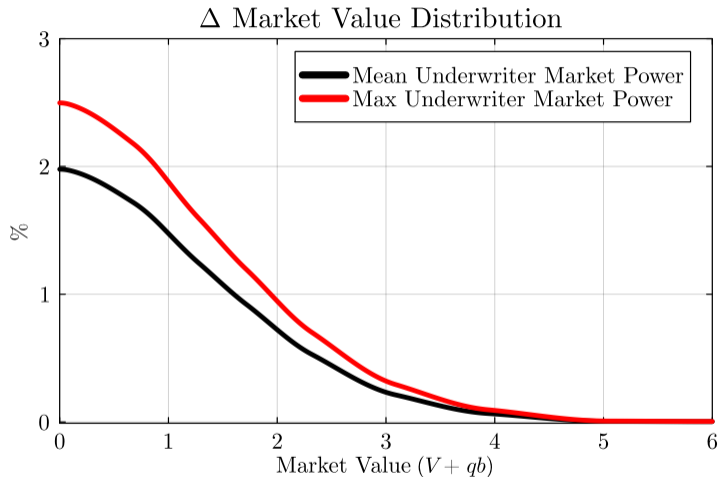


Counterfactual equilibrium: competitive underwriter market

Change Choice Probabilities of bundle (b' , s'_D)
— High Income Firm —



How does underwriter market power impact firm value?



- Average market value \uparrow 0.85% – 1.01%