Lecture 9
Endogenous Growth
Consumption and Savings

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Endogenous Growth Models

- Now briefly discuss some models which try to explain sources of growth endogenous growth models.
- An active research topic initiated in late 1980s. Romer (1986, 1990), Lucas (1988) most influential: models of R & D, human capital.
- More recently Acemoglu et al: role of institutions in growth.

What's in TFP? Institutions & Geography

- Aside from innovations (which we'll turn to next), infrastructure, institutions, and geography are also important.
- Interesting comparison: experiences of former colonies. Acemoglu, Johnson and Robinson (2001).
- Small initial differences in income.
- Differences in settlers mortality influenced whether colony was run for "extraction" or whether colonists developed institutions. Those colonies where institutions took hold developed faster.
- Large differences in outcomes – still today!

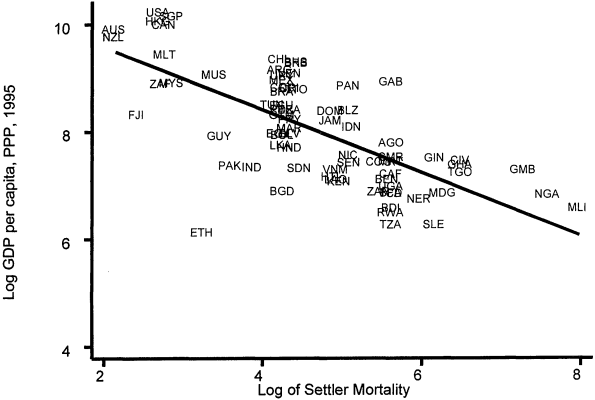


FIGURE 1. REDUCED-FORM RELATIONSHIP BETWEEN INCOME AND SETTLER MORTALITY

What's in TFP? Ideas and Human Capital

- Relatively new branch of economic theory: endogenous growth theory seeks to explain how technical change happens.
- Simple endogenous growth model (AK model): aggregate production function $Y = AK$. (Ignore labor and population growth, could think of this as per capita production.)
- Not subject to diminishing returns: MPK is constant

$$F_K = \frac{Y}{K} = A.$$

- Idea: Aggregate capital K captures not just increases in physical capital but changes in the makeup of that capital.

Human Capital as a Source of Growth

- Human capital: knowledge, skills, and training of individuals. As economies become richer they invest in human capital in the same proportion, offsetting the diminishing marginal product of physical capital alone.
- Explicitly: production depends on human capital H, physical capital K:

$$Y = zH^{\theta}K^{1-\theta}$$

- Say $H = hK$, so that human capital is *constant* fraction of physical, then letting $A = zh^{\theta}$:

$$Y = z(hK)^{\theta}K^{1-\theta} = \left[zh^{\theta}\right]K = AK$$

Other Interpretations

- Research and development programs are part of capital investment. They increase the stock of knowledge, which offsets diminishing marginal products of capital accumulation.
- Learning by doing: as economies produce more they learn better how to produce.

Implications of the Endogenous Growth Model

- Again savings constant fraction s of output. So:

$$\dot{K} = sAK - \delta K$$

- Since $Y = AK$,

$$\frac{\dot{Y}}{Y} = \frac{\dot{K}}{K} = sA - \delta$$

- Growth of output depends on the saving rate, even in the long run. No steady state.
- Higher savings $\Rightarrow$ more human capital, R&D, learning by doing. So higher savings leads to productivity improvements and higher growth.
- Important implication, some evidence that measured TFP does depend on savings, human capital.

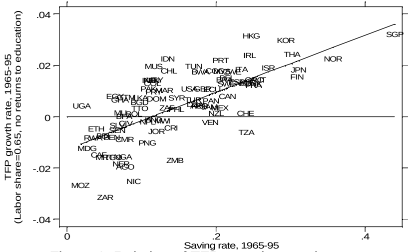


Figure 1: Relation of TFP growth to saving rate

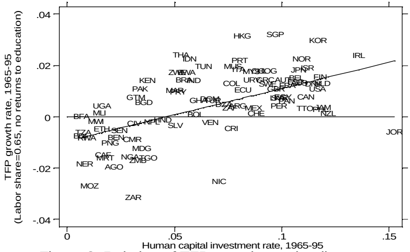


Figure 2: Relation of TFP growth to schooling rate

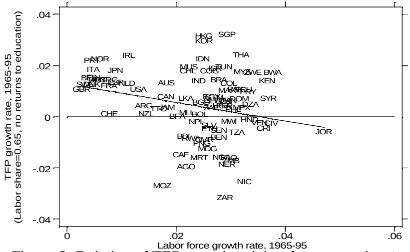


Figure 3: Relation of TFP growth to labor force growth rate

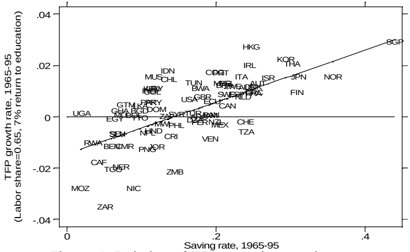


Figure 4: Relation of TFP growth to saving rate

Ideas

- Romer won(shared) 2018 Nobel Prize: "Paul Romer has demonstrated how knowledge can function as a driver of long-term economic growth. He showed how economic forces govern the willingness of firms to produce new ideas. Romer's central theory, which was published in 1990, explains how ideas are different to other goods and require specific conditions to thrive in a market."
- Properties of ideas:
  1. The accumulation of ideas is the source of long-run economic growth.
  2. Ideas are non-rival.
  3. A larger stock of ideas makes it easier to find new ideas.
  4. Ideas are created in a costly but purposeful activity.
  5. Ideas can be owned and the owner can sell the rights to use the ideas at a market price

Varieties of Goods

- Instead of having homogeneous capital as an input, production comes from labor and an interval of intermediate capital goods indexed by i, $x(i)$, and A is the endogenously determined length of this interval

$$Y_t = \left(\int_0^{A_t} x_t(i)^\alpha di \right) N_t^{1-\alpha}$$

- To produce one unit of $x(i)$, η units of general capital K_t are needed. This gives the constraint:

$$\int_0^{A_t} \eta x_t(i) di = K_t$$

- Given that each $x(i)$ has decreasing returns in final production, it is optimal to spread the general capital equally among the specialized goods:

$$x_t(i) = \frac{K_t}{\eta A_t} \quad \forall i$$

The Production of Ideas

- Suppose that labor can be used in research to produce ideas, in addition to directly in production.
- Assume cost of producing an idea is $1/\xi A_t$ units of labor. If we have N_t^R labor by researchers then the number of new ideas is given by:

$$A_{t+1} - A_t = \xi A_t N_t^R$$

  Note that this satisfies property 3 above.
- In this simple setting, all ideas are equally good from a production perspective and their unit costs of production are also identical.
- If we normalize the population at 1, then $N_t = 1 - N_t^R$ workers are supplied to production

Optimal Endogenous Growth

- The social planner's problem is to allocate resources as well as divide labor between production and research to solve:

$$\max_{\{C_t, K_{t+1}, N_t^R, A_{t+1}\}} \sum_{t=0}^{\infty} \beta^t U(C_t)$$

$$\text{subject to: } C_t = A_t \left(\frac{K_t}{\eta A_t} \right)^{\alpha} (1 - N_t^R)^{1-\alpha} - K_{t+1} + (1-\delta)K_t$$

$$\text{and: } A_{t+1} - A_t = \xi A_t N_t^R$$

$$\text{with } A_0,\ K_0 \text{ given}$$

- The growth rate of $A = \xi N_t^R$, is analogous to the exogenous rate g in the Solow model, but here it is endogenous: it is the result of a choice that trades off the use of workers in final output production against their use in research/ideas production.

The Market for Innovation

- Romer (1990) also considered a decentralized model with a market for ideas (R&D), with monopoly producers of the specialized goods $x_t(i)$ and patent protection gave them the right to their innovations.
- Patents necessary to provide incentives to do research.
- The level of research and investment is sub-optimally too low in equilibrium due to 2 forces: externalities (innovation today makes future production and innovation easier) and monopoly power (rents associated with ideas).
- Underprovision of research in equilibrium motivates a subsidy for it.

New Topic: Consumption and Savings

- Now start to analyze decentralized model, building toward dynamic general equilibrium.
- Start with household consumption-savings decisions. Previously in class analyzed labor-leisure decisions. Later put them together.
- Start today with two period model, extend later to infinite horizon.

A Two-Period Model of Consumption and Savings

- Household preferences:

$$U(c, c') = u(c) + \beta u(c')$$

- (Labor) income $y > 0$ in the first period of life and $y' \geq 0$ in the second period of life.
- Initial wealth $A \geq 0$, say received from parents.
- Household can save part of income or initial wealth in the first period, or it can borrow against future income y'. Interest rate on both savings and on loans is equal to r. Let s denote saving.
- Budget constraint in first period:

$$c + s = y + A$$

- Budget constraint in second period:

$$c' = y' + (1 + r)s$$

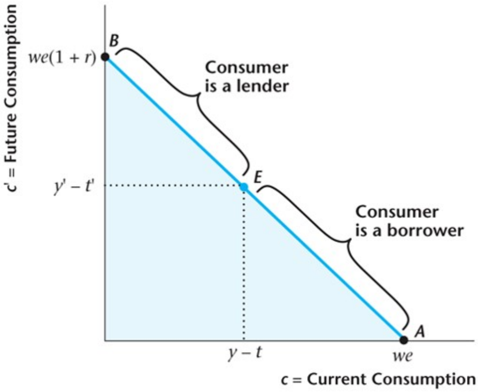
Budget Constraint II

- Summing both budget constraints

$$c + \frac{c'}{1+r} = y + \frac{y'}{1+r} + A \equiv y^{PV}$$

- We have normalized the price of the consumption good in the first period to 1. Price of the consumption good in period 2 is $\frac{1}{1+r}$, which is also the relative price of consumption in period 2, relative to consumption in period 1. Gross interest rate $1+r$ is the relative price of consumption goods today to consumption goods tomorrow.
- Called the present value budget constraint (PVBC).

Figure 9.1 Consumer's Lifetime Budget Constraint



Aside on Present Values

- Idea of PV extends more generally to any stream of payments or costs over time. Example: widely used in consulting to value a firm's assets and liabilities.
- General principle: income (or cost) in future is worth less than income today.
- General formula: for future income values $\{y_1, y_2, y_3, y_4, \ldots\}$

$$PV = \sum_{t=1}^{T} \frac{y_t}{(1+r)^t}.$$

- Distinction with discounting utility: β reflects subjective preference, here $1/(1+r)$ objective time value of money. (In equilibrium the two are linked.)

Present Value Examples

- Ex 1: Valuing a treasury bill/zero coupon bond. If I buy a treasury bill today, I get \$100 in six months.
  $PV = 100/(1+r)$, where r is the six-month interest rate.
  Note interest rates and bond prices are inversely related.
- Ex 2: Suppose invest \$5000 in a company today, it takes 3 years to become profitable, and thereafter gives \$2000 in profit for 3 years. If the interest rate is 4% is this a good investment?

  $$PV = -5000 + 0 + 0 + \frac{2000}{(1.04)^3} + \frac{2000}{(1.04)^4} + \frac{2000}{(1.04)^5} = \$131.46.$$

  What if $r = 6\%$? Can show $PV = -242.06$.
- Shows the importance of the interest rate for PV.

- Another example: Lottery winners always take the immediate payment over the annuity, even though the total value is less. In recent PowerBall jackpot of \$295 million, the 4 winners had option of \$2.95 million a year for the next 25 years ($4 \times 25 \times \$2.95 = \$295$ million, or \$73.75 million each), or an immediate \$41 million.
- All chose immediate payoff. Why?
- The present value is higher if interest rate is greater than 5.7% (Try it.)

Back to Household Problem

$$\max_{c,c'} u(c) + \beta u(c') \quad \text{s.t. } c + \frac{c'}{1+r} = y^{PV}$$

- Form Lagrangian with multiplier $\lambda > 0$.

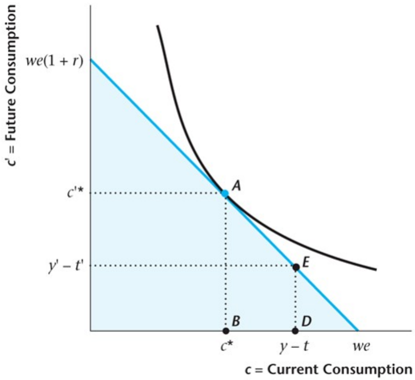
$$L = u(c) + \beta u(c') + \lambda \left(y^{PV} - c - \frac{c'}{1+r} \right)$$

$$\begin{aligned} \text{FOC: } u'(c) &= \lambda \\ \beta u'(c') &= \frac{\lambda}{1+r} \end{aligned}$$

- Combine them to get Euler Equation:

$$u'(c) = \beta (1+r) u'(c')$$

Figure 9.3 A Consumer Who Is a Lender



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A Parametric Example

- If $u(c) = \log c$, Euler Equation:

$$\frac{1}{c} = \beta\left(1+r\right)\frac{1}{c'} \Rightarrow c' = \beta\left(1+r\right)c$$

- Note that

$$c = y^{PV} - \frac{c'}{1+r} = y^{PV} - \beta c$$

So that:

$$\begin{aligned} c &= \frac{1}{1+\beta}y^{PV} \\ c' &= \frac{\beta\left(1+r\right)}{1+\beta}y^{PV} \\ s &= y + A - c = \frac{\beta}{1+\beta}\left(y+A\right) - \frac{1}{1+\beta}\left(\frac{y'}{1+r}\right) \end{aligned}$$

Comparative Statics: Income Changes

- What happens if y, y' or A increases? All matters is y^{PV}.
- *Both* c and c' increase (normal goods).
- If y or A increase, s increases to finance higher c'. Examples: increases in stock market or house prices – "wealth effect"
- If y' increases, s falls to finance higher current c. Examples: Announced layoffs, changing professions (or college majors).
- Sometimes discuss marginal propensity to consume (MPC). For example, MPC out of current income or wealth:

$$\frac{\partial c}{\partial A} = \frac{\partial c}{\partial y} = \frac{1}{1+\beta} > 0$$

Figure 9.5 The Effects of an Increase in Current Income for a Lender

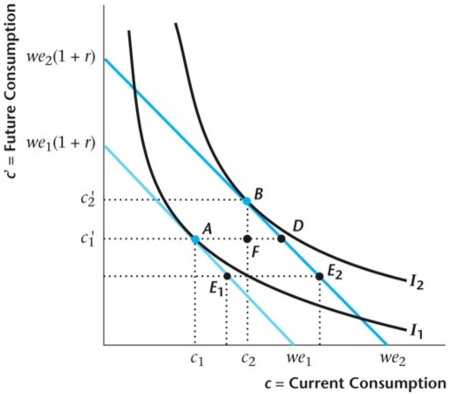

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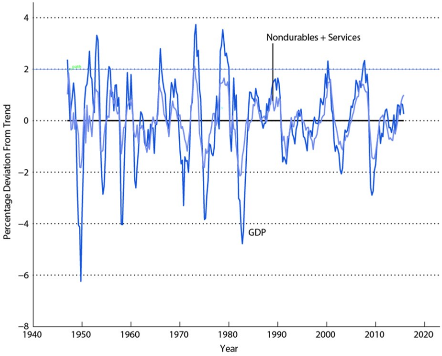
Figure 9.7 Percentage Deviations from Trend in Consumption of Nondurables and Services and Real GDP



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Comparative Statics: Changes in Interest Rate

- Income effect: if a saver $s > 0$, then higher interest rate increases income for given amount of saving. Increases consumption in first and second period. If borrower $s < 0$, then income effect negative.
- Substitution effect: gross interest rate $1 + r$ is relative price of consumption in period 1 to consumption in period 2. Current c becomes more expensive relative to c'. This increases c' and reduces c.
- Hence: for a saver an increase in r increases c' and may increase or decrease c. For a borrower an increase in r reduces c and may increase or decrease c'.

Figure 9.13 An Increase in the Real Interest Rate for a Lender

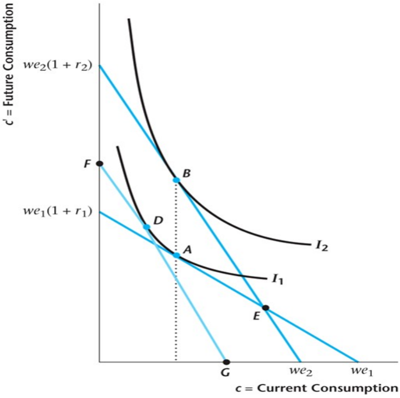
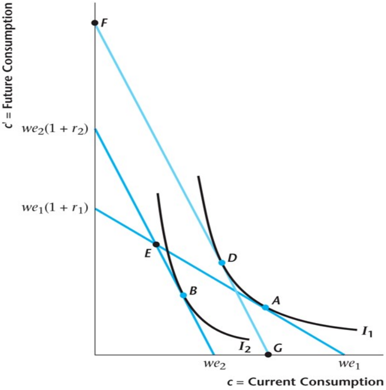


Figure 9.14 An Increase in the Real Interest Rate for a Borrower



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Savings Rate and Real Interest Rate