Noah Williams
Department of Economics
University of Wisconsin

Economics 714
Macroeconomic Theory
Spring 2018

Problem Set 1

Due in Class on 2/14

1. Consider an Lucas-type asset pricing model, in which a representative agent gets an endowment s_t in period t, where s_t follows a Markov process. In addition, the agent is subject to preference shocks ξ_t which also follow a (separate) Markov process, possibly correlated with s_t, and affect the marginal utility of consumption. That is, the agent seeks to maximize:

$$E_0 \sum_{t=0}^{\infty} \beta^t \xi_t u(c_t),$$

where the initial values (s_0, ξ_0) are given. The agent can trade in a full set of contingent securities.

(a) Find the expressions (Euler equations) which determine the equilibrium price p_t of a claim to the endowment process, and R the gross one-period risk free interest rate.

Now suppose that preferences are logarithmic $u(c) = \log c$, while the endowment and preference shock processes follow:

$$\begin{aligned} s_{t+1} &= (\mu + \varepsilon^s_{t+1}) s_t \\ \xi_{t+1} &= (\mu + \varepsilon^\xi_{t+1}) \xi_t \end{aligned}$$

where $\mu \geq 1$ is a constant, the ε^i_t are i.i.d. (but correlated across i) and $E_t \varepsilon^i_{t+1} = 0$ for $i = s, \xi$.

(b) First suppose that there are no endowment shocks at all $\varepsilon^s_t \equiv 0$. Find the equilibrium risk free interest rate. Interpret your answer.

(c) Now suppose that the shocks ε^i_t are perfectly correlated (i.e. $\varepsilon^s_t = \varepsilon^\xi_t$). Find the equilibrium risk free interest rate. How is it related to the stock price p_t? Interpret your answer.

(d) Next suppose that the shocks ε^i_t are perfectly negatively correlated (i.e. $\varepsilon^s_t = -\varepsilon^\xi_t$). In addition, suppose $\varepsilon^s_t \in \{-0.5\mu, 0.5\mu\}$ each with probability 0.5. Find the equilibrium risk free interest rate. Interpret your answer.

2. Suppose that we restrict attention to an infinite horizon model but where claims to future consumption are traded over a finite number of periods into the future, so rational bubbles are possible. Further suppose that agents are risk neutral and have subjective discount factor β.

(a) Derive the general solution for the price of an asset that pays no dividends. What is the fundamental price of the asset?

(b) Construct an equilibrium price process which with probability $1-p$ grows at some constant rate g each period and declines at rate b with probability p.

(c) Using your previous result, discuss how for a bubble the speed of a run-up in prices is related to the magnitude of a crash.

3. Consider an endowment economy where a representative agent has recursive preferences of the Epstein-Zin type. That is, the utility V_t of a consumption stream $\{c_s\}_{s=t}^{\infty}$ is evaluated recursively:

$$V_t = \left((1-\beta)c_t^{1-\rho} + \beta\left(E_t V_{t+1}^{1-\alpha}\right)^{\frac{1-\rho}{1-\alpha}}\right)^{\frac{1}{1-\rho}},$$

where $\rho > 0$ and $\alpha > 0$. Notice that this is a combination of a CES aggregate (with parameter ρ) of current utility of consumption and a risk-adjustment (with parameter α) of future utility.

(a) Show that when $\alpha = \rho$ these preferences collapse to standard expected utility with a power utility function.

(b) Epstein-Zin preferences allow us to disentangle risk aversion and intertemporal substitution. How are these properties characterized here?

(c) Find an expression for the intertemporal marginal rate of substitution (stochastic discount factor), which we can define here as:

$$S_t = \frac{\frac{\partial V_t}{\partial V_{t+1}}\frac{\partial V_{t+1}}{\partial C_{t+1}}}{\frac{\partial V_t}{\partial C_t}}.$$

Now suppose that the endowment process (fruit from the Lucas tree) has i.i.d. growth rates, that is:

$$\log\left(c_{t+1}/c_t\right) = g + \sigma_c \varepsilon_{t+1}$$

where $g > 0$ and $\sigma_c > 0$ are constants and $\varepsilon_t \sim N(0,1)$.

(d) Conjecture a Markov pricing function, then write down the Bellman equation for the representative agent and find his optimality conditions.

(e) Define a recursive competitive equilibrium, being specific about the objects which make it up.

(f) Show that the value function can be written $V(c_t) = vc_t$ for some constant v, and find an expression for $\log S_t$.

(g) Find an expression for the risk-free rate. How does this differ from the standard CRRA case?

(h) Find an expression for the return on the Lucas tree. How does this differ from the standard CRRA case?