Economics 702
Macroeconomics
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Problem Set 3

Due in class on March 11.

1. Consider the problem of a worker who is deciding between going back to school or continuing on the job. She supplies labor inelastically, and is indifferent between spending time in school or on the job. Thus the decision depends on the income profiles in each case. For simplicity, assume that the worker has a planning horizon of $t = 0, 1, \ldots, 20$. The worker's income if she works in period 0 is y_0. If she remains on the job, she will get a raise of g each year so will have income $y_t = (1+g)^t y_0$. If she goes to school, she pays the same tuition X in each of the first two years and has no other income. But when she gets out, she will earn a higher starting salary $y' > y_0$ and will also get larger raises $g' > g$ each year. She must decide at the start of date 0 whether to work or go back to school, and suppose that she has no savings or debt at that time but can borrow or lend at interest rate r.

   (a) Write expressions for the present value of lifetime income if the worker goes back to or if she stays on the job.

   (b) Suppose that $y_0 = \$50,000$, $y' = \$75,000$, $g = 0.01$, $g' = .02$ and $r = 0.05$. What is the maximal tuition that the worker is willing to pay to go back to school?

   (c) Suppose that the worker's preferences are given by:

   $$\sum_{t=0}^{20} \beta^t \log c_t$$

   and assume $\beta = 1/(1+r)$. Consider two arbitrary dates t and $t+1$ and find an expression relating the optimal consumption choices at these dates, c_t and c_{t+1}.

   (d) How does her consumption vary over time? How does she implement this consumption plan?

   (e) Suppose that $X = \$10,000$ and she chooses to go back to school. When she graduates at the end of period 1, how much debt/savings will she have?

2. In this problem we consider the finite horizon consumption-savings problem where a consumer has preferences:

$$E\sum_{t=0}^{T}\beta^t u(c_t)$$

where u is increasing, three times continuously differentiable, strictly concave, and has strictly convex marginal utility. The agent receives nonnegative labor income y_t each period and can invest only in a risk-free asset with constant gross return R. Relative to the model in class, we assume that no borrowing is allowed, and consumption is chosen before labor income is realized. Therefore, with assets a_t, the consumer faces the flow constraint:

$$a_{t+1} = R\left(a_t - c_t + y_{t+1}\right),$$

with $a_t \geq c_t \geq 0$, a_0 given.

(a) Derive the conditions for maximization, considering both interior and boundary solutions. Does the Euler equation still hold?

(b) Now consider a two period horizon $T = 1$. Suppose that instead of the agent receiving a random income, he receives a constant income equal to the mean of the income process $\overline{y} = Ey_t > 0$. Show that there exists a level of assets $\overline{A}$ such that for $a_t \leq \overline{A}$ the optimal policy is to set $c_t = a_t$. Find an implicit expression for $\overline{A}$.

(c) Again consider the two period horizon, but suppose that income y_t is an i.i.d. random variable that can take on the values of y_H with probability p and $y_L < y_H$ with probability $1-p$, but the mean remains constant $Ey_t = \overline{y}$. Show that there exists a level of assets $\widetilde{A}$ such that for $a_t \leq \widetilde{A}$ the optimal policy is to set $c_t = a_t$. Find an implicit expression for $\widetilde{A}$, and rank $\widetilde{A}$ and $\overline{A}$.

(d) Does the optimal savings policy under uncertainty lead to a greater or lesser accumulation of assets than the optimal policy under certainty? Interpret your results.

3. Consider a simple endowment economy where the total endowment grows constant rate g each period:

$$e_{t+1} = (1+g)e_t,$$

with $e_0 = 1$. Suppose that a representative agent has the following preferences:

$$\sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\gamma}}{1-\gamma}$$

where $\gamma > 1$ and suppose $\beta(1+g)^{1-\gamma} < 1$. In equilibrium $c_t = e_t$. Compute the prices of the following assets:

(a) A one-period risk free bond which has a current price of 1 and certain payoff of $1+r$ in the next period.

(b) A claim to the total endowment in the next period: this has a date t price of p_t and a payoff of e_{t+1} at $t+1$.

(c) A claim to the entire endowment process: this has a date 0 price of p_0 and a payoff of e_t for each future date $t \geq 0$.

4. Consider an endowment economy with one good and two assets. Asset 1 pays a constant amount R in each period. Asset 2 pays a stochastic amount $x_t \in \{R/2, 2R\}$. Assume that the x is i.i.d. and so $x = R/2$ with probability θ and $x = 2R$ with probability $1-\theta$. In equilibrium consumption of the nonstorable good is therefore $c = R + x$. The representative agent has preferences:

$$E \sum_{t=0}^{\infty} \beta^t \log c_t.$$

Find the equilibrium price/consumption ratios for the assets: $p_1(x)/c(x)$ and $p_2(x)/c(x)$. Which asset has the greater price/consumption ratio? Interpret your result.