SOLOW WITH LAND : HANDOUT DISTURBANCES

(1) $Y_t = B_t K_t^{\alpha} T^{\beta} L_t^{1-\alpha-\beta}$

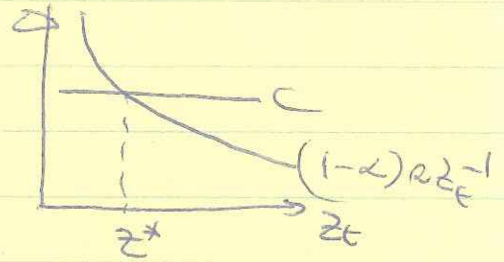
$\alpha + \beta < 1$

$0 < \alpha, \beta < 1$

$0 < d < 1$

(3) $\hat{K}_t = s \frac{Y_t}{K_t} - d$

(2) $\hat{L}_t = n$, $\hat{B}_t = g_B$



LET $z_t = K_t / Y_t$ THEN

(11) $\boxed{\hat{z}_t = (1-\alpha) s z_t^{-1} - c}$

WHERE $c = g_B + (1-\alpha-\beta) n + (1-\alpha) d = g_B + (1-\alpha)[n+d] - \beta n$

& $z^* = z_{BGP} = \boxed{\frac{(1-\alpha)}{c} s}$

(15) $y_t = B^{\frac{1}{1-\alpha}} z_t^{\frac{\alpha}{1-\alpha}} T^{\frac{\beta}{1-\alpha}} L^{-\frac{\beta}{1-\alpha}}$

$\Rightarrow$

$\hat{y}_t = (1/(1-\alpha)) g_B + \frac{\alpha}{1-\alpha} \hat{z}_t - \frac{\beta}{1-\alpha} n$

$= \frac{1}{1-\alpha} [g_B + \alpha \hat{z}_t - \beta n] = \frac{1}{1-\alpha} g_B - \frac{\beta}{1-\alpha} n + \frac{\alpha}{1-\alpha} \hat{z}_t$

AT BGP:

$\hat{y}_t^{BGP} = \frac{g_B}{1-\alpha} - \frac{\beta}{1-\alpha} n$

$\Rightarrow$

(20) EVERYWHERE $\boxed{\hat{y}_t = \frac{\alpha}{1-\alpha} \hat{z}_t + \underbrace{\frac{g_B}{1-\alpha} - \frac{\beta}{1-\alpha} n}_{\hat{y}^{BGP}}}$

WE CAN REWRITE (20) USING (11)

$$\boxed{\hat{y}_t} = \frac{\alpha}{1-\alpha}\left[(1-\alpha)a z^{-1} - c\right] + \frac{g_B}{1-\alpha} - \frac{\beta}{1-\alpha} n$$

$$= \alpha a z^{-1} + \frac{1}{1-\alpha}\left[g_B - \beta n - \alpha c\right]$$

$$= \alpha a z^{-1} + \frac{1}{1-\alpha}\left[g_B - \beta n - \alpha g_B - \alpha(1-\alpha)(n+d) + \alpha\beta n\right]$$

$$= \alpha a z^{-1} + \frac{1}{1-\alpha}\left[(1-\alpha) g_B - \beta n(1-\alpha) - \alpha(1-\alpha)n - (1-\alpha)\alpha d\right]$$

$$= \alpha a z^{-1} + g_B - \alpha d - n(\beta+\alpha)$$

$$\Rightarrow \quad (22) \quad \boxed{\hat{y}_t} = \boxed{\alpha a z_t^{-1} + g_B - [\alpha d + (\alpha+\beta) n]}$$

SUMMARY : LET $z_t = k_t/y_t$, $c = g_B + (1-\alpha-\beta)n + (1-\alpha)d$

(11) $\hat{z}_t = (1-\alpha) a z_t^{-1} - c$

(20) $\hat{y}_t = \frac{\alpha}{1-\alpha}\hat{z}_t + \underbrace{\frac{g_B}{1-\alpha} - \frac{\beta}{1-\alpha}n}_{\hat{y}_{BGP}}$

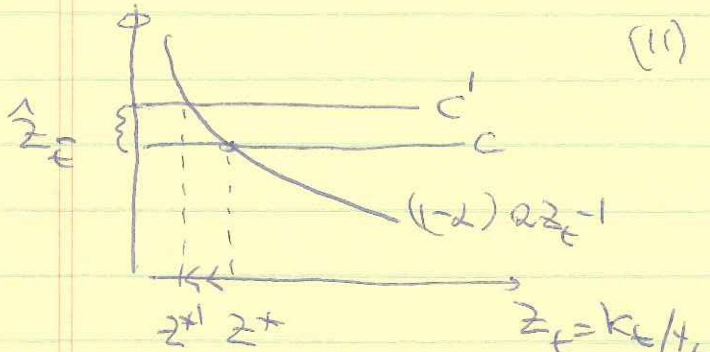
$z^* = z_t^{BGP} = \frac{1-\alpha}{c} a$

(22) $\hat{y}_t = \alpha a z_t^{-1} + g_B - [\alpha d + (\alpha+\beta) n]$

EXAMPLES DISTURBANCES

1) ASSUME COUNTRY AT BGP & AT $\bar{t}$:
↑d

SHOW EFFECTS ON $z, \hat{z}, \hat{y}, y$

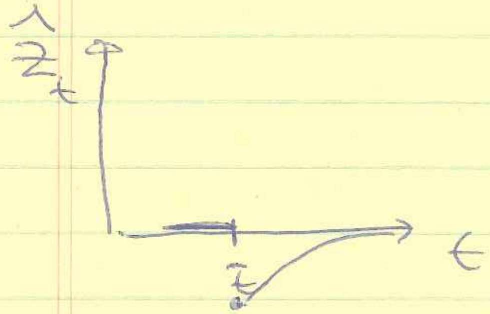


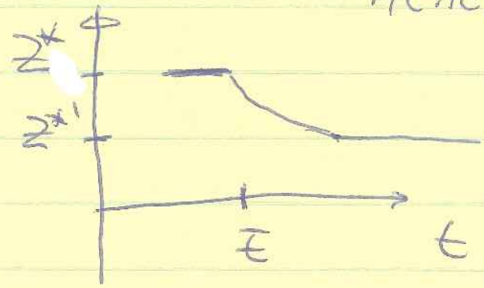
(11) $\hat{z}_t = (1-\alpha) a z_t^{-1} - c$

↑d ⇒ ↑c to c'

& $\hat{z}_{\bar t} < 0$

⇒ z_t ↓ to REACH $z^{*\prime}$





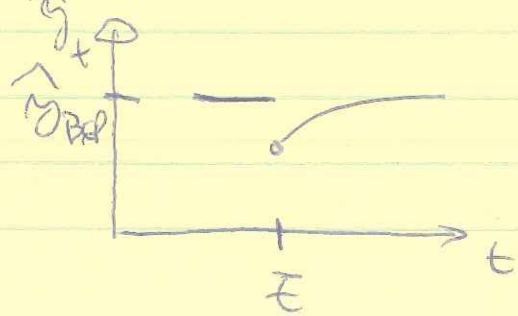
(20): $\hat{y}_t = \frac{\alpha}{1-\alpha}\hat{z}_t + \underbrace{\frac{g_B}{1-\alpha} - \frac{\beta}{1-\alpha} n}_{\hat{y}_{BGP}}$

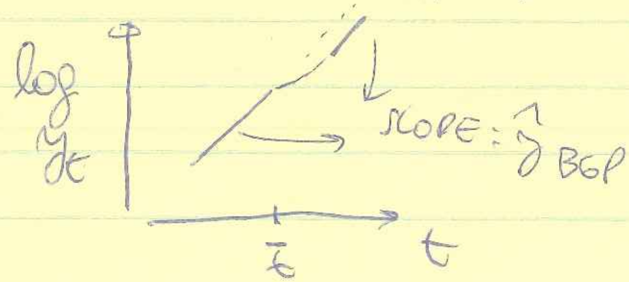
ASSUMING $\hat{y}_{BGP}$ POSITIVE

(a) ASSUME $\hat{y}_{BGP} > \frac{\alpha}{1-\alpha}\hat{z}_{\bar t}$

(b) ASSUME $\hat{y}_{BGP} < \frac{\alpha}{1-\alpha}\hat{z}_{\bar t}$

(a)





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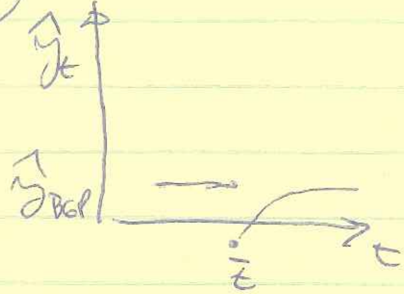
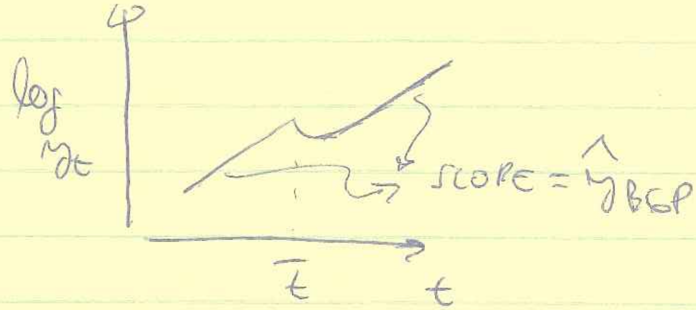




2) ASSUME COUNTRY AT BGP & AT $\bar{t}$:

$\downarrow \beta \Rightarrow$ EFFECT ON $z_{\bar{t}}$ (SEE BELOW)

$\Rightarrow \uparrow C$ TO C' $\Rightarrow z^* \downarrow$ TO $z^{*\prime}$

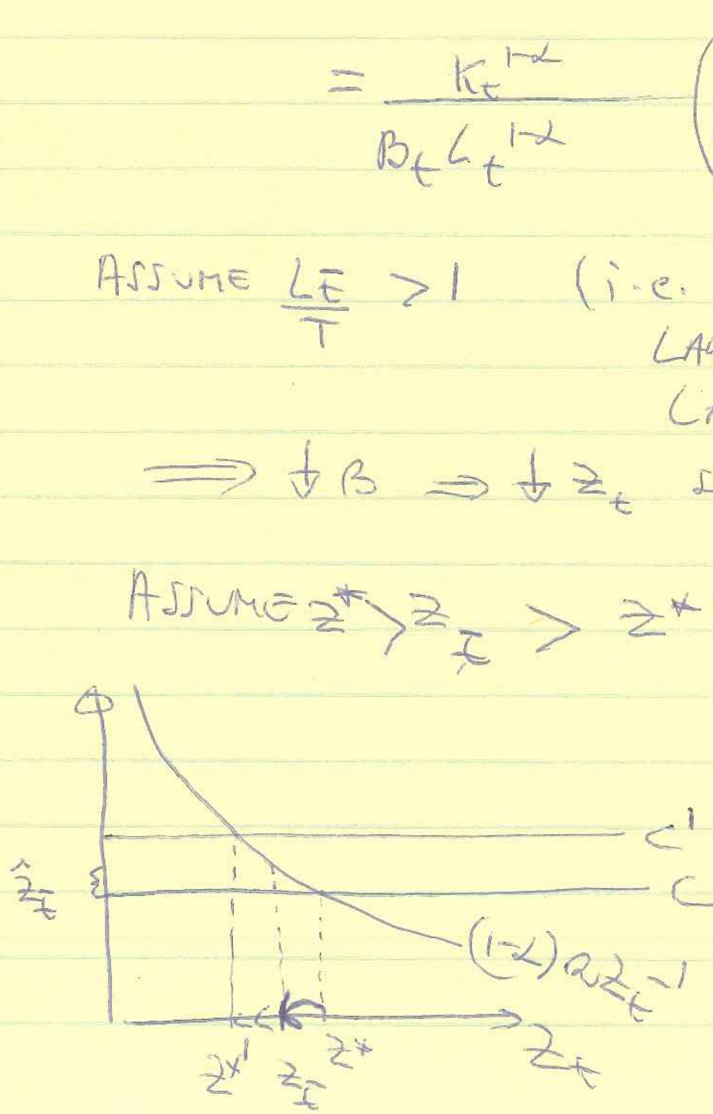
NOTICE $z_t = K_t/Y_t = \frac{1}{Y_t/K_t} = \frac{1}{B_t K_t^{\alpha-1} L_t^{1-\alpha} \left(\frac{T}{L_t}\right)^{\beta}}$

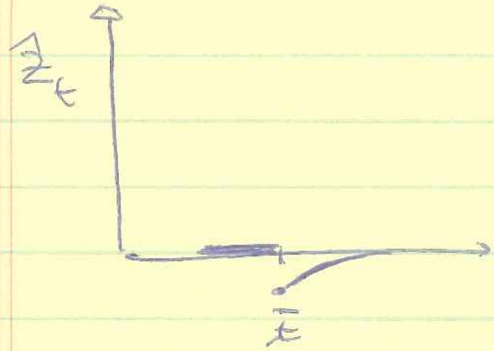
$$= \frac{K_t^{1-\alpha}}{B_t L_t^{1-\alpha}} \left(\frac{L_t}{T}\right)^{\beta} = \frac{1}{B_t}\left(\frac{K_t}{L_t}\right)^{1-\alpha}\left(\frac{L_t}{T}\right)^{\beta}$$

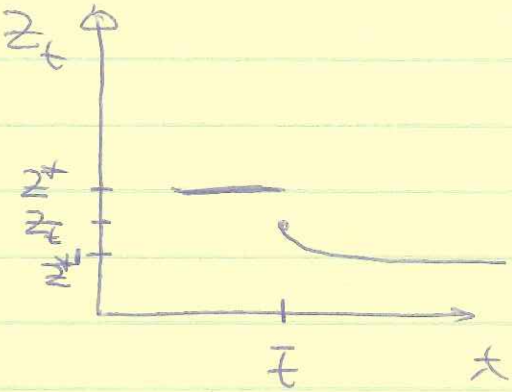
ASSUME $\frac{L_{\bar{t}}}{T} > 1$ (i.e. LABOR FORCE AT $\bar{t}$ LARGE COMPARED WITH STOCK OF LAND)

$\Rightarrow \downarrow \beta \Rightarrow \downarrow z_t$ SO $z_{\bar{t}} < z^*$

ASSUME $z^* > z_{\bar{t}} > z^{*\prime}$







EFFETS ON y_t, $\hat{y}_t$

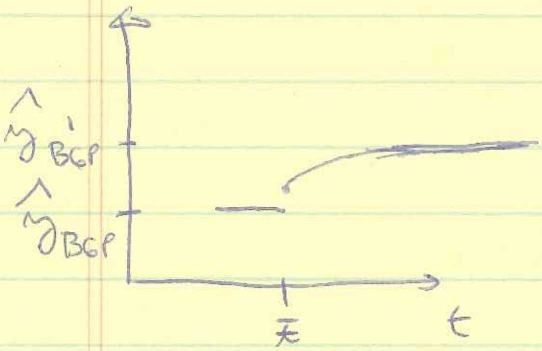
ASSUME $\hat{y}_{BGP} = \frac{g_B}{1-\alpha} - \frac{\beta}{1-\alpha} n > 0$

IF $\beta \downarrow \Rightarrow \hat{y}_{BGP} \uparrow$ TO $\hat{y}'_{BGP}$

& USING (20) $\hat{y}_t = \frac{\alpha}{1-\alpha} \hat{z}_t + \frac{g_B}{1-\alpha} - \frac{\beta}{1-\alpha} n$

ASSUMING

$\hat{y}'_{BGP} > \frac{\alpha}{1-\alpha} \hat{z}_{\bar{t}}$



HOW ABOUT y_t?

DIVIDING (1) BY L_t:

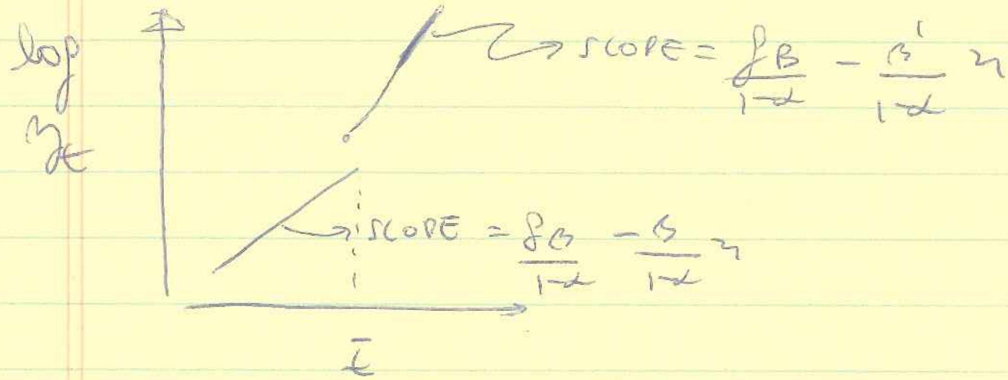
$$y_t = \frac{Y_t}{L_t} = \frac{B_t \cdot K_t^{\alpha} T^{\beta} L_t^{1-\alpha-\beta}}{L_t^{\alpha} L_t^{\beta} L_t^{1-\alpha-\beta}}$$

$$= \boxed{B_t \left(\frac{K_t}{L_t}\right)^{\alpha} \left(\frac{T}{L_t}\right)^{\beta}}$$

ASSUME $(T/L_t) < 1$

THEN

$\downarrow \beta \Rightarrow \uparrow \left(\frac{T}{L_t}\right)^{\beta} \Rightarrow$ y_t JUMPS AT $\bar{t}$


log y_t
SLOPE = $\frac{gB}{1-\alpha} - \frac{B'}{1-\alpha} n$
SLOPE = $\frac{gB}{1-\alpha} - \frac{B}{1-\alpha} n$
t