Deflationary Equilibrium under Uncertainty

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What We Do

Theoretical

- We theoretically analyze the so-called **deflationary equilibrium of the New Keynesian model with an interest-rate lower bound** and make a novel observation.

- **Novel observation**: Inflation is higher at the **R**isky **S**teady **S**tate (RSS) than at the **D**eterministic **S**teady **S**tate (DSS).
  - RSS takes into account uncertainty, while DSS does not.
  - Certainty Equivalence breaks down.

Empirical

- We relate the theory to the Japanese experience in 2010s.
  - Lack of deflation in 2010s while at the lower bound is more consistent with RSS than with DSS

Literature

Uncertainty in Macroeconomics

- Many

DSGE-based Analysis of the Japanese economy

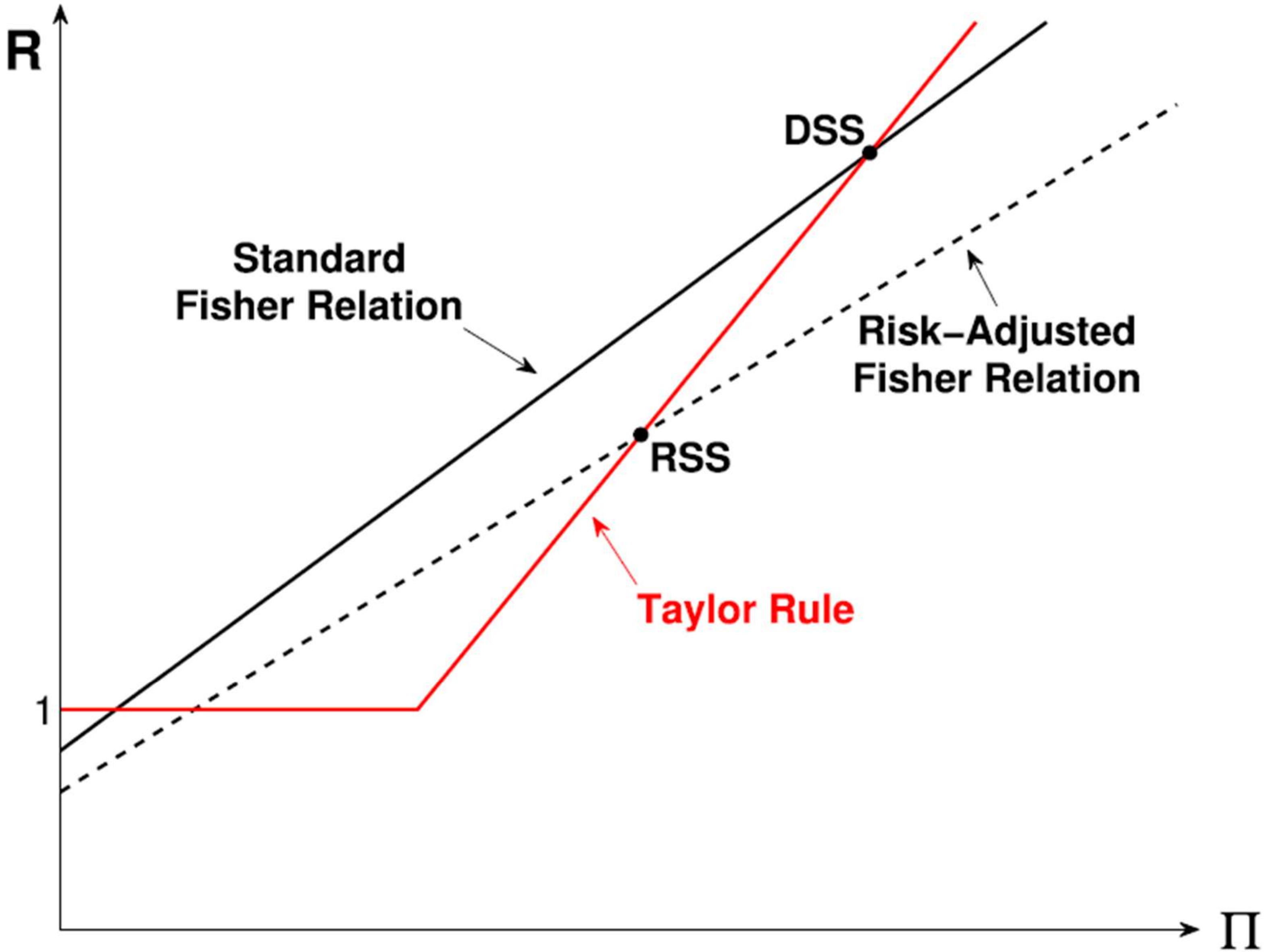
- Many

Interaction of Uncertainty and the ELB

- Uncertainty in a fundamental-driven liquidity trap
  - Basu and Bundick (2017), Nakata (2017), etc.
- Risky steady state of the target equilibrium in the NK model
  - Hills, Nakata, and Schmidt (2019), Nakata and Schmidt (2019)
  - An explanation of below-target inflation in the U.S. in the second half of 2010s
- Our paper: Risky steady state of the deflationary equilibrium

Illustrative figure (from Hills et al. (2019))

T.S. Hills, T. Nakata and S. Schmidt / European Economic Review 120 (2019) 103321



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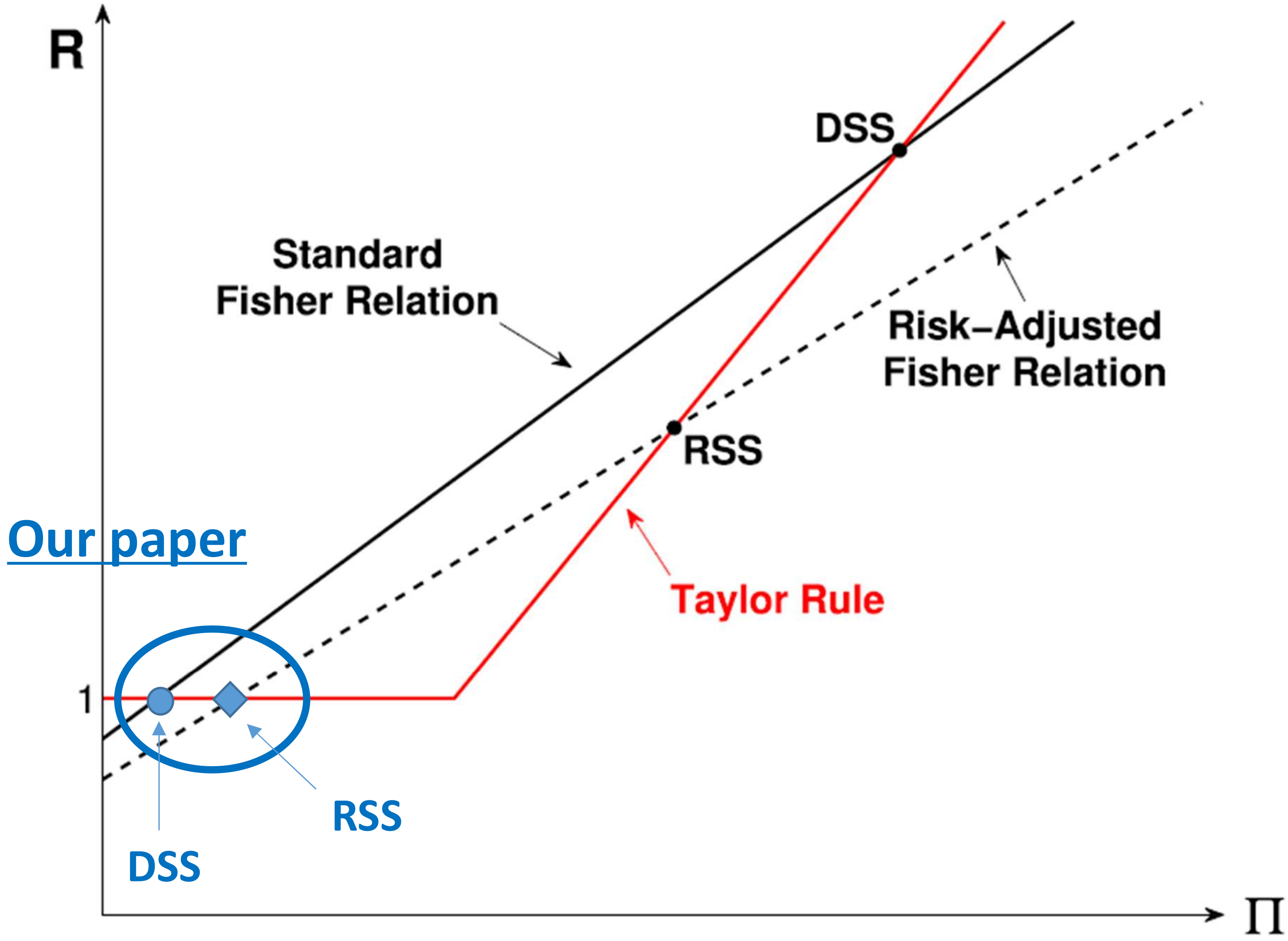


Outline

- Theory

- Empirics

Model

- Log-linearized NK model with an interest-rate lower bound

- …and with a natural-rate shock.
  - For analytical results: Three-state and i.i.d.
  - For numerical results: AR(1) process.

Model

$$y_t = Ey_{t+1} - \sigma\left(i_t - E\pi_{t+1} - (r^* + \boldsymbol{\delta_t})\right)$$

$$\pi_t = \kappa y_t + \beta E\pi_{t+1}$$

$$i_t = \max\left[0, r^* + \phi_\pi \pi_t\right]$$

- y_t : output
- π_t: inflation
- i_t : policy rate
- r^* : steady-state natural rate
- **δ_t : a shock to the natural rate**
- $\phi_\pi > 1$

Model

$$\delta_H = c, \qquad \delta_M = 0, \qquad \delta_L = -c$$

$$\mathrm{Prob}(\delta_t = \delta_H) = \frac{1 - p_M}{2}$$

$$\mathrm{Prob}(\delta_t = \delta_M) = p_M$$

$$\mathrm{Prob}(\delta_t = \delta_L) = \frac{1 - p_M}{2}$$

- $c \geq 0$
  - $c = 0$ is no-uncertainty case.
- $0 < p_M < 1$

Model

- Let $\{y(\cdot), \pi(\cdot), \text{and } i(\cdot)\}$ be the set of time-invariant policy functions for output, inflation, and the policy rate, respectively, satisfying EE, PC, and the Taylor rule.

- We call $\{y(\cdot), \pi(\cdot), \text{and } i(\cdot)\}$ "an equilibrium."

- In the model with a three-state shock, an equilibrium is characterized by
  - y_H, y_M, y_L
  - π_H, π_M, π_L
  - i_H, i_M, i_L

Model

- A Risky Steady States (RSS) of the model is given by $\{y_M, \pi_M, \text{and } i_M\}$.
  - ...because $\delta_M = 0$.
- A Deterministic Steady States (DSS) of the model is given by $\{y_M, \pi_M, \text{and } i_M\}$ **under $c = 0$**.

Model

- Provided that c is sufficiently small, there are two equilibria.
  - We call one with higher inflation as **the target equilibrium.**
  - We call one with lower inflation as **the deflationary equilibrium.**

- DSS and RSS for each of the two equilibria
  - One DSS and one RSS associated with the target equilibrium (focus of Hills et al. (2019)).
  - **One DSS and one RSS associated with the deflationary equilibrium** (focus of this paper).

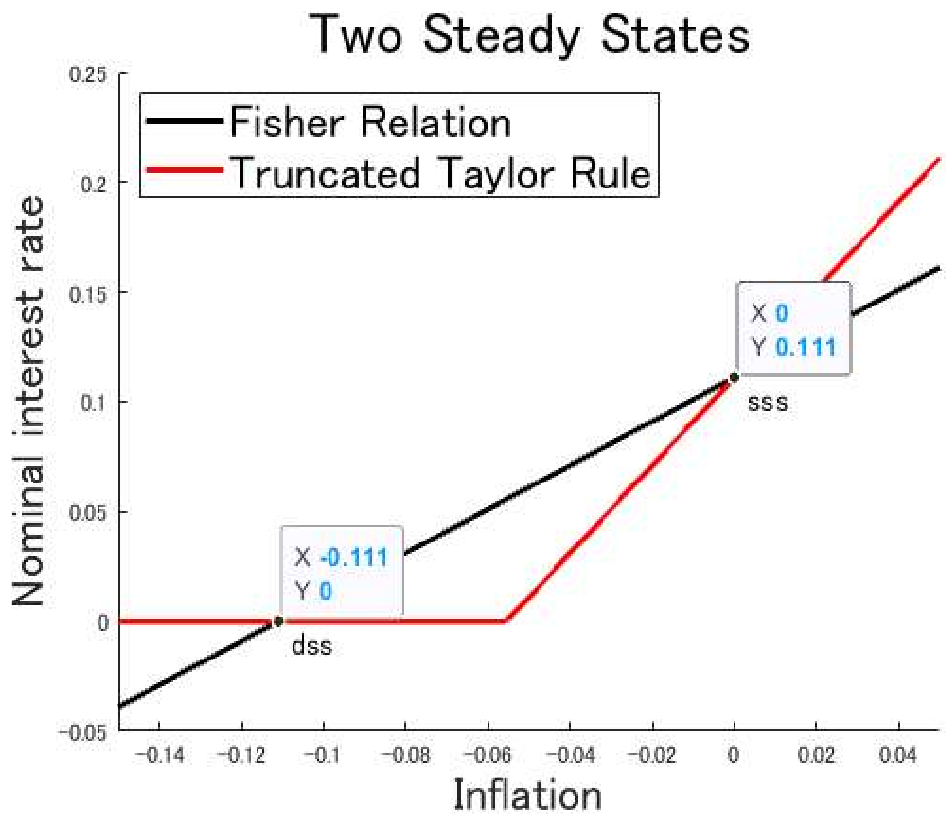
DSS (Deterministic Steady State) of Two Equilibria

DSS of the Targeted Equilibrium:

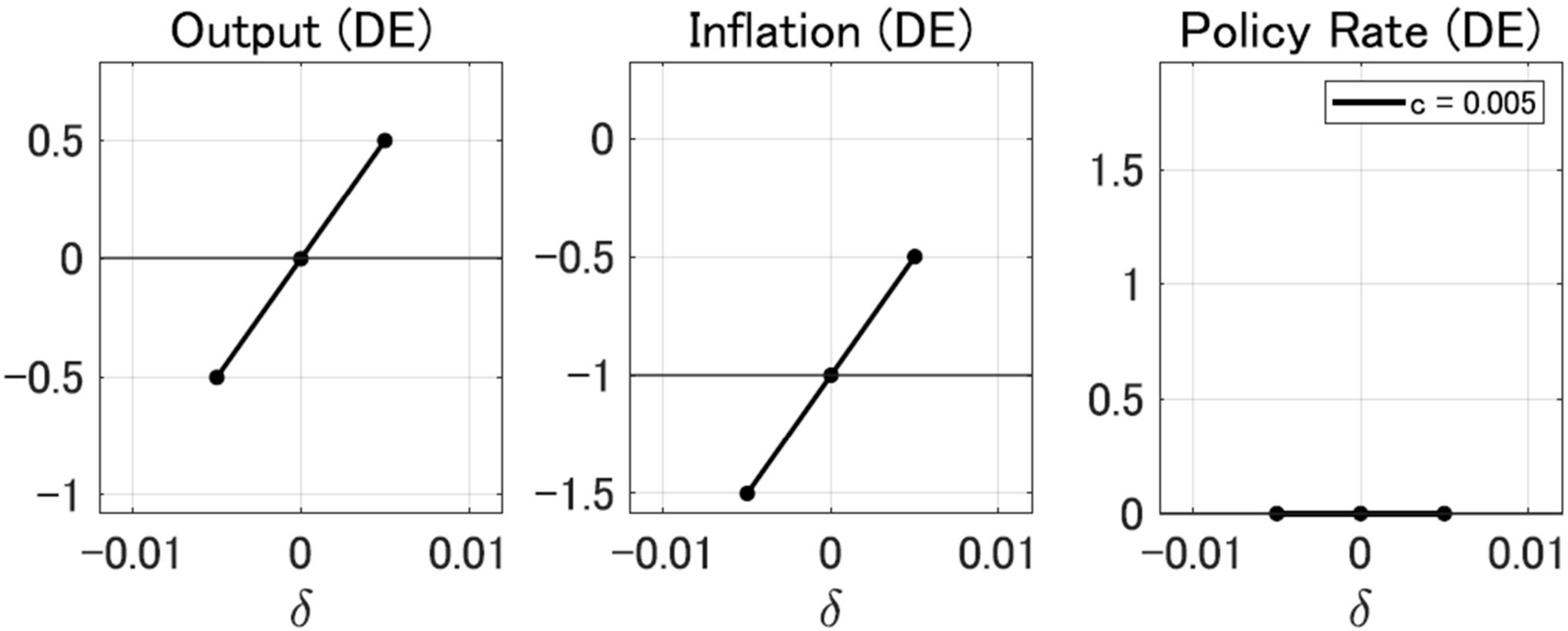
- $\pi_{ss} = 0, i_{ss} = r^* > 0$
- ELB does not bind.

DSS of the Deflationary Equilibrium:

- $\pi = -r^*, i_{ss} = 0$
- ELB binds.

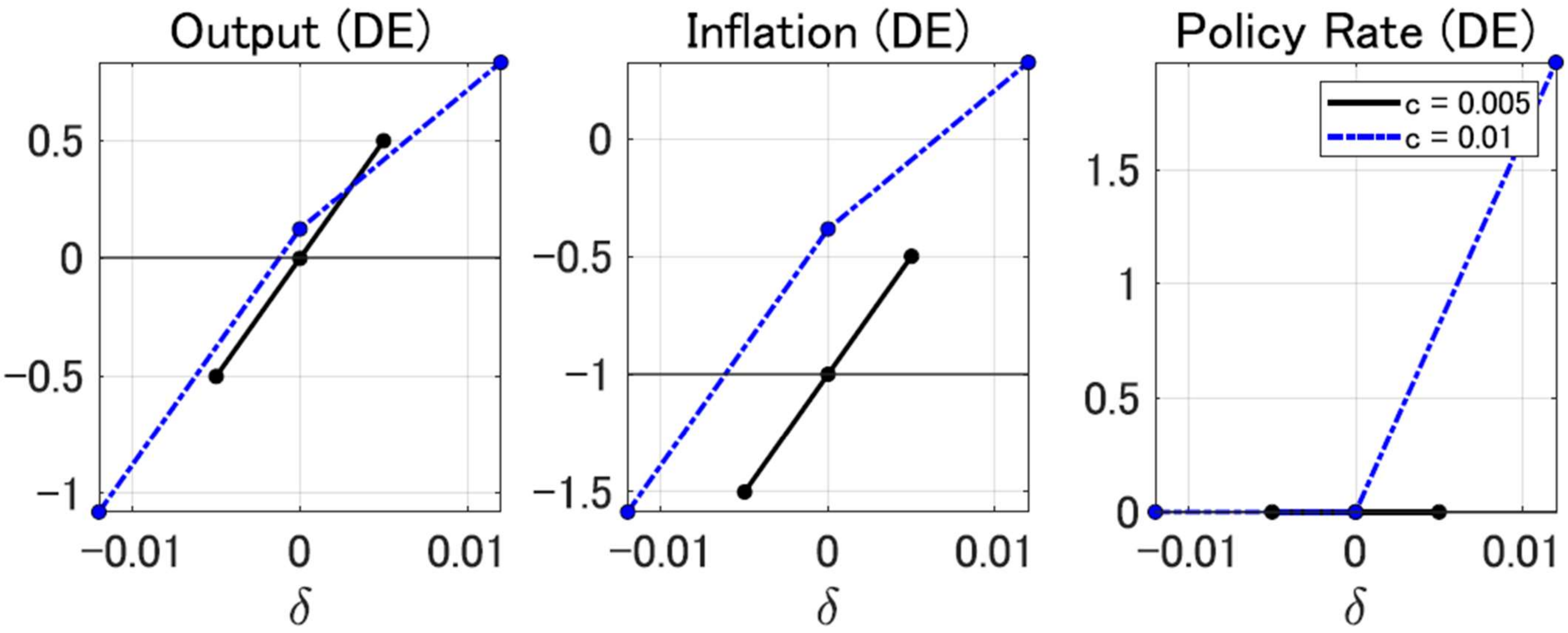


Deflationary Equilibrium with Low Uncertainty



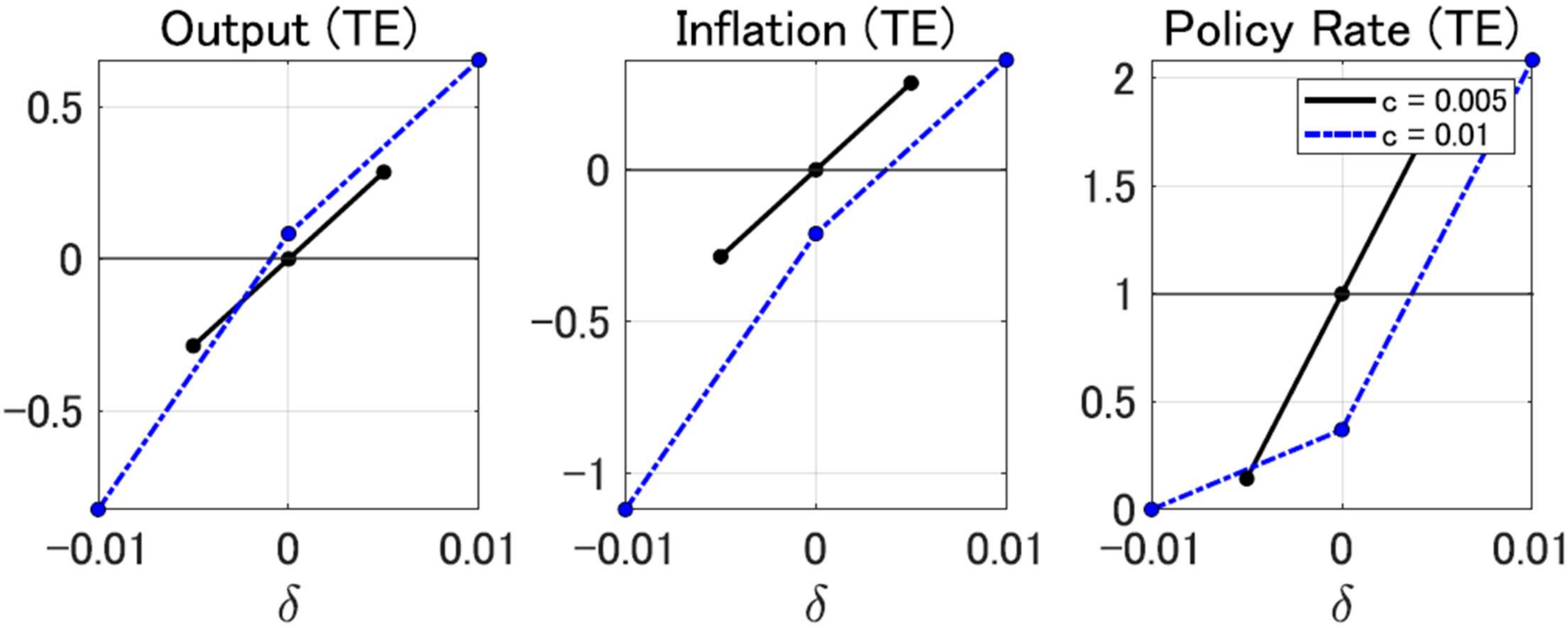
- For sufficiently small c, RSS inflation is the same as DSS inflation.
  - Key: The distributions of future y and π are symmetric, because ELB binds in all three states.
  - Thus, Certainty Equivalence holds.

Deflationary Equilibrium with High Uncertainty



- For sufficiently large c, inflation is higher at the RSS than at the DSS.
  - Key: The distribution of future y and π are NOT symmetric anymore, because ELB binds in L and M, but not in H state.
  - Thus, Certainty Equivalence breaks down.

...in contrast to what happens in the Target Equilibrium



- For a sufficiently large c, inflation is lower at the RSS than at the DSS
  - ...a result examined in Hills et al. (2019)

Understanding the results via risk-adjusted Fisher Relation

- Fisher relation: Relationship b/w i_M and π_M implied by the Euler equation without uncertainty.

$$y_M = y_M - \sigma\,(i_M - \pi_M - r^*)$$

$$\Rightarrow\ i_M = r^* + \pi_M$$

Understanding the results via risk-adjusted Fisher Relation

- Risk-adjusted Fisher Relation: Relationship b/w i_M and π_M implied by the Euler equation with uncertainty.

$$y_M = Ey - \sigma\,(i_M - E\pi - r^*)$$

- Where is π_M? Inside the expectation!

$$y_M = p_M y_M + (1 - p_M)\frac{y_H + y_L}{2}$$
$$-\sigma\,\left(i_M - \left(p_M \boldsymbol{\pi_M} + (1 - p_M)\frac{\pi_H + \pi_L}{2}\right) - r^*\right)$$

$$y_M = [p_M y_M + (1-p_M)\frac{\mathbf{y_H + y_L}}{\mathbf{2}}]$$
$$-\sigma \left(i_M - [p_M \pi_M + (1-p_M)\frac{\boldsymbol{\pi_H + \pi_L}}{\mathbf{2}}] - r^*\right)$$

- When c is sufficiently small, allocations are symmetric.

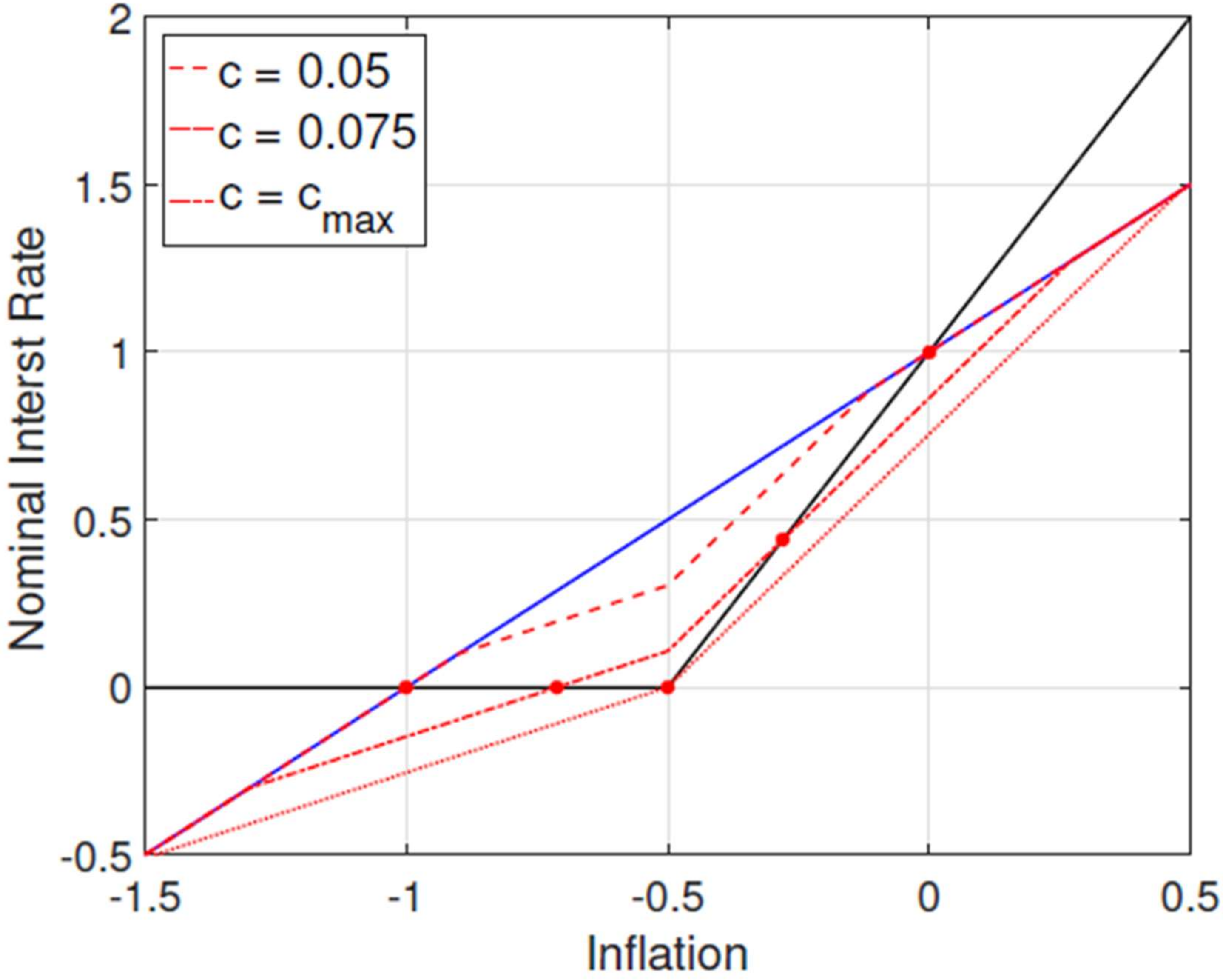
$$y_M = p_M y_M + (1-p_M)\mathbf{y_M} - \sigma\,(i_M - [p_M \pi_M + (1-p_M)\boldsymbol{\pi_M}]\; - r^*)$$

$$\Rightarrow\; i_M = r^* + \pi_M$$

- When c is sufficiently large, allocations are NOT symmetric.

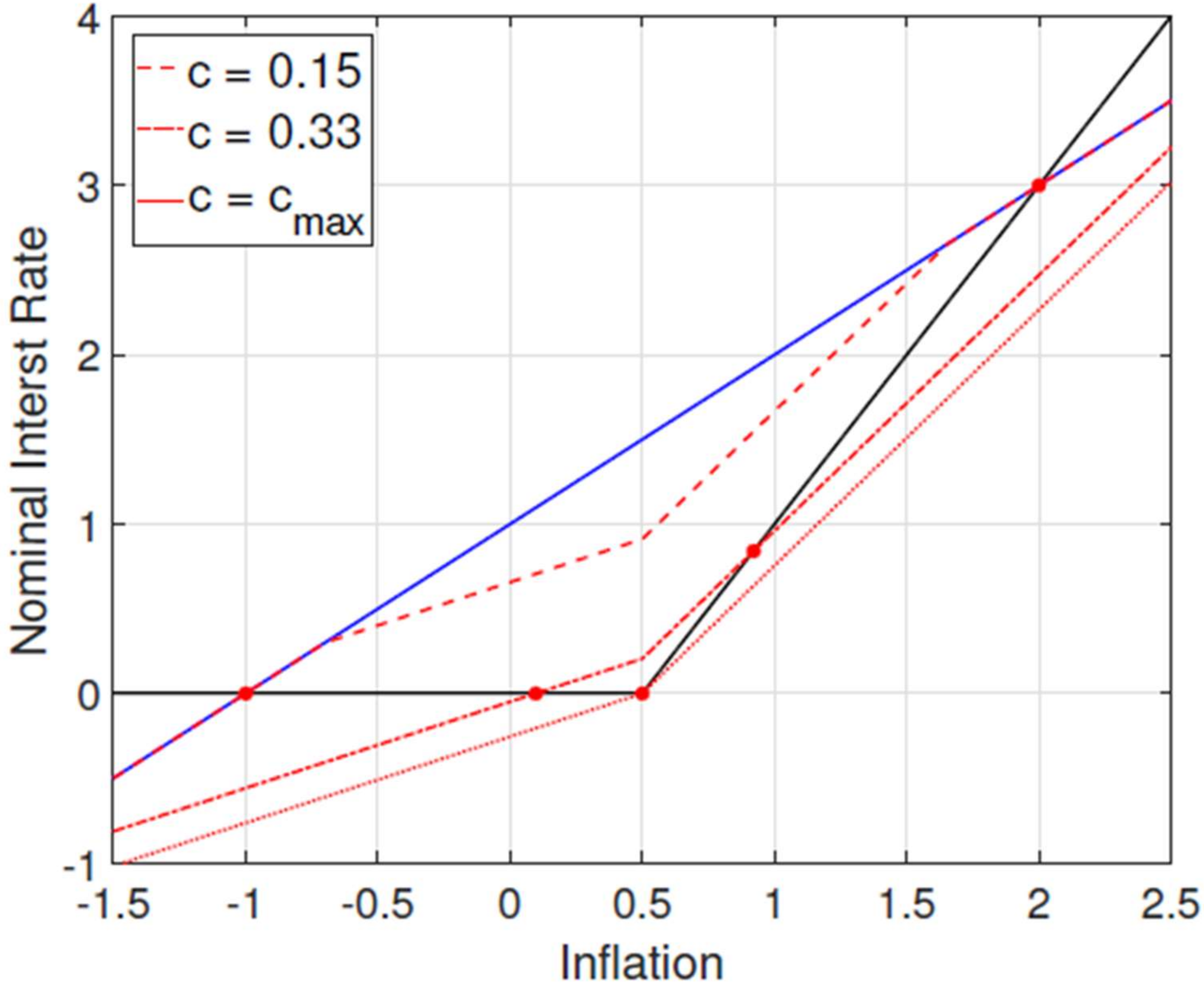
$$y_M = p_M y_M + (1-p_M)(\mathbf{y_M - h_x(\boldsymbol{\pi_M})})$$
$$-\sigma\,(i_M - p_M \boldsymbol{\pi_M} + (1-p_M)(\boldsymbol{\pi_M} - \mathbf{h}_{\boldsymbol{\pi}}(\boldsymbol{\pi_M})) - r^*)$$
$$\ldots$$
$$\Rightarrow i_M = (r^* + \boldsymbol{\alpha_1})\; + \boldsymbol{\alpha_2}\pi_M$$

Understanding the results via risk-adjusted Fisher Relation



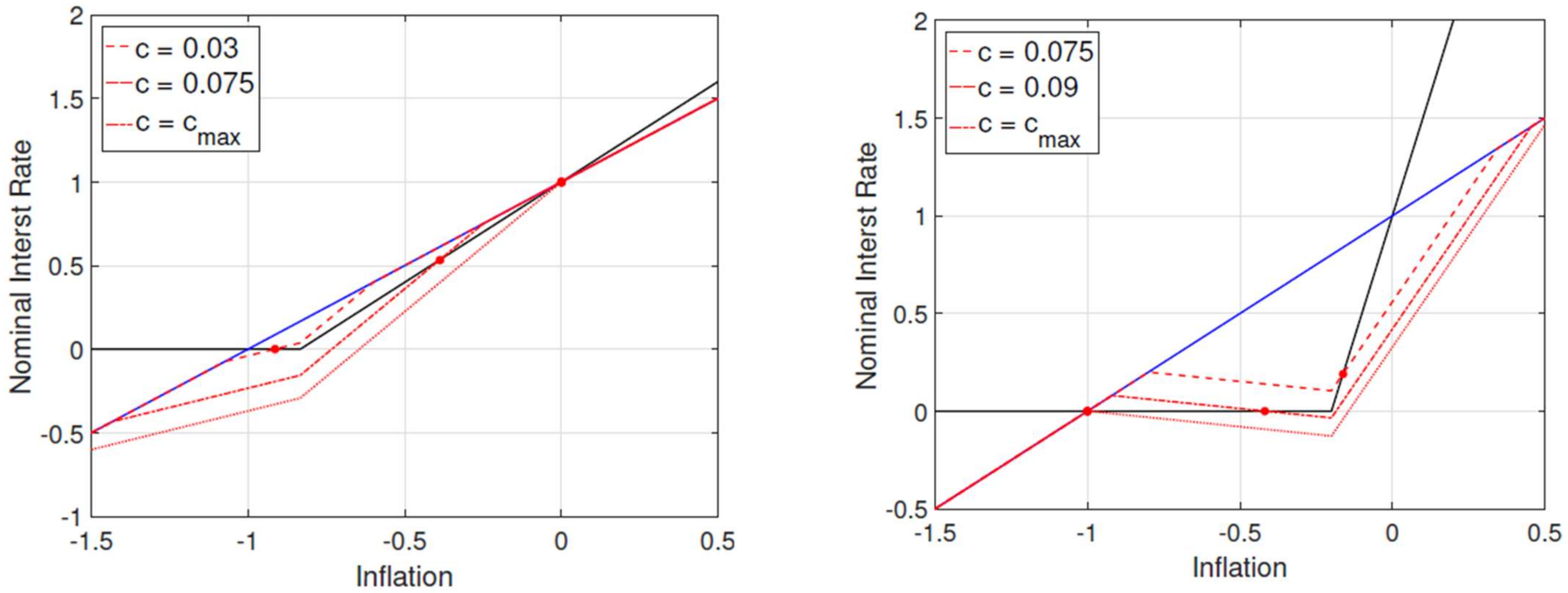
Interesting Cases

- At the RSS of the deflationary equilibrium, inflation is positive **(Left)**
  - (i) if the inflation target is positive, and (ii) if c is sufficiently high.



Interesting Cases

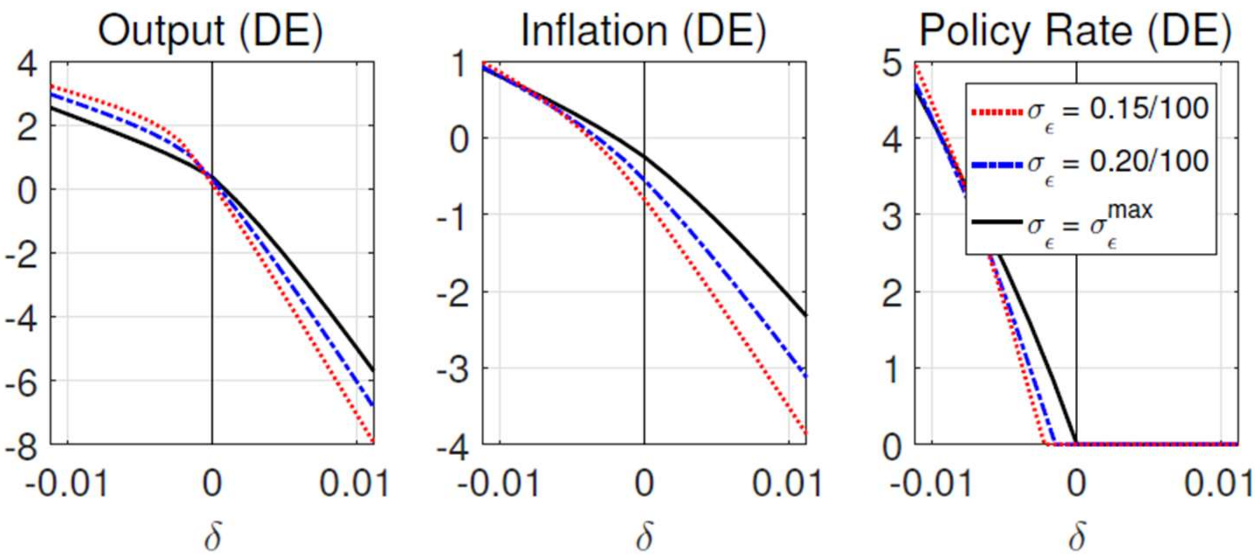
- At the RSS of the deflationary equilibrium, the interest rate is not constrained by the ELB **(Left)**
  - …if the inflation coefficient in the Taylor rule is sufficiently small.



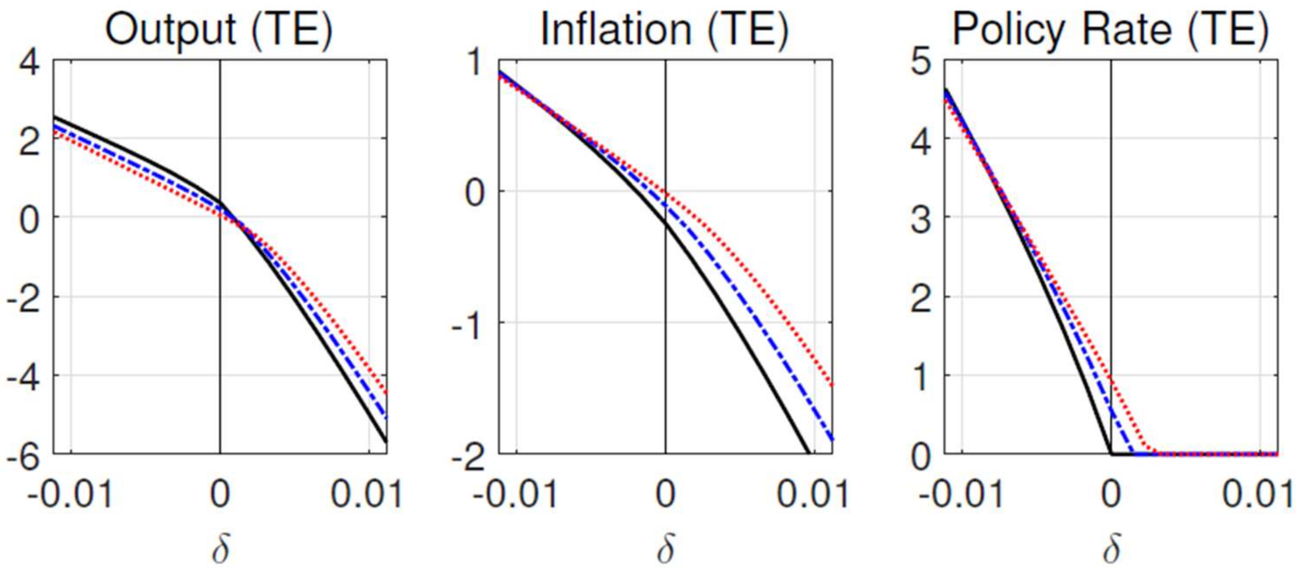
- At the RSS of the target equilibrium, the interest rate is constrained by the ELB **(Right)**
  - …if the inflation coefficient in the Taylor rule is sufficiently large.

Model with AR(1)

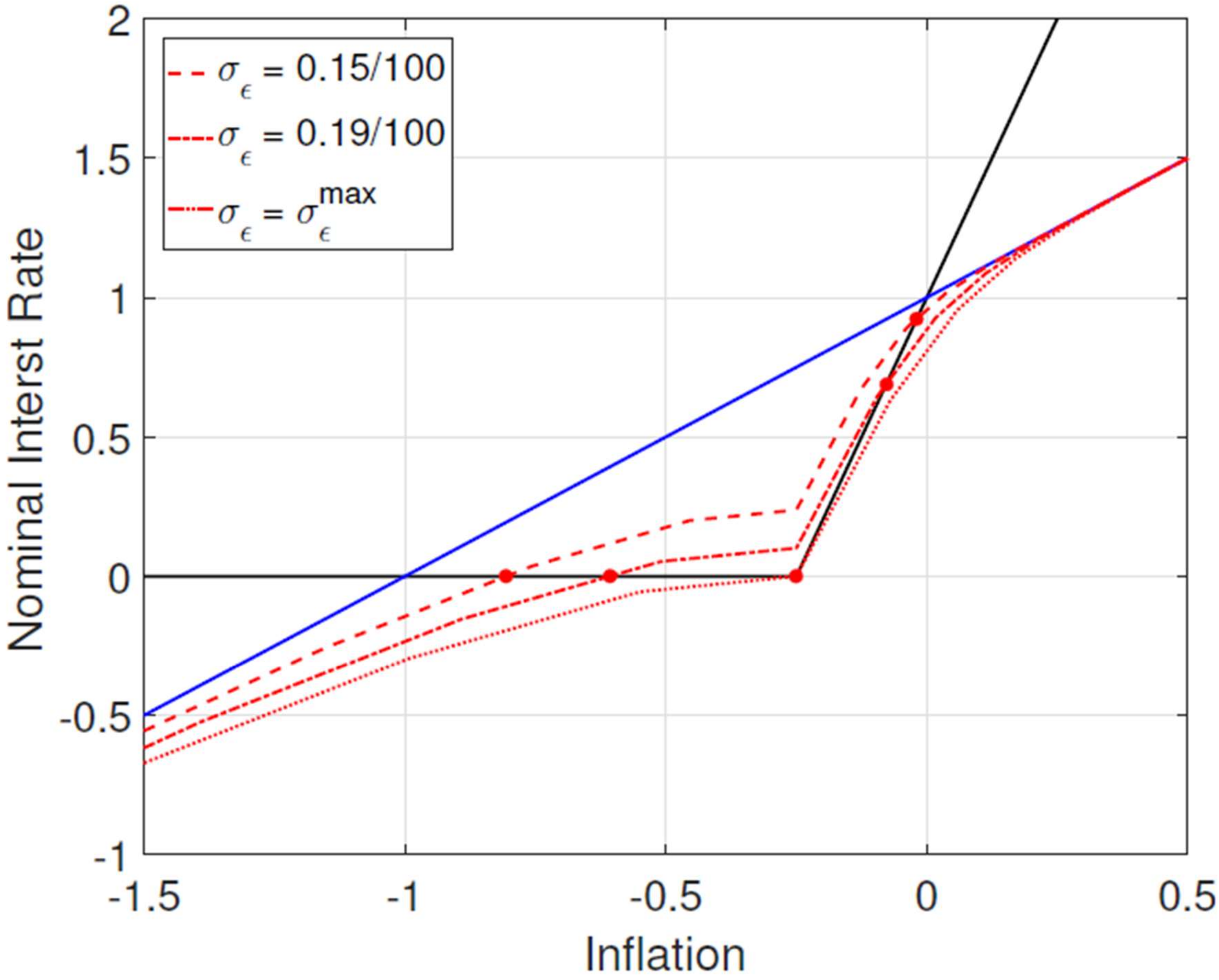
Policy functions for the Deflationary Equilibrium



Policy functions for the Target Equilibrium



Model with AR(1)



Outline

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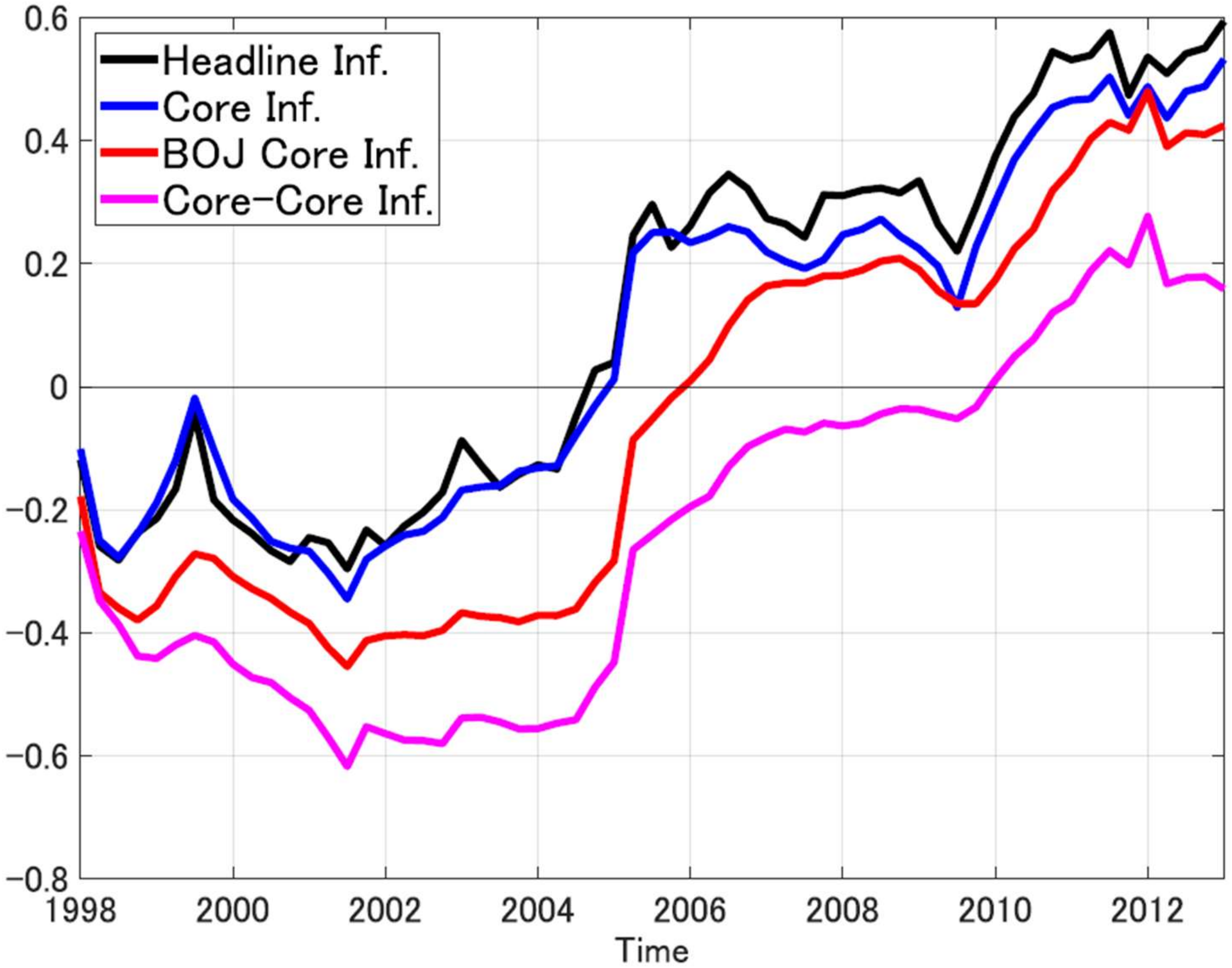
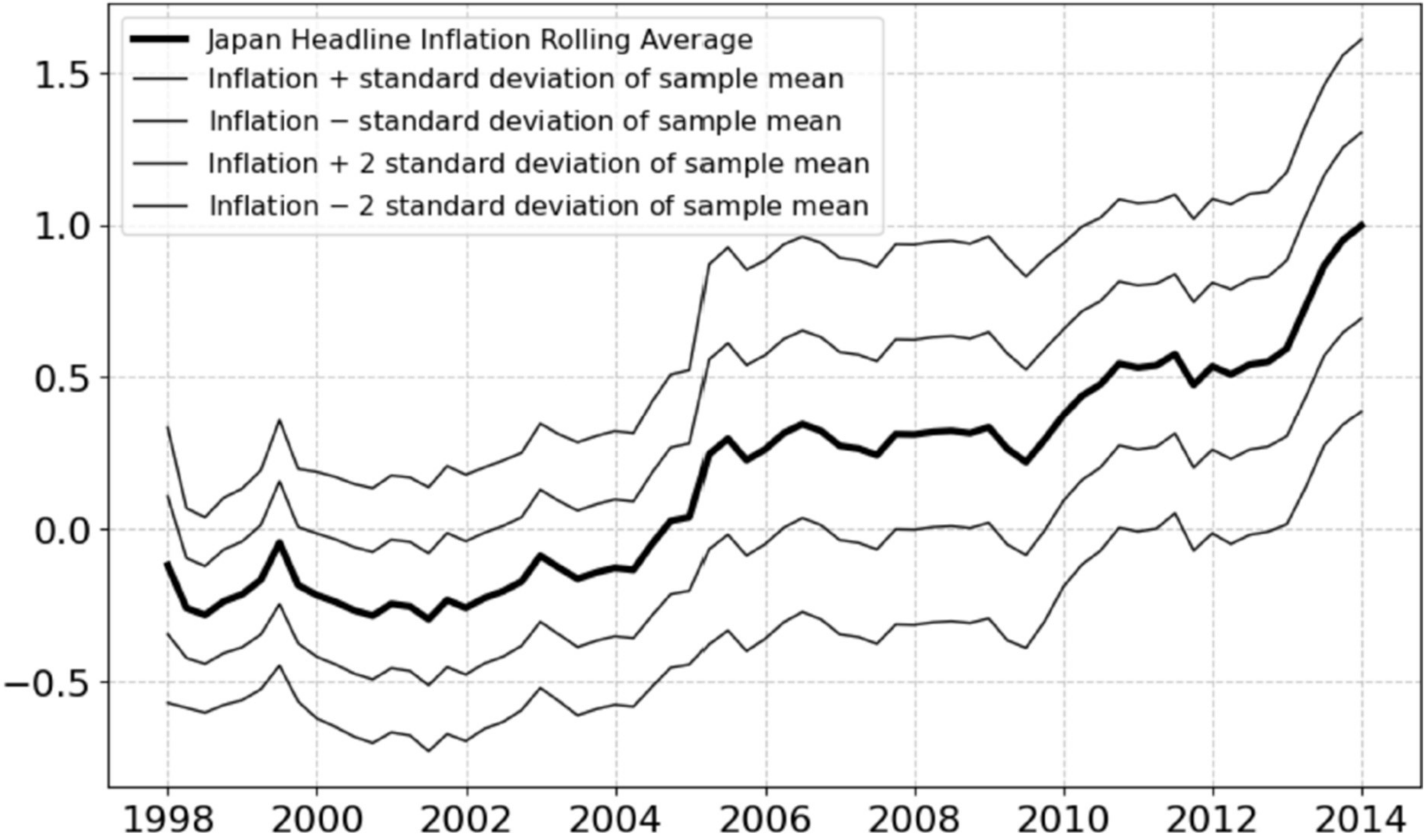
10-yr rolling average of four measures of inflation in Japan



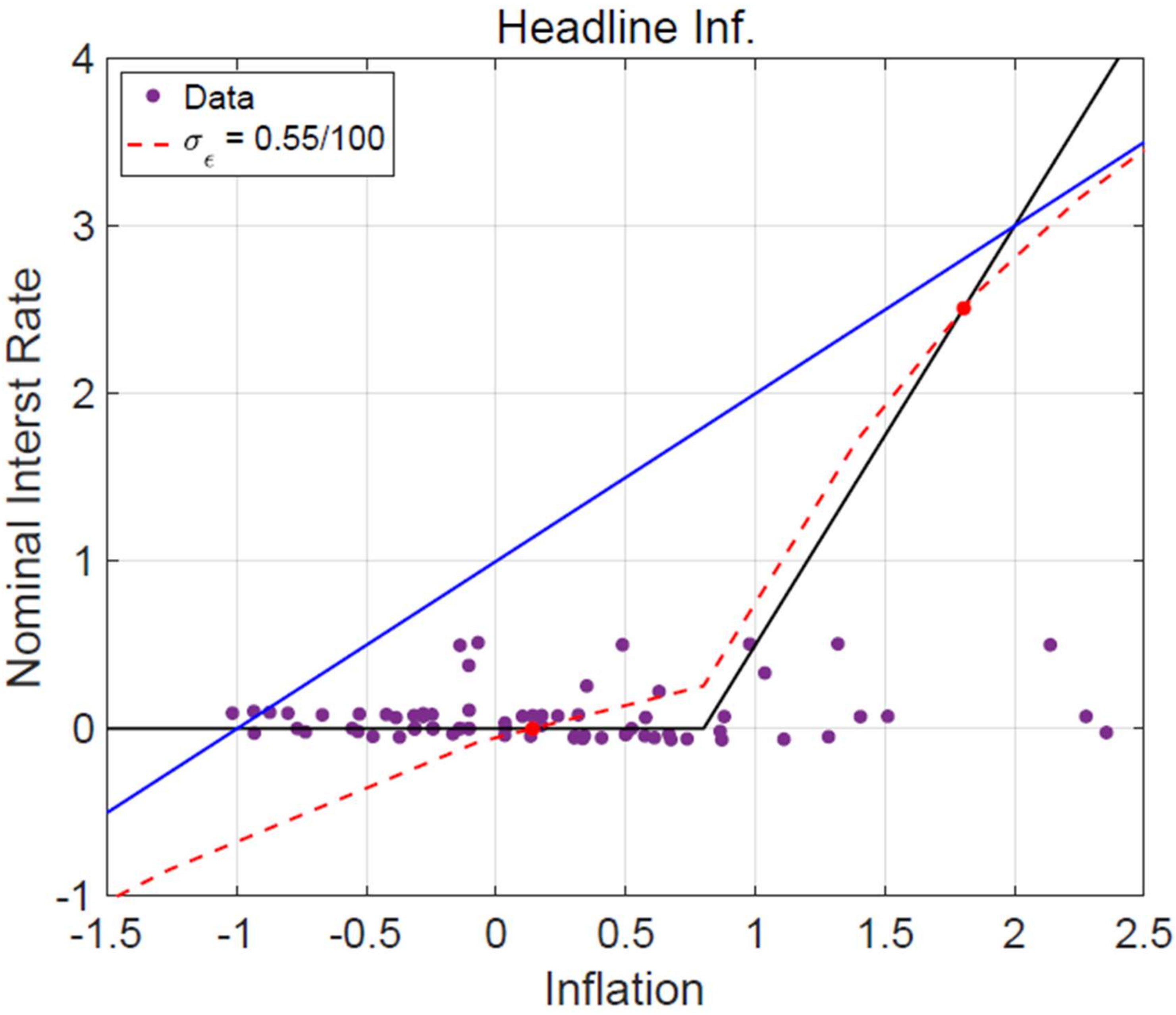
- Inflation in Japan seems to have been fluctuating around a positive rate in 2010s
  - …even since mid-2000s if focus on headline, core, and BOJ core.

Confidence bands for the 10-yr rolling average



- …or fluctuating around "zero or slightly positive."
  - Still, it does not look like inflation has been fluctuating around a negative rate.

- Once uncertainty is taken into account, deflationary equilibrium is consistent with Japanese experience in 2010s.



Summary

- In the deflationary equilibrium, inflation is higher at the **R**isky **S**teady **S**tate (RSS) than at the **D**eterministic **S**teady **S**tate (DSS).

- Lack of deflation in 2010s while at the lower bound is more consistent with RSS than with DSS.