Dynamic Thin Markets

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December 21, 2014

[0] We thank the editor Itay Goldstein and anonymous referees for many helpful suggestions. We would like to thank Robert Almgren, Zhiwu Chen, Peter DeMarzo, Steven Durlauf, Frank Fabozzi, Willie Fuchs, John Geanakoplos, Ananth Madhavan, Antonio Mello, Matthew Pritsker, Larry Samuelson, Robert Shimer, Bruno Strulovici, Dimitri Vayanos (AFA discussant), Elu Von Thaden, Jan Werner and Ji Hee Yoon for comments. We are also grateful for the suggestions offered by seminar participants at Chicago, Johns Hopkins, LSE (Mathematics), Mannheim (Finance), Northwestern, Oxford (Nuffield), Penn State, Princeton, Queen's, Southampton, UCL, Warwick (CRETA), Washington (St. Louis) and Wisconsin-Madison, and participants at numerous conferences.

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Abstract Large institutional investors dominate many financial markets. This paper develops a consumption-based model of markets in which all institutional traders recognize their impact on prices. Bilateral (buyer and seller) market power changes efficiency and arbitrage properties of equilibrium and gives rise to temporary and permanent price effects of supply shocks, order break-up, limits to arbitrage, non-neutrality of trading frequency, and real effects in periods other than event dates of shocks and announcements. Maximizing welfare and stabilizing liquidity through disclosure of information about fundamentals presents a tradeoff. Equilibrium representation as "trading against price impact" provides a link with the industry's practice.

JEL Classification: E21, G12, L13 Keywords: Liquidity, Institutional investors, Temporary and permanent price impact, Anticipated and unanticipated supply shocks, Public information releases, Trading frequency, Slow trading

Since trade-level data first became available two and a half decades ago, it has been well understood that, in many contemporaneous markets, trade is dominated by a relatively small group of large, institutional investors whose positions move prices, thereby adversely affecting their terms of trade. Trading costs associated with price impact are first order and exceed all explicit trading costs, such as commission, brokerage, and order-processing fees. Techniques to estimate price impact are widespread in the financial industry and are available to individual as well as institutional investors (e.g., software designed by Citigroup, EQ International, ITG, MCI Barra, and OptiMark). In financial jargon, markets in which individual trades are large relative to average daily volume and, hence, impact prices, are known as *thin markets*.[1]

This paper presents a dynamic equilibrium model of asset pricing, trade, and consumption in which all traders correctly recognize their price impact. We study a market with I *strategic* investors who trade risky assets. The market operates as the (repeated) standard double auction in which traders submit (net) demand schedules (or limit and market orders; i.e., the uniform-price auction, e.g., Kyle (1989); Vayanos (1999); Vives (2011)), variants of which are used in practice in many markets. Relaxing price taking in trader optimization is the sole departure from the competitive framework. To delineate how the presence of price impact affects financial markets, we adopt preferences and assets from the classic CARA-Normal setting in an infinite horizon model; the competitive infinite-horizon consumption-trade framework obtains if traders are price takers in equilibrium.

Main Results. Over the past decade, a large body of research has emerged to explain price behavior that is broadly interpreted as temporary departures of prices from their fundamental values as a reaction of prices to exogenous shocks in supply (or demand).[2] The *AFA Presidential Address* by Duffie (2010) provides an extensive overview. A typical price behavior exhibits a significant price

[1] Increase in institutional participation in the stock market over recent decades (continuing the trend since the World War II) has been extensively documented. Ownership of all outstanding financial assets in the U.S. rose from 6.1% in 1950 to 50.6% in 2009. Of the total market value of U.S. common stocks of $1.4 trillion in 1980, institutions held $473 billion (34%). By 2010, the total market value of common stocks had increased to $17.1 trillion and institutions had increased their holdings to $11.5 trillion (67%). Even in markets as deep as NYSE, a typical institutional package would represent more than 60% of the average daily trading volume, if traded at once. Institutional investors (e.g., hedge funds, mutual and pension funds, investment banks) dominate trading in traditional, public exchanges as well as alternative trading venues, such as intra-dealer markets and dark pools. (E.g., Holthausen, Leftwich, and Mayers (1987); Chan and Lakonishok (1993, 1995); Keim and Madhavan (1995, 1996, 1998); Biais, Bisière, and Spatt (2010); Blume and Keim (2012); `www.nysedata.com/factbook.`)

[2] Among the shocks examined by numerous studies for various securities are forced liquidations, issuance of new debt, selling Initial Public Offerings (IPOs), stock inclusions into or deletions from S&P 500 and other market indices, index weight changes, and fire sales. Index funds invest a constant fraction of wealth in the companies included in an index, regardless of the performance of an asset; therefore, an index weight change induces a demand shock that is not associated with new information about the fundamental value of an asset. Rather, weight changes are bureaucratic decisions, and data on ownership used for reweighting are publicly available prior to events.

change on the event date, followed by a partial reversal of the change in subsequent periods. Thus, apart from the *permanent* effect, the price dynamics feature a *temporary* component (also known as "long-run and short-run," "mispricing," or "asset price overshooting"). Estimation software used in the financial industry routinely distinguishes between the "permanent" and "temporary" price-impact effects of trades. Notably, even when a supply shock has been publicly pre-announced, the temporary price drop below the long-run level still occurs and, moreover, takes place on the actual event date, not on the date of the announcement.[3] From a theoretical point of view, the price behavior is striking: Trade announcements and trade-induced price effects are separated in time, and *anticipated* price changes are observed. In the standard competitive model, these features of price behavior are ruled out by no-arbitrage, which ties the equilibrium price to the fundamental value; any effects of announcements of future trades are reflected in price fully upon the announcement.

Price making changes some of the central equilibrium properties, relative to the competitive model. This paper develops the implications of non-competitive trading for price behavior, non-neutrality of trading frequencies, and the impact of public information disclosure:

We show that *any* unanticipated exogenous supply shock in thin markets has two effects on prices – *fundamental* and *liquidity* – which differ in origin, timing, persistence, and dynamics. The fundamental effect, which is permanent, reflects the change in the average market holdings (aggregate risk) and is present also in markets with price-taking traders. In a thin market, the fundamental effect is amplified by a temporary liquidity effect, which results from traders' order reduction in response to price impact (and occurs even when information is symmetric). Furthermore, market thinness introduces a *time separation of announcements and trade-induced price effects*, which are observed in periods *other* than those during which information about the shock becomes available. While price exhibits the permanent effect upon the announcement of the shock, the liquidity effect always occurs at the moment of trade. If anticipated, the liquidity effect of the shock results in an additional temporary effect, which is present in *all* periods between the announcement and the supply shock event, attaining the maximum at the moment of trade. In contrast to competitive markets, pre-

[3] Pre-announced weight changes in stock market indices have a significant price effect on the day of inclusion, observed for stocks and currencies or foreign equity. Such natural experiments facilitate controlling for the informational component of the price change. Newman and Rierson (2004) find that new bond issuance in the European telecommunication sector increased the yield spreads of other firms in the sector. The effect was transitory, significant, and peaked on the day of issuance, not on the day of announcement. Lou, Yan, and Zhang (2013) show that Treasury security prices in the secondary market decrease significantly during the few days prior to Treasury auctions and recover shortly thereafter, despite the time and size of each auction being announced in advance. The authors argue that the price effects are significant even in very liquid markets. Duffie (2010), Gromb and Vayanos (2010), and Lou, Yan, and Zhang (2013) review the evidence.

announcing the shock affects the distribution of consumption and wealth: Anticipated supply shocks redistribute wealth towards buyers, whereas demand shocks redistribute towards sellers.

Our model decouples the frequency of trade and asset payments by allowing for *multiple* trading rounds between the payments of assets. Equating the frequencies of trade and dividend payments would be without loss with price-taking investors. In contrast, trade frequencies impact the performance of thin markets.[4] While order break-up ("slow trading") per se has also been demonstrated in other studies of thin markets,[5] by allowing trade to take place more frequently than dividend payments, we show that a higher frequency of trading improves market performance: Absent non-trivial transaction costs, large investors take every opportunity to break up their orders more finely, thus lowering the total price impact costs, *even if their beliefs do not change (i.e., with perfect foresight)* and *without* asymmetric information, inference effects, or shocks to endowments or information occurring throughout trading. Investors' market power thus creates trading volume even in the absence of price changes.[6] The key mechanism in thin markets is that (anticipation of) market thinness/competitiveness in future rounds induces market thinness/competitiveness in earlier rounds, regardless of the number of traders and risk aversion.[7]

Finally, in competitive markets, given full diversification accomplished within the first round, market performance is invariant to releasing public information dynamically. For thin markets, we present a positive result: For any Gaussian dynamic multi-asset information structure, postponing releases of information increases consumption and *ex ante* welfare, as it improves diversification before the risks are resolved. However, the welfare-maximizing disclosure introduces episodes of low market liquidity. Thus, in thin markets, the objectives of stabilizing liquidity and maximizing welfare through disclosure involve a tradeoff. The mechanism through which public information impacts welfare and expected returns, *absent of private information*, is new and contrasts with competitive markets.[8]

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[4] Dividends of risky assets are typically paid semiannually or annually, whereas trading opportunities in financial markets are clearly more frequent.

[5] Like us, Vayanos (1999) shows order break-up as an equilibrium strategy with all traders optimizing dynamically. Also related is the literature that derives the optimal trade execution, treating the asset and quantity to trade as exogenously given, and calibrates numerical solutions (reviewed in Garleanu and Pedersen (2013)).

[6] This complements the predictions of the models of volume that are derived from asymmetric information or heterogeneous beliefs (see, e.g., the review in Bond, Edmans, and Goldstein (2012); Eyster, Rabin, and Vayanos (2013)). Indeed, in thin markets, liquidity becomes available over time.

[7] As an additional insight from allowing higher trade-to-payment frequency, the optimality of order break-up in dynamic markets does not follow directly from the optimality of order reduction by large traders in one-shot markets (e.g., Kyle (1989); Vives (2011)).

[8] The mechanism through which public information impacts welfare and expected returns, *absent of private information*, is new and contrasts with competitive markets. The question of the welfare effects of public information releases about fundamentals (e.g., central bank transparency) has received renewed attention in the literature. A major finding has been that, when agents have private information, releases can be welfare reducing due to a coordination motive

Related Literature; Contribution to Modeling Non-Competitive Trading. This paper is part of a large literature on non-competitive trading. The non-competitive models can be grouped around two ideas, depending on where the primary source of price impact is attributed: a finite number of risk averse liquidity providers (i.e., decreasing primitive marginal utility or limited risk bearing capacity) or frictions. One insight from our analysis is that the implications of price impact we report arise jointly on the equilibrium path solely as a result of the departure from the assumption of price taking in trader optimization, in an otherwise classical setting, for centralized markets with fully rational forward-looking agents without asymmetric information, search, cost, financial constraints, bounded rationality or capital mobility frictions (the *Handbook* chapter by Vayanos and Wang (2013) provides a recent review of the liquidity models). We assume away price-taking investors, whose presence does not affect the qualitative properties of equilibrium with price-making investors.

In the literature that attributes price impact to traders' limited risk bearing capacity, apart from the classic contribution of Kyle (1989), in which the primary source of price impact is risk aversion (asymmetric information being the derivative), our model is the closest to Vayanos (1999), who introduced the infinite horizon model of double-auction with non-competitive traders and *no* noise traders or agents who do not fully optimize dynamically. To the double-auction framework of non-competitive trading, we add (1) analysis of supply shocks in thin markets along with analysis of announcements of shocks or fundamentals and (2) separation of trade and asset payment frequencies, which also allows the incorporation of (3) general information structures for fundamentals (dividends). Each extension reveals a new economic mechanism present in the thin but not the competitive markets. Respectively, (1) Vayanos' (1999) model incorporates *exclusively* endowment shocks, and equilibrium price always coincides with the fundamental value;[9] thus, the model does not generate new predictions about price behavior in response to supply shocks, relative to a model with price takers (i.e., existence and characterization of the transitory price effect and the real effects of shock announcements). (2)

among agents (payoff externalities among agents, e.g., Morris and Shin (2002); or learning externalities, e.g., Amador and Weill (2010)). By contrast, in thin markets, the effects of releasing information about fundamentals are ambiguous, even in the absence of private information. A policy objective that takes into account both traders' welfare and market liquidity (e.g., volatility or welfare of occasional traders) of public releases of fundamental information involves a trade-off, and a policy of partial disclosure may be optimal. This is consistent with Angeletos and Pavan (2007), who, in a competitive setting, argue that the frictions which create a gap between equilibrium and first-best are more important than coordination in determining the welfare effects of public information.

[9] The fundamental value (equal to the average marginal utility) varies over time in Vayanos's model, due to endowment shocks, but the price path never departs from it. The key behind the distinct effects of supply and endowment shocks is whether or not the new supply needs to be absorbed by the liquidity providers through trading or it directly alters their holdings. If the former, but not the latter, is the case, the temporary liquidity effect is present, due to price impact. Endowment shocks do not result in temporary price effects, despite the nonnegligible price impacts.

Vayanos (1999) assumes, as does the standard competitive setup, that *both* trade and asset payments occur once in each period; thus, price impact and trading are stationary. Not imposing equal frequency of trade and asset payments reveals thin-market implications of non-stationary equilibrium dynamics of prices and liquidity, consumption-savings behavior, and welfare. Let us note that the dynamic equilibrium in our model directly gives rise to the representation of *a market as a sequence of short- and long-run demands*, commonly used in empirical studies of supply shocks (e.g., those following Greenwood (2005)).[10] (3) We offer a new result on the welfare-optimal disclosure of public information about fundamentals and the welfare-stability tradeoff of announcements. Additionally, we provide implications of the cross-asset price impact.

An early strand of literature on price impact attributed to risk aversion builds monopoly/Cournot-type models of *one-sided* market power with $I \geq 1$ large investors who trade with a fringe of price-taking traders (e.g., Ho and Stoll (1981); Grossman and Miller (1988); Vayanos (2001); Brunnermeier and Pedersen (2005); and DeMarzo and Urošević (2006), extended by Urošević (2005)). We show that the equilibrium implications of bilateral market power (i.e., of *both* buying and selling traders) differ markedly from markets with one-sided market power or with price-taking traders who optimize dynamically. Unless information happens to be revealed contemporaneously with the shocks' occurrence, shocks only have permanent and *no transitory effects* and price impact – and hence the liquidity effect of shocks – depends only on contemporaneous uncertainty. With bilateral market power, price impact in any given round depends on *future and current* uncertainty about asset payoffs; thus, bilateral price impact introduces a time separation between information disclosure and its effects. Bilateral price impact describes well markets in which large traders provide liquidity for and effectively trade with one another. The modeling contribution of the paper is relevant beyond financial markets for modeling durable good bilateral oligopoly with divisible goods like assets.

In our analysis, price impact does not result from asymmetric information, as we seek a source of market thinness that would also be present in perfect-foresight environments, as suggested by the data.[11] Some predictions could also be generated by persistent asymmetric information or systematic

[10] Recent contributions by Garleanu and Pedersen (2013) and Kyle, Obzihaeva and Wang (2013)) present (stationary) dynamic double auction models (respectively, with exogenous transaction costs and overconfidence) in continuous time, which formalizes that, mathematically, equilibrium with non-competitive trading can be seen as equilibrium in flows of trades with target portfolios being diffusions. In game theoretic terms, our characterization of equilibrium as a sequence of (non-stationary) short-run demands (trades) captures that each trader best responds by choosing a flow, given his residual market flow, which is determined by the flows submitted by others. Accordingly, our characterization of nonstationary equilibrium cannot exploit a recursive structure for trade and, instead, tackles trade sequences.

[11] In addition to evidence on pre-announced shocks, large institutional investors do not outperform fixed benchmark portfolios, which would be the case if they had superior information about asset fundamentals. Furthermore, for the average trade value, price impacts in downstairs markets do not differ significantly from those in upstairs markets, which

behavioral biases (e.g., beliefs heterogeneity). Market thinness contributes as follows: (i) it creates trading volume; the new prediction in thin markets is that agents trade *even in the absence of price changes* (Ft. 2 and 3); (ii) public information impacts welfare through slow trading – a new mechanism, relative to the literature on public disclosure *in the presence of private information* (Ft. 8); (iii) higher frequency of trading improves market performance even with perfect foresight, which contrasts with the arguments recently put forward in the literature based on asymmetric information and learning (Sections 4 and 6).

The double-auction model in demand schedules is the workhorse of the literature that was developed following Kyle (1989) and Vayanos (1999). We introduce an alternative representation of trading in the demand game that is consistent with the financial industry's practice. Namely, the Nash equilibrium can be represented as a result of optimization according to "trading against price impact function" by anonymous investors whose information about the market is summarized by their price impact (Kyle's lambda), which they correctly estimate. The equivalent representation result, which applies to other analyses of thin markets based on the demand game (e.g., Vayanos (1999); Garleanu and Pedersen (2013); Kyle, Obzihaeva and Wang (2013); Du and Zhu (2014)), offers a link with a typical practitioners' model used in the price-impact estimation software in the financial industry, which adopts the perspective of an agent who trades against a residual market, represented as an exogenously given price impact function, the functional form of which is motivated empirically.[12]

Moreover, the alternative formulation can be useful to other researchers working on thin markets, as it is more tractable; allows the accommodation of rich information structures (public and private information, see Vives (2011), and Rostek and Weretka (2012)); relates double-auction and industry techniques; and has a *dual* game-theoretic (Nash in demands) and general-equilibrium representation of thin markets, thereby also permitting a mapping of the predictions to the standard competitive model. These results apply to all models based on the demand game.[13] Equilibrium representation

are more transparent and less susceptible to informational asymmetries (e.g., Madhavan and Cheng (1997)). Thus, in certain types of markets, price impact need not be mainly driven by the asymmetry of information.

[12] E.g., Almgren and Chriss (2000); Almgren et al. (2005) implemented by Citigroup; Huberman and Stanzl (2004); see also a recent contribution by Garleanu and Pedersen (2013); for a review of market impact models, see, e.g., `http://www.eqimpact.com/main.asp?models`. While these models focus on single-agent optimization, they are closely related to the general equilibrium literature on price impact (see, e.g., an overview by Hart (1985)), which considered an exchange economy with the same optimization assumption as ours; each agent trades against a residual supply with an exogenously given price impact function. Relative to the model of Garleanu and Pedersen (2013) with exogenous transaction costs, deriving price impact from the equilibrium strategies of other traders allows us to link the equilibrium dynamics of temporary and permanent cost components to the traders' endogenously time-varying risk bearing capacity and dynamic information structure for fundamentals.

[13] The demand game is canonical in the finance literature (e.g., Kyle (1989), Vayanos (1999)). In industrial organization, the game of Nash in demands (or supplies) was introduced for an oligopolistic industry by Grossman (1981) and

in terms of "trading against price impact function" coincides in the CARA-Normal setting with the linear equilibrium (robust to uncertainty or noise; Kyle (1989), Vayanos (1999)) used essentially in all of the literature based on the demand game, but it also allows researchers to extend the existing theory of thin markets and calibration of temporary and permanent price impact models beyond the CARA-Normal settings to non-linear schedules, which the linear equilibrium cannot.

1 Model of Thin Markets

This section describes the market structure, trade and consumption problem, and introduces an alternative formulation of equilibrium.

1.1 Market

$I \geq 3$ traders trade assets to maximize the expected CARA utility function from consumption,

$$U = -E \sum\nolimits_{t=1}^{\infty} \beta^t \exp\left(-\alpha c_t^i\right), \tag{1}$$

where c_t^i is trader i's consumption in period $t = 1, 2, \ldots, \infty$, α is the absolute risk-aversion and β is the discount factor. The traders are interpreted as liquidity providers – financial intermediaries who stand ready to buy and sell assets. There are two investment opportunities: a storable consumption good with return r per period (a riskless asset) and a risky asset that pays a random dividend d_t in terms of the consumption good; d_t follows a random walk, $d_t = d_{t-1} + \delta_t$, $\delta_t \sim N\left(0, \sigma^2\right)$. Each trader i is initially endowed with $\left(w_0^i, \theta_0^i\right)$ of the riskless and risky asset, respectively. Risky endowments θ_0^i differ across traders and gains to trade in financial markets come from risk sharing. The standard infinite-horizon CARA-Normal framework for competitive (deep) markets is encompassed (as the number of traders grows, $I \to \infty$).

The timing in each period is as follows (Figure 1). Traders enter period t with the stocks of riskless consumption good w_{t-1}^i and risky asset shares θ_{t-1}^i. Agents trade the risky asset for $T \geq 1$ rounds indexed by $l = 1, ..., T$, after which the risky asset pays dividend d_t on the post-trade holdings θ_t^i and the riskless holdings w_t^i give interest r. Finally, traders choose consumption c_t^i.[14] Let $\gamma \equiv$

was further developed by Klemperer and Meyer (1989).

[14] The assumption that consumption is less frequent than trade or information disclosure is not essential for our results. Instead, the new effects of market thinness arise from decoupling the frequencies of trade and disclosure of information about dividends (fundamentals). As a modeling insight, imposing that trade and asset payments are equally frequent, while neutral in competitive markets, rules out non-trivial equilibrium dynamics of prices and liquidity, welfare and

$1 - 1/(I-1) \in (0,1)$ be an index of *market depth.* The portfolio held by all traders evaluated in *per capita* terms, $\bar{\theta} \equiv (1/I) \sum_{i \in I} \theta_0^i$, defines the *average holdings.* Given the identical risk aversion, holdings $\theta_t^i = \bar{\theta}$ for all i and t are the unique Pareto efficient allocation of risky assets.

[Figure 1 here]

In each trading round $l = 1, ..., T$ of period t, financial markets operate as the *uniform-price mechanism* (i.e., the demand submission game; e.g., Kyle (1989)). Each trader i submits a downward-sloping (net) demand schedule $\Delta_{t,l}^i(\cdot)$ that specifies demanded quantity of risky assets for every price $p_{t,l}$ (e.g., a profile of limit and stop orders). The aggregate net demand determines the market-clearing price $p_{t,l}^*$, $\sum_{i \in I} \Delta_{t,l}^i(p_{t,l}^*) = 0$, and trader i receives $\Delta_{t,l}^{i*} \equiv \Delta_{t,l}^i(p_{t,l}^*)$ shares of risky asset and pays $p_{t,l}^* \Delta_{t,l}^{i*}$ in the consumption good. We study an equilibrium in linear[15] demands and the solution concept is the (robust) Subgame Perfect Nash Equilibrium (hereafter, *equilibrium*). Throughout, "*" denotes equilibrium. While price impact measures market thinness, its inverse is a measure of market liquidity. All proofs are presented in the Appendix.

1.2 Equilibrium: Trade and Consumption Problem

In the first part of the paper, we characterize the (non-stationary) trade problem during period t. No information disclosure about dividends or other events occur throughout trading rounds $l = 1$ to T. In Appendix A we show that equilibrium trading decisions within each period t maximize a quasilinear-quadratic utility that depends on holdings of riskless and risky asset (w, θ) after the last round T, before assets pay,

$$V_T(w, \theta) = w + \bar{a}\theta - \frac{1}{2}\bar{\alpha}\sigma^2\theta^2. \tag{2}$$

The mean-variance utility (2) is a monotone transformation of the value function in the infinite horizon problem (see Equation (63)). In Section 2.4, we determine endogenous parameters $\bar{a}$ and $\bar{\alpha}$ in terms of primitives of the infinite horizon model by solving the (stationary) consumption problem.

allocation with price-making traders.

For the information structure in which no information is revealed between dividend payments (as in the first part of the paper), the assumed utility functions over consumption plans can be rationalized by the value functions derived from maximizing utility in continuous time; equilibria are the same. With information disclosure (as in the second part), consumption becomes a random process; still, the implications of price impact are expected to hold qualitatively, given the unambiguous reduction of consumption, relative to competitive trading.

[15] E.g., Kyle (1989), Vayanos (1999). Trader strategies are not restricted to linear bids. Rather, we analyze the equilibrium in which it is optimal for a trader to submit a linear demand, given that others do.

Marginal utility $\partial V_T/\partial\theta$ is downward sloping and reflects traders' aversion to dividend risk, given the equilibrium trade and consumption in the future periods. Coefficient $\bar{\alpha} > 0$ measures *effective risk aversion* at T. The *fundamental value* of a risky asset, defined as the marginal utility at the diversified (average) holdings, $\theta_t^i = \bar{\theta}$, is

$$\bar{v}_t(\bar{\theta}) \equiv \frac{\partial V_T(\bar{\theta})}{\partial\theta} = \bar{a} - \bar{\alpha}\sigma^2\bar{\theta}. \tag{3}$$

The fundamental value coincides with the price in a *competitive model* (i.e., the model with I traders with utilities (2) who are, by assumption, price takers). Its component $-\bar{\alpha}\sigma^2\bar{\theta}$ captures the reduction in asset value due to aggregate risk in $\bar{\theta}$ (see Section 2.4). $V_l(w,\theta)$ denotes the quasilinear-quadratic value function in round l obtained by solving for equilibrium trade in rounds $l+1$ through T, given the quasilinear-quadratic utility (2).

1.3 Trading against Price Impact

As is well known, the multiplicity of equilibria in games with demands or supplies as strategies makes modeling strategic behavior challenging, even in a static setting. As is standard in the literature, we focus on the linear equilibrium, which is robust to adding uncertainty in trade (e.g., Klemperer and Meyer (1989), Vayanos (1999)), which is unique. A profile of demand functions $\{\Delta_{t,l}^i(\cdot)\}_{i=1}^I$ is a *(robust) Nash equilibrium* in round l if $\{\Delta_{t,l}^i(\cdot)\}_{i=1}$ is a Nash equilibrium that is robust to adding noise in demand at l, where noise is independent across agents.[16]

In this paper, we use an alternative (equivalent) representation of trading behavior in the demand game, which links equilibrium in a double auction with practitioners' representation of trading and markets, is more tractable, and applies to the existing models based on the demand game. In a game with demands or supplies as strategies, it is useful to conceptualize a trader as trading against a residual market. From the perspective of trader i, in each round l, the demand schedules of traders $j \neq i$ and the market clearing condition define a residual supply faced by i with slope λ_l^i, which measures *price impact* of trader i. Lemma 1 shows that equilibrium (prices and trades) in a (dynamic) double auction can then be characterized by two conditions on traders' demand schedules and price impacts: (i) each trader submits a demand schedule that equalizes his marginal utility

[16] A profile of demand functions $\{\Delta_{t,l}^i(\cdot)\}_{i=1,l=1}^{I,T}$ is a *(robust) Subgame Perfect Nash equilibrium* if, for any $l = 1, ..., T$ and any subgame starting at l, $\{\Delta_{t,l'}^i(\cdot)\}_{i=1,l'\geq l}$ is a Nash equilibrium that is robust to adding noise in demand at l, where noise is additive and independent across agents and rounds. Treating negligible noise uncertainty as part of the description of the game, "robust" can be dropped from "robust Nash."

and the marginal payment, given his assumed price impact, and (ii) the price impact assumed by each trader is correct (i.e., it equals the slope of the residual inverse supply defined by aggregation of the schedules submitted by other traders). Consider the first-order condition for trader i who assumes that his price impact is λ_l^i. By equalizing, for each price $p_{t,l}$, his round$-l$ marginal utility with marginal payment (or revenue) given the assumed λ_l^i,

$$\frac{\partial V_l^i(\theta_{t,l-1}^i + \Delta_{t,l}^i)}{\partial \theta_{t,l}^i} = p_{t,l} + \lambda_l^i \Delta_{t,l}^i, \tag{4}$$

the trader can then construct his schedule of quantities demanded for each price (Figure 2). Let $\Delta_{t,l}^i\left(\cdot, \lambda_l^i\right)$ be trader i's demand optimal given his assumed price impact λ_l^i, defined by condition (4) for all prices.

Lemma 1 (Trading against Price Impact) *A profile of demand schedules and price impacts* $\{\Delta_{t,l}^i\left(\cdot, \lambda_l^i\right), \lambda_l^i\}_{i=1}^I$ *is a (robust) Nash equilibrium in round l if, and only if,*

(i) each $i \in I$ submits the demand schedule $\Delta_{t,l}^i\left(\cdot, \lambda_l^i\right)$ given his assumed price impact, such that

(ii) $\lambda_l^i = -(\sum_{j \neq i}(\partial \Delta_{t,l}^j(\cdot, \lambda_l^j)/\partial p))^{-1}$.

Note that the only information any trader i needs to respond optimally to all prices – not just the equilibrium price – and to arbitrary bids of others – and not just the equilibrium bids – is, apart from his own marginal utility, his own price impact λ_l^i. In particular, no information about the number, let alone the utility functions, identities, or trading strategies of other traders, is required. Appendix C shows that the model of behavior based on the demand game has a dual game-theoretic and general-equilibrium representation; this establishes a relation between competitive and non-competitive predictions (both strategic and general-equilibrium based on rational-expectations).

[Figure 2 here]

By capturing "trading against price impact" (slope taking behavior), the alternative characterization of equilibrium in a double-auction in Lemma 1 fits the practice of institutional traders who estimate their price impact functions treated as sufficient statistics for the payoff-relevant information about the residual market against which they each trade. This paper determines price impact functions as part of a Nash equilibrium, through a fixed-point condition (ii). The linear-in-trade, possibly time-dependent price impact function is the predominant assumption among practitioners (the "quadratic cost model"). Our model predictions are consistent with the stylized facts about the functional form of the empirical price impact functions and price effects of trades (Propositions 1 and 3).[17] For an overview, see, for example, Almgren and Chriss (2000) and Almgren at al. (2005), whose models are the basis of Citigroup's Best Execution Consulting Services software, or the *Handbook* chapter by Vayanos and Wang (2013).

In the quadratic utility (CARA-Normal) setting, the price impact is independent of the equilibrium quantity. The selection of Nash equilibrium defined by conditions (i) and (ii) exists outside of quadratic utilities, *unlike the selection of a linear (robust or Bayesian) Nash equilibrium* (used, e.g., by Kyle (1989) and Vayanos (1999)). It can thus be used to extend the predictions from this paper to a more general theory of thin markets, with quasilinear asymmetric utilities and *non-linear* bid schedules.[18]

2 Model Predictions

2.1 One-Round Market (l=T)

From Equation (2) and the first-order condition (4), $\bar{a} - \bar{\alpha}\sigma^2(\theta^i_{t,T-1} + \Delta^i_{t,T}) = p_{t,l} + \lambda^i_T \Delta^i_{t,T}$, trader i's best-response schedule given his assumed price impact λ^i_T is given by

$$\Delta^i_{t,T}(\cdot, \lambda^i_T) = (\lambda^i_T + \bar{\alpha}\sigma^2)^{-1}(\bar{a} - \bar{\alpha}\sigma^2\theta^i_{t,T-1} - p_{t,T}). \tag{5}$$

Given the profile of demands submitted by traders $j \neq i$, $\{\Delta^j_{t,T}\left(\cdot, \lambda^j_T\right)\}_{j\neq i}$, with slopes $\partial\Delta^j_{t,T}(p)/\partial p = -(\lambda^j_T + \bar{\alpha}\sigma^2)^{-1}$, the equilibrium price impact of trader i is by Lemma 1 characterized as

$$\lambda^i_T = \left(\sum_{j\neq i}\left(\lambda^j_T + \bar{\alpha}\sigma^2\right)^{-1}\right)^{-1}. \tag{6}$$

[17] Let us observe that the empirically documented concavity of price impact function as a function of trade size is implied by our characterization of dynamic equilibrium and, in fact, any model with non-stationary price impact in which larger trades are associated with smaller price impact.

[18] Weretka (2011) applies Lemma 1. The key insight from Lemma 1 is the game-theoretic definition: Formulating fixed-point conditions on *bid functions and slopes* (or, slope functions) rather than *levels* (prices, trades and price impacts, as in a general equilibrium) gives a game-theoretic solution concept that refines the multiplicity of (Subgame Perfect) Nash equilibria. The selection comes from the requirement that traders respond optimally to all prices, which refines the demand slopes and pins down the price impacts. (See Appendix C.)

By the equilibrium symmetry, $\lambda_T^i = \lambda_T^j \equiv \lambda_T$, and the unique solution to (6) gives the equilibrium price impact at T,[19]

$$\lambda_T^* = \frac{1-\gamma}{\gamma}\bar{\alpha}\sigma^2 > 0. \tag{7}$$

Given the finite number of trading partners ($\gamma < 1$) and effective risk aversion ($\bar{\alpha} > 0$), in equilibrium, each trader faces an imperfectly elastic residual supply. Thus, price-taking behavior does not satisfy equilibrium conditions. Equation (6) captures that a trader's price impact results from the decreasing marginal utility of other traders ($\bar{\alpha} > 0$); buying or selling affects a trader's (decreasing) marginal utility. Price impact of an agent's trade corresponds to the price concessions required by the other agents to trade and for the market to clear. Price impact strictly decreases in the number of traders I (market depth γ). In a larger market, the effect of any agent's trade on the average marginal utility of other traders (and, hence, price) weakens, as other traders absorb smaller fractions of the trades.

In response to price impact, each trader reduces his trade relative to the competitive trade – he demands less for any price (cf. Figure 2),

$$\Delta_{t,T}^{i*}(\cdot, \lambda_T^{i*}) = \gamma \underbrace{\left(\bar{\alpha}\sigma^2\right)^{-1}\left(\bar{a} - \bar{\alpha}\sigma^2\theta_{t,T-1}^i - p_{t,T}\right)}_{\text{Competitive demand schedule}}. \tag{8}$$

Geometrically, the equilibrium bid is a rotation of marginal utility around the zero-trade point. With identical risk aversion, buyers and sellers reduce demands and supplies by the same factor γ and, consequently, thin markets clear at the price that would also clear the market with price takers, equal to the average marginal utility (and the fundamental value),[20] $p_{t,T}^* = \bar{v}_t(\bar{\theta})$.

2.2 Dynamic Equilibrium in Thin Markets

Consider a market in which agents can trade for $T > 1$ rounds in each period t. Given that there are no shocks within the period, and hence the competitive price is constant throughout, it would be

[19] In the symmetric model, $\lambda_l^i = -(1-\gamma)\left(\partial\Delta_{t,l}^i(\cdot)/\partial p\right)^{-1}$, as by the symmetry of equilibrium, bid slopes coincide, $\partial\Delta_l^i(p)/\partial p = \partial\Delta_l^j(p)/\partial p$ for all $j \neq i$, and hence the system of I harmonic-mean conditions on price impacts $\{\lambda_l^i = -(\sum_{j\neq i}(\partial\Delta_{t,l}^j(\cdot)/\partial p))^{-1}\}_{i=1}^I$ becomes $\lambda_l^i = \lambda_l^j = \lambda_l = -(1/(I-1))\left(\partial\Delta_{t,l}^i(\cdot)/\partial p\right)^{-1}$. The non-existence of equilibrium with two traders is standard (e.g., Kyle (1989), Klemperer and Meyer (1989) with a vertical demand, and Vayanos (1999)). Alternatively, the outcome for markets with two traders can be interpreted as an equilibrium in which price impacts are infinite, in which case no trade is optimal.

[20] The price result relies on the symmetry of utilities. Market thinness *per se* does not necessarily affect equilibrium prices; yet, as we show, it alters trading, efficiency, how market prices respond to shocks, and valuation of assets that are not traded. The price result, which stems from the bilaterally oligopolistic market structure, enables isolation of the new mechanisms due to price impact without having to deal with additional effects due to price changes.

optimal for price-taking traders to sell all of their holdings θ^i_{t-1} to buy $\bar{\theta}$ in the first trading round, $l = 1$, and asset allocation would be Pareto efficient. With gains to trade exhausted in the first round, no trade would subsequently take place. By contrast, in a thin market, agents trade slowly – the equilibrium strategy involves breaking up an order into smaller trades in response to price impact, which is strictly positive in all rounds.

Proposition 1 (Dynamic Equilibrium) *In every trading round $l = 1, ..., T$, the equilibrium trade of trader i is*

$$\Delta^{i*}_{t,l} = \gamma(\bar{\theta} - \theta^{i*}_{t,l-1}) \tag{9}$$

and price impact is

$$\lambda^*_l = \frac{1-\gamma}{\gamma} \underbrace{(1-\gamma)^{2(T-l)}\bar{\alpha}}_{\text{Effective risk aversion at } l} \sigma^2 > 0. \tag{10}$$

In every round l, traders sell a fraction γ of (the remaining part of) their initial holdings $\theta^i_{t,l-1}$ and buy γ of the average holdings $\bar{\theta}$. Consequently, the risky part of holdings is a convex combination of the initial and average holdings, $\theta^{i*}_{t,l} = (1-\gamma)\,\theta^{i*}_{t,l-1} + \gamma\bar{\theta}$, $l = 1, ..., T$, and traders' risky holdings are heterogeneous in every round (Figure 3A).

[Figure 3 here]

With T trading rounds, idiosyncratic risk is not fully diversified. The speed of trade, equal to γ, is independent of risk aversion $\bar{\alpha}$ or asset riskiness σ^2: Higher effective risk aversion increases gains to trade, thus encouraging more aggressive trading, but it also amplifies price impact, thus making interactions less competitive and reducing the trade. With symmetric quadratic utilities, these two effects offset exactly. Thus, even if large institutional traders are almost risk-neutral, as is often assumed in the literature, they choose to trade slowly. This holds despite the market-clearing price being constant and equal to the fundamental value, $p^*_{t,l} = \bar{v}_t(\bar{\theta})$, $l = 1, ..., T$.[21] Order break-up of a block of shares into smaller orders, which are then traded sequentially, is a common practice among large investors. An average block in small companies amounts to two or three times the daily trade

[21] The result might seem reminiscent of predictions for the classical durable-good monopoly, but the economic mechanism is novel. With bilateral price impact, delay in trade is optimal despite the absence of discounting or heterogeneous beliefs about fundamentals, in deterministic (including constant) price settings as well as with information disclosure. Since, in response to their market power at $l < T$, traders reduce their orders in that round, the outcome is not efficient and gains to trade are not exhausted. This then leads to more trade in subsequent rounds.

volume, and even in the largest companies, an average block takes up 25% of the daily volume.[22] In our model, order break-up ("slow trading") is the equilibrium strategy for handling large orders in thin markets.

Thus, unlike in a competitive market, trade occurs in several rounds, even if no new gains to trade are generated by endowment shocks and no information about the asset dividend is revealed between rounds. In a thin market, the equilibrium trading strategy is uniquely pinned down even without introducing any preference for urgency. Market thinness provides an alternative to discounting meaning to "time is money": Traders trade whenever they have a trading opportunity.

2.3 Origins of Price Impact

In markets with strategic buyers and sellers, multiple trading opportunities introduce a new source of market power. Market power in a static market (or, at T) results solely from the traders' decreasing marginal utility (or, effective risk aversion; $\bar{\alpha} > 0$). The non-competitiveness of dynamic markets is, in turn, governed by a dynamic mechanism: *Future market thinness begets current market thinness*; this holds even in deterministic-price settings. To explain this new mechanism, we consider the following counterfactual experiment. Suppose the market at $l = T$ were competitive. What would then be endogenous price impact at $T-1$? Knowing that at T they can trade arbitrary amounts without price concessions, agents would become *effectively risk neutral* at $T-1$ and would be willing to trade arbitrary amounts at the $T-$round price. This is apparent in the value function: The marginal utility at $T-1$, $\partial V_{T-1}(\cdot)/\partial\theta^i$, would be perfectly elastic and equal to the $T-$round price. Since, on the equilibrium path, risk from any current trade would be fully diversified before assets pay, utility at the time of payment would then be independent from trade at $T-1$. In the absence of convexity of utility $V_{T-1}(\cdot)$ in $\theta^i_{t,T-1}$, the endogenous price impact of all traders at $T-1$ would equal 0, even though traders are strategic at $T-1$.

As shown in Section 2.2, with strategic traders, the equilibrium at T is not competitive. Traders anticipate that, on the equilibrium path, they will only partially diversify at T holdings resulting from $(T-1)-$round trades and, consequently, will be exposed to idiosyncratic risk when assets pay.

[22] Table 1 presents typical figures from the NYSE.

[Table 1 here]

Trade delay as a response to market power is present in other models (e.g., Vayanos (1999)). There, however, shocks to endowments or asymmetric information introduce new gains to trade in each trading round.

The value function at $T-1$ is strictly concave in $\theta^i_{t,T-1}$.[23] This happens even in the absence of any shocks between rounds. In a non-competitive market, a fraction $1-\gamma > 0$ of the post-trade holdings at $T-1$, $\theta^i_{t,T-1}$, survives in the post-trade portfolio at T (at the time of assets' payment) and thus the value function at $T-1$ is strictly concave in $\theta^i_{t,T-1}$ with the effective risk aversion of $(1-\gamma)^2\bar{\alpha}$; in a competitive market, $\theta^i_{t,T} = \bar{\theta}$, and the effective risk aversion at $T-1$ would be zero. At $T-1$, and by a recursive argument, in all rounds $l = 1, ..., T$, effective risk aversion is $(1-\gamma)^{2(T-l)}\bar{\alpha} > 0$ (cf. Equation (10)), traders' demands $\Delta^i_{t,l}(\cdot)$ are downward sloping, and price impacts are positive, λ^*_l.[24] On the other hand, with future trading opportunities, the impact of current trade on the risky holdings at the moment of payment is smaller, effective risk aversion decreases with time to asset payment $T-l$, and agents are willing to absorb trades of others at smaller price concessions. When information about dividends is disclosed only after the last trading round, the increasing over time price impact (Figure 3B) results from the reduced diversification possibilities prior to resolution of risk. More generally, price impact depends on how information about dividends becomes available over the course of trading rounds and can be non-monotone, which we study in Section 4.

2.4 Effective Risk Aversion and Exposure to Future Risk

The parameters of the value function in Equation (2), treated as fixed in the analysis so far, are affected by market thinness. In this section, we determine the parameters in terms of the primitives of the infinite horizon model. Of particular interest is traders' effective risk aversion in the last round $\bar{\alpha}$ – the determinant of the slope of the marginal utility for a risky asset and the equilibrium price impact.

It is useful first to consider the benchmark model of an infinitely lived agent who chooses consumption to maximize the utility in Equation (1) and who does *not* trade in financial markets and, thus, holds his initial holdings θ throughout a lifetime. The marginal utility of the agent in autarky

[23] This can be seen by substituting the (equilibrium) policy functions $w^i_{t,T} = w^i_{t,T-1} + p^*_{l,T}\gamma(\theta^i_{t,T-1} - \bar{\theta})$ and $\theta^i_{t,T} = \gamma\bar{\theta} + (1-\gamma)\,\theta^i_{t,T-1}$ in value function in Equation (2) to obtain the quasilinear-quadratic value function at $T-1$ as a function of holdings at $T-1$.

[24] Bilateral market power is central to the thinness of dynamic markets. As a modeling remark, in the same (perfect foresight) setting, dynamic equilibrium of the Cournot market structure is as in the competitive model: In the Cournot model, the residual supplies of strategic traders are defined by marginal utilities of price-taking traders who would arbitrage any price differentials between rounds. The residual supplies of strategic traders are, thus, perfectly elastic at $T-1$, their price impacts are equal to 0 and allocation is efficient.

(A) is, in terms of consumption good,

$$\frac{\partial V_T^A(\theta)}{\partial\theta} = \underbrace{\frac{d_{t-1}}{r}}_{\bar{a}^A} - \underbrace{\left(\frac{\alpha}{r^2} + \frac{\alpha}{r}\right)}_{\bar{\alpha}^A}\sigma^2\theta. \tag{11}$$

Slope $\bar{\alpha}^A$ reflects the exposure of long-lived shares θ to dividend shocks in *all*, current and future, periods. Its component α/r corresponds to risk associated with current-period shock δ_t while α/r^2 is associated with dividend risk in all subsequent periods. Next, let us consider an infinite horizon model with agents who have access to financial markets (competitive or thin) in which average holdings are $\bar{\theta}$. Since, if endowed with diversified holdings $\theta = \bar{\theta}$, strategic agents and price takers alike choose not to trade on the equilibrium path, their marginal utilities at $\bar{\theta}$ coincide with that of the agent in autarky,

$$\frac{\partial V_T^A(\bar{\theta})}{\partial\theta} = \frac{\partial V_T^{CM}(\bar{\theta})}{\partial\theta} = \frac{\partial V_T(\bar{\theta})}{\partial\theta} \equiv \bar{v}_t(\bar{\theta}) = \frac{d_{t-1}}{r} - \left(\frac{\alpha}{r^2} + \frac{\alpha}{r}\right)\sigma^2\bar{\theta}, \tag{12}$$

where $\partial V_T^{CM}(\cdot)/\partial\theta$ and $\partial V_T(\cdot)/\partial\theta$ stand for the marginal utility in the competitive and the thin-market model, respectively. Thus, regardless of market depth γ, the marginal utility of an agent who can trade in financial markets intersects schedule $\partial V_T^A(\cdot)/\partial\theta$ at $(\bar{\theta}, \bar{v}_t(\bar{\theta}))$. A competitive agent anticipates trading all of his initial holdings θ for the diversified holdings $\bar{\theta}$ at the first opportunity to trade. Since his current holdings θ expose him to dividend risk in period t, *but not* in the following periods, the trader is effectively less risk averse than the autarky agent – the slope of his marginal utility is smaller than in (11),

$$\frac{\partial V_T^{CM}(\theta)}{\partial\theta} = \underbrace{\frac{d_{t-1}}{r} - \frac{\alpha}{r^2}\sigma^2\bar{\theta}}_{\bar{a}^{CM}} - \underbrace{\frac{\alpha}{r}}_{\bar{\alpha}^{CM}}\sigma^2\theta. \tag{13}$$

Due to slow trading, holdings of long-lived assets at t expose a trader to dividend shocks in all future periods, with the exposure decreasing over time. Therefore, the thin-market effective risk aversion $\bar{\alpha}$ in Equation (2) is between those of the competitive and autarky agents,

$$\bar{\alpha} = \xi\frac{\alpha}{r^2} + \frac{\alpha}{r}. \tag{14}$$

Coefficient $\xi = \xi(\gamma, T) \in (0, 1)$ is derived in the proof of Proposition 1. The convexity (14) of a thin-market trader's value function decreases in the speed of trade γ and the number of trading opportunities T, which determine the extent to which idiosyncratic risk can be diversified in markets, as well as riskless interest rate r. Geometrically, the marginal utility of a thin-market trader is a rotation of the marginal utility of an autarky agent and a competitive agent around point $(\bar{\theta}, \bar{v}_t(\bar{\theta}))$, approaching the latter (i.e., $\xi \to 0$) as $\gamma \to 1$ or $T \to \infty$ (Figure 4). Not only does slow trading give rise to price impact within each period t, but it also positively affects its level by increasing traders' risk exposure beyond t. While, in the infinite horizon model, exposure to future risk does not alter the fundamental value, and therefore equilibrium price, it does increase price impact in all rounds.

[Figure 4 here]

Mathematically, the fundamental value $\bar{v}_t(\cdot)$ as the mapping $\mathbb{R} \to \mathbb{R}$ coincides with the marginal utility schedule of the autarky agent $\partial V_T^A(\cdot)/\partial\theta$, with domains of $\partial V_T^A(\cdot)/\partial\theta$ and $\bar{v}_t(\cdot)$ understood as the initial holdings and the diversified holdings, respectively. The fundamental value reflects the value of a risky asset given the aggregate risk in all periods, which cannot be diversified through trade in financial markets; in an efficient allocation, traders are exposed to risk associated with the average portfolio.

2.5 Consumption and Welfare

The slow trading resulting from market thinness has important implications for consumption behavior and welfare. Unlike in a competitive market, the extent of risk sharing through trading in financial markets depends on the number of trading opportunities between asset payments as well as the market depth γ.

Proposition 2 (Consumption and Welfare) *Consider a trader with holdings (w_t^i, θ_t^i) in period t who trades in thin markets with the average holdings $\bar{\theta}$ and assume that the risky holdings are not fully diversified, $\theta_t^i \neq \bar{\theta}$. The trader's consumption c_t^i and expected lifetime utility at t strictly increase in γ and T to their respective competitive levels.*

Market thinness alters consumption and savings behavior in any market in which traders' holdings are not fully diversified. The increased savings are not the standard precautionary effect of self-insurance in reaction to uninsurable idiosyncratic risk in illiquid markets – an individual decision-making phenomenon. Rather, the change in savings behavior results from a general equilibrium effect: Market thinness increases savings of all traders, including those whose initial holdings are below the average and, thus, whose risk exposure due to slow trading is *smaller* than in the competitive model.

Moreover, the limited insurance possibilities do not result from market incompleteness, but rather market thinness itself.

In a competitive market, one round suffices for traders to exhaust gains to trade, whereas in a thin market, the number of trading opportunities T itself has *real* effects – on consumption and welfare. The question arises, what does a 'trading round' represent in our model? Our model defines a trading round not in terms of calendar time, but in terms of what the traders can accomplish. In short, a trading round is the time needed for one price to be formed and all trades to take place at that price.

3 Shock Absorption in Thin Markets

A voluminous body of evidence demonstrates that exogenous shocks in asset supply or demand result in temporary price effects present in addition to their permanent impact on the price (see Footnote 2). In the data, an unanticipated supply shock results in an immediate and significant price drop followed by a partial reversal of the price change in subsequent periods. Even if the shock is pre-announced, the temporary price drop below the long-run level occurs on the actual event date and not on the date of the announcement and attains the long-run value only in subsequent periods. The price behavior is not consistent with the competitive model with deterministic price changes; indeed, price-taking traders could make infinite profits. What should be observed, instead, is that – regardless of when the shock occurs – the price adjusts to the new fundamental value at the shock announcement and remains there.

We show that transitory departures of prices from the fundamental values are an equilibrium response to shocks in thin markets. An unanticipated exogenous supply or demand shock has two effects on prices: a *fundamental effect*, which is permanent, and a *liquidity effect*, which is temporary. These two effects differ not only in their origin and persistence, but also in timing of occurrence and magnitude dynamics.

3.1 Transitory and Permanent Price Effects

Consider an unanticipated one-time shock in asset supply: In round $\hat{l}$, an order of $\hat{\theta} \times I > 0$ shares is liquidated by an outside investor in a market with I traders; for example, the shock is an index recomposition or an order placed by an investor who does not monitor prices continuously is not ready to respond to price differentials at any time. By permanently increasing *per capita* holdings of risky assets in the market to $\bar{\theta} + \hat{\theta}$, the supply shock increases aggregate risk. Accordingly, the fundamental price effect corresponds to the change in the fundamental value to the post-shock level of the average marginal utility,

$$\Delta^F = \bar{v}_t(\bar{\theta} + \hat{\theta}) - \bar{v}_t(\bar{\theta}) = -\left(\frac{\alpha\sigma^2}{r^2} + \frac{\alpha\sigma^2}{r}\right)\hat{\theta}. \tag{15}$$

Since all traders learn about the shocks in round $\hat{l}$, the change (15) in the post-shock fundamental value occurs at $\hat{l}$. The fundamental effect is also present in a model with price-taking agents, as long as their mass is finite so that the *per capita* shock $\hat{\theta}$ is not negligible.

It is the liquidity effect, which temporarily lowers the price at $\hat{l}$ below $\bar{v}_t(\bar{\theta} + \hat{\theta})$, that is due to the non-competitive nature of trade. Observe that in any round, with or without the shock, on the equilibrium path, each trader equalizes round$-l$ marginal utility and his marginal revenue from (payment for) a share. This also holds on average in round $\hat{l}$,

$$\frac{1}{I}\sum_{i=1}^{T} \frac{\partial V_{\hat{l}}(\theta^{i*}_{t,\hat{l}})}{\partial \theta^{i}_{t,\hat{l}}} = \bar{v}_t = p_{t,\hat{l}} + \lambda^*_{\hat{l}}\frac{1}{I}\sum_{i\in I}\Delta^{i*}_{t,\hat{l}}, \tag{16}$$

where $\bar{v}_t \equiv \bar{v}_t(\bar{\theta} + \hat{\theta})$. Without the shock (i.e., $\hat{\theta} = 0$), the net trade $(1/I)\sum_{i\in I}\Delta^i_{t,l}$ is equal to zero by market clearing, and the price equals the fundamental value. With a positive net supply of risky assets $\hat{\theta}$ at $\hat{l}$, traders are on average buying, and the average marginal payment (the right hand side of (16)) exceeds the market-clearing price by $\lambda^*_{\hat{l}}\hat{\theta} > 0$. The equilibrium price is below the average marginal utility as in a monopsony, $p_{t,\hat{l}} = \bar{v}_t - \lambda^*_{\hat{l}}\hat{\theta}$. The liquidity effect is proportional to price impact,

$$\Delta^L \equiv -\lambda^*_{\hat{l}}\hat{\theta} = -\frac{(1-\gamma)^{2(T-\hat{l})+1}}{\gamma}\bar{\alpha}\sigma^2\hat{\theta}. \tag{17}$$

Why does the liquidity effect not persist as does the fundamental effect? Since no other shocks occur after $\hat{l}$, in all rounds $l > \hat{l}$, the net trade of the strategic traders equals zero by market clearing, and the round$-l$ price attains the post-shock fundamental value $p^*_{t,l} = \bar{v}_t(\bar{\theta} + \hat{\theta})$. Proposition 3 describes the response of equilibrium price to a supply shock.

Proposition 3 (Transitory and Permanent Price Effects) *Following an unanticipated (net) supply shock $\hat{\theta}$ in round $\hat{l}$, equilibrium price adjusts at $\hat{l}$ by $\Delta^F + \Delta^L$. In round $\hat{l}+1$, price reverts by Δ^L to the post-shock fundamental value $\bar{v}_t$ and remains at this level in all subsequent rounds.*

While the magnitude of the fundamental effect Δ^F is independent of the timing of the shock, the liquidity effect Δ^L depends on $\hat{l}$. All of these features of price behavior have been documented (see, e.g., reviews by Duffie (2010), and Gromb and Vayanos (2010))[25] and, indeed, have motivated the methodology to estimate price impact.[26] Transaction price pressure away from the fundamental value is observed empirically even in the most liquid markets ("premium for immediacy"). Figure 5 depicts the time series of trade and price implied by our model for exogenous shock $\hat{\theta}$ in round $\hat{l}$.

[Figure 5 here]

Empirical research on the reaction of markets to shocks distinguishes between *short-* and *long-run market demand* (e.g., Greenwood (2005) and the research following Shleifer (1986)). The short-run (inverse) demand captures the price reaction observed immediately after an exogenous supply shock $\hat{\theta}$, while the long-run demand specifies the price for round$-l$ trade after all price adjustments have taken place. Our model provides an equilibrium microfoundation for such a reduced form representation of markets. The long-run demand is given by the fundamental value function $\bar{v}_t(\bar{\theta}+\hat{\theta})$ (cf. Equation (12)). The short-run demand at l is, in turn, determined as the (*per capita*) sum of demand schedules submitted by strategic traders at l; that is, the inverse of $(1/I)\sum_{i\in I}\Delta^i_{t,l}(\cdot)$. Its construction is depicted in Figure 6.

[Figure 6 here]

In the competitive model, shocks can have only a negligible effect on prices; with price-taking traders, anticipated price differentials would create infinite profit opportunities. Indeed, since the

[25] Brunnermeier and Pedersen (2005) explain price overshooting in a Cournot-based model in terms of "predatory trading." When an investor must quickly liquidate a portfolio, other traders sell and subsequently buy back the asset. This strategy lowers the price at which they can obtain the liquidated portfolio. The mechanism arises due to the presence of "long-run" traders who define a downward-sloping demand, buying assets when they are expensive and selling when assets are cheap. These traders, by assumption, do not take advantage of short-term price differentials. If the traders were optimizing dynamically, overshooting would not arise, for otherwise, the traders could make infinite profits by taking unbounded positions. The economic mechanism behind overshooting present in our model is complementary in that predatory trading does not occur, since all traders have perfect foresight and optimize dynamically. In addition, while predatory trading can result in price overshooting for unanticipated shocks, market thinness also gives rise to delayed (anticipated) overshooting as well as its time dependence.

[26] E.g., the program implemented by Citigroup estimates separately a permanent price impact component ("reflects the information transmitted to the market by the buy/sell imbalance"), which is believed to be roughly independent of trade scheduling; and temporary price impact component ("reflects the price concession needed to attract counterparts within a specified short time interval"), which is highly sensitive to trade scheduling (Almgren et al. (2005)). "The temporary impact affects only the execution price but has no effect on the 'fair value' or fundamental price. In contrast, the permanent impact directly affects the fair value of the security while having no direct effect on the execution price. Thus we can think of the temporary impact as connected to the liquidity cost faced by the agent ..." (Li and Almgren (2011), p.2). "The temporary impact component of cost is interpreted as the additional premium that must be paid for execution in a finite time, above a suitably prorated fraction of the permanent cost" (Almgren (2009), p. 1).

price path within t is deterministic, why do strategic traders not arbitrage the price reversal between $\hat{l}$ and $\hat{l}+1$, as they would in competitive models? In essence, the strategic traders are (on average) buying the *per capita* shock at $\hat{l} > 0$. By increasing buy orders in that round, they would increase the price and render their equilibrium trades less attractive. The potential marginal benefit from arbitrage is exactly offset by the marginal externality of the price increase on the units being traded in equilibrium.[27] Moreover, in contrast to competitive markets, profit opportunities from arbitraging anticipated price differentials by outside capital are only *finite* in a thin market. An unbounded position, or a round-trip involving a purchase at $\hat{l}$ of more shares than the shock $\hat{\theta} \times I$ sold in the next round results in a strictly negative profit. Thus, unlike the competitive model, sufficient fixed entry costs can discourage outside investors from arbitraging the liquidity effect. In practice, entry costs include not only explicit trading costs, but also costs associated with learning and monitoring characteristics of particular stocks.[28]

Once one acknowledges that traders who can place large orders – and these are the traders who determine the arbitrage properties of equilibrium – have price impact, limits to arbitrage arise endogenously. Note that the argument behind no-arbitrage with non-price-taking behavior differs in two ways from that in the competitive model. First, the externality of arbitrage on other trades introduces a difference in arbitrage possibilities between insiders and outsiders. Secondly, profits from arbitrage are bounded for any round-trip trade.

3.2 Time Separation of Trades and Price Effects

Many market events that involve a supply shock are publicly announced prior to their occurrence; for example, inclusions of new stocks into a stock market index or index weight changes typically are. In the data, pre-announced shocks have price effects not only on the day of the announcement,

[27] Suppose, for transparency, that traders' holdings of risky assets are fully diversified so that their only trade is from shock absorption. Suppose that, in round $\hat{l}$ when the shock occurs, an investor increases his trade by ε and sells the same amount in the next rounds. The round-trip trade would have a first-order benefit of the price differential induced by the shock for each share ε, $\lambda^*_{\hat{l}}\hat{\theta} \times \varepsilon$. At the same time, the additional demand created by arbitrage increases the price at $\hat{l}$ by $\lambda^*_{\hat{l}} \times \varepsilon$, which adversely affects the terms of trade for $\hat{\theta}$.

[28] Mitchell, Pedersen and Pulvino (2007) examine price behavior in the convertible bond market in 2005 and around the collapse of LTCM in 1998, and merger targets in the 1987 market crash. During these events, "natural" liquidity providers were themselves forced to liquidate their holdings, which depressed the prices below the fundamental values, despite the fact that there was little change in the overall fundamentals. In the convertible bond markets, the prices deviated from the fundamental values, reaching the maximum discount of 2.7% in 2005 (2.5 standard deviations from the historical average), and 4% in 1998 (4 standard deviations from the average). During the crash of 1987, the median merger arbitrage deal spreads increased to 15.1%. In all episodes, it took several months for traders to increase their capital or for better-capitalized traders to enter. The authors attribute the slow entry to information barriers and the costs of maintaining dormant financial and human capital in a state of readiness when arbitrage opportunities arise.

but also during the period between the announcement and the event date. Specifically, the transitory price drop below the long-run level takes place on the actual event date and not on the date of the announcement, only after which the price attains the long-run value. In addition, the price is depressed between the announcement day and the shock day when the transitory effect peaks. Thus, the observed price effects cannot be attributed to any revelation of information about the fundamental value, which should be incorporated upon the announcement. None of these is explained by a competitive model; pre-announcing the shock does not alter the price adjustment which involves the price change to the post-shock fundamental value at the announcement at which level the price would remain subsequently, regardless of when the shock occurs. In thin markets, trade announcements alter price behavior in periods *other* than the announcement.

Consider a public announcement at $\hat{l}$ that an extra supply of assets $\hat{\theta} \times I$ will be available at $\hat{l}' > \hat{l}$. Since the fundamental effect Δ^F, given by Equation (15), is a function of the (assumed) average holdings after round T, the adjustment to the post-shock level occurs *on the announcement date*. The additional liquidity effect of shock, Δ^L, given by Equation (17) evaluated at $\hat{l}'$, takes place *on the shock date*, $\hat{l}'$. This holds because the liquidity effect is not driven by information about increased aggregate risk in the market, but rather by the impact that absorption of the extra assets has on the average marginal payment, and it is on the event date that the net trade of strategic traders is positive. Pre-announcing a shock in a thin market thus separates the timing of the fundamental and liquidity effects of price adjustment.

In addition to the fundamental and liquidity effects, pre-announcing a shock changes price behavior in all rounds between the shock announcement and occurrence. Anticipation of Δ^L temporarily depresses the price by $\gamma\Delta^L$. The anticipation of the price drop at $\hat{l}'$ lowers traders' willingness to buy the assets in periods prior to $\hat{l}'$. The price response to an announced once-and-for-all shock is depicted in Figure 7. More generally, an anticipated sequence of shocks $\{\hat{\theta}_l \times I\}_{l=1}^{T}$, with the total liquidated portfolio *per capita* being $\hat{\theta} \equiv \sum_{l=1}^{T} \hat{\theta}_l$, has the following effects. As with a single shock, the fundamental value adjusts to the new level $\bar{v}_t \equiv \bar{v}_t(\bar{\theta}+\hat{\theta})$ upon the announcement of the sequence. The fundamental effect is independent of how the portfolio of shocks $\hat{\theta}$ is partitioned into smaller orders. Thus, with price-*taking* traders, the cash obtained by liquidating $\hat{\theta}$ is not affected by the partition. In a thin market, the price path, and hence the revenue obtained from liquidation, do depend on the order of trade sizes – unlike the permanent effect, the temporary effect is not additive. In any round l in which $\hat{\theta}_l \neq 0$, the price departs from the fundamental value by the current liquidity effect $\Delta_l^L = -\lambda_l^* \hat{\theta}_l$. In addition, in every round between the announcement and liquidation of the final order, the contemporaneous liquidity effect is amplified by the fraction γ of all the subsequent liquidity effects. As a result of two countervailing effects of anticipation, for any $h > l$ and order $\hat{\theta}_h$, the impact of the cumulative price effect, γ, is the same regardless of how far in the future the liquidity shock occurs. The farther in the future the liquidity shock occurs, the smaller the fraction of the current round's trade that maintains a lower price until the shock period. On the other hand, anticipation of a liquidity effect influences all prices between the announcement and the shock round, which increases the weight. The lack of incentive to arbitrage deterministic price changes in thin markets extends to anticipated events. Since the traders are, on average, buying on each shock day, the average marginal payment – and the fundamental value with which it coincides – exceeds the equilibrium price.

Corollary 1 (Price Behavior with Anticipated Shocks) *For an anticipated sequence of shocks* $\{\hat{\theta}_l \times I\}_{l=1}^T$, *with the total liquidated portfolio* per capita $\hat{\theta} \equiv \sum_{l=1}^T \hat{\theta}_l$, *the equilibrium price is*

$$p_{t,l}^* = \bar{v}_t + \Delta_l^L + \gamma \sum_{h=l+1}^{T} \Delta_h^L. \tag{18}$$

In sum, just as when shocks are not anticipated, long-run prices are not affected by how the trade is divided into smaller orders or the time at which the trades take place. Nevertheless, the price path in a thin market is sensitive to the partitioning of the trade. This occurs because a future sale depresses prices during the whole period between the announcement and the shock occurrence and because the effects of multiple sales on prices are cumulative.

[Figure 7 here]

3.3 Endowment Shocks

In contrast to competitive markets, in thin markets, price response to endowment shocks differs from the response to exogenous shocks in supply. Specifically, if the shares $I \times \hat{\theta}$ constituted an increase in the endowments of the strategic traders, distributed arbitrarily among the agents, then equilibrium price dynamics would follow that of the fundamental value; only the permanent effect but no temporary effects would be observed. Thus, with endowment shocks, price dynamics is not affected by the timing of the shocks and a full price adjustment occurs on the date of the shocks' announcement. While endowment shocks change *per capita* holdings, as does a supply shock of size $I \times \hat{\theta}$, the net trade of liquidity providers is zero in every round, and the price impact of buyers and sellers does not create a wedge between the equilibrium price and the average marginal utility.

This suggests that traders who liquidate risky positions have incentives to bypass centralized markets by selling shares over the counter and, potentially, avoiding the price concessions resulting from the liquidity effect. Understanding of the price impact associated with bargaining in over-the-counter markets would illuminate investors' incentives to choose to trade over the counter.

4 Disclosure of Information about Fundamentals

In the previous sections, we made a strong assumption that traders learn all the information about dividends only after the last trading round. More realistically, information about fundamentals becomes available to market participants during trading. As this section shows, revelation of fundamental information impacts welfare in thin – but not competitive – markets. To this end, we enrich the information structure by introducing a sequence of public noisy signals about fundamentals d_t.

Let $s_{t,l}$ be the vector of all signals observed by traders between rounds l and $l+1$ of period t. The informativeness and dimensionality of signal vectors $s_{t,l}$ can differ across rounds l, and signals can be correlated within and across rounds. Since, after round $l = T$, traders learn the realization of the current-period dividend, we adopt the convention that δ_t is part of vector $s_{t,T}$. To preserve tractability, and in particular the stationarity across periods, we assume that the *information structure* $\left(\{s_{t,l}\}_{l=1}^{T}\right)$, given by c.d.f. F, is the same in all periods t and jointly Normal, and the marginal distribution of δ_t has zero mean and variance $\sigma^2 > 0$. Let the expectation and variance conditional on the information $(\{s_{t,h}\}_{h=1}^{l-1}, d_{t-1})$ available to traders in round l be $E_l(\cdot) \equiv E(\cdot|\{s_{t,h}\}_{h=1}^{l-1}, d_{t-1})$ and $Var_l(\cdot) \equiv Var(\cdot|\{s_{t,h}\}_{h=1}^{l-1}, d_{t-1})$, respectively, so that $\sigma_l^2 \equiv Var_l(d_t) - Var_{l+1}(d_t) \geq 0$ measures the informativeness of all signals observed between rounds l and $l+1$; $\sum_{l=1}^{T} \sigma_l^2 = \sigma^2$. This class of information structures nests that from the previous sections, where $s_{t,l}$ is uninformative for all $l < T$.

With dividend information revealed during trading, the fundamental value of the risky asset (the marginal utility of an agent who holds the diversified holdings throughout lifetime) is random within period t,

$$\bar{v}_{t,l}(\bar{\theta}) \equiv \frac{\partial V^R_{t,l}(\bar{\theta})}{\partial \theta} = \underbrace{\frac{E_l(d_t)}{r}}_{\bar{a}^R_l} - \underbrace{\left(\frac{\alpha}{r^2} + \frac{\alpha \sum_{h=l}^T \sigma_h^2/\sigma^2}{r}\right)}_{\bar{\alpha}^R_l}\sigma^2\theta. \quad (19)$$

Consequently, apart from dividend risk, $Var_l(d_T)$, traders face price risk $\{Var_l(p_h)\}_{h=l+1}^T$ resulting from the randomness of the conditional expectations $E_l(d_t)$. With price takers, such price risk is neutral for consumption and welfare; traders diversify idiosyncratic risk in the first round and subsequent price variation does not alter the distributions of risky and riskless holdings. With slow trading, the way in which traders learn about dividends affects liquidity and welfare through price and dividend risk.

All of the thin-market mechanisms characterized in previous sections operate in the model with the general information structure (the Appendix presents proofs for the general information structure). In particular, for any information structure F, dynamic equilibrium trading strategy involves the order break-up as described by Proposition 1, and price dynamics coincides with that of the fundamental value (19) in the absence of exogenous supply shocks. Determined by effective risk aversion – a function of dividend and price risk remaining in subsequent rounds – price impact does depend on F. With the general information structure, equilibrium price impact can exhibit *arbitrary, potentially non-monotone* behavior, depending on how the information about fundamentals becomes available throughout trading. Specifically, round$-l$ price impact depends on the extent of the lack of diversification before price and dividend risk is resolved – measured by the weighted informativeness of *future* signals, σ_h^2, $h \geq l$ (see Equation (25) and Lemma 5),

$$\lambda_l^* = \frac{1-\gamma}{\gamma}\underbrace{\left(\frac{r(1-\gamma)^{2(T-l)}}{1+r-(1-\gamma)^{2T}}\sum_{h=1}^T (1-\gamma)^{2h}\frac{\sigma_h^2}{\sigma^2}\frac{\alpha}{r^2} + \sum_{h=l}^T (1-\gamma)^{2(h-l)}\frac{\sigma_h^2}{\sigma^2}\frac{\alpha}{r}\right)}_{\text{Effective risk aversion at } l}\sigma^2 > 0. \quad (20)$$

In a model with price takers, effective risk aversion in round l (the limit of effective risk aversion in (20) as $\gamma \to 1$ equal to $\frac{\sigma_l^2}{\sigma^2}\frac{\alpha}{r}$) is determined exclusively by the informativeness of the signals observed between the current and the next round and, thus, is negligible if the next-round price risk σ_l^2/r is close to zero. By contrast, in markets where agents trade slowly, all of the price risk in subsequent rounds and periods as well as dividend risk contribute to the convexity of round$-l$ value function, and therefore price impact.[29] The contribution of price uncertainty in round $h \geq l$ to

[29] With full disclosure at T, $\sigma_l^2 = 0$ for all $l < T$, price-taking traders are effectively risk neutral in all but the last round. Thus, on the equilibrium path, they arbitrage away any price differential between current and subsequent

the slope of marginal utility in l depends on diversification possibilities between rounds l and $h \geq l$; fraction $(1-\gamma)^{2(h-l)+1}$ of trade at l will remain undiversified at h and the weight associated with σ_h^2 in (20) is exponentially decreasing in $h-l$. Notably, in contrast to models with price-taking liquidity providers (e.g., Grossman and Miller (1988)), with price-making traders, exogenous supply shocks have temporary price effects even when shocks occur at times when no information regarding dividends is revealed, $\sigma_{\hat{l}}^2 = 0$.[30] For any information structure, the fundamental effect $\Delta_{\hat{l}}^F$ of a *per capita* supply shock $\hat{\theta}$ in round $\hat{l}$ reflects the adjustment in the fundamental value (19) due to the change of average holdings in all future rounds and periods; generically in information structure, it is decreasing over time. The corresponding temporary liquidity effect $\Delta_{\hat{l}}^L = -\lambda_{\hat{l}}^* \hat{\theta}$ depends on price impact on the shock day, given by Equation (20), and hence the number of trading opportunities left and information disclosure in the contemporaneous and all future rounds.

Insofar as disclosure of information is a choice variable, which dynamic strategy to disclose information F is best for markets? In thin markets, the objectives of stabilizing liquidity and welfare maximization yield distinct recommendations, which using characterization (20), we illustrate in Figures 8 and 9.

[Figure 8 here]

Infrequent disclosure of fundamental information, such as the information structure from the previous sections, gives rise to temporary endogenous market freezes – episodes with abnormally low market liquidity prior to disclosure. That is, as long as *public* information is not revealed in a continuous way – a dividend payment is an example of a discontinuous informational event – there exist trading rounds in which market depth γ is bounded away from zero, as $T \to \infty$, even without asymmetric information (Figure 8).[31] Higher frequency of trade relative to asset payments,

trading rounds, which makes price impacts equal to zero for all other traders. The dependence of price impact in thin markets on future as well as contemporaneous uncertainty arises because of its dynamic origins, as captured by endogenous convexification of value function (Section 2.3).

[30] In the Grossman and Miller (1988) model, the assumption $Var_1(E_2\bar{P}_2) \equiv \sigma_2^2 > 0$ is necessary for a round-trip trade to have an effect on the equilibrium price. In particular, in the version of the model in which all of the information regarding dividends is revealed after the last trading round, equilibrium prices in the first two rounds are equal $p_1 = p_2$ (see Equation (12) in Grossman-Miller). With strategic traders, non-zero variance is not necessary for price effects to occur. In short, the central insights from our thin-markets model are implications of slow trading on the equilibrium path and would not be present in the Grossman-Miller model, including temporary departures of equilibrium price from the fundamental value, real effects of announcements, the impact of future uncertainty on current liquidity (in contrast to markets with price takers or one-sided market power, with strategic traders, price impact is strictly positive even without any price risk), higher savings, and lower welfare.

[31] The stationarity assumption requires that signal informativeness per trading round goes to zero uniformly and so does their weighed sum. Thus, the prediction of zero price impact is an implication of the assumption that the informativeness becomes negligible in each round. In a model where information revelation is infrequent (i.e., there is a finite and fixed number of times when the information is revealed), the signal informativeness, and therefore price

measured by T, introduces an exponential-in-time component into equilibrium price impact, even if the number of trading opportunities T is large so that markets are essentially competitive, except prior to disclosure. Gradual disclosure smooths liquidity, as it lowers asset riskiness in subsequent rounds and, hence, effective risk aversion.

Stabilizing market liquidity ($\lambda_l = \lambda_{l'}$ for all l, l') requires an increasing signal informativeness. With equally informative signals in each round, $\sigma_l^2 = \sigma^2/T$ for all l, price impact monotonically decreases over time so that markets are less liquid in earlier rounds (Figure 9).

[Figure 9 here]

Our model also offers strong normative predictions about information disclosure, taking *ex ante* welfare as the objective. We say that information structure F' *withholds information* relative to F if the residual uncertainty $Var_l(d_t)$ associated with F' is not smaller than under F in each round l and is strictly higher for some.

Proposition 4 (Information Disclosure and Welfare) *Consider a trader with holdings (w_t^i, θ_t^i) in period t who trades in markets with the average holdings $\bar{\theta}$.*

(1) In the limit as $\gamma \to 1$, consumption c_t^i and expected lifetime utility of each trader at t are neutral with respect to dynamic strategies to disclose information F.

(2) Assume $\gamma < 1$. Consider information structures F and F', such that F' withholds information relative to F. Then, under F', consumption c_t^i and expected lifetime utility of each trader at t are strictly higher for all traders whose portfolios are not fully diversified (i.e., $\theta_t^i \neq \bar{\theta}_t$).

While information withholding induces only a partial order over the set of information structures, the model identifies the welfare-maximizing information structure as one in which information is withheld until after trade in the last round (studied in Section 3); this information structure is strictly preferred by all traders to any other information structure. Information disclosure prior to asset payments lowers the degree to which equilibrium trading strategy enables traders to diversify idiosyncratic risk before price uncertainty is resolved. The welfare implication of disclosure can be seen as a thin-market counterpart of the competitive Hirshleifer effect (Hirshleifer (1971), which concerns full and no disclosure for $\gamma \to 1$. Namely, postponing information helps agents realize gains to trade, given slow trading. Whereas it would be neutral in a competitive market, maximally

impact, in rounds prior to revelation is bounded away from zero and so is price impact.

postponing dynamically is optimal for welfare in a thin market. Since one expects at least one trading round before dividends pay in a financial market setting, in this sense, Hirshleifer effect does not affect welfare in competitive markets; by contrast, in a thin market, it is inherent and arises endogenously because of slow trading. In general, the objectives of maximizing welfare and smoothing liquidity through information disclosure cannot be attained at the same time and, thus, a policy maker necessarily faces a tradeoff.[32] Maximization of *ex ante* welfare, which requires that releases of fundamental information are infrequent, introduces episodes of low liquidity. Moreover, by reducing insurance opportunities, early revelation of information provides additional incentives to increase (precautionary) savings.

5 Cross-Asset Effects

We report cross-asset effects of market thinness. (The Appendix presents all proofs for the general, multiple-asset model.) Consider an economy with $N > 1$ assets whose dividends $d_t = d_{t-1} + \delta_t$ are an $N-$dimensional random vector that follows a random walk, where shocks δ_t are zero-mean and with variance-covariance matrix Σ, jointly Normally distributed and independent across periods (not necessarily across assets). Information structure F admits (an arbitrary number of) possibly correlated public signals for different assets. The vector of the fundamental values of risky assets depends on cross-asset correlations, $\bar{v}_{t,l}(\bar{\theta}) \equiv \frac{1}{r}E_l(d_t) - \frac{\alpha}{r}(\sum_{h=l}^{T}\Sigma_h + \frac{1}{r}\Sigma)\bar{\theta}$.

- *Thin market 'fund separation'*: In the competitive CAPM, traders invest in a combination of the market portfolio and the riskless asset (Two-Fund Separation Theorem). Extended to multiple risky assets, Proposition 1 (with $\bar{\theta}$ interpreted as the market portfolio) provides the thin market counterpart. Order break-up takes an easy-to-execute form. In every trading round l, the risky part of the optimal portfolio is held in the initial and the average portfolios, θ_t^i and $\bar{\theta}$, with the weight assigned to $\bar{\theta}$ monotonically increasing over time, $\theta_{t,l}^i = (1-\gamma)^l\theta_{l-1}^i + (1-(1-\gamma)^l)\bar{\theta}$; the remaining wealth is invested in the riskless asset. Traders thus keep their wealth in endowments, market portfolio and riskless asset. Slow trading gives rise to inefficiency, precautionary savings, and higher consumption and lifetime utility for information structures that withhold information.

- *Cross-market price effects*: They are present so long as the dividend payoffs of stocks are not independent. For example, in a model with disclosure only after T, $\Sigma_T = \Sigma$, the price impact (an

[32] With natural definitions of increasing the number of rounds, given a fixed information structure, more rounds are welfare-improving.

$N \times N$ matrix) is

$$\lambda_l^* = \frac{1-\gamma}{\gamma}(1-\gamma)^{2(T-l)}\bar{\alpha}\Sigma. \tag{21}$$

The impact of sales of asset n on price of asset m is proportional to the covariance of the assets' dividends. The sale of shares of one asset can exert downward or upward pressure on the price of the other assets (e.g., Newman and Rierson (2004); Duffie (2010); Gromb and Vayanos (2010)).

• *Security Market Line and spillover effects*: Without exogenous shocks, expected asset returns are spanned by the riskless return and the return on the average portfolio, as in the competitive model. In thin markets with exogenous supply shocks, the market beta $\beta_l^{\bar{\theta}}$ is not sufficient for the expected returns during shock episodes: On the day of the shock $\hat{\theta}$, the expected returns on any portfolio θ temporarily depart from the Line by the amount of the liquidity effect. To illustrate, when information about dividends is revealed after the last round, for any portfolio θ with equilibrium price $p^\theta = \theta \cdot p_{t,l}^*$, let $R^\theta \equiv \theta \cdot d_t/p^\theta$ denote its per-dollar return, let $R^{\bar{\theta}}$ be the return on the average portfolio $\bar{\theta}$, and let $\beta^{\bar{\theta}}$ be the round-specific market beta for portfolio θ. Beta coefficients are defined in the standard way, $\beta^{\bar{\theta}} \equiv \left(\frac{\theta}{p^\theta} \cdot \Sigma \frac{\bar{\theta}}{p^{\bar{\theta}}}\right) / \left(\frac{\bar{\theta}}{p^{\bar{\theta}}} \cdot \Sigma \frac{\bar{\theta}}{p^{\bar{\theta}}}\right)$ and $\beta^{\hat{\theta}} \equiv \left(\frac{\theta}{p^\theta} \cdot \Sigma \frac{\hat{\theta}}{p^{\hat{\theta}}}\right) / \left(\frac{\hat{\theta}}{p^{\hat{\theta}}} \cdot \Sigma \frac{\hat{\theta}}{p^{\hat{\theta}}}\right)$. The expected returns to θ are

$$E(R^\theta) - r = \beta^{\bar{\theta}}(E(R^{\hat{\theta}}) - r) + \underbrace{\beta^{\hat{\theta}}(E(R^{\hat{\theta}}) - r) - (\beta^{\bar{\theta}} a_1 + \beta^{\hat{\theta}} a_2) cov\left(R^{\bar{\theta}}, R^{\hat{\theta}}\right)}_{\text{Temporary departures}}, \tag{22}$$

where $a_1, a_2 > 0$ are the portfolio-specific constants (Proposition 5, Appendix B).[33] As observed in the data on supply shocks, when asset returns are correlated, overshooting of a return in one asset market "spills over" to other markets (e.g., the survey by Amihud, Mendelson and Pedersen (2005)).

• *Asset Valuation in Thin Markets*: Formalizing appraisal and the *fair value* of assets in thin markets is challenging, as it requires counterfactual reasoning; *assets are often transferred outside of the market or transfer is hypothetical* (e.g., a transfer of a property in the case of a divorce). Techniques used in appraisal businesses and valuation consulting are based on heuristic methods (e.g., Estabrook (1999)); valuation specialists apply an instrument of *blockage discount*, recognized by the IRS since 1937, defined as a "deduction from the actively traded price of a stock because the block of stock to be valued is so large relative to the volume of actual sales on the existing market

[33] The magnitude of the temporary departures of the expected return from the Security Market Line depends on the return correlations of θ and $\hat{\theta}$. In particular, in the model with uncorrelated returns $R^{\bar{\theta}}$ and $R^{\hat{\theta}}$, the expected returns are characterized completely by the two-factor model, where $\beta^{\bar{\theta}}$ determines the long-run expected return and $\beta^{\hat{\theta}}$ corrects for the short-run effect. More generally, temporary deviations depend on the covariance $cov(R^{\bar{\theta}}, R^{\hat{\theta}})$. If the returns on portfolios $\theta, \bar{\theta}$ and $\hat{\theta}$ are positively correlated, component $\beta^{\hat{\theta}}(E(R^{\hat{\theta}}) - r)$ overestimates the temporary departure; hence, the third term in (22).

that the block could not be liquidated within a reasonable time without depressing the market price" (Handbook of Advanced Business Valuation, p. 140). Blockage discounts are employed not only for stocks, but also for real estate, personal property (e.g., collections of art, antiques and manuscripts), and charitable gifts. The discounts have typically been estimated to range between 0 and 15 percent. In a competitive market, an asset can be unambiguously priced; the cash value of a block of shares, $\hat{\theta}\times I$ is determined by the currently observed market prices, $\hat{\theta}\cdot p^*_{t,l}$. In thin markets, the market price no longer reflects the actual amount of cash that would be obtained by selling block $\hat{\theta}$. Our model permits counterfactual asset valuation in thin markets: Determining the value of an asset if it were sold on the market (even though it will *not* be) corresponds to how price impact is determined in our model, through (i) optimization by all traders who correctly recognize their price impact in equilibrium allocations *as well as for additional shares* and (ii) market clearing. Let $p^*_{t,\hat{l}}$ be the observed market price in round $\hat{l}$ and let $p_{t,\hat{l}}$ be the hypothetical price that would be obtained if the block were offered on the market in that round. The loss in value, equal to $\hat{\theta}\cdot(p^*_{t,\hat{l}}-p_{t,\hat{l}})$, where $p^*_{t,\hat{l}}-p_{t,\hat{l}}=\lambda^*_{\hat{l}}\hat{\theta}$ results from the liquidity effect from Proposition 3, gives a blockage discount.

6 Discussion

This paper shows that market thinness affects consumption, savings, and utility, in any market in which traders face idiosyncratic risk. Accounting for the very presence of price impact helps understand temporary and permanent price effects of shocks, limits to arbitrage, cross-market liquidity effects, the existence of asset valuation instruments, and order break-up. Our model shows that the *frequency of trade* and the *information* about fundamentals, along with limited risk-bearing capacity, are key determinants of traders' price impact and welfare. Our analysis also provides a perspective for the predictions of the infinite horizon models of double auction with *stationary* price impact (Vayanos (1999); Kyle, Obizhaeva and Wang (2013); Du and Zhu (2014)). In trading environments where trade is more frequent than payments, price impact (and, hence, liquidity effects of shocks) is inherently non-stationary and can be essentially arbitrary, non-monotone throughout trading, even if fundamental information arrives via equally informative signals across trading rounds. Moreover, higher frequency of trade-to-payments introduces an exponential component into price impact.

The implications of investors' market power are relevant to the much-debated increase in trading frequency associated with institutional trading. The impact of trading frequency on market performance is attracting increasing attention of researchers and regulators.[34] The literature has emphasized the asymmetric information arguments for or against high frequency. Cramton, Budish and Shim (2013) advocate for frequent batch (discrete) auctions – the mechanism studied in our paper – to improve upon continuous limit order book design, as they transform competition on the speed on information into competition on price. The authors assume that large orders are placed at once, as in competitive markets; hence, our results offer complementary insights, given the optimality of order break-up. An early contribution of Vayanos (1999) demonstrates that when information about endowments is private (and independent across traders), higher frequency may decrease liquidity and lower welfare. Du and Zhu (2014) argue that with the arrival of private (interdependent) information, low trading frequency is optimal for scheduled but not continuous arrival.[35] In symmetric information markets like ours, allowing for higher trading frequency increases welfare. We show, however, that, as long as *public* information does not arrive continuously (e.g., a dividend payment is a discontinuous informational event), for some trading rounds, market depth is bounded away from zero, even as the number of trading rounds between asset payments grows to infinity, and even without asymmetric information.[36] Thus, in thin markets, even with arbitrary high trade frequency, discontinuous information arrival itself introduces short episodes of low liquidity, with an otherwise essentially negligible price impact. Given the non-stationary component introduced in asset returns by higher-than-payments trade frequency in thin markets, it would be worthwhile to re-evaluate the welfare impact of the breakdown of market correlations at higher-frequency time horizons documented by of Cramton, Budish and Shim (2013).

[34] The Security and Exchange Commission is evaluating the impact of trading frequencies to develop recommendations concerning regulation of trading (e.g., the speech by the SEC Chair (June 5, 2014) `http://www.sec.gov/News/Speech/Detail/Speech/1370542004312`).

[35] Fuchs and Skrzypacz (2013) argue that, in the presence of adverse selection, a continuous trading design can be improved in welfare terms by closing the market for some (small) time window followed by continuous trading. Biais, Foucault and Moinas (2013) show that the ability to privately acquire information leads to an overinvestment in speed. Again, the thin-market mechanisms in this paper hold even in the absence of private information and, thus, point to additional considerations. Moreover, given that demand schedules enable agents to make choices fully contingent on prices, the *interaction between market power and private information* (and its acquisition) can improve or hinder efficiency under greater frequencies (the results in Rostek and Weretka (2014) suggest that this depends on the type of (interdependence among traders') private information).

[36] Thus, our results point to a welfare-liquidity smoothing tradeoff under symmetric information, while other authors discuss a welfare-liquidity tradeoff in the presence of asymmetric information.

In Vayanos (1999), as the number of trading opportunities grows large, price impact becomes zero in *all* rounds. In each round, price impact is a weighted sum of signal informativeness (variance) in subsequent rounds, with weights decaying in the time distance. The stationarity assumption requires that signal informativeness per trading round goes to zero uniformly as the number of rounds grows to infinity and so does their weighed sum. Thus, the prediction of zero price impact is an implication of the assumption that the informativeness becomes negligible in each round. In a model in which information revelation is infrequent (i.e., there is a finite and fixed number of dates when the information is revealed), the signal informativeness, and therefore price impact, in rounds prior to disclosure, is bounded away from zero, as is price impact. (See also Figure **??**.)

Finally, as this paper shows, in the thin (but not the competitive) markets, it matters for price behavior whether the risky holdings of liquidity providers change through supply or endowment shocks. Namely, endowment shocks result only in permanent (fundamental) effects, and no transitory liquidity effects on the equilibrium price path. Thus, traders who liquidate risky positions may have incentives to bypass centralized markets by selling shares over the counter and potentially avoiding the price concessions resulting from the liquidity effect on the market price of the assets. Understanding of the bargaining in over-the-counter markets, which creates its own price impact, will shed light on investors' incentives to choose to trade over the counter.

Appendix

We derive equilibrium and prove results in a model that encompasses all settings analyzed in the main body of the paper – with multiple assets, arbitrary information disclosure and an arbitrary sequence of unanticipated or anticipated *per capita* shocks $\{\hat{\theta}_l\}_{l=1}^T$ (Appendices A and B). Consider an economy with $N > 1$ assets whose dividends $d_t = d_{t-1} + \delta_t$ are an $N-$dimensional random vector that follows a random walk, where shocks δ_t are zero-mean and with variance-covariance matrix Σ, jointly Normally distributed and independent across periods (not necessarily across assets). In each trading round l, traders submit schedules $\Delta_{t,l}^i$: $R^N \rightarrow R^N$ and the market-clearing price vector $p_{t,l}^*$ is determined by aggregate net demands, $\sum_{i=1}^I \Delta_{t,l}^i(p_{t,l}^*) = 0$. The price impact λ_l^* is an $N \times N$ matrix, element (n, m) of which is the price change of asset m that results from a marginal increase in demand for asset n. Information structure F admits (an arbitrary number of) possibly correlated public signals for different assets. Informativeness of signals observed between rounds l and $l+1$ is measured by a positive semidefinite matrix $\Sigma_l \equiv Var_l(d_t) - Var_{l+1}(d_t)$. Information structure F withholds information relative to F', if it does so for any asset portfolio $\theta \neq 0$.

A. Equilibrium in Thin Markets

Lemmas 2 and 3 characterize equilibrium in the non-stationary trade problem. Lemma 2 shows that, without loss of generality, one can restrict attention to information structures induced by $N-$dimensional signals that are conditionally independent across rounds. Lemma 3 derives equilibrium in the non-stationary trade problem in rounds $l = 1, ..., T$ of each period t, assuming a value function after trade in round T. Lemmas 4 and 5 then determine the coefficients of the value function in terms of primitives, and equilibrium consumption in the stationary consumption problem across periods $t = 1, 2, ...$

We say that the information structures F and F' with corresponding signals $\{s_{t,l}\}_{l=1}^T$ and $\{s'_{t,l}\}_{l=1}^T$ respectively, are *equivalent for* d_t, if the distribution of d_t conditional on $(\{s_{t,h}\}_{h=1}^{l-1}, d_{t-1})$ coincides with the distribution of d_t conditional on $(\{s'_{t,h}\}_{h=1}^{l-1}, d_{t-1})$, in each round $l = 1, 2, ..., T$. In round l, traders observe all signal realizations up to round $l-1$. Recall that $E_l(\cdot) \equiv E(\cdot|\{s_{t,h}\}_{h=1}^{l-1}, d_{t-1})$ and $Var_l(\cdot) \equiv Var(\cdot|\{s_{t,h}\}_{h=1}^{l-1}, d_{t-1})$. Without loss of generality, assume that the variance-covariance matrix of the joint distribution of signals $\{s_{t,h}\}_{h=1}^{l}$ is positive definite for any l (i.e., there are no redundant signals).

Lemma 2 (Simple Informational Strategies) *For every jointly Normal information structure F with signal vectors $\{s_{t,l}\}_{l=1}^{T}$ of arbitrary dimension, for each l, there exists an equivalent for d_t information structure F' with signals $\{\delta_{t,l}\}_{l=1}^{T}$ such that: (1) in each round l, $\delta_{t,l}$ is a jointly Normally distributed $N-$dimensional random vector with a conditional mean of zero, $E_l(\delta_{t,h}) = 0$ for all $h \geq l$; (2) vectors $\delta_{t,l}$ are independent conditionally on information available to traders at l (i.e., for each l, for all $h, k \geq l$, $h \neq k$, $\delta_{t,h}$ and $\delta_{t,k}$ are independent conditionally on $(\{s_{t,h}\}_{h=1}^{l-1}, d_{t-1})$); and (3) $d_t = d_{t-1} + \sum_{h=1}^{T} \delta_{t,h}$.*

Proof 1 (Lemma 2) *For any $\{s_{t,l}\}_{l=1}^{T}$, define $\delta_{t,l} \equiv E_{l+1}(d_t) - E_l(d_t)$. Conditional on $\{s_{t,h}\}_{h=1}^{l-1}$ and d_{t-1}, by the projection theorem, d_t is jointly Normally distributed and thus described completely by the first two moments. Since $E_l(d_t) = d_{t-1} + \sum_{h=1}^{l-1} \delta_{t,h}$ and $Var_l(d_t)$ does not depend on realizations of $(\{s_{t,h}\}_{h=1}^{l-1}, d_{t-1})$, signal profile $(\{\delta_{t,h}\}_{h=1}^{l-1}, d_{t-1})$ is sufficient for $(\{s_{t,h}\}_{h=1}^{l-1}, d_{t-1})$. On the other hand, $(\{\delta_{t,h}\}_{h=1}^{l-1}, d_{t-1})$ can be derived from $(\{s_{t,h}\}_{h=1}^{l-1}, d_{t-1})$. Thus, the signal profiles are equivalent; in particular, the distributions of d_t conditional on either profile have the same moments. Moreover, (1) by the law of iterated expectations, for any $h > l$, $E_l(\delta_{t,h}) = E_l(E_{h+1}(d_t) - E_h(d_t)) = E_l(d_t) - E_l(d_t) = 0$. Being linear functions of jointly Normally distributed signals $\{s_{t,l}\}_{l=1}^{T}$, for any k and h, $\delta_{t,k}$ and $\delta_{t,h}$ are jointly Normally distributed as well; (2) since expectations $\{E_l(d_t)\}_{l=1}^{T+1}$ are conditional on nested information sets, $\delta_{t,h}$ and $\delta_{t,k}$ are conditionally independent; and (3) since after the last trading round T, dividends are known, that is $E(d_t|\{s_{t,h}\}_{l=1}^{T}, d_{t-1}) = d_t$, and $E(d_t|d_{t-1}) = d_{t-1}$ by the random walk assumption, it follows that $d_t = d_{t-1} + \sum_{h=1}^{T} \delta_{t,l}$.*

By Lemma 2, without loss of generality, we focus on simple information structures, which are induced by signals with properties (1)-(3) from Lemma 2. Hereafter, we identify signals $\{s_{t,l}\}_{l=1}^{T}$ with variances $\{\Sigma_l\}_{l=1}^{T}$, where $\Sigma_l \equiv Var_l(d_t) - Var_{l+1}(d_t) = Var(\delta_{t,l})$ measures dividend uncertainty resolved between rounds l and $l+1$ or, given Lemma 2, the informativeness of signal $\delta_{t,l}$. The marginal distribution of $\delta_t = \sum_{l=1}^{T} \delta_{t,l}$ is Normal with variance $\Sigma = \sum_{l=1}^{T} \Sigma_l$. Assume the following value function after trade at T, after assets pay and before consumption takes place,

$$V(w_t^i, \theta_t^i, \bar{\theta}_t, d_t) = -\exp(a_w w_t^i + a_{\theta d}\theta_t^i \cdot d_t + \bar{\theta}_t \cdot A_{\theta\bar{\theta}}\theta_t^i + \frac{1}{2}\theta_t^i \cdot A_{\theta^2}\theta_t^i + \frac{1}{2}\bar{\theta}_t \cdot A_{\bar{\theta}^2}\bar{\theta}_t + c), \tag{23}$$

where w_t^i and θ_t^i are the holdings of riskless and risky assets, respectively, of trader i after trade in round

T, $\bar{\theta}_t \equiv \frac{1}{I}\sum_{i=1}^{I}\theta_t^i$ is the average risky portfolio after round T, $a_w, a_{\theta d} < 0$ and c are scalars, and $A_{\theta\bar{\theta}}, A_{\theta^2}$ and $A_{\bar{\theta}^2}$ are $N \times N$ symmetric matrices, where A_{θ^2} is positive semidefinite. Given value function (23), the *fundamental value* of risky assets in round l, $\bar{v}_{t,l} \in R^N$, is defined as the marginal conditional expected utility of the risky assets in terms of the riskless asset (the consumption good) of an agent who holds the average portfolio throughout the lifetime (see Section 2.4),

$$\bar{v}_{t,l} \equiv D_{\theta_t^i} E_l\left[V\left(\bar{\theta}_t\right)\right]\left(\frac{\partial E_l\left[V\left(\bar{\theta}_t\right)\right]}{\partial w_t^i}\right)^{-1} = \frac{a_{\theta d}}{a_w}(d_{t-1} + \sum_{h=1}^{l-1}\delta_{t,h}) + \frac{a_{\theta d}^2}{a_w}\sum_{h=l}^{T}\Sigma_h\bar{\theta}_{t,h} + \frac{1}{a_w}\left(A_{\theta\bar{\theta}} + A_{\theta^2}\right)\bar{\theta}_t. \quad (24)$$

Lemma 3 (Within-Period Equilibrium) *The profile of trades* $\Delta_{t,l}^{i*} = \gamma(\bar{\theta}_{t,l-1} - \theta_{t,l-1}^{i*}) + \hat{\theta}_l$, *prices* $p_{t,l}^* = \bar{v}_{t,l} - \lambda_l^*\hat{\theta}_l - \gamma\sum_{h=l+1}^{T}\lambda_h^*\hat{\theta}_h$, *and price impact matrices*

$$\lambda_l^* = -\frac{1-\gamma}{\gamma}\left(\frac{(1-\gamma)^{2(T-l)}}{a_w}A_{\theta^2} + \frac{a_{\theta d}^2}{a_w}\sum_{h=l}^{T}(1-\gamma)^{2(h-l)}\Sigma_h\right), \quad (25)$$

for each trader i and each round l of period t, characterize the unique (robust) Subgame Perfect Nash equilibrium.

By Lemma 3, given $\theta_{t,l-1}^i$, trader i's portfolio in round l is $\theta_{t,l}^{i*} = \gamma\bar{\theta}_{t,l-1} + (1-\gamma)\,\theta_{t,l-1}^{i*} + \hat{\theta}_l$.

Proof 2 (Lemma 3) *Consider the last trading round, $l = T$. Given trade in this round $\Delta_{t,T}^i$, the risky portfolio at the moment of dividend payment is $\theta_t^i \equiv \theta_{t,T}^i = \theta_{t,T-1}^i + \Delta_{t,T}^i$ and riskless wealth is $w_t^i \equiv w_{t,T}^i = w_{t,T-1}^i - \Delta_{t,T}^i \cdot p_{t,T}(\Delta_{t,T}^i)$, where $p_{t,T}(\cdot)$ is the residual supply function faced by trader i in round$-T$ equilibrium. Define*

$$X \equiv a_w w_t^i + a_{\theta d}\theta_t^i \cdot d_t + \bar{\theta}_t \cdot A_{\theta\bar{\theta}}\theta_t^i + \frac{1}{2}\theta_t^i \cdot A_{\theta^2}\theta_t^i + \frac{1}{2}\bar{\theta}_t \cdot A_{\bar{\theta}^2}\bar{\theta}_t + c. \quad (26)$$

Maximization of round$-T$ value function (23), $V_T(X) \equiv E_T V = -\exp\left\{E_T X + \frac{1}{2}Var_T X\right\}$ gives the (necessary and sufficient) N first-order conditions: For all $p_{t,T}$,

$$D_{\Delta_{t,T}^i} E_T X + \frac{1}{2} D_{\Delta_{t,T}^i} Var_T X = 0. \quad (27)$$

Since traders submit schedules contingent on prices $p_{t,T}$, uncertainty faced by trader i when choosing trade $\Delta_{t,T}^i$ concerns only d_t and the vectors of derivatives of expected value $E_T X$ and variance $Var_T X$ are

$$D_{\Delta_{t,T}^i} E_T X = -a_w(p_{t,T} + \lambda_T\Delta_{t,T}^i) + a_{\theta d}(d_{t-1} + \sum_{h=1}^{T-1}\delta_{t,h}) + A_{\theta\bar{\theta}}\bar{\theta}_t + A_{\theta^2}(\theta_{t,T-1}^i + \Delta_{t,T}^i), \quad (28)$$

and $D_{\Delta_{t,T}^i} Var_T X = 2a_{\theta d}^2\Sigma_T\left(\theta_{t,T-1}^i + \Delta_{t,T}\right)$. *The first-order conditions (27) determine a linear demand* *schedule submitted by trader i,* $\Delta^i_{t,T}(\cdot):\mathbb{R}^N \to \mathbb{R}^N$*, with slope*

$$D_{p_{t,T}}\Delta^i_{t,T}(\cdot) = (-\lambda_T + \frac{a^2_{\theta d}}{a_w}\Sigma_T + \frac{1}{a_w}A_{\theta^2})^{-1}. \tag{29}$$

By Lemma 1 (straightforwardly extended to economies with exogenous supply shocks), the equilibrium price impact – the slope of a trader's residual supply – is determined by condition $\lambda^*_T = (1-\gamma)(-D_{p_{t,T}}\Delta^i_{t,T}(\cdot))^{-1}$,

$$\lambda^*_T = -\frac{1-\gamma}{\gamma}\left(\frac{a^2_{\theta d}}{a_w}\Sigma_T + \frac{1}{a_w}A_{\theta^2}\right). \tag{30}$$

The first-order conditions (27) *averaged across all traders and the market clearing condition* $\frac{1}{I}\sum_{i=1}^{I}\Delta^i_{t,T}(p^*_{t,T}) = \hat{\theta}_T$ *determine equilibrium prices*

$$p^*_{t,T} = \frac{a_{\theta d}}{a_w}(d_{t-1} + \sum_{h=1}^{T-1}\delta_{t,h}) + \frac{1}{a_w}\left(a^2_{\theta d}\Sigma_T + A_{\theta\bar{\theta}} + A_{\theta^2}\right)\bar{\theta}_t - \lambda^*_T\hat{\theta}_T. \tag{31}$$

Substituting equilibrium prices (31) *into the first-order conditions* (27) *determines equilibrium trades in the last round,* $\Delta^{i*}_{t,T} = \gamma(\bar{\theta}_{t,T-1} - \theta^i_{t,T-1}) + \hat{\theta}_T$ *and i's risky portfolio*

$$\theta^{i*}_t = \theta^i_{t,T-1} + \Delta^i_T = \gamma\bar{\theta}_{t,T-1} + (1-\gamma)\theta^i_{t,T-1} + \hat{\theta}_T. \tag{32}$$

Equations (30), (31) *and* (32) *prove Lemma 3 for round* $l = T$.

Consider round $l < T$*, assuming that the assertion of Lemma 3 holds for rounds* $h > l$*. In round–*l *value function* $V_l \equiv E_l V = -\exp(E_l X + \frac{1}{2}Var_l X)$*,* X *is now a function of trade in round* l*,* $\Delta^i_{t,l}$*, in Equation* (26)*. On the equilibrium path,*

$$w^i_t \equiv w^i_{t,T} = w^i_{t,l-1} - \Delta^i_{t,l}\cdot p_{t,l}\left(\Delta^i_{t,l}\right) - \sum_{h=l+1}^{T}\Delta^{i*}_{t,h}\cdot p^*_{t,h}, \tag{33}$$

where, by Lemma 3, for $h > l$*,*

$$p^*_{t,h} = \frac{a_{\theta d}}{a_w}(d_{t-1} + \sum_{k=1}^{h-1}\delta_{t,k}) + \frac{a^2_{\theta d}}{a_w}\sum_{k=h}^{T}\Sigma_k\bar{\theta}_{t,k} + \frac{1}{a_w}\left(A_{\theta\bar{\theta}} + A_{\theta^2}\right)\bar{\theta}_t - \lambda^*_h\hat{\theta}_h - \gamma\sum_{k=h+1}^{T}\lambda^*_k\hat{\theta}_k \tag{34}$$

and

$$\Delta^{i*}_{t,h} = \gamma(1-\gamma)^{h-l-1}(\bar{\theta}_{t,l} - \theta^i_{t,l-1} - \Delta^i_{t,l}) + \hat{\theta}_h. \tag{35}$$

Risky holdings of trader i *after the last trading round, in terms of round–*l *trade, are*

$$\theta^i_t \equiv \theta^i_{t,T} = (1-(1-\gamma)^{T-l})\bar{\theta}_{t,l} + (1-\gamma)^{T-l}(\theta^i_{t,l-1} + \Delta^i_{t,l}) + \sum_{k=l+1}^{T}\hat{\theta}_k. \tag{36}$$

Maximization of round–l expectation of value function (23), $V_l(X) \equiv E_l V = -\exp(E_l X + \frac{1}{2} Var_l X)$, gives N first-order conditions for trader i in round l: For all $p_{t,l}$,

$$D_{\Delta^i_{t,l}} E_l X + \frac{1}{2} D_{\Delta^i_{t,l}} Var_l X = 0. \tag{37}$$

Expectation of X conditional on round–l information is

$$\begin{aligned} E_l X &= a_w (w^i_{t,l-1} - \Delta^i_{t,l} \cdot p_{t,l}(\cdot) - \sum_{h=l+1}^{T} \Delta^{i*}_{t,h} \cdot E_l\left(p^*_{t,h}\right)) + a_{\theta d} \theta^i_t \cdot (d_{t-1} + \sum_{k=1}^{l-1} \delta_{t,k}) && (38) \\ & + \bar{\theta}_t \cdot A_{\theta\bar{\theta}} \theta^i_t + \frac{1}{2} \theta^i_t \cdot A_{\theta^2} \theta^i_t + \frac{1}{2} \bar{\theta}_t \cdot A_{\bar{\theta}^2} \bar{\theta}_t + c, && (39) \end{aligned}$$

and its derivative with respect to $\Delta^i_{t,l}$ is

$$\begin{aligned} D_{\Delta^i_{t,l}} E_l X &= -a_w (p_{t,l} + \lambda^*_l \Delta^i_{t,l}) + a_{\theta d} (1-\gamma)^{T-l} (d_{t-1} + \sum_{k=1}^{l-1} \delta_{t,k}) && (40) \\ & + (1-\gamma)^{T-l} A_{\theta\bar{\theta}} \bar{\theta}_t + (1-\gamma)^{T-l} A_{\theta^2} \theta^i_t && (41) \\ & + \gamma \sum_{h=l+1}^{T} (1-\gamma)^{h-l-1} (a_{\theta d}(d_{t-1} + \sum_{k=1}^{l-1} \delta_{t,k}) + (a^2_{\theta d} \sum_{k=h}^{T} \Sigma_k \bar{\theta}_{t,k} + (A_{\theta\bar{\theta}} + A_{\theta^2}) \bar{\theta}_t)) && (42) \\ & - \sum_{h=l+1}^{T} \gamma (1-\gamma)^{h-l-1} (a_w \lambda^*_h \hat{\theta}_h + a_w \gamma \sum_{k=h+1}^{T} \lambda^*_k \hat{\theta}_k). && (43) \end{aligned}$$

Using that

$$\gamma \sum_{h=l+1}^{T} (1-\gamma)^{h-l-1} = 1 - (1-\gamma)^{T-l}, \tag{44}$$

$$\sum_{h=l+1}^{T} \gamma (1-\gamma)^{h-l-1} \sum_{k=h}^{T} \Sigma_k \bar{\theta}_{t,k} = \sum_{h=l+1}^{T} \left(1 - (1-\gamma)^{h-l}\right) \Sigma_h \bar{\theta}_{t,h}, \tag{45}$$

$$\sum_{h=l+1}^{T} \gamma (1-\gamma)^{h-l-1} (\lambda^*_h \hat{\theta}_h + \gamma \sum_{k=h+1}^{T} \lambda^*_k \hat{\theta}_k) = \gamma \sum_{h=l+1}^{T} \lambda^*_h \hat{\theta}_h, \tag{46}$$

gives

$$\begin{aligned} D_{\Delta^i_{t,l}} E_l X &= -a_w (p_{t,l} + \lambda^*_l \Delta^i_{t,l}) + a_{\theta d} (d_{t-1} + \sum_{k=1}^{l-1} \delta_{t,k}) + a^2_{\theta d} \sum_{h=l+1}^{T} (1 - (1-\gamma)^{h-l}) \Sigma_h \bar{\theta}_{t,h} && (47) \\ & + A_{\theta\bar{\theta}} \bar{\theta}_t + A_{\theta^2} \left(\left(1 - (1-\gamma)^{T-l}\right) \bar{\theta}_t + (1-\gamma)^{T-l} \theta^i_t\right) - a_w \gamma \sum_{h=l+1}^{T} \lambda^*_h \hat{\theta}_h. && (48) \end{aligned}$$

Variance of X conditional on round–l information is

$$Var_l X = Var_l \left(a_{\theta d} \left(\sum_{h=l}^{T} \theta^i_t \cdot \delta_{t,h} - \sum_{h=l+1}^{T} \Delta^{i*}_{t,h} \cdot \sum_{k=l}^{h-1} \delta_{t,k} \right) \right). \tag{49}$$

Since $\sum_{h=l+1}^{T}(\Delta_{t,h}^{i*}\cdot\sum_{k=l}^{h-1}\delta_{t,k}) = \sum_{k=l}^{T-1}(\delta_{t,k}\cdot\sum_{h=k+1}^{T}\Delta_{t,h}^{i*})$*, it follows that*

$$Var_l X = Var\left(a_{\theta d}\sum_{k=l}^{T}\left(\theta_t^i - \sum_{h=k+1}^{T}\Delta_{t,h}^{i*}\right)\cdot\delta_{t,k}\right) = Var(a_{\theta d}\sum_{k=l}^{T}\theta_{t,k}^i\cdot\delta_{t,k}) = a_{\theta d}^2\sum_{k=l}^{T}\theta_{t,k}^i\cdot\Sigma_h\theta_{t,k}^i, \tag{50}$$

where the last equality uses the conditional independence of $\delta_{t,k}$ *for all* $k \geq l$*. Consequently,*

$$D_{\Delta_{t,l}^i} Var_l X = 2a_{\theta d}^2\sum_{h=l}^{T}(1-\gamma)^{h-l}\,\Sigma_h\theta_{t,h}^i. \tag{51}$$

Substituting derivatives of expectations (48) *and variance* (51) *into the first-order conditions* (37)*,*

$$0 = -a_w(p_{t,l} + \lambda_l^*\Delta_{t,l}^i) + a_{\theta d}(d_{t-1} + \sum_{h=1}^{l-1}\delta_{t,h}) + a_{\theta d}^2\sum_{h=l+1}^{T}(1-(1-\gamma)^{h-l})\Sigma_h\bar{\theta}_{t,h} \tag{52}$$

$$+A_{\theta\bar{\theta}}\bar{\theta}_t + A_{\theta^2}((1-(1-\gamma)^{T-l})\bar{\theta}_t + (1-\gamma)^{T-l}\,\theta_t^i) - a_w\gamma\sum_{h=l+1}^{T}\lambda_h^*\hat{\theta}_h + a_{\theta d}^2\sum_{h=l}^{T}(1-\gamma)^{h-l}\,\Sigma_h\theta_{t,h}^i. \tag{53}$$

Averaging the first-order conditions (52) *across all traders and using market clearing* $\frac{1}{I}\sum_{i=1}^{I}\Delta_{t,l}^i = \hat{\theta}_l$ *determines round–*l *equilibrium prices*

$$p_{t,l}^* = \frac{a_{\theta d}}{a_w}(d_{t-1} + \sum_{h=1}^{l-1}\delta_{t,h}) + \frac{a_{\theta d}^2}{a_w}\sum_{h=l}^{T}\Sigma_h\bar{\theta}_{t,h} + \frac{1}{a_w}(A_{\theta\bar{\theta}} + A_{\theta^2})\,\bar{\theta}_t - \lambda_l^*\hat{\theta}_l - \gamma\sum_{h=l+1}^{T}\lambda_h^*\hat{\theta}_h \tag{54}$$

$$= \bar{v}_{t,l} - \lambda_l^*\hat{\theta}_l - \gamma\sum_{h=l+1}^{T}\lambda_h^*\hat{\theta}_h. \tag{55}$$

The first-order conditions (52) *determine a linear demand schedule* $\Delta_{t,l}^i(\cdot) : \mathbb{R}^N \to \mathbb{R}^N$ *submitted by trader* i*, the slope of which is*

$$D_{p_{t,l}}\Delta_{t,l}^i(\cdot) = \left(-\lambda_l + \frac{(1-\gamma)^{2(T-l)}}{a_w}A_{\theta^2} + \frac{a_{\theta d}^2}{a_w}\sum_{h=l}^{T}(1-\gamma)^{2(h-l)}\,\Sigma_h\right)^{-1}. \tag{56}$$

By Lemma 1, condition $\lambda_l^* = (1-\gamma)(-D_{p_{t,l}}\Delta_{t,l}^i(\cdot))^{-1}$ *determines the equilibrium price impact in round* l*,*

$$\lambda_l^* = -\frac{1-\gamma}{\gamma}\frac{1}{a_w}\left(A_{\theta^2}\,(1-\gamma)^{2(T-l)} + a_{\theta d}^2\sum_{h=l}^{T}(1-\gamma)^{2(h-l)}\,\Sigma_h\right). \tag{57}$$

Substituting (55) *and* (57) *into* (52) *and using* $\bar{\theta}_{t,h} - \theta_{t,h}^{i*} = (1-\gamma)(\bar{\theta}_{t,h-1} - \theta_{t,h-1}^{i*})$ *for* $h > l$ *gives equilibrium trade,* $\Delta_{t,l}^{i*} = \gamma(\bar{\theta}_{t,l-1} - \theta_{t,l-1}^i) + \hat{\theta}_l$*. It follows that risky holdings after round* l *are* $\theta_{t,l}^{i*} = \gamma\bar{\theta}_{t,l-1} + (1-\gamma)\theta_{t,l-1}^i + \hat{\theta}_l$*.*

Lemma 4 (Bellman Equation) *Assume that the Bellman equation that defines the value function after* *assets pay can be written as*

$$V = \max_{c_t^i} \left\{ -\exp\left(-\alpha c_t^i\right) - \beta \exp\left(-a_w c_t^i + Y\right) \right\}, \tag{58}$$

where $a_w < 0$ *and* Y *does not depend on* c_t^i*. Then, value function after assets pay is given by*

$$V = -\exp\left(\frac{\alpha}{\alpha - a_w} Y\right) \left(\exp\left(-\frac{\alpha}{\alpha - a_w} \ln \frac{\alpha}{-\beta a_w}\right) + \beta \exp\left(-\frac{a_w}{\alpha - a_w} \ln \frac{\alpha}{-\beta a_w}\right)\right). \tag{59}$$

Proof 3 (Lemma 4) *The first-order condition of the Bellman equation* (58) *with respect to consumption* c_t^i *gives policy function*

$$c_t^i = \frac{1}{\alpha - a_w} (\ln \frac{\alpha}{-\beta a_w} - Y), \tag{60}$$

which substituted into (58) *gives the value function*

$$\begin{aligned} V &= -\exp\left(\frac{\alpha}{\alpha - a_w} Y - \frac{\alpha}{\alpha - a_w} \ln \frac{\alpha}{-\beta a_w}\right) - \beta \exp\left(\frac{\alpha}{\alpha - a_w} Y - \frac{a_w}{\alpha - a_w} \ln \frac{\alpha}{-\beta a_w}\right) && (61) \\ &= -\exp\left(\frac{\alpha}{\alpha - a_w} Y\right) \left(\exp\left(-\frac{\alpha}{\alpha - a_w} \ln \frac{\alpha}{-\beta a_w}\right) + \beta \exp\left(-\frac{a_w}{\alpha - a_w} \ln \frac{\alpha}{-\beta a_w}\right)\right). && (62) \end{aligned}$$

Lemma 5 (Value Function Coefficients) *Assume* $\hat{\theta}_l = 0$ *for all* l *in every period* t*. Function*

$$V\left(w_t^i, \theta_t^i, d_t, \bar{\theta}_t\right) = -\exp(a_w w_t^i + a_{\theta d} \theta_t^i \cdot d_t + \bar{\theta} \cdot A_{\theta\bar{\theta}} \theta_t^i + \frac{1}{2} \theta_t^i \cdot A_{\theta^2} \theta_t^i + \frac{1}{2} \bar{\theta}_t \cdot A_{\bar{\theta}^2} \bar{\theta}_t + c), \tag{63}$$

where $a_w = -\alpha r$*,* $a_{\theta d} = -\alpha$*,*

$$A_{\theta^2} = \frac{\alpha^2}{1 + r - (1-\gamma)^{2T}} \sum_{l=1}^{T} (1-\gamma)^{2l} \Sigma_l, \; A_{\theta\bar{\theta}} = \frac{\alpha^2}{r} \Sigma - A_{\theta^2}, \; A_{\bar{\theta}^2} = A_{\theta^2} - \frac{\alpha^2}{r} \Sigma, \tag{64}$$

and constant

$$c = \frac{1}{r} \ln\left(\beta^{\frac{1}{1+r}} (r^{\frac{r}{1+r}} + r^{-\frac{r}{1+r}})\right) \tag{65}$$

characterize the value function in a (robust) Subgame Perfect Nash equilibrium.

Proof 4 (Lemma 5) *Consider value function* (23)*. By the stationarity of the problem across periods* $t = 1, 2, \ldots,$

$$\begin{aligned} & V\left(w_{t+1}^i, \theta_{t+1}^i, \bar{\theta}_{t+1}, d_{t+1}\right) && (66) \\ = \; & -\exp(a_w w_{t+1}^i + a_{\theta d} \theta_{t+1}^i \cdot d_{t+1} + \bar{\theta}_{t+1} \cdot A_{\theta\bar{\theta}} \theta_{t+1}^i + \frac{1}{2} \theta_{t+1}^i \cdot A_{\theta^2} \theta_{t+1}^i + \frac{1}{2} \bar{\theta}_{t+1} \cdot A_{\bar{\theta}^2} \bar{\theta}_{t+1} + c). && (67) \end{aligned}$$

With $\hat{\theta}_l = 0$ *for all* l *in every period* t*,* $\bar{\theta}_t = \bar{\theta}_{t+1} = \bar{\theta}_{t+1,h}$ *and it follows from Lemma 3 that, on the equilibrium* *path, riskless holdings of trader i in $t+1$ in terms of previous-period holdings w_t^i and θ_t^i are*

$$w_{t+1}^i = w_t^i (1+r) + \theta_t^i \cdot d_t - c_t^i - (\bar{\theta}_t - \theta_t^i) \cdot \sum_{l=1}^{T} \gamma (1-\gamma)^{l-1} p_{t+1,l}^* \tag{68}$$

$$= w_{t+1}^{i-} - (\bar{\theta}_t - \theta_t^i) \cdot \frac{a_{\theta d}}{a_w} \sum_{l=1}^{T} ((1-\gamma)^l - (1-\gamma)^T) \delta_{t+1,l}, \tag{69}$$

where

$$w_{t+1}^{i-} = w_t^i (1+r) - (\bar{\theta}_t - \theta_t^i) \cdot (1-(1-\gamma)^T) \left(\frac{a_{\theta d}}{a_w} d_t + \frac{1}{a_w} (A_{\theta\bar{\theta}} + A_{\theta^2}) \bar{\theta}_t \right) \tag{70}$$

$$+\theta_t^i \cdot d_t - c_t - (\bar{\theta}_t - \theta_t^i) \cdot \frac{a_{\theta d}^2}{a_w} \sum_{l=1}^{T} (1-(1-\gamma)^l) \Sigma_l \bar{\theta}_t, \tag{71}$$

and portfolio of risky assets is

$$\theta_{t+1}^i = (1-\gamma)^T \theta_t^i + (1-(1-\gamma)^T) \bar{\theta}_t. \tag{72}$$

Substitute (68) *and* (72) *into* (66)*. Before taking expectations with respect to period$-t$ information to find the Bellman equation, separate stochastic and deterministic components of w_{t+1}^i, and consolidate stochastic terms $\delta_{t+1,l}$ in w_{t+1}^i, and component $\theta_{t+1} \cdot d_{t+1}$ in value function* (66) *gives*

$$a_{\theta d} \sum_{l=1}^{T} ((1-(1-\gamma)^l) \bar{\theta}_t + (1-\gamma)^l \theta_t^i) \cdot \delta_{t+1,l} = a_{\theta d} \sum_{l=1}^{T} \theta_{t+1,l}^i \cdot \delta_{t+1,l}. \tag{73}$$

Taking expectations of (66) *at the moment of period$-t$ consumption choice,*

$$E_t V = -\exp(a_w w_{t+1}^{i-} + a_{\theta d}^2 \frac{1}{2} \sum_{l=1}^{T} \theta_{t+1,l}^i \cdot \Sigma_l \theta_{t+1,l}^i + \bar{\theta}_t \cdot A_{\theta\bar{\theta}} \theta_{t+1}^i + \frac{1}{2} \theta_{t+1}^i \cdot A_{\theta^2} \theta_{t+1}^i + \frac{1}{2} \bar{\theta}_t \cdot A_{\bar{\theta}^2} \bar{\theta}_t + c), \tag{74}$$

we observe that

$$Y = a_w w_t^i (1+r) - (\bar{\theta} - \theta_t^i) \cdot (1-(1-\gamma)^T) (A_{\theta\bar{\theta}} + A_{\theta^2}) \bar{\theta} + (a_{\theta d} + a_w) \theta_t^i \cdot d_t + \tag{75}$$

$$-(\bar{\theta} - \theta_t^i) \cdot a_{\theta d}^2 \sum_{l=1}^{T} (1-(1-\gamma)^l) \Sigma_l \bar{\theta} + a_{\theta d}^2 \frac{1}{2} \sum_{l=1}^{T} \theta_{t+1,l}^i \cdot \Sigma_l \theta_{t+1,l}^i \tag{76}$$

$$+\bar{\theta}_{t+1} \cdot A_{\theta\bar{\theta}} \theta_{t+1}^i + \frac{1}{2} \theta_{t+1}^i \cdot A_{\theta^2} \theta_{t+1}^i + \frac{1}{2} \bar{\theta}_{t+1} \cdot A_{\bar{\theta}^2} \bar{\theta}_{t+1} + c \tag{77}$$

and the Bellman equation is as assumed in Lemma 4 (note that quadratic terms involving cross terms $\bar{\theta}_t \cdot d_t$ disappear). We match the coefficients of the value function from Lemma 4, equations (59)*, and* (66) *via the method of undetermined coefficients. Equation $a_w = \frac{\alpha}{\alpha - a_w} a_w (1+r)$ gives $a_w = -\alpha r$ and $a_{\theta d} = \frac{1}{1+r} (a_{\theta d} + a_w)$ gives $a_{\theta d} = -\alpha$. To find A_{θ^2}, $A_{\bar{\theta}\theta}$, and $A_{\bar{\theta}^2}$, we use $\theta_{t+1,l}^i = (1-(1-\gamma)^l) \bar{\theta}_t + (1-\gamma)^l \theta_t^i$. Matching the coefficient* *matrices of* (59) *and* (66), $\frac{1}{2}A_{\theta^2} = \frac{1}{1+r}\left(\frac{1}{2}(1-\gamma)^{2T} A_{\theta^2} + \frac{1}{2}a_{\theta d}^2 \sum_{l=1}^{T}(1-\gamma)^{2l}\Sigma_l\right)$ *and, hence,*

$$A_{\theta^2} = \frac{\alpha^2}{1+r-(1-\gamma)^{2T}} \sum_{l=1}^{T} (1-\gamma)^{2l} \Sigma_l. \tag{78}$$

Similarly, $A_{\theta\bar{\theta}} = \frac{1}{1+r}(A_{\theta\bar{\theta}} + (1-(1-\gamma)^{2T}) \cdot A_{\theta^2} + a_{\theta d}^2 \sum_{l=1}^{T}(1-(1-\gamma)^{2l})\Sigma_l)$*, which gives*

$$A_{\theta\bar{\theta}} = \frac{\alpha^2}{r}(\Sigma - \frac{r}{1+r-(1-\gamma)^{2T}} \sum_{l=1}^{T} (1-\gamma)^{2l} \Sigma_l). \tag{79}$$

Observe that $A_{\theta\bar{\theta}}=\frac{\alpha^2}{r}\Sigma - A_{\theta^2}$*. Finally,* $\frac{1}{2}A_{\bar{\theta}^2} = \frac{1}{1+r}(-\frac{1}{2}(1-(1-\gamma)^{2T})A_{\theta^2} + \frac{1}{2}A_{\bar{\theta}^2} + \frac{1}{2}a_{\theta d}^2 \sum_{l=1}^{T}(1-\gamma)^{2l}\Sigma_l - \frac{1}{2}a_{\theta d}^2\Sigma)$ *and, hence,*

$$A_{\bar{\theta}^2} = -\frac{\alpha^2}{r}\left(\Sigma - \left(\frac{r}{1+r-(1-\gamma)^{2T}}\right) \sum_{l=1}^{T} (1-\gamma)^{2l} \Sigma_l\right). \tag{80}$$

Observe that $A_{\bar{\theta}^2}= A_{\theta^2}-\frac{\alpha^2}{r}\Sigma$*. By Lemma 4,*

$$c = \frac{1}{r}\ln\left(\exp\left(-\frac{\alpha}{\alpha-a_w}\ln\frac{\alpha}{-\beta a_w}\right) + \beta\exp\left\{-\frac{a_w}{\alpha-a_w}\ln\frac{\alpha}{-\beta a_w}\right\}\right), \tag{81}$$

which, given $a_w = -\alpha r$*, yields the constant as in* (65).

B. Comparative Statics

Lemma 6 and Propositions 2 and 4 characterize comparative statics with respect to market depth γ, trade frequency T and information structure, for welfare properties and the Security Market Line.

Lemma 6 (Positive Definiteness) *Consider information structures* F *and* F' *such that* F' *withholds information relative to* F*. Then matrix* $\sum_{l=1}^{T}(1-\gamma)^{2l}(\Sigma_l - \Sigma_l')$ *is positive definite.*

Proof 5 (Lemma 6) *Let* $\geq_{pd}$ *denote the positive semidefinite order (i.e.,* $A \geq_{pd} B$ *if* $A-B$ *is positive semidefinite). Define* $\eta_l \equiv \Sigma_l - \Sigma_l'$*. Since* $\sum_{l=1}^{T}\Sigma_l = \sum_{l=1}^{T}\Sigma_l' = \Sigma$*, for any* k*,* $\sum_{l=1}^{k}\eta_l = -\sum_{l=k+1}^{T}\eta_l$*. By the assumption that* F' *withholds information relative to* F*, matrix* $\sum_{h=1}^{k}\eta_l$ *is positive semidefinite for all* k *and for any* $\theta \neq 0$ *there exists round* k *for which* $\theta \cdot \left(\sum_{l=1}^{k}\eta_l\right)\theta > 0$*. For all* $k = 1, ..., T$*, matrix*

$$\sum_{l=1}^{k}\left((1-\gamma)^{2l} - (1-\gamma)^{2k}\right)\eta_l \tag{82}$$

is positive semidefinite. For $k=1$*, the statement is straightforward. For* $k \geq 1$*, assume that* (82) *holds for all*

$l = 1, ..., k$.

$$\sum_{l=1}^{k+1} (1-\gamma)^{2l} \eta_l = \sum_{l=1}^{k} (1-\gamma)^{2l} \eta_l + (1-\gamma)^{2(k+1)} \eta_{k+1} \tag{83}$$

$$\geq_{pd} (1-\gamma)^{2k} \sum_{l=1}^{k} \eta_l + (1-\gamma)^{2(k+1)} \eta_{k+1} \tag{84}$$

$$\geq_{pd} (1-\gamma)^{2(k+1)} \sum_{l=1}^{k} \eta_l + (1-\gamma)^{2(k+1)} \eta_{k+1} = (1-\gamma)^{2(k+1)} \sum_{l=1}^{k+1} \eta_l \tag{85}$$

and (82) *is positive semidefinite for* $k+1$ *and, by induction, for all* l*. Moreover,*

$$\sum_{l=k}^{T} \left((1-\gamma)^{2l} - (1-\gamma)^{2k} \right) \eta_l \tag{86}$$

is positive semidefinite. Again, for $k = T$*, the assertion is immediate. Assume* (86) *holds for all* l *between* $k+1$ *and* T*. Then,*

$$\sum_{l=k}^{T} (1-\gamma)^{2l} \eta_l = \sum_{l=k+1}^{T} (1-\gamma)^{2l} \eta_l + (1-\gamma)^{2k} \eta_k \tag{87}$$

$$\geq_{pd} (1-\gamma)^{2(k+1)} \sum_{l=k+1}^{T} \eta_l + (1-\gamma)^{2k} \eta_k \tag{88}$$

$$\geq_{pd} (1-\gamma)^{2k} \sum_{l=k+1}^{T} \eta_l + (1-\gamma)^{2k} \eta_k = (1-\gamma)^{2k} \sum_{l=k}^{T} \eta_l \tag{89}$$

and (86) *is positive semidefinite for* k *and, by induction, for all* l*.*

Finally, for any $\theta \neq 0$ *and corresponding* k *for which* $\theta \cdot \sum_{l=1}^{k} \eta_l \theta > 0$,

$$\theta \cdot \left(\sum_{l=1}^{k} (1-\gamma)^{2l} \eta_l \right) \theta \geq \theta \cdot \left((1-\gamma)^{2k} \sum_{l=1}^{k} \eta_l \right) \theta > \theta \cdot \left((1-\gamma)^{2(k+1)} \sum_{l=1}^{k} \eta_l \right) \theta \tag{90}$$

$$= \theta \cdot \left(-(1-\gamma)^{2(k+1)} \sum_{l=k+1}^{T} \eta_h \right) \theta \geq -\theta \cdot \left(\sum_{l=k+1}^{T} (1-\gamma)^{2l} \eta_l \right) \theta, \tag{91}$$

where the first inequality is by the positive semidefiniteness result (82)*, the second inequality holds by the assumption* $\theta \cdot \sum_{l=1}^{k} \eta_l \theta > 0$*, and the last inequality holds by equation* (86)*. Inequality* (90) *implies* $\theta \cdot \sum_{l=1}^{T} (1-\gamma)^{2l} \eta_l \theta > 0$*. Since the inequality is strict for any* $\theta \neq 0$, $\sum_{l=1}^{T} (1-\gamma)^{2l} (\Sigma_l - \Sigma'_l)$ *is positive definite.*

Proof 6 (Propositions 2 and 4) *We prove the result for the general model with gradual revelation of information and many assets. Let* V *and* V' *be the value function of a trader* i *who trades in a market with the average holdings* $\bar{\theta}$*, evaluated at period–*t *risky portfolio* θ_t^i *and riskless asset* w_t^i *in models characterized by*

(γ, T, F) and (γ', T', F'), respectively. By Lemma 5,

$$\ln(-V) - \ln(-V') = \bar{\theta} \cdot (A'_{\theta^2} - A_{\theta^2})\, \theta^i_t + \frac{1}{2} \theta^i_t \cdot (A_{\theta^2} - A'_{\theta^2})\, \theta^i_t + \frac{1}{2} \bar{\theta} \cdot (A_{\theta^2} - A'_{\theta^2})\, \bar{\theta} \quad (92)$$

$$= \frac{1}{2} (\theta^i_t - \bar{\theta}) \cdot (A_{\theta^2} - A'_{\theta^2})\, (\theta^i_t - \bar{\theta}). \quad (93)$$

Observe that for a competitive model $\gamma' \cong 1$, matrix $A'_{\theta^2} = 0$ and $\ln(-V) - \ln(-V') = \frac{1}{2}(\theta^i_t - \bar{\theta}) \cdot A_{\theta^2} (\theta^i_t - \bar{\theta})$. Similarly, let Y and Y' be defined as in equation (59), for the two markets. Their difference is

$$Y - Y' = \frac{1}{2} (1 + r)\, (\theta^i_t - \bar{\theta}) \cdot (A_{\theta^2} - A'_{\theta^2})\, (\theta^i_t - \bar{\theta}). \quad (94)$$

(The number of trading opportunities per period, T) Consider two markets characterized by (γ, T, F) and (γ, T', F), $T' > T$, for which F is such that all the information is revealed after the respective last trading round. Since $\frac{\alpha^2 (1-\gamma)^{2T}}{1+r-(1-\gamma)^{2T}}$ is monotonically decreasing in T, using that $A_{\theta^2} = \frac{\alpha^2 (1-\gamma)^{2T}}{1+r-(1-\gamma)^{2T}} \Sigma$ (by Lemma 5), matrix $A_{\theta^2} - A'_{\theta^2}$ is positive definite. It follows from (93) and (94) that, for a trader with $\theta^i_t \neq \bar{\theta}$, inequalities $V' > V$ and $Y > Y'$ hold. Thus, any trader who faces idiosyncratic risk has strictly higher lifetime utility and consumption for T' than T. Moreover, $\lim_{T \to \infty} A_{\theta^2} = \lim_{T \to \infty} \frac{\alpha^2 (1-\gamma)^{2T}}{1+r-(1-\gamma)^{2T}} \Sigma = 0$, and hence by Lemma 5, as $T \to \infty$, Y, consumption c_t and lifetime utility V coincide with those with price taking traders.

(Market depth, γ) Consider two markets characterized by (γ, T, F) and (γ', T, F), $\gamma' > \gamma$, where F is an arbitrary information structure. Since $\sum_{l=1}^{T} (1 - \gamma)^{2l} \Sigma_l >_{pd} \sum_{l=1}^{T} (1 - \gamma')^{2l} \Sigma_l$, matrix $A_{\theta^2} - A'_{\theta^2}$ is positive definite. It follows from (93) and (94) that, for a trader with $\theta^i_t \neq \bar{\theta}$, inequalities $V' > V$ and $Y > Y'$ hold. Thus, any trader who faces idiosyncratic risk has strictly higher lifetime utility and consumption c_t for γ' than γ. Moreover, $\lim_{\gamma \to 1} A_{\theta^2} = \lim_{\gamma \to 1} \sum_{l=1}^{T} (1 - \gamma)^{2l} \Sigma_l = 0$ and, by Lemma 5, Y, consumption c_t and lifetime utility V coincide with those with price taking traders.

(Information structure, F) Consider two markets characterized by (γ, T, F) and (γ, T, F'). For any F' that withholds information relative to F, by Lemma 6, $\sum_{h=1}^{T} (1 - \gamma)^{2h} (\Sigma_l - \Sigma'_l)$ is positive definite and, hence, matrix $A_{\theta^2} - A'_{\theta^2}$ is positive definite. Thus, for a trader with $\theta^i_t \neq \bar{\theta}$, it follows by Lemma 5 that $V' > V$ and $Y > Y'$, and hence consumption c_t and lifetime utility V are higher under F'.

Proof 7 (Proposition 1) *In the general model, assume $\Sigma_l = 0$ for all $l < T$, $\Sigma_T = \sigma^2$ and zero supply shocks, $\hat{\theta}_l = 0$, $l = 1, ..., T$. Lemma 3 then gives $\Delta^{i*}_{t,l} = \gamma(\bar{\theta} - \theta^i_{t,l-1})$. Moreover, from Lemma 5, effective risk aversion is $\bar{\alpha} = \xi \frac{\alpha}{r^2} + \frac{\alpha}{r}$, where $\xi = \frac{r(1-\gamma)^{2T}}{1+r-(1-\gamma)^{2T}} \in (0, 1)$. (Taking the monotone transformation of the* *expectations of the value function with respect to information at T gives the functional form as in Equation* (2)*). Price impact* (25) *with coefficients from Lemma 5 gives price impact* (10)*.*

Proof 8 (Proposition 3) *In the general model, assume$\Sigma_l = 0$ for all $l < T$ and $\Sigma_T = \sigma^2$. Take the sequence of shocks $\{0, ..., \hat{\theta}_{\hat{l}}, \dots, 0\}$ and apply the results from Appendix A.*

Proposition 5 (Security Market Line) *Consider information structures in which information is revealed only in the last round and assume an exogenous shock $\hat{\theta}$ in round l. Then in this round, the expected returns of all assets satisfy*

$$E(R^{\theta}) - r = \beta^{\bar{\theta}}\left(E(R^{\bar{\theta}}) - r\right) + \beta^{\hat{\theta}}\left(E(R^{\hat{\theta}}) - r\right) - (\beta^{\bar{\theta}}a_1 + \beta^{\hat{\theta}}a_1)cov\left(R^{\bar{\theta}}, R^{\hat{\theta}}\right), \tag{95}$$

where $\beta^{\bar{\theta}} = \left(\frac{\theta}{p^{\theta}} \cdot \Sigma \frac{\bar{\theta}}{p^{\bar{\theta}}}\right) / \left(\frac{\bar{\theta}}{p^{\bar{\theta}}} \cdot \Sigma \frac{\bar{\theta}}{p^{\bar{\theta}}}\right)$, $\beta^{\hat{\theta}} = \left(\frac{\theta}{p^{\theta}} \cdot \Sigma \frac{\hat{\theta}}{p^{\hat{\theta}}}\right) / \left(\frac{\hat{\theta}}{p^{\hat{\theta}}} \cdot \Sigma \frac{\hat{\theta}}{p^{\hat{\theta}}}\right)$ and the constants are $a_1 \equiv r\frac{1-\gamma}{\gamma}(1-\gamma)^{2(T-l)}\bar{\alpha}p^{\hat{\theta}}$ and $a_2 \equiv \alpha\frac{1+r}{r}p^{\bar{\theta}}$.

Proof 9 (Proposition 5) *For information structures in which information is revealed only in the last round, from* (24) *and Lemma 5, the value of portfolio θ in round l is given by $p^{\theta} = \theta \cdot p^*_{t,l} = \frac{\theta \cdot E(d_t)}{r} - \frac{\alpha}{r}\theta \cdot \Sigma\bar{\theta} - \frac{\alpha}{r^2}\theta \cdot \Sigma\bar{\theta} - \theta \cdot \lambda^*_l\hat{\theta}$ and, hence,*

$$\frac{\theta \cdot E(d_t)}{p^{\theta}} - r = \alpha\frac{1+r}{r}\frac{\theta}{p^{\theta}} \cdot \Sigma\frac{\bar{\theta}}{p^{\bar{\theta}}}p^{\bar{\theta}} + r\frac{\theta}{p^{\theta}} \cdot \lambda^*_l\frac{\hat{\theta}}{p^{\hat{\theta}}}p^{\hat{\theta}}. \tag{96}$$

In particular, for the average and shock portfolios,

$$\alpha\frac{1+r}{r}\frac{\bar{\theta}}{p^{\bar{\theta}}} \cdot \Sigma\frac{\bar{\theta}}{p^{\bar{\theta}}}p^{\bar{\theta}} = E(R^{\bar{\theta}}) - r - r\frac{\bar{\theta}}{p^{\bar{\theta}}} \cdot \lambda^*_l\frac{\hat{\theta}}{p^{\hat{\theta}}}p^{\hat{\theta}} \tag{97}$$

$$r\frac{\hat{\theta}}{p^{\hat{\theta}}} \cdot \lambda^*_l\frac{\hat{\theta}}{p^{\hat{\theta}}}p^{\hat{\theta}} = E(R^{\hat{\theta}}) - r - \alpha\frac{1+r}{r}\frac{\hat{\theta}}{p^{\hat{\theta}}} \cdot \Sigma\frac{\bar{\theta}}{p^{\bar{\theta}}}p^{\bar{\theta}}. \tag{98}$$

Thus,

$$E(R^{\theta}) - r = \beta^{\bar{\theta}}\left(E(R^{\bar{\theta}}) - r\right) + \beta^{\hat{\theta}}\left(E(R^{\hat{\theta}}) - r\right) - \beta^{\bar{\theta}}\left(r\frac{\bar{\theta}}{p^{\bar{\theta}}} \cdot \lambda^*_l\frac{\hat{\theta}}{p^{\hat{\theta}}}p^{\hat{\theta}}\right) - \beta^{\hat{\theta}}\left(\alpha\frac{1+r}{r}\frac{\hat{\theta}}{p^{\hat{\theta}}} \cdot \Sigma\frac{\bar{\theta}}{p^{\bar{\theta}}}p^{\bar{\theta}}\right). \tag{99}$$

Then, Security Market Line (95) *follows from $\lambda^*_l = \frac{1-\gamma}{\gamma}(1-\gamma)^{2(T-l)}\bar{\alpha}\Sigma$.*

C. Trading against Price Impact

The proof of the first part of Lemma 1 does not assume that the value function $V^i_l(\cdot)$ is (quasilinear-)quadratic.

Proof 10 (Lemma 1) *Given the quasilinear value function in round l, $V_l^i(\cdot)$, let $\Delta_{t,l}^i\left(\cdot,\lambda_l^i\right)$ be trader i's optimal demand given his assumed price impact λ_l^i, defined by equalization of marginal utility and marginal payment for all prices $p_{t,l}$,*

$$D_{\Delta_{t,l}^i}V_l^i\left(\theta_{t,l-1}^i+\Delta_{t,l}^i\right)=p_{t,l}+\lambda_l^i\Delta_{t,l}^i. \tag{100}$$

(Equation (4) in the text.)

("if") Fix demand schedules $\{\Delta_{t,l}^i(\cdot,\tilde{\lambda}_l^i)\}_{i=1}^I$ submitted by all traders given their assumed price impact $\tilde{\lambda}_l^i$ in round l. Assume that the slope of each trader's residual supply, defined by $\{\Delta_{t,l}^j(\cdot,\tilde{\lambda}_l^j)\}_{j\neq i}$, is $\tilde{\lambda}_l^i=-(\sum_{j\neq i}D_{p_{t,l}}\Delta_{t,l}^j(\cdot,\tilde{\lambda}_l^j))^{-1}$, for each i. The market-clearing price is determined by $\sum_{i=1}^I\Delta_{t,l}^i(\tilde{p}_{t,l},\tilde{\lambda}_l^i)=0$. Since for each i, demand $\Delta_{t,l}^i(\cdot,\tilde{\lambda}_l^i)$ satisfies condition (100) for all prices $p_{t,l}$, it does so for the market-clearing price $\tilde{p}_{t,l}$. Given the global concavity of the maximization problem of each trader, demand functions $\{\Delta_{t,l}^i(\cdot,\tilde{\lambda}_l^i)\}_{i=1}^I$ are mutual best responses at $\tilde{p}_{t,l}$.

Assume that $V_l^i(\cdot)$ is quasilinear-quadratic. With an additive perturbation in demand (e.g., exogenous noise), trader i's residual supply with slope $\tilde{\lambda}_l^i$ has a stochastic intercept. Since, for each i, condition (100) holds for all prices $p_{t,l}$, it holds for each realization of the residual supply (or noise). Hence, given the global concavity, $\{\Delta_{t,l}^j(\cdot,\tilde{\lambda}_l^j\}_{i=1}^I$ is a Nash equilibrium with an arbitrary additive noise and, thus, a robust Nash Equilibrium.

("only if") Suppose that, in round l, traders submit demand functions $\{\Delta_{t,l}^i(\cdot,\tilde{\lambda}_l^i)\}_{i=1}^I$ such that price impact $\tilde{\lambda}_l^i\neq-(\sum_{j\neq i}D_{p_{t,l}}\Delta_{t,l}^j(\cdot,\tilde{\lambda}_l^j))^{-1}$ for some i. Then, schedule $\Delta_{t,l}^i(\cdot,\tilde{\lambda}_l^i)$ is not a best response to $\{\Delta_{t,l}^j(\cdot,\tilde{\lambda}_l^j)\}_{j\neq i}$ at the market-clearing price $\tilde{p}_{t,l}$, defined by $\sum_{i=1}^I\Delta_{t,l}^i(\tilde{p}_{t,l},\tilde{\lambda}_l^i)=0$. Assume that $V_l^i(\cdot)$ is quasilinear-quadratic. With an additive perturbation, by the linearity of demands, for each trader i, for almost all prices $p_{t,l}$, $D_{\Delta_{t,l}^i}V_l^i(\theta_{t,l-1}^i+\Delta_{t,l}^i)\neq p_{t,l}+\lambda_l^i\Delta_{t,l}^i$, where λ_l^i is the slope of i's residual supply. For any additive, absolutely continuous noise trade, the prices for which the equality is violated have measure one and the bid that equalizes marginal utility with marginal payment, $D_{\Delta_{t,l}^i}V_l^i(\theta_{t,l-1}^i+\Delta_{t,l}^i)=p_{t,l}+\lambda_l^i\Delta_{t,l}^i$ for all $p_{t,l}$, gives a strictly higher utility for measure one of noise realizations (and, hence, a strictly higher expected utility). It follows that $\Delta_{t,l}^i(\cdot,\tilde{\lambda}_l^i)$ is not a robust best response and noise exists for which $\{\Delta_{t,l}^i(\cdot,\tilde{\lambda}_l^i)\}_{i=1}^I$ is not a robust Nash equilibrium.

Corollary 2 shows that the (robust Subgame Perfect Nash) equilibrium in demand functions $\{\Delta_{t,l}^{i*}(\cdot)\}_{i=1,l=1}^{I,T}$ can be represented as a reduced form. For any equilibrium profile of demand functions $\{\Delta_{t,l}^{i*}(\cdot)\}_{i=1,l=1}^{I,T}$, let $\{p_{t,l}^*,\Delta_{t,l}^{i*},\lambda_l^{i*}\}_{i=1,l=1}^{I,T}$ be an *associated profile* of prices, trades and price impacts defined by conditions (i'), (ii') and (iii') from Section 1.3. The associated profile exists and is unique and, by Lemma 1, is also the associated profile of $\{\Delta_{t,l}^i\left(\cdot,\lambda_l^i\right),\lambda_l^i\}_{i=1,l=1}^{I,T}$ with $\lambda_l^i=\lambda_l^{*i}$ for all i and l.

Corollary 2 (Equivalence of Equilibrium Representation) *For a profile* $\{\Delta^{i*}_{t,l}(\cdot)\}^{I,T}_{i=1,l=1}$ *that constitutes a robust Subgame Perfect Nash equilibrium, the associated profile of prices, trades and price impacts* $\{p^*_{t,l}, \Delta^{i*}_{t,l}, \lambda^{i*}_l\}^{I,T}_{i=1,l=1}$ *satisfies conditions (i'), (ii') and (iii'). Conversely, for any profile* $\{\bar{p}_{t,l}, \bar{\Delta}^i_{t,l}, \bar{\lambda}^i_l\}^{I,T}_{i=1,l=1}$ *that satisfies conditions (i'), (ii') and (iii'), functions* $\{\Delta^i_{t,l}(\cdot) \equiv \bar{\Delta}^i_{t,l}(\cdot, \bar{\lambda}^i_l)\}^{I,T}_{i=1,l=1}$ *constitute a robust Subgame Perfect Nash equilibrium with which* $\{\bar{p}_{t,l}, \bar{\Delta}^i_{t,l}, \bar{\lambda}^i_l\}^{I,T}_{i=1,l=1}$ *is associated.*

Proof 11 (Corollary 2) *(Profile* $\{p^*_{t,l}, \Delta^{i*}_{t,l}, \lambda^{i*}_l\}^{I,T}_{i=1,l=1}$ *associated with equilibrium* $\{\Delta^{i*}_{t,l}(\cdot)\}^{I,T}_{i=1,l=1}$ *satisfies conditions (i'), (ii') and (iii').) Assume that the value function* $V_l(\cdot)$ *has the functional form as in Section 1.2. Let the profile of demand schedules* $\{\Delta^{i*}_{t,l}(\cdot)\}^{I,T}_{i=1,l=1}$ *be a robust Subgame Perfect Nash equilibrium. By Lemma 1, demand schedules* $\Delta^{i*}_{t,l}(\cdot)$ *can each be represented as* $\Delta^i_{t,l}\left(\cdot, \lambda^{i*}_l\right)$, *defined by condition* (100) *for all prices* $p_{t,l}$, *where for each* l, $\left\{\lambda^{i*}_l\right\}^I_{i=1}$ *is a fixed point of the system* $\{\lambda^i_l = -(\sum_{j\neq i} D_{p_{t,l}}\Delta^j_{t,l}(\cdot, \tilde{\lambda}^j_l))^{-1}\}^I_{i=1}$; *thus, condition (iii') holds. For conditions (i') and (ii'), by definition,* $\Delta^{i*}_{t,l} \equiv \Delta^{i*}_{t,l}(p^*_{t,l})$ *and price* $p^*_{t,l}$ *is defined as the market-clearing price* $\sum_l \Delta^i_{t,l}(p^*_{t,l}) = 0$; *hence,* $\sum_i \Delta^{i*}_{t,l} = 0$ *for each* l.

(Functions $\{\Delta^i_{t,l}(\cdot) \equiv \bar{\Delta}^i_{t,l}(\cdot, \bar{\lambda}^i_l)\}^{I,T}_{i=1,l=1}$ *associated with* $\{p^*_{t,l}, \Delta^{i*}_{t,l}, \lambda^{i*}_l\}^{I,T}_{i=1,l=1}$ *constitute a robust Subgame Perfect Nash equilibrium.) By definition.*

(The value functions coincide.) In round T, *the value function is the quasilinear-quadratic utility in Equation* (2). *If, for an arbitrary additive distribution of noise, the equivalence holds from* $l+1$ *to* T, *the value functions at* l *are the same, and they are given by* $V_l(\cdot)$ *(derived in Appendix B).*

Lemma 1 suggests that the following relation between the non-competitive and competitive equilibria can be established in a demand game (and the Walrasian auction) when viewed as a model of "trade given assumed price impact". In the (robust Nash) equilibrium, (i) each trader optimizes given his assumed price impact λ^i_l, that is, submits $\Delta^i_{t,l}\left(\cdot, \lambda^i_l\right)$; and (ii) for each trader i, his assumed price impact is correct, that is, $\lambda^i_l = -(\sum_{j\neq i} \partial\Delta^j_{t,l}(\cdot)/\partial p)^{-1}$. In a Nash equilibrium, the first-order condition in Equation (4) is required to hold only *at* the equilibrium price. In Lemma 1, traders respond optimally given their price impact for *all* prices rather than just the equilibrium price. (Analogously, in the robust Nash equilibrium, traders respond optimally for each realization of the residual supply whose intercept depends on the realization of noise trade.)[37] That strengthening is embedded in Lemma 1. As is apparent from condition (ii), traders are slope takers rather than price takers; each trader optimizes (condition (i)), assuming that his trade will not affect the slope of his residual supply (condition (ii)), determined by the schedules of other traders. Trade will affect prices as long as

[37] In a one-shot game, the selection through uncertainty deriving from noisy trade or independent private endowments coincide.

the slope is not zero. If the endogenously derived price impacts are equal to zero, the competitive equilibrium is obtained.

Just like the competitive equilibrium is defined as a profile of prices and trades such that prices clear markets and trades maximize utilities, one can define a non-game theoretic counterpart of conditions (i) and (ii) as a profile of prices, trades and price impacts such that, for all l and i, (i') price clears the market: $\sum_{i\in I}\Delta^i_{t,l}(p^*_{t,l})=0$; (ii') trade is optimal given price impact λ^{i*}_l: $\partial V^i_l(\cdot)/\partial\theta^i_{t,l}=p^*_{t,l}+\lambda^{i*}_l\Delta^{i*}_{t,l}$; and (iii') price impact is correct: $\lambda^{i*}_l=-(\sum_{j\neq i}\partial\Delta^j_{t,l}(\cdot)/\partial p)^{-1}$, $i\in I$. This reduced form can be used in a general-equilibrium setting (in a deterministic model, see, e.g., Weretka (2011)[38]; or, as follows from the results above, in a general-equilibrium *non-competitive and competitive* rational expectations equilibrium, in a setting with uncertainty, which may allow interdependent values (given the results of Rostek and Weretka (2012)). Given the feature of the model that each agent has mass equal to one, since the market clearing condition is not well defined in the limit with a continuum of traders, we characterize the limit of equilibria in the sequence of finite auctions.). By Corollary 2, we can represent the (robust Subgame Perfect Nash) equilibrium as the profile of demand functions $\{\Delta^i_{t,l}(\cdot,\lambda^i_l)\}^{I,T}_{i=1,l=1}$ and price impacts $\{\lambda^i_l\}^{I,T}_{i=1,l=1}$ that satisfy conditions (i) and (ii) or, equivalently, a profile of prices, trades and price impacts for all traders that satisfy (i'), (ii') and (iii'), $\{(p^*_{t,l},\Delta^{i*}_{t,l},\lambda^{i*}_l)\}^{I,T}_{i=1,l=1}$.

[38] The (one-period) general-equilibrium definition of Weretka (2011) before applying Lemma 1 from this paper is as follows:

Definition 1 *A profile* $\{(p^*,\Delta^{i*},\lambda^{i*})\}_i$ *is an* equilibrium *if (i) Asset markets clear,* $\sum_{i\in I}\Delta^{i*}=0$*; (ii) For any* i*, the trade* Δ^{i*} *is optimal, given demand function* $p_{p^*,\Delta^{i*},\lambda^{i*}}(\cdot)$*; (iii) For any* i*, price impact* λ^{i*} *is consistent with* λ^{-i*}.

To endogenize price impacts, fix an arbitrary profile of price impacts of all traders but i, λ^{-i*} and consider how the market reacts to an investor's i deviation from his equilibrium trade Δ^{i*} to any trade Δ^i. Any deviation Δ^i by trader i triggers a subequilibrium that is defined as follows:

Definition 2 *Given* λ^{-i*}*, vector* $(\bar p,\bar\Delta^{-i},\lambda^{-i*})$ *is a* subequilibrium *triggered by trade* $\bar\Delta^i$ *if: (i) Markets clear with the deviation,* $\bar\Delta^i+\sum_{j\neq i}\bar\Delta^j=0$*; (ii) For any* $j\neq i$*, trade* $\bar\Delta^j$ *is optimal given demand functions* $p_{\bar p,\bar\Delta^j,\lambda^{*j}}(\cdot)$.

If every deviation $\bar\Delta^i$ of i triggers a unique subequilibrium, then trader i is effectively facing a downward sloping residual demand $p^i(\bar\Delta^i)$, which assigns the market clearing price to any $\bar\Delta^i$, and the slope of which measures i's price impact. Consistent price impact reflects the price change needed to clear the market for any possible deviation $\bar\Delta^i$, given that the other traders respond optimally to market prices.

Definition 3 λ^{i*} *is* consistent *with* λ^{-i*} *if, for any deviation* $\bar\Delta^i$ *of* i*, there exists a unique subequilibrium* $(\bar p,\bar\Delta^{-i},\lambda^{-i*})$ *such that* $\bar p-p^*=\lambda^{-i*}\left(\bar\Delta^i-\Delta^{i*}\right)$.