

# More on the New Keynesian Model

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## Real marginal cost and the output gap

- Recall that we had an expression for real marginal cost

$$\hat{\varphi}_t = (\eta + \sigma) (\hat{y}_t - \hat{y}_t^f). \quad (1)$$

- Where we recall that  $\hat{y}_t = \hat{n}_t + \hat{z}_t$  and  $\hat{y}_t = \hat{c}_t$ , flexible-price equilibrium output  $\hat{y}_t^f$  can be expressed as

$$\hat{y}_t^f = \left[ \frac{1 + \eta}{\eta + \sigma} \right] \hat{z}_t. \quad (2)$$

- Using these results, the inflation adjustment equation is:

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t \quad (3)$$

where  $\kappa = (\eta + \sigma) \tilde{\kappa} = (\eta + \sigma) (1 - \omega) [1 - \beta\omega] / \omega$  and  $x_t \equiv \hat{y}_t - \hat{y}_t^f$  is the gap between actual output and the flexible-price equilibrium output.

# The demand side of the model

- Start with Euler condition for optimal consumption choice

$$C_t^{-\sigma} = \beta R_t E_t \left( \frac{P_t}{P_{t+1}} \right) C_{t+1}^{-\sigma}$$

- Linearize around steady-state:

$$-\sigma \hat{c}_t = (\hat{i}_t - E_t p_{t+1} + p_t) - \sigma E_t \hat{c}_{t+1}$$

or

$$\hat{c}_t = E_t \hat{c}_{t+1} - \left( \frac{1}{\sigma} \right) (\hat{i}_t - E_t p_{t+1} + p_t).$$

- Goods market equilibrium (no capital)

$$Y_t = C_t.$$

# The demand side of the model

## Linearization

- Euler condition becomes

$$\hat{y}_t = E_t \hat{y}_{t+1} - \left( \frac{1}{\sigma} \right) (\hat{i}_t - E_t p_{t+1} + p_t).$$

- This is often called an “expectational IS curve”, to make the comparisons with old-style Keynesian models clear.

## Demand and the output gap

- Express in terms of the output gap  $x_t = \hat{y}_t - \hat{y}_t^f$ :

$$\hat{y}_t - \hat{y}_t^f = E_t \left( \hat{y}_{t+1} - \hat{y}_{t+1}^f \right) - \left( \frac{1}{\sigma} \right) (\hat{i}_t - E_t p_{t+1} + p_t) + \left( E_t \hat{y}_{t+1}^f - \hat{y}_t^f \right)$$

or

$$x_t = E_t x_{t+1} - \left( \frac{1}{\sigma} \right) (r_t - r_t^n),$$

where  $r_t = \hat{i}_t - E_t p_{t+1} + p_t$  and

$$r_t^n \equiv \sigma \left( E_t \hat{y}_{t+1}^f - \hat{y}_t^f \right).$$

- Notice that the nominal interest rate affects output through the interest rate gap  $r_t - r_t^n$ .

# The general equilibrium model

- Two equation system

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa x_t$$

$$x_t = \mathbb{E}_t x_{t+1} - \left( \frac{1}{\sigma} \right) (\hat{i}_t - \mathbb{E}_t \pi_{t+1} - r_t^n)$$

# The general equilibrium model

- Consistent with
  - ▶ optimizing behavior by households and firms
  - ▶ budget constraints
  - ▶ market equilibrium
- Two equations but three unknowns:  $x_t$ ,  $\pi_t$ , and  $i_t$  – need to specify monetary policy

# Solving the model for the rational expectations equilibrium

- Suppose  $i$  is exogenous.
- Write system as

$$\begin{bmatrix} \beta & 0 \\ \frac{1}{\sigma} & 1 \end{bmatrix} \begin{bmatrix} E_t \pi_{t+1} \\ E_t x_{t+1} \end{bmatrix} = \begin{bmatrix} 1 & -\kappa \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \pi_t \\ x_t \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{\sigma} \end{bmatrix} (i_t - r_t^n)$$

- or

$$\begin{bmatrix} E_t \pi_{t+1} \\ E_t x_{t+1} \end{bmatrix} = \begin{bmatrix} \frac{1}{\beta} & -\frac{\kappa}{\beta} \\ -\frac{1}{\sigma\beta} & 1 + \frac{\kappa}{\sigma\beta} \end{bmatrix} \begin{bmatrix} \pi_t \\ x_t \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{\sigma} \end{bmatrix} (i_t - r_t^n)$$

- or

$$E_t Z_{t+1} = MZ_t + N(i_t - r_t^n)$$



# Solving the model for the rational expectations equilibrium

- There exists a unique, stationary rational expectations equilibrium if and only if the number of eigenvalues of  $M$  outside the unit circle is equal to the number of forward-looking variables (two).
- Condition is not satisfied!
- So a policy that just sets  $i_t = r_t^n$  exogenously does not result in a unique rational expectations equilibrium.
- Self-fulfilling increase in expected inflation is possible.

# Solving the model for the rational expectations equilibrium

- Suppose  $i_t = r_t^n + \delta\pi_t$ .
- Write system as

$$\begin{bmatrix} \beta & 0 \\ \frac{1}{\sigma} & 1 \end{bmatrix} \begin{bmatrix} E_t \pi_{t+1} \\ E_t x_{t+1} \end{bmatrix} = \begin{bmatrix} 1 & -\kappa \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \pi_t \\ x_t \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{\sigma} \end{bmatrix} \delta\pi_t$$

- or

$$\begin{bmatrix} E_t \pi_{t+1} \\ E_t x_{t+1} \end{bmatrix} = \begin{bmatrix} \frac{1}{\beta} & -\frac{\kappa}{\beta} \\ \frac{\beta\delta-1}{\sigma\beta} & 1 + \frac{\kappa}{\sigma\beta} \end{bmatrix} \begin{bmatrix} \pi_t \\ x_t \end{bmatrix}$$

- Two eigenvalues outside the unit circle if and only if

$$\delta > 1$$

# The Taylor Principle

- Policy must respond sufficiently strongly to inflation.

## Definition

The condition that the nominal interest rate respond more than one-for-one to inflation is called the Taylor Principle.

# Lessons

- Policy based on responding to exogenous disturbances does not ensure a unique equilibrium.
- Policy must respond to endogenous variables.
- In particular, the Taylor Principle needs to be satisfied.
  - ▶ If policy also responds to the output gap, then Bullard and Mitra show condition becomes

$$\kappa(\delta_{\pi} - 1) + (1 - \beta)\delta_x > 0.$$

# The Wicksellian interest rate

- Basic model:

$$x_t = E_t x_{t+1} - \left( \frac{1}{\sigma} \right) (i_t - E_t \pi_{t+1} - r_t^n)$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t$$

- The impact of monetary policy on output and inflation operates through the real rate of interest;
- Wicksellian interest rate gap  $i_t - E_t \pi_{t+1} - r_t^n$  summarizes impact of monetary policy.

# The Wicksellian interest rate

## Definition

Woodford (2003) has labelled  $r_t^n$  as the Wicksellian real interest rate. It is the interest rate consistent with output equal to the flexible-price equilibrium level.  $r^n$  is also called the natural rate of interest.

# The Wicksellian interest rate

- Output is affected by expected current and future one-period real interest rates. The presence of expected future output implies that the future path of the one-period real rate matters for current demand.
  - ▶ To see that this is the case, let  $r_t \equiv i_t - E_t \pi_{t+1}$  be the one-period real interest rate and then recursively solve the Euler condition forward to yield (assume  $C = Y$ )

$$x_t = -\frac{1}{\sigma} \sum_{i=0}^{\infty} E_t (r_{t+i} - r_{t+i}^n).$$

- ▶ Changes in the one-period rate that are persistent, so that they also influence expectations of future interest rates, will have stronger effects on  $x_t$  than more temporary changes in  $r$ .

# Other channels of monetary transmission

## The role of money

- So far, monetary policy only works via the Wicksellian interest rate gap.
- No direct role for money.
- Direct effects of the quantity of money: if utility is not separable, then changes in the real quantity of money would alter the marginal utility of consumption. The absence of money constitutes a special case.
  - ▶ The real money stock would appear in the household's Euler condition.
  - ▶ To replace real marginal cost with a measure of the output gap in the inflation equation, the real wage was equated to the marginal rate of substitution between leisure and consumption, and this will involve real money balances.



## Adding lagged inflation

- To capture the inflation persistence found in the data, it is common to augment the basic forward-looking inflation adjustment equation with the addition of lagged inflation:

$$\pi_t = (1 - \phi)\beta E_t \pi_{t+1} + \kappa x_t + \phi \pi_{t-1} + \varepsilon_t. \quad (4)$$

- In this formulation, the parameter  $\phi$  is often described as a measure of the degree of backward-looking price setting behavior.
  - ▶ Fuhrer (1997) finds little role for future inflation once lagged inflation is added to the inflation adjustment equation.
  - ▶ Rudebusch (2000) estimates (4) using U. S. data and argues that  $\phi$  is on the order of 0.7, suggesting that inflation is predominantly backward-looking.

# Indexation

- Christiano, Eichenbaum, and Evans (2001) make a distinction between firms that reoptimize it setting their price and those that do not:
  - ▶ each period a fraction  $1 - \omega$  of all firms optimally set their price;
  - ▶ the remaining firms either simply adjust their price based on the average rate of inflation, so that  $p_{jt} = \bar{\pi} p_{jt-1}$  where  $\bar{\pi}$  is the average inflation rate, or they adjust based on the most recently observed rate of inflation, so that  $p_{jt} = \pi_{t-1} p_{jt-1}$ .
- Costly to optimize

# Indexation and decision lag

- This specification results in an inflation adjustment equation of the form

$$\pi_t = \left( \frac{\beta}{1 + \beta} \right) \mathbb{E}_t \pi_{t+1} + \left( \frac{1}{1 + \beta} \right) \pi_{t-1} + \tilde{\kappa} \hat{\varphi}_t.$$

The presence of lagged inflation in this equation introduces inertia into the inflation process.

- CEE also assume prices set before time  $t$  information is available:

$$\pi_t = \left( \frac{\beta}{1 + \beta} \right) \mathbb{E}_{t-1} \pi_{t+1} + \left( \frac{1}{1 + \beta} \right) \pi_{t-1} + \tilde{\kappa} \mathbb{E}_{t-1} \hat{\varphi}_t.$$

# Structural estimates of the frequency of adjusting

- Estimates of new Keynesian Phillips curve yield values of  $\omega$  that may be too high.
- Estimates range from 0.758 to 0.911 (Dennis 2006)
- Value of 0.8 implies prices adjusted on average every  $(1 - 0.8)^{-1} = 5$  quarters.
- Micro evidence for U.S. suggests duration between price changes closer to 2 quarters, implying  $\omega = 0.5$ . Steinsson-Nakamamura suggest slightly larger.

# The sensitivity of marginal cost to output

- Empirically, inflation does not seem to respond strongly to the output gap:  $\kappa$  is small.
- In basic theory,

$$\kappa = (\eta + \sigma) \frac{(1 - \omega) [1 - \beta\omega]}{\omega}$$

where  $1 - \omega$  is the fraction of adjusting firms,  $\sigma$  is the coefficient of relative risk aversion, and  $\eta$  is the (inverse) of the wage elasticity of labor supply.

# The sensitivity of marginal cost to output

So  $\kappa$  small if

- $\omega$  large – high degree of price rigidity (estimates often imply unrealistic values around 0.8)
- $\sigma$  small – very little risk aversion
- $\eta$  is small – high degree of labor supply elasticity.

# The sensitivity of marginal cost to output

- Researchers have extended basic model to make marginal cost less sensitive to output.
- Christiano, Eichenbaum, and Evans (2006) – variable capital utilization
- Basic idea:
  - ▶ In standard model, increase in demand can only increase production if real wage rises to induce an increase in labor supply. If wage elasticity of labor supply is small, the real wage has to rise a lot. This boosts real marginal cost and inflation.
  - ▶ If output can increase by utilizing capital more intensely, wages and marginal cost will rise less.

# Demand persistence

## The trouble with Euler conditions

- Euler condition is purely forward looking – same problems arise as with inflation equation.
- Output is discounted value of future interest rate gaps:

$$x_t = - \left( \frac{1}{\sigma} \right) E_t \sum_{i=0}^{\infty} (r_{t+i} - r_{t+i}^n).$$



# Habit persistence

- To match the hump shaped response of output seen in the data, habit persistence has become a standard component of new Keynesian models (Christiano, Eichenbaum, and Evans 2006).
- External Habit persistence: Marginal utility of current consumption depends on past aggregate consumption.
- Internal Habit Persistence: Marginal utility of current consumption depends on household's past consumption.

# General equilibrium, estimated models

- Christiano, Eichenbaum, and Evans (2005)
- Smets and Wouters (2003, 2007)
- Levin, Onatski, Williams, and Williams (2006)
  - ▶ Components:
    - ★ Habit persistence
    - ★ Variable capital utilization
    - ★ Investment with 2nd-order adjustment costs
    - ★ Price adjustment at start of period (based on expectations – information delay)
    - ★ Wage and price stickiness

# Conclusions

- Basic model fails poorly when faced with data – too forward-looking;
- Habit persistence, variable capital utilization, firm specific capital, sticky wages all help.
- Models fit data, but decomposition into flexible-price and gap may miss major historical episodes.

# Policy analysis

## Key issues

- What are the objectives of optimal policy
- Is the policy environment one of commitment or discretion?
- What instrument rule implements the optimal policy?
- What are the properties of the resulting equilibrium?

# Welfare

- Given the specification of the economic environment, what are the appropriate objectives of the central bank?
- Standard to assume central bank is concerned with minimizing a quadratic loss function that depended on output and inflation – plausible, but ultimately *ad hoc*. Common in the Barro-Gordon tradition
- Woodford (2003) has provided the most detailed analysis of the link between a welfare criteria derived as a log-linear approximation to the utility of the representative agent and the type of quadratic loss functions so common in the literature.

# Welfare

## Quadratic approximation

- Woodford demonstrates that deviations of the expected discounted utility of the representative agent around the level of steady-state utility can be approximated by

$$E_t \sum_{i=0}^{\infty} \beta^i V_{t+i} \approx -\Omega E_t \sum_{i=0}^{\infty} \beta^i \left[ \pi_{t+i}^2 + \lambda (x_{t+i} - x^*)^2 \right]. \quad (5)$$

- $x_t$  is the gap between output and the output level that would arise under flexible prices, and  $x^*$  is the gap between the steady-state efficient level of output (in the absence of the monopolistic distortions) and the steady-state level of output.

## Comparison to a standard loss function

- This looks a lot like the standard quadratic loss function. There are, however, two critical differences.
  - 1 The output gap is measured relative to the rate of output under flexible prices.
  - 2 Inflation variability enters because, with price rigidity, higher inflation results in an inefficient dispersion of output among the individual producers.
    - ★ Because prices are sticky, higher inflation results in an increase in overall price dispersion.

## Policy weights

- Theory says something about the weights in the loss function:

$$E_t \sum_{i=0}^{\infty} \beta^i V_{t+i} \approx -\Omega E_t \sum_{i=0}^{\infty} \beta^i \left[ \pi_{t+i}^2 + \lambda (x_{t+i} - x^*)^2 \right],$$

where

$$\Omega = \frac{1}{2} \bar{Y} U_c \left[ \frac{\omega}{(1-\omega)(1-\omega\beta)} \right] (\theta^{-1} + \eta) \theta^2$$

and

$$\lambda = \left[ \frac{(1-\omega)(1-\omega\beta)}{\omega} \right] \frac{(\sigma + \eta)}{(1 + \eta\theta) \theta}.$$

- Greater nominal rigidity (larger  $\omega$ ) reduces  $\lambda$ .
- Loss function endogenous.
- Calvo specification implies  $\lambda$  is small – Taylor specification leads to larger weight on output gap.



# Instrument Rules

- A common approach to “optimal” policy is in terms of simple rules.
- The most famous of such instrument rules is the Taylor Rule (Taylor 1993):

$$i_t = \pi_t + 0.5x_t + 0.5 \left( \pi_t - \pi^T \right) + r^*,$$

where  $\pi^T$  was the target level of average inflation (Taylor assumed it to be 2%) and  $r^*$  was the equilibrium real rate of interest (Taylor assumed this too was equal to 2%).

- The Taylor Rule for general coefficients is

$$i_t = r^* + \pi^T + \delta_x x_t + \delta_\pi \left( \pi_t - \pi^T \right). \quad (6)$$

# Taylor rules

- A larger literature has now developed that has estimated the Taylor Rule, or similar simple rules, for a variety of countries and time periods.
- In general, the basic Taylor Rule, when supplemented by the addition of the lagged nominal interest rate, does quite well in matching the actual behavior of the policy interest rate.
- The argument for simple rules relies not on their optimality but on their simplicity; they may serve as a useful benchmark for policy or aid in promoting policy transparency.

## Policy Implication of forward-looking models

- The basic new Keynesian inflation adjustment equation took the form

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t.$$

- That is, there is no additional disturbance term.

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t \Rightarrow \pi_t = \kappa \sum_{i=0}^{\infty} \beta^i E_t x_{t+i}$$

- The absence of a stochastic disturbance implies there is no conflict between a policy designed to maintain inflation at zero and a policy designed to keep the output gap equal to zero.
- Just set  $x_{t+i} = 0$  for all  $i$ ; keeps inflation equal to zero.

## Optimal policy in forward-looking models

- Thus, the key implication of the basic new Keynesian model is that price stability is the appropriate objective of monetary policy.
- No policy conflicts.
- When prices are sticky but wages are flexible, the nominal wage can adjust to ensure labor market equilibrium is maintained in the face of productivity shocks. Optimal policy should then aim to keep the price level stable.

## Policy implications of price stickiness

- Models that combine optimizing agents and sticky prices have very strong policy implications.
- When the price level fluctuates, and not all firms are able to adjust, price dispersion results. This causes the relative prices of the different goods to vary. If the price level rises, for example, two things happen.
  - 1 The relative price of firms who have not set their prices for a while falls. They experience an increase in demand and raise output, while firms who have just reset their prices reduce output. This production dispersion is inefficient.
  - 2 Consumers increase their consumption of the goods whose relative price has fallen and reduce consumption of those goods whose relative price has risen. This dispersion in consumption reduces welfare.

# Woodford versus Friedman

- The basic new Keynesian model suggests price stability (i.e., zero inflation) is optimal.
  - ▶ Zero inflation eliminates inefficient price dispersion.
- Friedman rule: zero nominal rate of interest is optimal.
  - ▶ Zero nominal rate eliminates inefficiency in money holdings.
  - ▶ Optimal inflation is negative (deflation) at rate equal to real rate of interest.
- Khan, King, and Wolman (2000) analysis model with both distortions.
- They conclude optimal inflation is closer to zero than to the Friedman rule.

# Cost shocks

- Assume

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + e_t$$

where  $e$  represents an inflation or cost shock.

- Then

$$\pi_t = \kappa \sum_{i=0}^{\infty} \beta^i E_t x_{t+i} + \sum_{i=0}^{\infty} \beta^i E_t e_{t+i}$$

- Cannot keep both  $x$  and  $\pi$  equal to zero.
- Trade-offs must be made.

# Policy Implication of forward-looking models

## Discretion versus commitment

- If  $x^* = 0$ , is there any difference between discretion and commitment?
- In forward-looking models, the answer is yes.
- Discretion leads to a stabilization bias.



## Basic model

- When forward-looking expectations play a role, discretion leads to a stabilization bias even though there is no average inflation bias.
- Minimize

$$-\Omega E_t \sum_{i=0}^{\infty} \beta^i [\pi_{t+i}^2 + \lambda x_{t+i}^2]$$

subject to

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + e_t.$$

- Notice the Euler/IS equation imposes no constraint – use it to solve for  $i_t$  once optimal  $\pi_t$  and  $x_t$  have been determined.

## Basic model – eliminating the steady-state distortion

- Note that  $x^*$  has been set equal to zero in loss function

$$-\Omega E_t \sum_{i=0}^{\infty} \beta^i [\pi_{t+i}^2 + \lambda x_{t+i}^2].$$

- Fiscal subsidy to offset distortion from monopolistic competition.
- If  $x^* \neq 0$ , can't use first order approximations to structural equations to obtain a correct second order approximation to the representative agent's welfare.

# Discretion

- When the central bank operates with discretion, it acts each period to minimize the loss function subject to the inflation adjustment equation.
- Because the decisions of the central bank at date  $t$  do not bind it at any future dates, the central bank is unable to affect the private sector's expectations about future inflation.
- Thus, the decision problem of the central bank becomes the single period problem of minimizing  $\pi_t^2 + \lambda x_t^2$  subject to the inflation adjustment equation.

# Discretion

## The policy problem

- Central bank problem is to pick  $\pi_t$  and  $x_t$  to minimize

$$\pi_t^2 + \lambda x_t^2 + \psi_t (\pi_t - \beta \pi_{t+1} - \kappa x_t - e_t)$$

taking  $E_t \pi_{t+1}$  as given.

- The first order conditions can be written as

$$\pi_t + \psi_t = 0 \tag{7}$$

$$\lambda x_t - \kappa \psi_t = 0. \tag{8}$$

- Eliminating  $\psi_t$ ,  $\lambda x_t + \kappa \pi_t = 0$ .

# Discretion

## Equilibrium

- $x_t$  and  $\pi_t$  satisfy

$$\lambda x_t + \kappa \pi_t = 0.$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + e_t.$$

- Then

$$\pi_t = \beta E_t \pi_{t+1} - \frac{\kappa^2}{\lambda} \pi_t + e_t \Rightarrow \pi_t = \frac{\lambda \beta E_t \pi_{t+1} + \lambda e_t}{\lambda + \kappa^2}.$$

# Discretion

## Equilibrium

- Suppose

$$e_t = \rho e_{t-1} + \varepsilon_t.$$

and

$$\pi_t = A e_t.$$

- Then,  $E_t \pi_{t+1} = A E_t e_{t+1} = A \rho e_t$  and

$$\pi_t = \left( \frac{\lambda \beta A \rho + \lambda}{\lambda + \kappa^2} \right) e_t \Rightarrow A = \left( \frac{\lambda \beta A \rho + \lambda}{\lambda + \kappa^2} \right) = \frac{\lambda}{\lambda(1 - \beta \rho) + \kappa^2}.$$

- Zero average inflation bias.

# Discretion

## Behavior of the interest rate

- From the IS equation,

$$i_t = E_t \pi_{t+1} + \sigma (E_t x_{t+1} - x_t) + r_t^n.$$

- Using solution,

$$i_t = \left[ A\rho - \sigma \left( \frac{\kappa}{\lambda} \right) (\rho - 1) \right] e_t + r_t^n = B e_t + r_t^n.$$

- Shifts in natural rate of interest  $r^n$  are fully offset.
- So optimal policy involves  $i$  responding to shocks, but adopting a rule of the form

$$i_t = B e_t + r_t^n$$

does not ensure a unique rational expectations equilibrium.

## Precommitment

- When forward-looking expectations play a role, discretion leads to a stabilization bias even though there is no average inflation bias.
- Under optimal commitment, central bank at time  $t$  chooses both current and expected future values of inflation and the output gap.
- Minimize

$$-\Omega E_t \sum_{i=0}^{\infty} \beta^i \left[ \pi_{t+i}^2 + \lambda (x_{t+i} - x^*)^2 \right]$$

subject to

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + e_t.$$



## Optimal precommitment

- The central bank's problem is to pick  $\pi_{t+i}$  and  $x_{t+i}$  to minimize

$$\mathbb{E}_t \sum_{i=0}^{\infty} \beta^i [\pi_{t+i}^2 + \lambda x_{t+i}^2 + \psi_{t+i} (\pi_{t+i} - \beta \pi_{t+i+1} - \kappa x_{t+i} - e_{t+i})].$$

- The first order conditions can be written as

$$\pi_t + \psi_t = 0 \quad (9)$$

$$\mathbb{E}_t (\pi_{t+i} + \psi_{t+i} - \psi_{t+i-1}) = 0 \quad i \geq 1 \quad (10)$$

$$\mathbb{E}_t (\lambda x_{t+i} - \kappa \psi_{t+i}) = 0 \quad i \geq 0. \quad (11)$$

- Dynamic inconsistency – at time  $t$ , the central bank sets  $\pi_t = -\psi_t$  and promises to set  $\pi_{t+1} = -(\mathbb{E}_t \psi_{t+1} - \psi_t)$ . When  $t+1$  arrives, a central bank that reoptimizes will again obtain  $\pi_{t+1} = -\psi_{t+1}$  – the first order condition (9) updated to  $t+1$  will reappear.

## Timeless precommitment

- An alternative definition of an optimal precommitment policy requires the central bank to implement conditions (10) and (11) for all periods, including the current period so that

$$\pi_{t+i} + \psi_{t+i} - \psi_{t+i-1} = 0 \quad i \geq 0$$

$$\lambda x_{t+i} - \kappa \psi_{t+i} = 0 \quad i \geq 0.$$

- Woodford (1999) has labeled this the “timeless perspective” approach to precommitment.

## Timeless precommitment

- Under the timeless perspective optimal commitment policy, inflation and the output gap satisfy

$$\pi_{t+i} = - \left( \frac{\lambda}{\kappa} \right) (x_{t+i} - x_{t+i-1}) \quad (12)$$

for all  $i \geq 0$ .

- Woodford (1999) has stressed that, even if  $\rho = 0$ , so that there is no natural source of persistence in the model itself,  $a > 0$  and the precommitment policy introduces inertia into the output gap and inflation processes.
- This commitment to inertia implies that the central bank's actions at date  $t$  allow it to influence expected future inflation. Doing so leads to a better trade-off between gap and inflation variability than would arise if policy did not react to the lagged gap.

## Improved trade-off under commitment

- The difference in the stabilization response under commitment and discretion is the stabilization bias due to discretion.
- Consider a positive inflation shock,  $e > 0$ .
- A given change in current inflation can be achieved with a smaller fall in  $x$  if expected future inflation can be reduced:

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + e_t$$

- Requires a commitment to future deflation.
- By keeping output below potential (a negative output gap) for several periods into the future after a positive cost shock, the central bank is able to lower expectations of future inflation. A fall in  $E_t \pi_{t+1}$  at the time of the positive inflation shock improves the trade-off between inflation and output gap stabilization faced by the central bank.