Lecture 5: Complete and Incomplete Markets

Economics 714, Spring 2018

1 Implications of Complete Markets

We have the Euler equation for the asset paying a unit of consumption one period ahead in state s_i:

$$u'(c)q(s,s_i) = \beta u'(c(s_i))Q(s,s_i)$$

Along the same lines, we can derive the more general expression for the agent's optimality conditions for any future date $t+j$ and state:

$$u'(c_t)q^j(s_t,s^j) = \beta^j u'(c_{t+j}(s^j))Q^j(s_t,s^j)$$

1.1 Allowing heterogeneity

We have focused on a representative agent setting, but now suppose we allow for heterogeneity in preferences and endowments. There are $i = 1,\ldots,I$ consumers who have different preferences $(u_i(c),\beta_i)$ and endowments $e_t^i = e^i(s_t)$ where $\sum_i e^i(s_t) = s_t$.

Note then that the same optimality condition holds for any consumer. Let's look from date 0:

$$u_i'(c_0^i)q^t(s_0,s^t) = \beta_i^t u_i'(c_t^i(s^t))Q^t(s_0,s^t)$$

Since a similar expression holds for any consumer j as well, we have:

$$\frac{u_i'(c_0^i)q^t(s_0,s^t)}{u_j'(c_0^j)q^t(s_0,s^t)} = \frac{\beta_i^t u'(c_t^i(s^t))Q^t(s_0,s^t)}{\beta_j^t u_j'(c_t^j(s^t))Q^t(s_0,s^t)}$$

or

$$\frac{u_i'(c_0^i)}{u_j'(c_0^j)} = \left(\frac{\beta_i}{\beta_j}\right)^t \frac{u'(c_t^i(s^t))}{u_j'(c_t^j(s^t))}$$

We can use this to derive a few simple implications:

- Therefore if $\beta_i > \beta_j$, as $t \to \infty$ we must have $\frac{u'(c_t^i(s^t))}{u_j'(c_t^j(s^t))} \to 0$ and thus $c_t^j \to 0$. The same holds for any pairwise comparison, so in the long run the most patient agent will consume the entire aggregate endowment.

- Now suppose that we have preference homogeneity $\beta_i = \beta$ and $u_i(c) = \frac{c^{1-\gamma}}{1-\gamma}$. Then we have:

$$\frac{(c_0^i)^{-\gamma}}{(c_0^j)^{-\gamma}} = \frac{(c_t^i)^{-\gamma}}{(c_t^j)^{-\gamma}}$$

  That is, each agent's consumption is a constant fraction of any other's. Equivalently, each agent consumes a constant fraction of the aggregate endowment. Thus we have **perfect risk sharing**: no agent bears any idiosyncratic risk .

2 Incomplete Markets Model

2.1 Theoretical Implications

Put consumption-savings model with borrowing constraint into general equilibrium.

Only asset is risk free bond, agents face idiosyncratic (labor) income risk. Assume a law of large numbers so no aggregate risk. A direct implication of market incompleteness is that agents will bear idiosyncratic risk.

$$\max_{\{c_t, a_{t+1}\}} E_0 \sum_{t=0}^{\infty} \beta^t u(c_t)$$

with a_0, l_0 given and subject to:

$$c_t + a_{t+1} = Ra_t + wl_t$$

Huggett: $wl_t = y_t$, specify income directly.

Aiyagari: Representative firm, so w endogenous.

Constraints: $c_t \geq 0, \quad a_t \geq \underline{a}$. Debt limit.

l follows Markov process w/transition function Q on L

Agent optimization with borrowing constraint gives a policy function $a' = a'(a, l)$, Euler inequality:

$$u'(c_t) \geq \beta R E_t u'(c_{t+1})$$

with equality if $a_{t+1} > \underline{a}$

Define $\theta_t = \beta^t R^t u'(c_t) \geq 0$. Euler inequality: $\theta_t \geq E_t \theta_{t+1}$.

Theorem (Martingale Convergence) Let $\{X_t\}$ be a submartingale. If $K = \sup_t E(|X_t|) < \infty$, then $X_t \to x$ with probability 1, where X is a random variable that satisfies $E(|X|) \leq K$.

Here $-\theta_t$ is a submartingale, and $\sup_t E(|-\theta_t|) = E(u'(c_0)) < \infty$. So $\theta_t \to \overline{\theta}$.

Implies if $\beta R > 1$ then since $\lim_{t\to\infty} \beta^t R^t = +\infty$ then $\lim_{t\to\infty} u'(c_t) = 0$ with probability 1, so $c_t \to \infty$. This implies that $a_t \to \infty$ if $\beta R \geq 1$.

Same conclusions hold with more delicate argument if $\beta R = 1$.

So we have $\beta R < 1$, and thus interest rate is lower than under complete markets.

Agents build up a stock of assets to insure against their income risk, which drives down equilibrium interest rates.

2.2 Quantitative Implications

1. Model generates lower risk free rate than complete markets, but effect not substantial.

2. Precautionary saving effect not very large

3. Model generates heterogeneity in wealth and income, but not (nearly) enough to match US data

4. Welfare costs of borrowing constraints and market incompleteness relatively small: self-insurance via saving able to smooth consumption relatively well.

3 Incomplete Markets with Aggregate Risk

In the Aiyagari framework, there is no aggregate risk and so we can't discuss fluctuations or risky asset returns. Krusell and Smith (1998) introduced aggregate risk into an incomplete markets model.

Aggregate production function now has Markov productivity shock $z_t \sim Q_z$:

$$Y_t = z_t F(K_t, N_t)$$

Gives usual marginal productivity conditions: $w = zF_N(K, N)$, $r = zF_K(K, N) - \delta$

Idiosyncratic labor shocks are Markov conditional on z: $Q(l, dl'|z)$.

Joint distribution of (l, z) is Γ.

Similar to our previous approach, we could attempt a recursive formulation in a "big K-little k" approach for the individual agent problem:

$$V(a, l, z, K, N) = \max_{c,a'} \{u(c) + \beta E[V(a', l', z', K', N')|l, z, K, N]\}$$

subject to:

$$\begin{aligned} c + a' &= R(z, K, N)a + w(z, K, N)l - c \\ a \geq \underline{\mathrm{a}} & \\ (K', N') &= G(z, K, N) \end{aligned}$$

Problem: (z, K, N) not (in general) sufficient statistic for K', which depends on the **distribution** of assets

Correct recursive formulation with state $\mu(a, l)$:

$$V(a, l, z, \mu) = \max_{c,a'} \{u(c) + \beta E[V(a', l', z', \mu')|l, z, \mu]\}$$

subject to:

$$\begin{aligned} c + a' &= R(z, K, N)a + w(z, K, N)l - c \\ a \geq \underline{\mathrm{a}} & \\ \mu' &= H(z, \mu) \end{aligned}$$

and then $K = \int a\mu(da, dl)$, $N = \int l\mu(da, dl)$.

Problem: infinite dimensional state μ. Also unknown law of motion H maps distributions into distributions

Krusell and Smith approximate μ by moments, assume H is log-linear function mapping moments to moments. Show that to forecast w,r, essentially enough to consider law of motion for K. "Approximate aggregation" holds in this model, and has been shown to work in many other settings as well.