Money in a Neoclassical Framework

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Money

- Two basic questions:
  1. Modern economies use money. Why?
  2. How/why do changes in the amount of money affect nominal and real variables in the economy?
- Uses/functions of money: (1) Unit of account: contracts are usually denominated in terms of money.
- (2) Store of value: money allows consumers to trade current goods for future goods.
- (3) Medium of exchange: facilitates transactions.

Money in Exchange

- Non-monetary transactions require **barter**: one good or service is exchanged directly for another.
- Requires a *double coincidence of wants:* each party must want what other has.
- Other objects, like stocks and bonds, can be store of value and medium of exchange. Can dominate as store of value since they give a positive rate of return.
- But stocks and bonds are not efficient in exchange:
  1. Agents not well-informed about the exact value of stocks.
  2. It is not always easy to sell these assets. (Liquidity)
- Money very efficient in exchange. Facilitates specialization.
- In absence of regular money, other objects appear as media of exchange (cigarettes in POW camps).

Incorporating money

- To employ the neoclassical framework to analyze monetary issues, a role for money must be specified so that the agents will wish to hold positive quantities of money. **Money dominated as an asset.**
- Three general approaches to incorporating money into general equilibrium models have been followed:
  1. Assume that money yields direct utility by incorporating money balances directly into the utility functions of the agents of the model (Sidrauski 1967, Brock 1974).
  2. Impose transactions costs
     1. Make asset exchanges costly (Baumol 1952, Tobin 1956)
     2. Require that money be used for certain types of transactions (Clower 1967, Lucas 1982)
     3. Assume time and money can be combined to produce transaction services that are necessary for obtaining consumption goods
     4. Assume decentralized markets where direct barter or credit not feasible (Kiyotaki and Wright 1989).
  3. Treat money like any other asset used to transfer resources intertemporally (Samuelson 1958).

A basic MIU model

First approach: MIU

- Real money balances enter directly in the utility function.
- Given suitable restrictions on the utility function, such an approach can guarantee that, in equilibrium, agents choose to hold positive amounts of money so that money will be positively valued.
- Representative household takes prices as given and maximizes

$$\mathrm{E}_t \sum_{i=0}^{\infty} \beta^i U\left(C_{t+i}, m_{t+i}, L_{t+i}\right)$$

subject to a budget constraint.

A basic MIU model

- Budget constraint, in nominal terms:

$$\begin{array}{c} W_t(1-L_t) + (r_t + 1 - \delta)\, P_t K_{t-1} \\ +(1+i_{t-1})B_{t-1} + M_{t-1} + P_t \tau_t \end{array} = P_t C_t + P_t K_t + B_t + M_t$$

where τ are real, lump-sum transfers to the household (can be negative).

- In real terms:

$$\begin{array}{c} w_t(1-L_t) + (r_t + 1 - \delta)\, K_{t-1} \\ +(1+i_{t-1})\frac{B_{t-1}}{P_t} + \frac{M_{t-1}}{P_t} + \tau_t \end{array} = C_t + K_t + \frac{B_t}{P_t} + \frac{M_t}{P_t}$$

or

$$\begin{array}{c} w_t(1-L_t) + (r_t + 1 - \delta)\, K_{t-1} \\ +(1+i_{t-1})\frac{P_{t-1}}{P_t} b_{t-1} + \frac{P_{t-1}}{P_t} m_{t-1} + \tau_t \end{array} = C_t + K_t + b_t + m_t$$

A basic MIU model

- First order conditions:

$$U_C - \lambda_t = 0$$

$$U_L - w_t \lambda_t = 0 \Rightarrow \frac{U_L}{U_C} = w_t$$

$$-\lambda_t + \beta \mathrm{E}_t \left(r_{t+1} + 1 - \delta \right) \lambda_{t+1} = 0$$

$$-\lambda_t + \beta(1 + i_t) \mathrm{E}_t \left(\frac{P_t}{P_{t+1}} \right) \lambda_{t+1} = 0$$

$$U_m - \lambda_t + \beta \mathrm{E}_t \left(\frac{P_t}{P_{t+1}} \right) \lambda_{t+1} = 0$$

- First three look just as before (except U_C and U_L may depend on m).

A basic MIU model

- The FOC for money can be written

$$\frac{\lambda_t}{P_t} = \frac{U_m}{P_t} + \beta \mathrm{E}_t \left(\frac{\lambda_{t+1}}{P_{t+1}} \right)$$

- Solve forward to yield

$$\frac{1}{P_t} = \left(\frac{1}{\lambda_t} \right) \mathrm{E}_t \sum_{i=0}^{\infty} \beta^i \left[\frac{U_m \left(C_{t+i}, m_{t+i}, L_{t+i} \right)}{P_{t+i}} \right]$$

- Value of money equals expected present value of discounted utility return.

A basic MIU model

- Combining the first and third conditions yields the Euler condition:

$$U_C(C_t, m_t, L_t) = \beta \mathrm{E}_t \left(r_{t+1} + 1 - \delta\right) U_C(C_{t+1}, m_{t+1}, L_{t+1})$$

- Similarly,

$$U_C(C_t, m_t, L_t) = \beta(1 + i_t)\mathrm{E}_t \left(\frac{P_t}{P_{t+1}}\right) U_C(C_{t+1}, m_{t+1}, L_{t+1})$$

- From fourth and fifth,

$$\frac{U_m}{U_C} = \beta i_t \mathrm{E}_t \left(\frac{P_t}{P_{t+1}}\right) \frac{U_C(C_{t+1}, m_{t+1}, L_{t+1})}{U_C(C_t, m_t, L_t)}$$

- But

$$1 = \beta(1 + i_t)\mathrm{E}_t \left(\frac{P_t}{P_{t+1}}\right) \frac{U_C(C_{t+1}, m_{t+1}, L_{t+1})}{U_C(C_t, m_t, L_t)}$$

so

$$\frac{U_m}{U_C} = \beta i_t \mathrm{E}_t \left(\frac{P_t}{P_{t+1}}\right) \frac{U_C(C_{t+1}, m_{t+1}, L_{t+1})}{U_C(C_t, m_t, L_t)} = \frac{i_t}{1 + i_t}.$$

Monetary Neutrality

- Firm side of model is same as in basic RBC model.
- If U_L and U_C independent of m, real side identical to basic RBC model. (Steady state and dynamics)
  - Neutrality: One time changes in the nominal quantity of money affect only the price level.
  - Superneutrality: Changes in growth rate of money do not affect real variables.
- Changes in the rate of growth of money affect only the inflation rate, the nominal interest rate, and real money balances.
- Basic intuition: the private opportunity cost of holding money depends on the nominal rate of interest. The social marginal cost of producing money is essentially zero.
- Friedman Rule: This inefficiency would be eliminated if the nominal rate of interest were zero. So the optimal rate of inflation is a rate of *deflation* approximately equal to the real return on capital.

Cash-in-advance models

- Clower (1967): goods buy money and money buys goods, but goods don't buy goods.
- And because goods don't buy goods, a medium of exchange that serves to aid the process of transacting will have value.
- The demand for money is then determined by the nature of the economy's transactions technology.
- Rather than allowing substitutability between time and money in carrying out transactions, *cash-in-advance* (CIA) models simply require that money balances be held to finance certain types of purchases; without money, these purchases cannot be made.

Household Problem: Timing

- At the start of the period, the household has three assets: Money M_{t-1}, nominal bonds B_{t-1}, and capital K_{t-1}.
- Household consists of two members, a worker and a shopper.
- Worker supplies labor to firm, earns wage.
- Shopper takes M_{t-1} to store and purchases consumption goods
- At the **end of the period** household members pool resources, rebalance portfolio of assets.

Cash-in-advance models

The representative agent

- Maximizes present discounted value of expected utility subject to a sequence of new constraints.
- If the goods markets open first and the agent enters the period with money holdings M_{t-1} and receives a lump-sum transfer T_t (in nominal terms), the CIA constraint takes the form

$$P_t c_t \leq M_{t-1} + T_t,$$

  where c is real consumption, P is the aggregate price level, and T is the nominal lump-sum transfer.
- In real terms,

$$c_t \leq \frac{M_{t-1}}{P_t} + \frac{T_t}{P_t} = \frac{m_{t-1}}{\Pi_t} + \tau_t,$$

  where $m_{t-1} = M_{t-1}/P_{t-1}$, $\Pi_t = P_t/P_{t-1}$ is 1 plus the inflation rate, and $\tau_t = T_t/P_t$.
- Income from production during period t will not be available for consumption purchases during period t.

Cash-in-advance models

- Budget constraint in real terms, $\omega_t \geq c_t + m_t + b_t + k_t$ where

$$\omega_t \equiv f(k_{t-1}) + (1-\delta)k_{t-1} + \tau_t + \frac{m_{t-1} + (1+i_{t-1})b_{t-1}}{\Pi_t},$$

  and m and b are real cash and bond holdings.
- Let $a_t = b_t + m_t$. Then budget constraint implies

$$\omega_{t+1} = f(k_t) + (1-\delta)k_t + \tau_{t+1} + R_t a_t - \left(\frac{i_t}{\Pi_{t+1}}\right) m_t.$$

- This form highlights that there is a cost to holding money when the nominal interest rate is positive. This cost is $(1+i_t)/\Pi_{t+1}$.
- This is the same expression for the opportunity cost of money in an MIU model.

Cash-in-advance models

The decision problem

- Define the value function

$$V(\omega_t, m_{t-1}) = \max\{u(c_t) + \beta V(\omega_{t+1}, m_t)\},$$

where the maximization is subject to the budget constraint

$$\omega_t \geq c_t + m_t + b_t + k_t,$$

the CIA constraint, and the definition of ω_{t+1}.

Cash-in-advance models

The decision problem

- Letting λ_t (μ_t) denote the Lagrange multiplier associated with the budget constraint (the CIA constraint), the first order conditions for the agent's choice of consumption, capital, bond, and money holdings take the form

$$u_c(c_t) - \lambda_t - \mu_t = 0$$

$$\beta \left[f_k(k_t) + 1 - \delta \right] V_\omega(\omega_{t+1}, m_t) - \lambda_t = 0$$

$$\beta R_t V_\omega(\omega_{t+1}, m_t) - \lambda_t = 0$$

$$\beta \left[R_t - \frac{i_t}{\Pi_{t+1}} \right] V_\omega(\omega_{t+1}, m_t) + \beta V_m(\omega_{t+1}, m_t) - \lambda_t = 0.$$

Cash-in-advance models

From the envelope theorem,

$$V_\omega(\omega_t, m_{t-1}) = \lambda_t$$

$$V_m(\omega_t, m_{t-1}) = \left(\frac{1}{\Pi_t}\right)\mu_t.$$

- λ_t is equal to the marginal utility of wealth.
- The marginal utility of consumption exceeds the marginal utility of wealth by the value of liquidity services, μ_t.
- The individual must hold money in order to purchase consumption, so the "cost," to which the marginal utility of consumption is set equal, is the marginal utility of wealth plus the cost of the liquidity services needed to finance the transaction.

Cash-in-advance models

The first order conditions imply

$$\lambda_t = \beta \left(\frac{\lambda_{t+1} + \mu_{t+1}}{\Pi_{t+1}} \right) \Rightarrow \frac{1}{P_t} = \left(\frac{1}{\lambda_t} \right) \beta \left(\frac{\lambda_{t+1} + \mu_{t+1}}{P_{t+1}} \right).$$

- This equation can also be interpreted as an asset pricing equation for money.
- The price of a unit of money in terms of goods is just $1/P_t$ at time t; its value in utility terms is λ_t/P_t.
- Solving this equation forward implies that

$$\frac{1}{P_t} = \left(\frac{1}{\lambda_t} \right) \sum_{i=1}^{\infty} \beta^i \left(\frac{\mu_{t+i}}{P_{t+i}} \right).$$

- Value of money related to present discounted value of Lagrangian multipliers on CIA constraint.

Cash-in-advance models

The nominal rate of interest

- Since $\lambda_t = \beta R_t \lambda_{t+1} = \beta \left(\lambda_{t+1} + \mu_{t+1} \right) / \Pi_{t+1}$, or $R_t \Pi_{t+1} \lambda_{t+1} = \left(\lambda_{t+1} + \mu_{t+1} \right)$ and $1 + i_t = R_t \Pi_{t+1}$, the nominal interest rate is given by

$$i_t = \left(\frac{\lambda_{t+1} + \mu_{t+1}}{\lambda_{t+1}} \right) - 1 = \frac{\mu_{t+1}}{\lambda_{t+1}}.$$

- The nominal rate of interest is positive if and only if money yields liquidity services ($\mu_{t+1} > 0$).
- In particular, if the nominal interest rate is positive, the CIA constraint is binding ($\mu > 0$).

Cash-in-advance models

Marginal utility of consumption

- We can use the relationship between the nominal rate of interest and the Lagrangian multipliers to rewrite the expression for the marginal utility of consumption as $u_c = \lambda(1 + \mu/\lambda) = \lambda(1 + i) \geq \lambda$.
- Since λ represents the marginal value of income, the marginal utility of consumption exceeds that of income whenever the nominal interest rate is positive.
- Even though the economy's technology allows output to be directly transformed into consumption, the "price" of consumption is not equal to 1; it is $1 + i$ since the household must hold money to finance consumption.
- Thus, in this CIA model, a positive nominal interest rate acts as a tax on consumption; it raises the price of consumption above its production cost.

Welfare costs of inflation

- In CIA models, inflation acts as a tax on goods or activities whose purchase requires cash.
- This tax then introduces a distortion by creating a wedge between the marginal rates of transformation implied by the economy's technology and the marginal rates of substitution faced by consumers.
- Since the CIA model, like the MIU model, offers no reason for such a distortion to be introduced (there is no inefficiency that calls for Pigovian taxes or subsidies on particular activities, and the government's revenue needs can be met through lump-sum taxation), optimality calls for setting the inflation tax equal to zero.
- The inflation tax is directly related to the nominal rate of interest; a zero inflation tax is achieved when the nominal rate of interest is equal to zero.