The New Keynesian Model

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Basic new Keynesian model

Three basic components

1. An expectational “IS” curve (Euler equation)
2. An inflation adjustment equation (Phillips curve/price setting)
3. A specification of policy behavior
Overview of the Model

* The model consists of households who supply labor, purchase goods for consumption, and hold money and bonds, and firms who hire labor and produce and sell differentiated products in monopolistically competitive goods markets.

* The basic model of monopolistic competition is drawn from Dixit and Stiglitz (1977).

* Each firm set the price of the good it produces, but not all firms reset their price each period.

* Households and firms behave optimally: households maximize the expected present value of utility and firms maximize profits.
Households

- The preferences of a representative household defined over a composite consumption good $C_t$, real money balances $M_t/P_t$, and leisure $1 - N_t$, where $N_t$ is the time devoted to market employment.

- Households maximize

$$E_t \sum_{i=0}^{\infty} \beta^i \left[ \frac{C_{t+i}^{1-\sigma}}{1-\sigma} + \frac{\gamma}{1-b} \left( \frac{M_{t+i}}{P_{t+i}} \right)^{1-b} - \chi \frac{N_{t+i}^{1+\eta}}{1+\eta} \right]. \quad (1)$$

- The composite consumption good consists of differentiate products produced by monopolistically competitive final goods producers (firms). There are a continuum of such firms of measure 1, and firm $j$ produces good $c_j$. 
Households

- The composite consumption good that enters the household’s utility function is defined as

\[
C_t = \left[ \int_0^1 c_{jt}^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} \quad \theta > 1. \tag{2}
\]

The parameter \( \theta \) governs the price elasticity of demand for the individual goods.
The household’s decision problem can be dealt with in two stages.

1. Regardless of the level of $C_t$, it will always be optimal to purchase the combination of the individual goods that minimize the cost of achieving this level of the composite good.

2. Given the cost of achieving any given level of $C_t$, the household chooses $C_t$, $N_t$, and $M_t$ optimally.
Households

Dealing first with the problem of minimizing the cost of buying $C_t$, the household’s decision problem is to

$$\min_{c_{jt}} \int_0^1 p_{jt} c_{jt} \, dj$$

subject to

$$\left[ \int_0^1 \frac{\theta - 1}{\theta} c_{jt}^{\theta - 1} \, dj \right]^{\frac{\theta}{\theta - 1}} \geq C_t, \quad (3)$$

where $p_{jt}$ is the price of good $j$. Letting $\psi_t$ be the Lagrange multiplier on the constraint, the first order condition for good $j$ is

$$p_{jt} - \psi_t \left[ \int_0^1 \frac{\theta - 1}{\theta} c_{jt}^{\theta - 1} \, dj \right]^{\frac{1}{\theta - 1}} c_{jt}^{-\frac{1}{\theta}} = 0.$$
Households

- Rearranging, \( c_{jt} = \left( \frac{p_{jt}}{\psi_t} \right)^{-\theta} C_t \). From the definition of the composite level of consumption (2), this implies

\[
C_t = \left[ \int_0^1 \left( \frac{p_{jt}}{\psi_t} \right)^{-\theta} C_t \right]^{\frac{\theta}{\theta - 1}} \left[ \int_0^1 \left( \frac{1}{\psi_t} \right)^{-\theta} p_{jt}^{1-\theta} dj \right]^{\frac{\theta}{\theta - 1}} C_t.
\]

Solving for \( \psi_t \),

\[
\psi_t = \left[ \int_0^1 p_{jt}^{1-\theta} dj \right]^{\frac{1}{1-\theta}} \equiv P_t.
\]
Households

- The Lagrange multiplier is the appropriately aggregated price index for consumption.
- The demand for good $j$ can then be written as

$$c_{jt} = \left( \frac{p_{jt}}{P_t} \right)^{-\theta} C_t. \quad (5)$$

The price elasticity of demand for good $j$ is equal to $\theta$. As $\theta \to \infty$, the individual goods become closer and closer substitutes, and, as a consequence, individual firms will have less market power.
Households

- Given the definition of the aggregate price index in (4), the budget constraint of the household is, in real terms,

\[ C_t + \frac{M_t}{P_t} + \frac{B_t}{P_t} = \left( \frac{W_t}{P_t} \right) N_t + \frac{M_{t-1}}{P_t} + R_{t-1} \left( \frac{B_{t-1}}{P_t} \right) + \Pi_t, \quad (6) \]

where \( M_t \) (\( B_t \)) is the household’s nominal holdings of money (one period bonds). Bonds pay a gross nominal rate of interest \( R_t \). Real profits received from firms are equal to \( \Pi_t \).

- In the second stage of the household’s decision problem, consumption, labor supply, money, and bond holdings are chosen to maximize (1) subject to (6).
Households

The following conditions must also hold in equilibrium

1. the Euler condition for the optimal intertemporal allocation of consumption

\[ C_t^{-\sigma} = \beta R_t E_t \left( \frac{P_t}{P_{t+1}} \right) C_{t+1} \]  

(7)

2. the condition for optimal money holdings:

\[ \gamma \left( \frac{M_t}{P_t} \right)^{-b} \frac{C_t^{-\sigma}}{C_t^{-\sigma}} = R_t - 1 \frac{R_t}{R_t} \]  

(8)

3. the condition for optimal labor supply:

\[ \chi \frac{N_t^\eta}{C_t^{-\sigma}} = \frac{W_t}{P_t} \]  

(9)
Firms maximize profits, subject to three constraints:

1. The first is the production function summarizing the technology available for production. For simplicity, we have ignored capital, so output is a function solely of labor input $N_{jt}$ and an aggregate productivity disturbance $Z_t$:

$$c_{jt} = Z_t N_{jt}, \quad \mathbb{E}(Z_t) = 1.$$

2. The second constraint on the firm is the demand curve each faces. This is given by equation (5).

3. The third constraint is that each period some firms are not able to adjust their price. The specific model of price stickiness we will use is due to Calvo (1983).
Price adjustment

- Each period, the firms that adjust their price are randomly selected: a fraction $1 - \omega$ of all firms adjust while the remaining $\omega$ fraction do not adjust.
  - The parameter $\omega$ is a measure of the degree of nominal rigidity; a larger $\omega$ implies fewer firms adjust each period and the expected time between price changes is longer.

- For those firms who do adjust their price at time $t$, they do so to maximize the expected discounted value of current and future profits.
  - Profits at some future date $t + s$ are affected by the choice of price at time $t$ only if the firm has not received another opportunity to adjust between $t$ and $t + s$. The probability of this is $\omega^s$. 
Price adjustment
The firm’s decision problem

First consider the firm’s cost minimization problem, which involves minimizing $W_t N_{jt}$ subject to producing $c_{jt} = Z_t N_{jt}$. This problem can be written as

$$\min_{N_{jt}} W_t N_{jt} + \phi^n_t (c_{jt} - Z_t N_{jt}) .$$

where $\phi^n_t$ is equal to the firm’s nominal marginal cost. The first order condition implies

$$W_t = \phi^n_t Z_t ,$$

or $\phi^n_t = W_t / Z_t$. Dividing by $P_t$ yields real marginal cost as

$$\phi_t = W_t / (P_t Z_t) .$$
The firm’s decision problem

The firm’s pricing decision problem then involves picking \( p_{jt} \) to maximize

\[
E_t \sum_{i=0}^{\infty} \omega^i \Delta_{i,t+i} \Pi \left( \frac{p_{jt}}{P_{t+i}}, \phi_{t+i}, c_{t+i} \right) =

E_t \sum_{i=0}^{\infty} \omega^i \Delta_{i,t+i} \left[ \left( \frac{p_{jt}}{P_{t+i}} \right)^{1-\theta} - \phi_{t+i} \left( \frac{p_{jt}}{P_{t+i}} \right)^{-\theta} \right] C_{t+i},
\]

where the discount factor \( \Delta_{i,t+i} \) is given by \( \beta^i (C_{t+i}/C_t)^{-\sigma} \) and profits are

\[
\Pi(p_{jt}) = \left[ \left( \frac{p_{jt}}{P_{t+i}} \right)^{c_{jt+i}} - \phi_{t+i} c_{jt+i} \right]
\]
Price adjustment

- All firms adjusting in period \( t \) face the same problem, so all adjusting firms will set the same price.

- Let \( p^*_t \) be the optimal price chosen by all firms adjusting at time \( t \). The first order condition for the optimal choice of \( p^*_t \) is

\[
E_t \sum_{i=0}^{\infty} \omega^i \Delta_{i,t+i} \left[ (1 - \theta) \left( \frac{1}{p_{jt}} \right) \left( \frac{p^*_t}{P_{t+i}} \right)^{1-\theta} + \theta \varphi_{t+i} \left( \frac{1}{p^*_t} \right) \left( \frac{p^*_t}{P_{t+i}} \right)^{-\theta} \right]
\]

- Using the definition of \( \Delta_{i,t+i} \),

\[
\left( \frac{p^*_t}{P_t} \right) = \left( \frac{\theta}{\theta - 1} \right) \frac{E_t \sum_{i=0}^{\infty} \omega^i \beta^i \mathcal{C}_{t+i}^{1-\sigma} \varphi_{t+i} \left( \frac{P_{t+i}}{P_t} \right)^{\theta}}{E_t \sum_{i=0}^{\infty} \omega^i \beta^i \mathcal{C}_{t+i}^{1-\sigma} \left( \frac{P_{t+i}}{P_t} \right)^{\theta-1}}. \tag{10}
\]
The case of flexible prices

- If all firms are able to adjust their prices every period ($\omega = 0$):

\[
\left( \frac{p^*_t}{P_t} \right) = \left( \frac{\theta}{\theta - 1} \right) \varphi_t = \mu \varphi_t. \tag{11}
\]

- Each firm sets its price $p^*_t$ equal to a markup $\mu > 1$ over nominal marginal cost $P_t \varphi_t$.
- When prices are flexible, all firms charge the same price, and $\varphi_t = \mu^{-1}$. 
The case of flexible prices

- Using the definition of real marginal cost, this means

\[ \frac{W_t}{P_t} = \frac{Z_t}{\mu}. \]

- However, the real wage must also equal the marginal rate of substitution between leisure and consumption to be consistent with household optimization:

\[ \frac{\chi N_t^\eta}{C_t^{-\sigma}} = \frac{Z_t}{\mu}. \]  

(12)
The case of flexible prices

Flexible-price output

- Let a $\hat{x}_t$ denote the percent deviation of a variable $X_t$ around its steady-state. Then, the steady-state yields

$$\eta \hat{n}_t + \sigma \hat{c}_t = \hat{z}_t.$$  

- Now using the fact that $\hat{y}_t = \hat{n}_t + \hat{z}_t$ and $\hat{y}_t = \hat{c}_t$, flexible-price equilibrium output $\hat{y}_t^f$ can be expressed as

$$\hat{y}_t^f = \left[ \frac{1 + \eta}{\eta + \sigma} \right] \hat{z}_t.$$  

(13)
The case of sticky prices

- When prices are sticky ($\omega > 0$), the firm must take into account expected future marginal cost as well as current marginal cost when setting $p_t^*$. 

- The aggregate price index is an average of the price charged by the fraction $1 - \omega$ of firms setting their price in period $t$ and the average of the remaining fraction $\omega$ of all firms who set prices in earlier periods.

- Because the adjusting firms were selected randomly from among all firms, the average price of the non-adjusters is just the average price of all firms that was prevailing in period $t - 1$.

- Thus, the average price in period $t$ satisfies

$$P_t^{1-\theta} = (1 - \omega)(p_t^*)^{1-\theta} + \omega P_{t-1}^{1-\theta}. \quad (14)$$
Inflation adjustment

- Using the first order condition for $p_t^*$ and approximating around a zero average inflation, flexible-price equilibrium,

$$\pi_t = \beta E_t \pi_{t+1} + \tilde{\kappa} \hat{\phi}_t$$  \hspace{1cm} (15)

where

$$\tilde{\kappa} = \frac{(1 - \omega)(1 - \beta \omega)}{\omega}$$

- Equation (15) is often referred to as the New Keynesian Phillips curve.
The New Keynesian Phillips curve is forward-looking; when a firm sets its price, it must be concerned with inflation in the future because it may be unable to adjust its price for several periods.

Solving forward,

\[
\pi_t = \bar{\kappa} \sum_{i=0}^{\infty} \beta^i E_t \hat{\phi}_{t+i},
\]

Inflation is a function of the present discounted value of current and future real marginal cost.

Inflation depends on real marginal cost and not directly on a measure of the gap between actual output and some measure of potential output or on a measure of unemployment relative to the natural rate, as is typical in traditional Phillips curves.
Real marginal cost and the output gap

- The firm’s real marginal cost is equal to the real wage it faces divided by the marginal product of labor: $\varphi_t = W_t / P_t Z_t$.
- Because nominal wages have been assumed to be completely flexible, the real wage must equal the marginal rate of substitution between leisure and consumption.
- In a flexible price equilibrium, all firms set the same price, so (11) implies that $\varphi = \mu^{-1}$. From equation (9), $\hat{w}_t - \hat{p}_t = \eta \hat{n}_t + \sigma \hat{y}_t$

Recalling that $\hat{c}_t = \hat{y}_t$, $\hat{y}_t = \hat{n}_t + \hat{z}_t$, the percentage deviation of real marginal cost around the flexible price equilibrium is

$$\hat{\varphi}_t = [\eta \hat{n}_t + \sigma \hat{y}_t] - \hat{z}_t = (\eta + \sigma) \left[ \hat{y}_t - \left( \frac{1 + \eta}{\eta + \sigma} \right) \hat{z}_t \right].$$
Real marginal cost and the output gap

- But from (13), this can be written as

\[
\hat{\phi}_t = (\eta + \sigma) \left( \hat{y}_t - \hat{y}_t^f \right). \tag{16}
\]

- Using these results, the inflation adjustment equation (15) becomes

\[
\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa x_t \tag{17}
\]

where \(\kappa = (\eta + \sigma) \tilde{\kappa} = (\eta + \sigma) (1 - \omega) \left[ 1 - \beta \omega \right] / \omega\) and \(x_t \equiv \hat{y}_t - \hat{y}_t^f\) is the gap between actual output and the flexible-price equilibrium output.

- This inflation adjustment or forward-looking Phillips curve relates output, in the form of the deviation around the level of output that would occur in the absence of nominal price rigidity, to inflation.
The demand side of the model

- Start with Euler condition for optimal consumption choice

\[ C_t^{-\sigma} = \beta R_t E_t \left( \frac{P_t}{P_{t+1}} \right) C_{t+1} \]

- Linearize around steady-state:

\[ -\sigma \dot{c}_t = (\dot{i}_t - E_t p_{t+1} + p_t) - \sigma E_t \dot{c}_{t+1} \]

or

\[ \dot{c}_t = E_t \dot{c}_{t+1} - \left( \frac{1}{\sigma} \right) (\dot{i}_t - E_t p_{t+1} + p_t). \]

- Goods market equilibrium (no capital)

\[ Y_t = C_t. \]
The demand side of the model

Euler condition becomes

\[ \hat{y}_t = E_t \hat{y}_{t+1} - \left( \frac{1}{\sigma} \right) (\hat{i}_t - E_t \rho_{t+1} + \rho_t). \]

This is often called an “expectational IS curve”, to make the comparisons with old-style Keynesian models clear.
Demand and the output gap

- Express in terms of the output gap \( x_t = \hat{y}_t - \hat{y}_t^f \):

\[
\hat{y}_t - \hat{y}_t^f = E_t \left( \hat{y}_{t+1} - \hat{y}_{t+1}^f \right) - \left( \frac{1}{\sigma} \right) \left( \hat{i}_t - E_t p_{t+1} + p_t \right) + \left( E_t \hat{y}_{t+1}^f - \hat{y}_t^f \right)
\]

or

\[
x_t = E_t x_{t+1} - \left( \frac{1}{\sigma} \right) \left( r_t - r^n_t \right),
\]

where \( r_t = \hat{i}_t - E_t p_{t+1} + p_t \) and

\[
r^n_t \equiv \sigma \left( E_t \hat{y}_{t+1}^f - \hat{y}_t^f \right).
\]

- Notice that the nominal interest rate affects output through the interest rate gap \( r_t - r^n_t \).
The general equilibrium model

- Two equation system

\[
\pi_t = E_t \pi_{t+1} + \kappa x_t
\]

\[
x_t = E_t x_{t+1} - \left( \frac{1}{\sigma} \right) (\hat{i}_t - E_t \pi_{t+1} - r^n_t)
\]
The general equilibrium model

Consistent with

- optimizing behavior by households and firms
- budget constraints
- market equilibrium

Two equations but three unknowns: $x_t$, $\pi_t$, and $i_t$ – need to specify monetary policy
Solving the model for the rational expectations equilibrium

• Suppose $i_t = r^n_t + \delta \pi_t$.

• Write system as

$$\begin{bmatrix} \beta & 0 \\ \frac{1}{\sigma} & 1 \end{bmatrix} \begin{bmatrix} E_t \pi_{t+1} \\ E_t x_{t+1} \end{bmatrix} = \begin{bmatrix} 1 & -\kappa \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \pi_t \\ x_t \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{\sigma} \end{bmatrix} \delta \pi_t$$

or

$$\begin{bmatrix} E_t \pi_{t+1} \\ E_t x_{t+1} \end{bmatrix} = \begin{bmatrix} \frac{1}{\beta} & -\kappa/eta \\ \beta \delta - 1/\sigma \beta & 1 + \kappa / \sigma \beta \end{bmatrix} \begin{bmatrix} \pi_t \\ x_t \end{bmatrix}$$

• Two eigenvalues outside the unit circle if and only if

$$\delta > 1$$
The Taylor Principle

- Policy must respond sufficiently strongly to inflation.

**Definition**

The condition that the nominal interest rate respond more than one-for-one to inflation is called the Taylor Principle.
Lessons

- Policy based on responding to exogenous disturbances does not ensure a unique equilibrium.
- Policy must respond to endogenous variables.
- In particular, the Taylor Principle needs to be satisfied.
Woodford demonstrates that deviations of the expected discounted utility of the representative agent around the level of steady-state utility can be approximated by

\[
E_t \sum_{i=0}^{\infty} \beta^i V_{t+i} \approx -\Omega E_t \sum_{i=0}^{\infty} \beta^i \left[ \pi_{t+i}^2 + \lambda (x_{t+i} - x^*)^2 \right].
\] (18)

- $x_t$ is the gap between output and the output level that would arise under flexible prices, and $x^*$ is the gap between the steady-state efficient level of output (in the absence of the monopolistic distortions) and the steady-state level of output.
Comparison to a standard loss function

- This looks a lot like the standard quadratic loss function. There are, however, two critical differences.

1. The output gap is measured relative to the rate of output under flexible prices.
2. Inflation variability enters because, with price rigidity, higher inflation results in an inefficient dispersion of output among the individual producers.
   - Because prices are sticky, higher inflation results in an increase in overall price dispersion.
Policy weights

- Theory says something about the weights in the loss function:

\[
E_t \sum_{i=0}^{\infty} \beta^i V_{t+i} \approx -\Omega E_t \sum_{i=0}^{\infty} \beta^i \left[ \pi_{t+i}^2 + \lambda (x_{t+i} - x^*)^2 \right],
\]

where

\[
\Omega = \frac{1}{2} \bar{Y} U_c \left[ \frac{\omega}{(1 - \omega)(1 - \omega \beta)} \right] (\theta^{-1} + \eta) \theta^2
\]

and

\[
\lambda = \left[ \frac{(1 - \omega)(1 - \omega \beta)}{\omega} \right] \frac{(\sigma + \eta)}{(1 + \eta \theta)} \theta.
\]

- Greater nominal rigidity (larger \( \omega \)) reduces \( \lambda \).
- Loss function endogenous.
- Calvo specification implies \( \lambda \) is small – Taylor specification leads to larger weight on output gap.
The basic new Keynesian inflation adjustment equation took the form

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t.$$ 

That is, there is no additional disturbance term.

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t \Rightarrow \pi_t = \kappa \sum_{i=0}^{\infty} \beta^i E_t x_{t+i}$$

The absence of a stochastic disturbance implies there is no conflict between a policy designed to maintain inflation at zero and a policy designed to keep the output gap equal to zero.

Just set $x_{t+i} = 0$ for all $i$; keeps inflation equal to zero.
Thus, the key implication of the basic new Keynesian model is that price stability is the appropriate objective of monetary policy.

No policy conflicts.

When prices are sticky but wages are flexible, the nominal wage can adjust to ensure labor market equilibrium is maintained in the face of productivity shocks. Optimal policy should then aim to keep the price level stable.
Models that combine optimizing agents and sticky prices have very strong policy implications.

When the price level fluctuates, and not all firms are able to adjust, price dispersion results. This causes the relative prices of the different goods to vary. If the price level rises, for example, two things happen.

1. The relative price of firms who have not set their prices for a while falls. They experience in increase in demand and raise output, while firms who have just reset their prices reduce output. This production dispersion is inefficient.

2. Consumers increase their consumption of the goods whose relative price has fallen and reduce consumption of those goods whose relative price has risen. This dispersion in consumption reduces welfare.
Cost shocks

Assume

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + e_t$$

where $e$ represents an inflation or cost shock.

Then

$$\pi_t = \kappa \sum_{i=0}^{\infty} \beta^i E_t x_{t+i} + \sum_{i=0}^{\infty} \beta^i E_t e_{t+i}$$

Cannot keep both $x$ and $\pi$ equal to zero.

Trade-offs must be made.
Basic model

- When forward-looking expectations play a role, discretion leads to a stabilization bias even though there is no average inflation bias.

- Minimize

\[-\Omega E_t \sum_{i=0}^{\infty} \beta^i [\pi_{t+i}^2 + \lambda x_{t+i}^2]\]

subject to

\[\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + e_t.\]

- Notice the Euler/IS equation imposes no constraint – use it to solve for \(i_t\) once optimal \(\pi_t\) and \(x_t\) have been determined.
Basic model – eliminating the steady-state distortion

- Note that $x^*$ has been set equal to zero in loss function

$$-\Omega E_t \sum_{i=0}^{\infty} \beta^i \left[ \pi_{t+i}^2 + \lambda x_{t+i}^2 \right].$$

- Fiscal subsidy to offset distortion from monopolistic competition.
- If $x^* \neq 0$, can't use first order approximations to structural equations to obtain a correct second order approximation to the representative agent's welfare.
Discretion

- When the central bank operates with discretion, it acts each period to minimize the loss function subject to the inflation adjustment equation.
- Because the decisions of the central bank at date $t$ do not bind it at any future dates, the central bank is unable to affect the private sector’s expectations about future inflation.
- Thus, the decision problem of the central bank becomes the single period problem of minimizing $\pi_t^2 + \lambda x_t^2$ subject to the inflation adjustment equation.
Central bank problem is to pick $\pi_t$ and $x_t$ to minimize

$$\pi_t^2 + \lambda x_t^2 + \psi_t (\pi_t - \beta \pi_{t+1} - \kappa x_t - e_t)$$

taking $E_t \pi_{t+1}$ as given.

The first order conditions can be written as

$$\pi_t + \psi_t = 0 \quad (19)$$

$$\lambda x_t - \kappa \psi_t = 0. \quad (20)$$

Eliminating $\psi_t$, $\lambda x_t + \kappa \pi_t = 0$. 
Discretion

Equilibrium

- $x_t$ and $\pi_t$ satisfy

$$\lambda x_t + \kappa \pi_t = 0.$$  

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + e_t.$$  

- Then

$$\pi_t = \beta E_t \pi_{t+1} - \frac{\kappa^2}{\lambda} \pi_t + e_t \Rightarrow \pi_t = \frac{\lambda \beta E_t \pi_{t+1} + \lambda e_t}{\lambda + \kappa^2}.$$
Suppose

\[ e_t = \rho e_{t-1} + \varepsilon_t. \]

and

\[ \pi_t = A e_t. \]

Then, \( E_t \pi_{t+1} = A E_t e_{t+1} = A \rho e_t \) and

\[ \pi_t = \left( \frac{\lambda \beta A \rho + \lambda}{\lambda + \kappa^2} \right) e_t \Rightarrow A = \left( \frac{\lambda \beta A \rho + \lambda}{\lambda + \kappa^2} \right) = \frac{\lambda}{\lambda(1 - \beta \rho) + \kappa^2}. \]

Zero average inflation bias.
Discretion

Behavior of the interest rate

- From the IS equation,

\[ i_t = E_t \pi_{t+1} + \sigma (E_t x_{t+1} - x_t) + r^n_t. \]

- Using solution,

\[ i_t = \left[ A \rho - \sigma \left( \frac{\kappa}{\lambda} \right) (\rho - 1) \right] e_t + r^n_t = B e_t + r^n_t. \]

- Shifts in natural rate of interest \( r^n \) are fully offset.

- So optimal policy involves \( i \) responding to shocks, but adopting a rule of the form

\[ i_t = B e_t + r^n_t \]

does not ensure a unique rational expectations equilibrium.
Precommitment

- When forward-looking expectations play a role, discretion leads to a stabilization bias even though there is no average inflation bias.
- Under optimal commitment, central bank at time $t$ chooses both current and expected future values of inflation and the output gap.
- Minimize
  \[-\Omega E_t \sum_{i=0}^{\infty} \beta^i \left[ \pi_{t+i}^2 + \lambda (x_{t+i} - x^*)^2 \right]\]
  subject to
  \[\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + e_t.\]
Optimal precommitment

- The central bank’s problem is to pick $\pi_{t+i}$ and $x_{t+i}$ to minimize

$$E_t \sum_{i=0}^{\infty} \beta^i \left[ \pi_{t+i}^2 + \lambda x_{t+i}^2 + \psi_{t+i} \left( \pi_{t+i} - \beta \pi_{t+i+1} - \kappa x_{t+i} - e_{t+i} \right) \right].$$

- The first order conditions can be written as

$$\pi_t + \psi_t = 0 \quad (21)$$
$$E_t \left( \pi_{t+i} + \psi_{t+i} - \psi_{t+i-1} \right) = 0 \quad i \geq 1 \quad (22)$$
$$E_t \left( \lambda x_{t+i} - \kappa \psi_{t+i} \right) = 0 \quad i \geq 0. \quad (23)$$

- Dynamic inconsistency – at time $t$, the central bank sets $\pi_t = -\psi_t$ and promises to set $\pi_{t+1} = - \left( E_t \psi_{t+1} - \psi_t \right)$. When $t+1$ arrives, a central bank that reoptimizes will again obtains $\pi_{t+1} = -\psi_{t+1}$ – the first order condition (21) updated to $t+1$ will reappear.
An alternative definition of an optimal precommitment policy requires the central bank to implement conditions (22) and (23) for all periods, including the current period so that

\[ \pi_{t+i} + \psi_{t+i} - \psi_{t+i-1} = 0 \quad i \geq 0 \]

\[ \lambda x_{t+i} - \kappa \psi_{t+i} = 0 \quad i \geq 0. \]

Woodford (1999) has labeled this the “timeless perspective” approach to precommitment.
Timeless precommitment

- Under the timeless perspective optimal commitment policy, inflation and the output gap satisfy

\[ \pi_{t+i} = - \left( \frac{\lambda}{\kappa} \right) (x_{t+i} - x_{t+i-1}) \] (24)

for all \( i \geq 0 \).

- Woodford (1999) has stressed that, even if \( \rho = 0 \), so that there is no natural source of persistence in the model itself, \( a > 0 \) and the precommitment policy introduces inertia into the output gap and inflation processes.

- This commitment to inertia implies that the central bank’s actions at date \( t \) allow it to influence expected future inflation. Doing so leads to a better trade-off between gap and inflation variability than would arise if policy did not react to the lagged gap.
Improved trade-off under commitment

- The difference in the stabilization response under commitment and discretion is the stabilization bias due to discretion.
- Consider a positive inflation shock, \( e > 0 \).
- A given change in current inflation can be achieved with a smaller fall in \( x \) if expected future inflation can be reduced:

\[
\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + e_t
\]

- Requires a commitment to future deflation.
- By keeping output below potential (a negative output gap) for several periods into the future after a positive cost shock, the central bank is able to lower expectations of future inflation. A fall in \( E_t \pi_{t+1} \) at the time of the positive inflation shock improves the trade-off between inflation and output gap stabilization faced by the central bank.