# Lecture 8: Fiscal Policy in the Growth Model

## Economics 714, Spring 2018

## 1 Fiscal Policy in the Growth Model

Focus on deterministic neoclassical growth model with fiscal policy, date-0 sequence formulation

A government policy is a sequence of spending $\{G_t\}$ and taxes $\{T_t\}$ that satisfy the government budget constraint:

$$\sum_{t=0}^{\infty} q_t G_t = \sum_{t=0} q_t T_t$$

Government spending not valued, will consider different tax instruments to raise revenue $T_t$.

Households face budget constraint:

$$\sum_{t=0}^{\infty} q_t [C_t + I_t] = \sum_{t=0}^{\infty} q_t [r_t K_t + w_t N_t - T_t]$$

Capital law of motion:

$$K_{t+1} = (1-\delta)K_t + I_t$$

Goods market/ aggregate feasibility:

$$C_t + I_t + G_t = F(K_t, N_t)$$

A **competitive equilibrium** is a price system $\{q_t, r_t, w_t\}$, an allocation $\{C_t, K_t, N_t\}$, and a government policy $\{G_t, T_t\}$ s.t. (i) households optimize, (ii) firms optimize, (iii) markets clear

Ricardian equivalence: If $T_t$ is lump sum (i.e. independent of household choices) then timing of taxes is irrelevant, all that matters is date-0 present value.

Level of taxes and spending (even if lump sum) matter because of wealth effects.

Assume $U(C_t, N_t) = U(C_t)$, so $N_t \equiv 1$, define $F(K, 1) = f(K)$.

Equilibrium conditions:

$$\begin{aligned} u'(C_t) &= \beta u'(C_{t+1})[r_{t+1} + 1 - \delta] \\ &= \beta u'(C_{t+1})[f'(K_{t+1}) + 1 - \delta] \\ K_{t+1} &= f(K_t) + 1 - \delta K_t - C_t - G_t \end{aligned}$$

Dynamics:

$$\begin{aligned} \Delta c = 0 \quad &\Rightarrow \quad f'(K) = \rho + \delta \\ \Delta k = 0 \quad &\Rightarrow \quad f(K) = C + \delta K + G \end{aligned}$$

## 1.1 Distorting Taxes

Now consider linear tax $\tau_t^N$ on labor income, $\tau_t^K$ on capital income:

$$T_t = \tau_t^N w_t N_t + \tau_t^K (r_t - \delta) K_t$$

Government budget constraint as before with this definition of revenue.

Define after-tax gross return $R_t^K = 1 + (1 - \tau_t^K)(r_t - \delta)$.

Household budget constraint, using capital law of motion:

$$\sum_{t=0}^{\infty} q_t [C_t + K_{t+1}] = \sum_{t=0}^{\infty} q_t [(1 - \tau_t^N) w_t N_t + R_t^K K_t]$$

Firm problem unaffected: $w_t = F_N$, $r_t = F_K$.

Households: now allow elastic labor supply, so solve

$$\max_{\{C_t, K_{t+1}, N_t\}} \sum_{t=0}^{\infty} \beta^t u(C_t, 1 - N_t) \text{ s.t. BC}$$

Equilibrium conditions:

$$\begin{aligned}
u_C(C_t, 1 - N_t) &= \beta u_C(C_{t+1}, 1 - N_{t+1})[1 + (1 - \tau_{t+1}^K)(F_K(K_{t+1}, N_{t+1}) - \delta)] \\
\frac{u_L(C_t, 1 - N_t)}{u_C(C_t, 1 - N_t)} &= (1 - \tau_t^N) F_N(K_t, N_t) \\
K_{t+1} &= F(K_t, N_t) + 1 - \delta K_t - C_t - G_t
\end{aligned}$$

# 2 Ramsey Optimal Taxation

## 2.1 Setup

Look for linear taxes that fund given $\{G_t\}$ and maximize household welfare

First, find implementability constraint summarizing equilibria. Rewrite HH BC:

$$\sum_{t=0}^{\infty} q_t[C_t - (1 - \tau_t^N) w_t N_t] = \sum_{t=0}^{\infty} q_t[R_t^K K_t - K_{t+1}] = q_0 R_0^K K_0$$

Then use HH first order conditions to substitute out for prices

$$\sum_{t=0}^{\infty} \beta^t [U_C(C_t, 1 - N_t) C_t - U_L(C_t, 1 - N_t) N_t] = U_C(C_0, 1 - N_0) K_0 [1 + (1 - \tau_0^K)(F_K(K_0, N_0) - \delta)]$$

Primal approach: solve for allocation first, back out supporting taxes from equilibrium conditions:

$$\begin{aligned}
\left( \frac{u_C(C_t, 1 - N_t)}{\beta u_C(C_{t+1}, 1 - N_{t+1})} - 1 \right) \frac{1}{F_K(K_{t+1}, N_{t+1}) - \delta} &= 1 - \tau_{t+1}^K \\
\frac{u_L(C_t, 1 - N_t)}{u_C(C_t, 1 - N_t) F_N(K_t, N_t)} &= 1 - \tau_t^N
\end{aligned}$$

$\tau_0^K$ only affects period zero: initial capital is inelastic, tax it as much as possible. To make problem interesting, restrict $\tau_0^K \leq \overline{\tau}^K$.

Define $U(C, N) = U(C, 1 - N)$ so $U_N = -U_L$. Also define:

$$W(C_t, N_t; \lambda) = U(C_t, N_t) + \lambda[U_C(C, N)C + U_N(C, 1 - N)N]$$

Then Ramsey problem can be written:

$$\max_{\{C_t, K_{t+1}, N_t\}} \sum_{t=0}^{\infty} \beta^t W(C_t, N_t; \lambda) - \lambda U_C(C_0, N_0)K_0[1 + (1 - \tau_0^K)(F_K(K_0, N_0) - \delta)]$$

subject to (multiplier $\mu_t$):

$$C_t + G_t + K_{t+1} = F(K_t, N_t) + 1 - \delta K_t$$