ECON 475

JONES CH.5 MODEL

PLANNER / AGGREGATE

(1) $Y = K^{\alpha}(AL_Y)^{1-\alpha}$

(2) $\hat{K} = \theta_K \frac{Y}{K} - d$

(3) $\hat{A} = \frac{\delta L_A^{\lambda}}{A^{1-\phi}}$ $\quad 0<\lambda<1$, $\quad 1<\phi<1$

(4) $L_Y + L_A = L$, $\hat{L} = n$

(5) ASSUME $\frac{L_Y}{L} = \text{CONSTANT} = \theta_Y$, $\frac{L_A}{L} = \theta_A = \text{CONSTANT}$

& $\Rightarrow \hat{L}_Y = \hat{L}_A = \hat{L} = n$

TO ANALYZE MODEL NEED TO FOCUS ON

$\frac{Y}{K}$ & $\hat{A}$

LET $X = \frac{Y}{K}$ & (*) IDENTIFY BGP/SS. VALUES.

(I) $\hat{A}$

AT BGP $\hat{A} = \text{CONSTANT} \Rightarrow$ (USING (3)):

$\widehat{\delta L_A^{\lambda}} = \widehat{A^{1-\phi}} \Rightarrow$

$$\lambda \hat{C}_A = (1-\phi)\hat{A}^*$$

USING (5) → $= n$

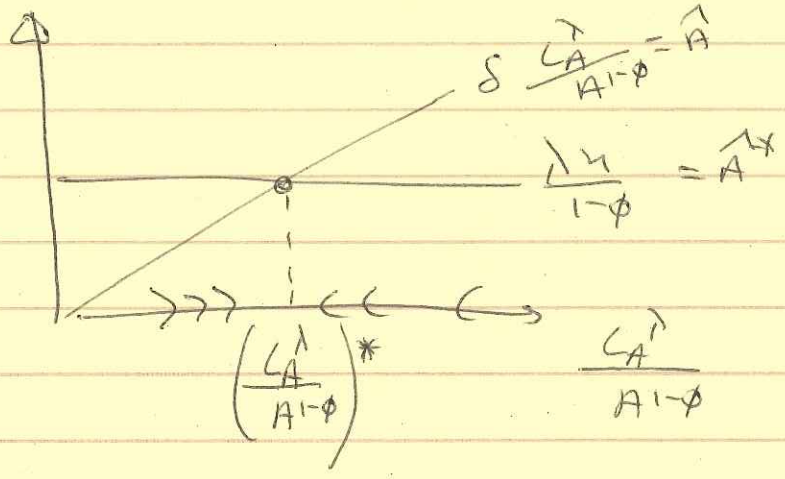
$\Rightarrow$

(6) $\hat{A}^* = \dfrac{\lambda n}{1-\phi}$ AT BGP

DIAGRAM ((3) & (6))

(3) $\hat{A} = s\left(\hat{C}_A / A^{1-\phi}\right)$ EVERYWHERE

(6) $\hat{A}^* = \lambda n / 1-\phi$



(II) $X = Y/K$

→ CALCULATE $\hat{Y}$ FIRST STEP

USING (1) $\Rightarrow \hat{Y} = \alpha \hat{K} + (1-\alpha)\left[\hat{A} + \hat{C}_Y\right]$

$\Rightarrow$ (7) $\hat{Y} = \alpha \hat{k} + (1-\alpha)\left[\hat{A} + n\right]$

USING (5)

THEN USING (7) AND (2)

$$\hat{X} = \hat{Y} - \hat{k} = \alpha \hat{k} + (1-\alpha)\left[\hat{A} + n\right] - \hat{k}$$

$$= (\alpha - 1)\hat{K} + (1-\alpha)[\hat{A} + n]$$

USING (2) $= (\alpha-1)[\Omega_K\, Y/K - d] + (1-\alpha)[\hat{A} + n]$

$$= (1-\alpha)[\hat{A} + n + d] - (1-\alpha)\Omega_K X$$

$\Rightarrow$

(8) $\boxed{\hat{X} = (1-\alpha)[\hat{A} + n + d] - (1-\alpha)\Omega_K X}$ EVERY WHERE

AT BGP, $\hat{K}$ CONSTANT $\Rightarrow$ Y/K CONSTANT

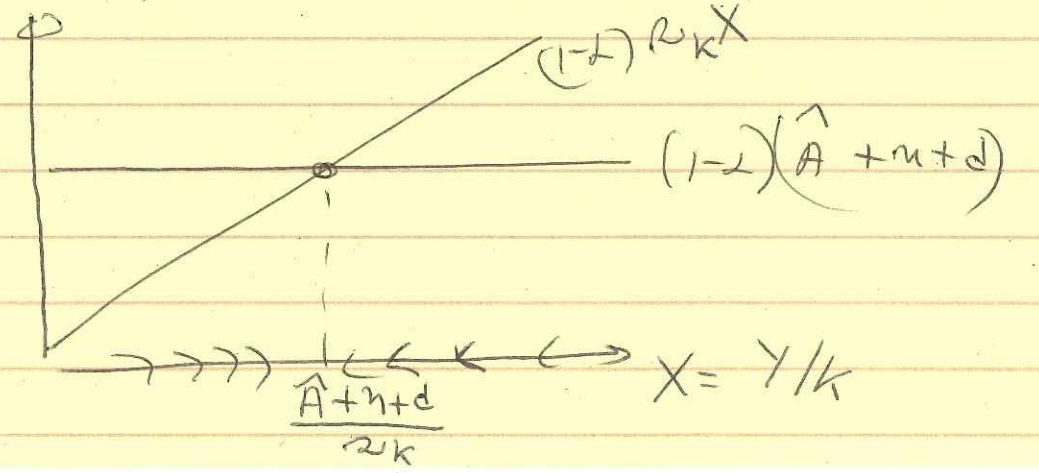
(USING (2))

(i.e. X CONSTANT) $\Rightarrow \left(\frac{\hat{Y}}{K}\right) = 0$

$\Rightarrow$ (9) $\boxed{\hat{X}^* = 0}$

USING (8) $\Rightarrow \hat{X}^* = 0 \Rightarrow \hat{A} + n + d = \Omega_K X$

$\Rightarrow$ (10) $\boxed{X^* = \dfrac{\hat{A} + n + d}{\Omega_K}}$



(III) USEFUL EQUATION

IT CAN BE SHOWN THAT

$$(11) \quad \hat{\eta} = \left(\frac{\hat{Y}}{L}\right) = \alpha\, \rho_k (X - X^*) + (1-\alpha)\left(\hat{A} - \hat{A}^*\right) + \hat{A}^*$$

THIS EQUATION IS HELPFUL FOR COMPARATIVE STATICS ANALYSIS.