# Lecture 10
# Consumption and Savings

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Economics 702
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# A Parametric Example

- If $u(c) = \log c$, Euler Equation:

$$\frac{1}{c} = \beta (1+r) \frac{1}{c'} \Rightarrow c' = \beta (1+r) c$$

- Note that

$$c = y^{PV} - \frac{c'}{1+r} = y^{PV} - \beta c$$

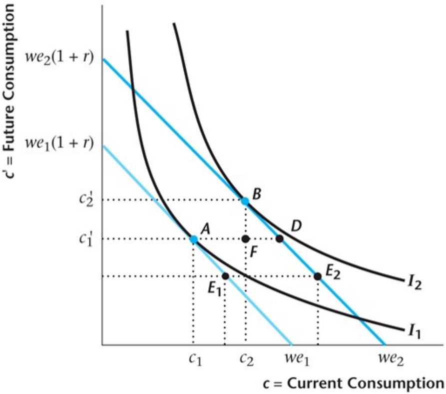
So that:

$$\begin{aligned} c &= \frac{1}{1+\beta} y^{PV} \\ c' &= \frac{\beta (1+r)}{1+\beta} y^{PV} \\ s &= y + A - c = \frac{\beta}{1+\beta} (y + A) - \frac{1}{1+\beta} \left( \frac{y'}{1+r} \right) \end{aligned}$$

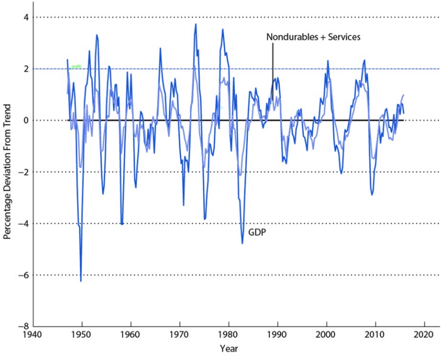
# Comparative Statics: Income Changes

- What happens if $y, y'$ or $A$ increases? All matters is $y^{PV}$.
- *Both* $c$ and $c'$ increase (normal goods).
- If $y$ or $A$ increase, $s$ increases to finance higher $c'$. Examples: increases in stock market or house prices – "wealth effect"
- If $y'$ increases, $s$ falls to finance higher current $c$. Examples: Announced layoffs, changing professions (or college majors).

# Figure 9.5 The Effects of an Increase in Current Income for a Lender



# Figure 9.7 Percentage Deviations from Trend in Consumption of Nondurables and Services and Real GDP

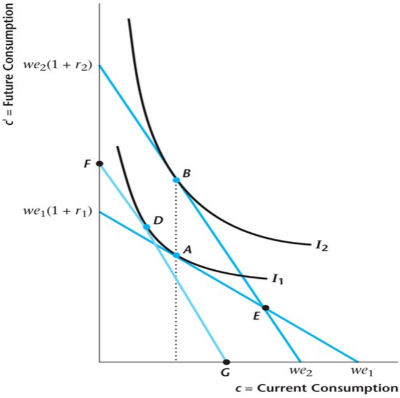



Pearson

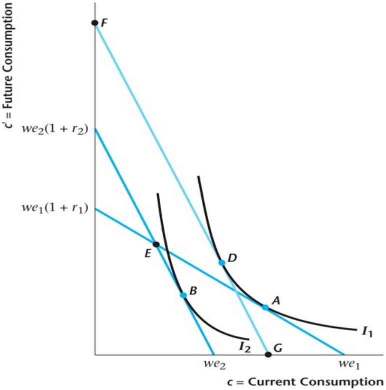
# Comparative Statics: Changes in Interest Rate

- Income effect: if a saver $s > 0$, then higher interest rate increases income for given amount of saving. Increases consumption in first and second period. If borrower $s < 0$, then income effect negative.
- Substitution effect: gross interest rate $1 + r$ is relative price of consumption in period 1 to consumption in period 2. Current $c$ becomes more expensive relative to $c'$. This increases $c'$ and reduces $c$.
- Hence: for a saver an increase in $r$ increases $c'$ and may increase or decrease $c$. For a borrower an increase in $r$ reduces $c$ and may increase or decrease $c'$.

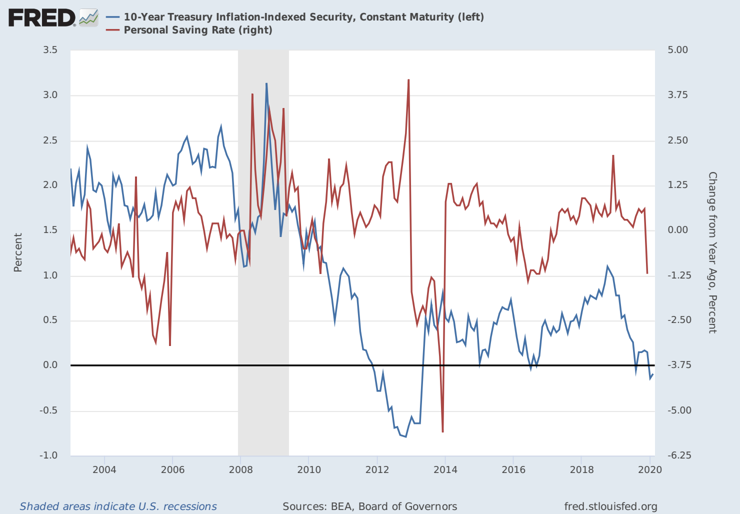
# Figure 9.13 An Increase in the Real Interest Rate for a Lender



# Figure 9.14 An Increase in the Real Interest Rate for a Borrower



# Savings Rate and Real Interest Rate



# Infinite Horizon Consumption-Savings Model

- Now extend the consumption-savings model from 2 periods to an infinite horizon. Many of the same implications.
- Slightly different timing/notation following Wickens.
- Flow budget constraint: $c_t$ consumption at date $t$, $a_t$ assets on hand at start of $t$. $a_{t+1}$ assets chosen at $t$, carried over to $t+1$, $r_t$ interest rate between $t-1$ and $t$, $x_t$ income:

$$c_t + a_{t+1} = x_t + (1 + r_t)a_t$$

- Derive intertemporal budget constraint, with $r_0 = 0$:

$$\begin{aligned}
c_0 &= x_0 - a_1 + a_0 \\
&= x_0 - \frac{c_1 - x_1}{1 + r_1} - \frac{a_2}{1 + r_1} + a_0 \\
&= x_0 - \frac{c_1 - x_1}{1 + r_1} - \frac{c_2 - x_2}{(1 + r_1)(1 + r_2)} - \frac{a_3}{(1 + r_1)(1 + r_2)} + a_0
\end{aligned}$$

$$\begin{aligned}
c_0 + \frac{c_1}{1 + r_1} &+ \frac{c_2}{(1 + r_1)(1 + r_2)} = \\
x_0 &+ \frac{x_1}{1 + r_1} + \frac{x_2}{(1 + r_1)(1 + r_2)} - \frac{a_3}{(1 + r_1)(1 + r_2)} + a_0
\end{aligned}$$

# Intertemporal Budget Constraint

- Continue same process for any horizon $T$:

$$\sum_{t=0}^{T} \frac{c_t}{\prod_{s=0}^{t}(1+r_s)} = \sum_{t=0}^{T} \frac{x_t}{\prod_{s=0}^{t}(1+r_s)} + a_0 - \frac{a_{T+1}}{\prod_{s=0}^{T}(1+r_s)}$$

- For any finite horizon $T$ we would have $a_{T+1} = 0$. No reason to save, and more importantly no one would lend.
- For infinite horizon, need to rule out the possibility of borrowing forever and never repaying principal.
- A Ponzi game occurs when agents borrow, repaying existing debt obligations by borrowing more. We impose the No Ponzi Game (NPG) restriction:

$$\lim_{T\to\infty} \frac{a_{T+1}}{\prod_{s=0}^{T}(1+r_s)} \geq 0$$

- This rules out borrowing indefinitely. Household won't want to have strictly positive assets in limit, so NPG will hold with equality.

# Household Problem: Infinite Horizon

- Under the NPG restriction we can take limits as $T \to \infty$:

$$\sum_{t=0}^{\infty} \frac{c_t}{\prod_{s=0}^{t}(1+r_s)} = \sum_{t=0}^{\infty} \frac{x_t}{\prod_{s=0}^{t}(1+r_s)} + a_0 \equiv x^{PV}$$

- The household problem is now to choose $\{c_t\}_{t=0}^{\infty}$ to maximize utility subject to the present value budget constraint. Single optimization problem, choosing plan for consumption for entire future.
- Lagrangian:

$$L = \sum_{t=0}^{\infty} \beta^t u(c_t) + \lambda \left( x^{PV} - \sum_{t=0}^{\infty} \frac{c_t}{\prod_{s=0}^{t}(1+r_s)} \right)$$

# Household Problem: Optimality Conditions

- First order condition for consumption at any dates $t, t+1$:

$$\begin{aligned} \beta^t u'(c_t) &= \frac{\lambda}{\prod_{s=0}^{t}(1+r_s)} \\ \beta^{t+1} u'(c_{t+1}) &= \frac{\lambda}{\prod_{s=0}^{t+1}(1+r_s)} \end{aligned}$$

- Divide these two equations:

$$\frac{u'(c_t)}{\beta u'(c_{t+1})} = \frac{\prod_{s=0}^{t+1}(1+r_s)}{\prod_{s=0}^{t}(1+r_s)} = 1 + r_{t+1}$$

- So once again we get the consumption Euler equation:

$$u'(c_t) = \beta u'(c_{t+1})(1 + r_{t+1})$$

- This governs behavior of consumption for any dates $t, t+1$.

# The Life Cycle Hypothesis

- One application: Franco Modigliani's life-cycle hypothesis of consumption
- Individuals want smooth consumption profile over their life. Labor income varies substantially over lifetime, starting out low, increasing until around the 50th year of a person's life and then declining until retirement around 65, with no labor income after retirement.
- Life-cycle hypothesis: by saving and borrowing individuals turn a very non-smooth labor income profile into a very smooth consumption profile.

# Life-Cycle Hypothesis: An Example

- Suppose that $r_t = r \ \forall t$, and $\beta(1+r) = 1$. Then Euler equation implies $c_t = c_{t+1} = \bar{c}$.
- Use present value budget constraint to work out consumption level:

$$\begin{aligned} \sum_{t=0}^{\infty} \frac{c_t}{(1+r)^t} &= x^{PV} \\ \Rightarrow \bar{c} \sum_{t=0}^{\infty} \frac{1}{(1+r)^t} &= \frac{\bar{c}(1+r)}{r} = x^{PV} \end{aligned}$$

So $c_t = \frac{r}{1+r} x^{PV}$ for all $t$.

- If $x_t = \frac{r}{1+r} x^{PV}$ for all $t$ then $a_t = 0$ for all $t$.

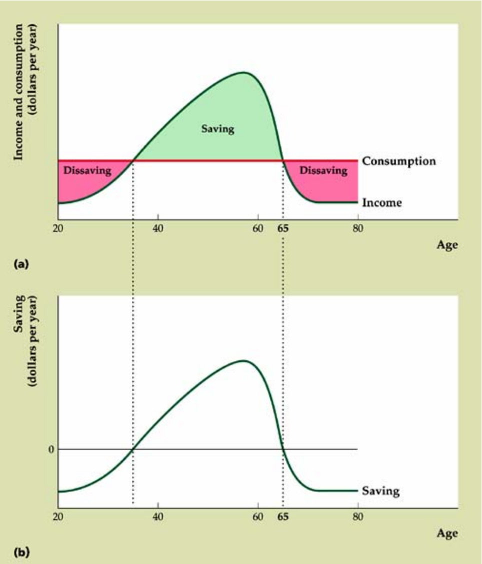
# Life-Cycle Predictions

- In general, consumption is constant but income $x_t$ varies. How is this implemented?

$$
\begin{aligned}
c_0 &= x_0 - a_1 + a_0 \Rightarrow a_1 = x_0 - c_0 + a_0 \\
a_1 &= x_0 + a_0 - \frac{r}{1+r} x^{PV}
\end{aligned}
$$

- If current income $x_0 + a_0$ is low relative to $\frac{r}{1+r} x^{PV}$, borrow $a_1 < 0$.
  If $x_0 + a_0$ is high relative to $\frac{r}{1+r} x^{PV}$, save $a_1 > 0$.
- These same general implications extend to varying $r_t$, $\beta(1 + r_t) \neq 1$.
- Main predictions: current consumption depends on total lifetime income and initial wealth. Saving should follow a very pronounced life-cycle pattern with borrowing in the early periods of an economic life, significant saving in the high earning years from 35-50 and dissaving in retirement years.

# Figure 4.A.5
Life-cycle consumption, income, and saving



# Life-Cycle Evidence

- This pattern of life-cycle savings is generally consistent with the data
- One empirical puzzle: Older household do not dissave to the extent predicted by the theory. Several explanations:
  1. Individuals are altruistic and want to leave bequests to their children.
  2. Uncertainty with respect to length of life and health status.
- Important in aggregate as population ages (Japan).

# Case of Japan

- Japanese saving rate fell from 23% of personal income in 1975 to 14% in 1990 down to 5% in 2000.
- Over same horizon, US saving rate roughly flat around 6%.
- Why? One reason: aging of the population in Japan.
- Ratio of Japanese over age of 65 to those of working age rose from 15% in 1980 to 28% in 2000. Forecast to increase further to 38% by 2010 and 50% by 2020.
- Estimates by HSBC that demographic shift can account for half of the decline in the savings rate.
- Effects of inflation, slower growth rates, changes in government debt are other factors contributing to savings decline.

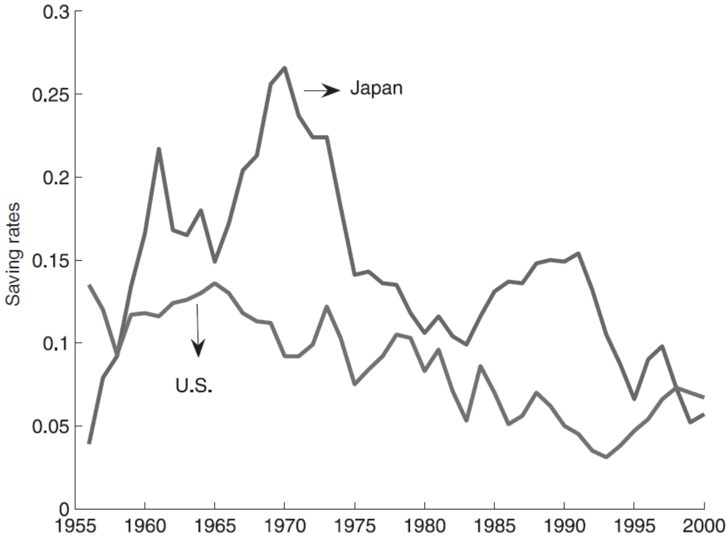



FIGURE 1. NET NATIONAL SAVING RATES

# Permanent Income Hypothesis

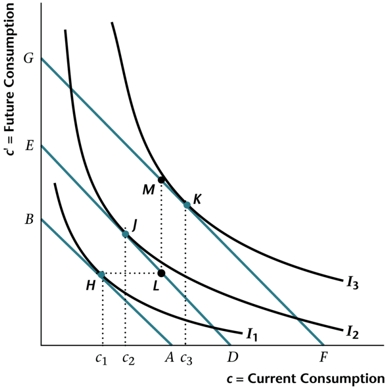
- Future income is uncertain.
- Income of an individual household, $x_t$ consists of a permanent part, $x^p$ and a transitory part $v_t$

$$x_t = x^p + v_t$$

- Permanent part $x^p$: expected average future income (usual salary)
- Transitory part $v_t$: random fluctuations around this average income (bonus)
- In two period model from last time, permanent means $y$ and $y'$ change. Transitory: only $y$ changes.

- Friedman (1956): Individuals react differently to an increase in permanent and an increase in transitory income.
- Increase in the permanent component of income brings about an (almost) equal response in consumption. Large increase in $x^{PV}$.
- Individuals smooth out transitory income shocks over time. Little effect on $x^{PV}$. Greater fraction of increase is saved.
- It follows that individual consumption is almost entirely determined by permanent income. So consumption should be smoother than income.
- Data suggests it is so, but not as smooth as theory suggests. Effects of credit market imperfections and borrowing constraints.

# **Figure 8.8** Temporary Versus Permanent Increases in Income



# Consumption-Savings Under Uncertainty

- Now $x_{t+1}, r_{t+1}$ are random, unknown at $t$.
- Agents form expectations of future income, maximize expected utility.
- Can derive an Euler equation of the same form, but now must have expectations over $c_{t+1}$ and $r_{t+1}$:

$$u'(c_t) = \beta E_t \left[ u'(c_{t+1})(1 + r_{t+1}) \right]$$

- Here $E_t(\cdot)$ represents the agent's expectations, conditional on all information available at date $t$.

- Suppose again that $r_t = r$ and $\beta(1+r) = 1$, so the Euler equation is:

$$u'(c_t) = E_t u'(c_{t+1})$$

- Also suppose that agents have quadratic preferences, where $a > 0$ is a constant:

$$u(c_t) = c_t - \frac{a}{2} c_t^2,$$

- So $u'(c_t) = 1 - ac_t$ and the Euler equation becomes:

$$c_t = E_t c_{t+1}$$

- Also by the law of iterated expectations:

$$c_t = E_t c_{t+1} = E_t(E_{t+1} c_{t+2}) = E_t c_{t+2}$$

# Implications

- With these preferences consumption is a random walk:

$$c_{t+1} = c_t + \varepsilon_{t+1}, \quad E_t \varepsilon_{t+1} = 0$$

- The best predictor of consumption one period ahead is current consumption. No other variables which are known at date $t$ help predict consumption at $t+1$.
- To express this another way, note that the present value budget constraint holds for any date $t$:

$$\sum_{s=0}^{\infty} \frac{E_t c_{t+s}}{(1+r)^s} = \sum_{s=0}^{\infty} \frac{E_t x_{t+s}}{(1+r)^s} + a_t(1+r)$$

# Permanent Income Theory Example

- Then note that $E_t c_{t+s} = c_t$ for all $s$. So then we have:

$$
\begin{aligned}
c_t \sum_{s=0}^{\infty} \frac{1}{(1+r)^s} &= \sum_{s=0}^{\infty} \frac{E_t x_{t+s}}{(1+r)^s} + a_t(1+r) \\
c_t &= \frac{r}{1+r} \sum_{s=0}^{\infty} \frac{E_t x_{t+s}}{(1+r)^s} + r a_t
\end{aligned}
$$

  Consumption depends on expectations of all future income.

- Changes in consumption over time are driven by changes in expectations of future income. Information revealed about future income is the driver of consumption.

$$
c_{t-1} = E_{t-1} c_t = \frac{r}{1+r} \sum_{s=0}^{\infty} \frac{E_{t-1} x_{t+s}}{(1+r)^s} + r a_t
$$

# Permanent and Transitory Shocks

- A pure transitory income shock reveals at date $t$ that $x_t > E_{t-1}x_t$ is higher than anticipated, but $E_t x_{t+s}$ is unaffected for $s \geq 1$. Example: $x_t = x_{t-1} + v_t$, $x_{t+s} = x_{t-1}$.

$$c_t = c_{t-1} + \frac{r}{1+r} v_t$$

- A permanent income shock reveals at date $t$ that $x_t > E_{t-1}x_t$ is higher than anticipated, and $E_t x_{t+s}$ is also higher for $s \geq 1$. Example: $x_{t+s} = x_{t-1} + \Delta$

$$c_t = c_{t-1} + \Delta$$

# Extensions of Permanent Income Theory

- With quadratic utility, uncertainty in income does not affect decisions:

$$c_t = \frac{r}{1+r} \sum_{s=0}^{\infty} \frac{E_t x_{t+s}}{(1+r)^s} + r a_t$$

- This is a property known as **certainty equivalence**. Decisions are the same as if $x_t$ took on its expected value with certainty.
- With more general preferences, variability of income would matter.
- Suppose again that $r_t = r$ and $\beta(1+r) = 1$, so the Euler equation is:

$$u'(c_t) = E_t u'(c_{t+1})$$

- If $u'(c)$ is convex ($u'''(c) > 0$), then more uncertain income will lead to lower consumption today, more savings: **precautionary savings**

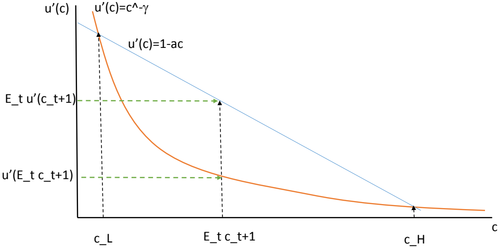
# Precautionary Savings

- Quadratic utility: $u'(c) = 1 - ac$, $u''(c) = -a < 0$, $u'''(c) = 0$.

$$u'(c_t) = E_t u'(c_{t+1}) \Rightarrow c_t = E_t c_{t+1}$$

- Power utility: $u'(c) = c^{-\gamma}$, $u''(c) = -\gamma c^{-\gamma-1} < 0$, $u'''(c) = -\gamma(-\gamma - 1)c^{-\gamma-2} > 0$.

$$u'(c_t) = E_t u'(c_{t+1}) > u'(E_t c_{t+1}) \Rightarrow c_t < E_t c_{t+1}$$



# Implications for Consumption

- Uncertainty about future income will lead to more savings, to allow households to smooth potential consumption fluctuations.
- Periods of increased uncertainty will be characterized by reductions in household consumption.
- Another complication we've abstracted from is borrowing constraints. These affect consumption in two ways:
  1. When household is constrained, consumption will closely follow income. Unable to smooth.
  2. Household will build up stock of assets to diminish the impact of the constraint.
- There is significant micro evidence for these effects on household consumption. Macro effects are less clear.