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PRICE INFERENCE IN SMALL MARKETS

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PRICE INFERENCE IN SMALL MARKETS

BY MARZENA ROSTEK AND MAREK WERETKA¹

This paper investigates the effects of market size on the ability of price to aggregate traders' private information. To account for heterogeneity in correlation of trader values, a Gaussian model of double auction is introduced that departs from the standard information structure based on a common (fundamental) shock. The paper shows that markets are informationally efficient only if correlations of values coincide across all bidder pairs. As a result, with heterogeneously interdependent values, price informativeness may not increase monotonically with market size. As a necessary and sufficient condition for the monotonicity, price informativeness increases with the number of traders if the implied reduction in (the absolute value of) an average correlation statistic of an information structure is sufficiently small.

KEYWORDS: Information aggregation, double auction, divisible good auction, heterogeneous correlations, commonality.

1. INTRODUCTION

UNPRECEDENTED GROWTH in contemporaneous markets has raised questions regarding how market size impacts the ability of price to aggregate the private information dispersed among traders.² The literature on information aggregation suggests that market growth unambiguously improves the informativeness of market price: Markets are informationally efficient in that *all* payoff-relevant information in the economic system is revealed in prices. Consequently, price informativeness builds as information is introduced by new participants. The existing literature assumes that the values of a good for all traders are determined by an underlying common shock (fundamental value). The common shock assumption abstracts from a feature of growth inherent in

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²For example, in futures markets, electronic trading (80% of total exchange volume in 2007) facilitates trades from widely dispersed geographic locations. In just the last two decades, the number of traders has doubled in the top four futures markets. The Commodity Futures Trading Commission (CFTC) is concerned with how the unprecedented growth in futures markets, which increases trader diversity, will impact the traditional roles of markets in price discovery and efficiency (CFTC (2007)).

many economic settings: By increasing diversity in the population of traders, whose values for the good traded are subject to different shocks, market expansion affects heterogeneity in preference covariance among traders.³ This paper shows that information aggregation in markets in which trader values are heterogeneously correlated differs qualitatively from that of markets with a common shock. In particular, smaller markets may offer opportunities to learn from prices that are not available in large markets.

We present a model of a uniform-price double auction with an arbitrary number of traders, cast in a linear-normal setting. Permitting shock structures with heterogeneous correlations in values distinguishes our model from other strategic (small- and large-market) models of information aggregation, in particular, that of Vives (2009), and is central to this paper's results.⁴ We allow all shock structures for which the average correlation of each bidder's value with the other bidders' values, which we dub *commonality*, is the same across bidders. The model of an *equicommonal auction* accommodates a variety of aspects of heterogeneity, including preference interdependence that varies with "distance," such as geographical or social proximity; group dependence in values; and certain asymmetries in composition of trader population. Moreover, negative dependence of bidder values is permitted.

We demonstrate that in equicommonal auctions, prices convey to traders all information available in the market *only if* the correlation between values is the same for all pairs of traders (for example, as under the common shock assumption).⁵ We establish the necessary and sufficient condition for price informativeness to be monotone in the number of traders in equicommonal auctions: reduction in the absolute value of the commonality must not exceed a threshold determined by auction primitives. Focal examples in the paper use this condition to examine the growth impact for empirically motivated aspects of preference heterogeneity, including spatial or group dependence.

One lesson from the small-market literature (Dubey, Geanakoplos, and Shubik (1987), Ostrovsky (2009), and Vives (2009)) is that the nonnegligibility of individual signals in price, per se, does not obscure information aggregation.

³In many markets, trading strategies depend strongly on spatial proximity, social identity (cultural or linguistic), professional membership, or, more abstractly, shocks that affect groups of traders, but not the market as a whole. (Veldkamp (2011) provided an overview of the literature. See also Section 2.2.)

⁴There are strategic models of markets with a finite number of traders that allow for more general, nonquadratic utilities and nonnormal distributions (Dubey, Geanakoplos, and Shubik (1987) and Ostrovsky (2009)). However, these models are based on the fundamental value assumption. Departure from the fundamental value formulation of preferences also distinguishes our model from information aggregation models in the linear-normal setting, in particular, those of Kyle (1989), Vives (2009), and Colla and Mele (2010); see also Vives (2008). We postpone discussion of related literature until after full development of our model.

⁵In a model with identical correlations for all trader pairs, Vives (2009) demonstrated informational efficiency and, hence, established the "if" counterpart of our result.

An insight from our analysis is that it is not the nonnegligibility of an individual signal as such, but rather its interaction with heterogeneity in preference correlation that gives rise to nonmonotone price informativeness.

2. EQUICOMMONAL AUCTIONS

2.1. *A Double Auction Model*

Consider a market of a divisible good with $I \geq 2$ traders. We model the market as a double auction in the linear-normal setting. Trader i has a quasilinear and quadratic utility function

$$(1) \quad U_i(q_i) = \theta_i q_i - \frac{\mu}{2} q_i^2,$$

where q_i is the obtained quantity of the good auctioned and $\mu > 0$. Each trader is uncertain about how much the good is worth. Trader uncertainty is captured by the randomness of the intercepts of marginal utility functions $\{\theta_i\}_{i \in I}$, referred to as *values*. Randomness in θ_i is interpreted as arising from shocks to preferences, endowment, or other shocks that shift the marginal utility of a trader. The key novel feature of the model is that it permits heterogeneous interdependencies among values $\{\theta_i\}_{i \in I}$, as described next.

Information Structure

Prior to trading, each trader i observes a noisy signal about his true value θ_i , $s_i = \theta_i + \varepsilon_i$. We adopt an affine information structure: Random vector $\{\theta_i, \varepsilon_i\}_{i \in I}$ is jointly normally distributed; noise ε_i is mean-zero independent and identically distributed (i.i.d.) with variance σ_ε^2 , and the expectation $E(\theta_i)$ and the variance σ_θ^2 of θ_i are the same for all i . The variance ratio $\sigma^2 \equiv \sigma_\varepsilon^2 / \sigma_\theta^2$ measures the relative importance of noise in the signal. The $I \times I$ variance-covariance matrix of the joint distribution of values $\{\theta_i\}_{i \in I}$, normalized by variance σ_θ^2 , specifies the matrix of correlations

$$C \equiv \begin{pmatrix} 1 & \rho_{1,2} & \cdots & \rho_{1,I} \\ \rho_{2,1} & 1 & \cdots & \rho_{2,I} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{I,1} & \rho_{I,2} & \cdots & 1 \end{pmatrix}.$$

Lack of any correlation among values corresponds to the *independent (private) value* model, $\rho_{i,j} = 0$ for all $j \neq i$. At the other extreme, perfect correlation of values for all bidders, $\rho_{i,j} = 1$ for all $j \neq i$, gives the *pure common value* model of a double auction (e.g., the classic model of Kyle (1989), which also includes noise traders). Vives (2009) relaxed this strong dependence, while still requiring the values of all trader pairs in the market to covary in the same way,

$\rho_{i,j} = \bar{\rho} \geq 0$ for all $j \neq i$.⁶ The present paper allows correlations of values $\rho_{i,j}$ to be heterogeneous across all pairs of bidders in the market. We impose one restriction: for each trader i , his value θ_i is *on average* correlated with other traders' values θ_j , $j \neq i$, in the same way: for each i ,

$$(2) \quad \frac{1}{I-1} \sum_{j \neq i} \rho_{i,j} = \bar{\rho}$$

for some $\bar{\rho} \in [-1, 1]$; that is, in each row in \mathcal{C} , the average of the off-diagonal elements is the same. Statistic $\bar{\rho}$ measures how a trader value correlates on average with the values of all other traders in the market. Given the same average correlation across traders, $\bar{\rho}$ can be viewed as a measure of the *commonality* in values of the traded good to all market participants. We call the family of all auctions that satisfy condition (2) *equicommonal*.

Analysis is carried out at the level of correlations among values $\{\theta_i\}_{i \in I}$ specified by matrix \mathcal{C} , rather than the underlying shocks that determine the joint distribution of values. The results developed in this paper hold for all jointly normal data generating processes that give rise to an equicommonal correlation matrix.

A Sequence of Auctions

Since our primary interest is market-size effects, we analyze sequences of auctions indexed by market size $\{A^I\}_{I=1}^\infty$. In a sequence, the utility function remains the same for all auctions. What changes is the number of traders I and, crucially, the equicommonal matrix \mathcal{C} may vary with market size in an arbitrary way; commonality $\bar{\rho}$ itself may change with market size and so may other details of the correlation matrix. (In Section 4.3, it will be natural to make a stronger assumption that, when adding a trader, the correlation matrix among the remaining traders is preserved.) Define a measure of *market size* as a monotone function of the number of traders, $\gamma \equiv 1 - 1/(I - 1)$; γ ranges from 0 for $I = 2$ to 1 as $I \rightarrow \infty$. Throughout, we refer to auctions with $\gamma < 1$ as *finite* and reserve the term *infinite* for limits as $\gamma \rightarrow 1$. For a sequence of equicommonal auctions $\{A^I\}_{I=1}^\infty$, *commonality function* $\bar{\rho}(\gamma)$ specifies commonality for any market size.

Double Auction

We study double auctions based on the canonical uniform-price mechanism.⁷ Bidders submit strictly downward-sloping (net) demand schedules $\{q_i(p)\}_{i \in I}$;

⁶In a later version of Vives (2009), $\rho_{i,j} = \bar{\rho} < 0$, $i \neq j$, is also permitted. The analyses for negative correlations in this paper and Vives (2009) were developed independently.

⁷In this paper, as in Vives (2009), the results extend to a larger class of models: competitive and strategic, including one-sided auctions (with an elastic or inelastic demand) and nonmarket settings, in which I Bayesian agents each make inference about a random variable θ_i based on the observed signal s_i and a statistic that is a deterministic function of the average signal.

the part of a bid with negative quantities is interpreted as a supply schedule. The market clearing price p^* is that for which aggregate demand equals zero, $\sum_{i \in I} q_i(p^*) = 0$.⁸ Bidder i obtains the quantity determined by his submitted bid evaluated at the equilibrium price, $q_i^* = q_i(p^*)$, for which he pays $q_i^* \cdot p^*$. Bidder payoff is given by $U_i(q_i^*) - q_i^* \cdot p^*$. The symmetric linear⁹ Bayesian Nash equilibrium (henceforth, equilibrium) is used as a solution concept.

A noteworthy feature of our double auction model is that all traders—buyers and sellers—are Bayesian and strategic. (In particular, there are no noise traders.)

2.2. Examples

While restrictive, the class of equicommonal auctions subsumes a variety of economic environments beyond those with common $\rho_{i,j} = \bar{\rho} \geq 0$. Let us introduce two examples of equicommonal auctions that capture various aspects of preference interdependence that are common to many economic settings. As a benchmark, we also consider the standard model based on the fundamental value assumption.

EXAMPLE 1—Fundamental Value Model: A common (fundamental) shock determines the values of all bidders who are, in addition, subject to idiosyncratic (i.i.d.) shocks. As a result, values are equally correlated for all pairs of bidders in the auction; that is, $\rho_{i,j} = \bar{\rho} > 0$ for all $j \neq i$. Correlation of a new bidder's value with each incumbent's value is equal to $\bar{\rho}$ and the commonality function $\bar{\rho}^{\text{FV}}(\gamma) = \bar{\rho}$ is constant.

A stochastic process with fundamental and idiosyncratic shocks is often assumed in the macroeconomics and finance literature. Nevertheless, the fundamental value assumption precludes markets in which shocks affect subgroups of traders and, thus, the values of some traders covary more closely than others.

Electronic trading, trade liberalization, and globalization trends in contemporaneous markets all encourage participation from diverse geographic locations. Increased trader diversity translates into greater heterogeneity in preference correlations.¹⁰ Heterogeneity that results from spatial considerations motivates the next model.¹¹

⁸The definition of the game can be completed in the usual way: If there is no such price or if multiple prices exist, then no trade takes place. The assumption that bids are strictly downward-sloping rules out trivial (no-trade) equilibria.

⁹The term “symmetric linear” is understood as bids having the functional form of $q_i(p) = \alpha_0 + \alpha_s s_i + \alpha_p p$, where the coefficients α_s and α_p are the same across bidders.

¹⁰As empirical evidence demonstrates, trading preferences or endowments depend strongly on geographical and cultural proximity or educational networks (e.g., Coval and Moskowitz (2001), Hong, Kubik, and Stein (2004), Cohen, Frazzini, and Malloy (2008); see also Veldkamp (2011)).

¹¹Malinova and Smith (2006) and Colla and Mele (2010) studied spatial informational linkages in a linear-normal setting: While the asset has a *fundamental value*, traders pool signals with their

EXAMPLE 2—Spatial Model: I bidders are located on a circle. The distance between any two immediate neighbors is normalized to 1. Let $d_{i,j}$ be the shorter of the two distances between bidders i and j (measured along the circle). To capture that values of closer neighbors covary more strongly, correlation between any two bidders $\rho_{i,j}$ is assumed to decay with distance, $\rho_{i,j} = \beta^{d_{i,j}}$, where $\beta \in (0, 1)$ is a decay rate. The model takes as a primitive the decay rate β and assumes that a new bidder enlarges the auction by increasing the circle circumference by 1.¹² The commonality function $\bar{\rho}^S(\gamma) = 2(1 - \gamma)\beta(1 - \beta^{(1/2)1/(1-\gamma)})/(1 - \beta)$ (assuming that I is odd) is decreasing.

In many markets, one can identify groups of traders with distinct preferences—sectors, industries, countries, clubs, or social affiliations. Since the income, endowment, and liquidity needs of traders from different groups are governed by different shocks, values tend to covary more strongly within than across groups. The group dependence of correlations in values is captured by the group model.

EXAMPLE 3—Group Model: There are two groups, each of size $I/2$. The values that members of each group derive from the traded good are perfectly correlated ($\rho_{i,j} = 1$); cross-group correlation can be positive or negative, or values can be independent ($\rho_{i,j} = \alpha$; $\alpha \in [-1, 1]$). Additional bidders increase the populations of both groups and the commonality function $\bar{\rho}^G(\gamma) = (\gamma + (2 - \gamma)\alpha)/2$ (assuming that I is even) is increasing.

Unlike the fundamental value and spatial models, the group model permits negative correlation of values.

3. CHARACTERIZATION OF EQUILIBRIUM

In a finite double auction, bidders shade their bids relative to the bids they would submit if they were price-takers and values were independent and private ($\rho_{i,j} = 0$ for $j \neq i$). It is well known (e.g., from Kyle (1989)) that equilibrium existence requires that the resulting bid shading not be too strong. Correspondingly, equilibrium existence in equicommonal auctions requires an upper bound on commonality, $\bar{\rho}^+(\gamma, \sigma^2)$ (derived in the proof of Proposition 1).

neighbors. These models differ from our spatial model in Example 2 in two main respects: In terms of matrix C , the models assume $\rho_{i,j} = \bar{\rho}$ for all $j \neq i$ and, there, noise is correlated as a function of distance between traders. In our paper, heterogeneity in preference correlations derives from the interdependence of values rather than noise and all traders are Bayesian (in particular, there are no noise traders).

¹²Without loss of generality a bidder can be added at an arbitrary position on a circle. Alternatively, one can assume that the circumference is fixed and additional bidders increase population density. Such a formulation implies that values covary more closely in pairs of traders in larger markets.

For the fundamental value model, the bound $\bar{\rho}^+(\gamma, \sigma^2)$ coincides with that of Vives (2009)¹³ and otherwise weakens the Vives bound in that $\bar{\rho}^+(\gamma, \sigma^2)$ involves only the *average* correlation. Moreover, for negative correlations, commonality has to be strictly above the lower bound of $\bar{\rho}^-(\gamma) = -(1 - \gamma) < 0$.¹⁴ Proposition 1 demonstrates that a symmetric linear Bayesian Nash equilibrium exists in equicommonal double auctions under quite general conditions. The necessary and sufficient conditions are assumed thereafter.

PROPOSITION 1—Existence of Equilibrium: *There exist bounds $\bar{\rho}^-(\gamma)$ and $\bar{\rho}^+(\gamma, \sigma^2)$ such that, in an equicommonal double auction characterized by $(\gamma, \bar{\rho})$, a symmetric linear Bayesian Nash equilibrium exists if and only if $\bar{\rho}^-(\gamma) < \bar{\rho} < \bar{\rho}^+(\gamma, \sigma^2)$. The symmetric equilibrium is unique.*

The bounds $\bar{\rho}^-(\gamma)$ and $\bar{\rho}^+(\gamma, \sigma^2)$ are tight; that is, for any pair $(\gamma, \bar{\rho})$ that strictly satisfies the two bounds, there is an auction (i.e., a data generating process $\{\theta_i\}_{i \in I}$ with $1 - 1/(I - 1) = \gamma$ and an equicommonal correlation matrix \mathcal{C} characterized by $\bar{\rho}$) for which an equilibrium exists. Proposition 1 contributes to the equilibrium existence for divisible goods by accommodating markets with heterogeneously interdependent values (pairwise correlations of values in the auction can be arbitrary), negative (individual and average) dependence, and auctions with two bidders, as long as $\bar{\rho} < 0$. (In the classic models by Wilson (1979) and Kyle (1989), an equilibrium fails to exist with $I = 2$.¹⁵)

Proposition 2 derives the equilibrium bids for a class of auctions characterized by $(\gamma, \bar{\rho})$. Given a profile of linear bids of bidders $j \neq i$, the best response of bidder i with utility (1) is given by the first-order (necessary and sufficient) condition: for any p ,

$$(3) \quad E(\theta_i | s_i, p) - \mu q_i = p - \frac{(\partial q_i(p) / \partial p)^{-1}}{I - 1} q_i,$$

where $-(\partial q_i(p) / \partial p)^{-1} / (I - 1)$ is the slope of the (aggregate) supply defined for bidder i by symmetric bids $j \neq i$, the realization of which depends on signals $\{s_j\}_{j \neq i}$. By the first-order condition (3) and market clearing, the equilibrium

¹³Assuming an inelastic demand and downward-sloping bids in Vives (2009).

¹⁴For the stochastic process that generates a joint distribution of values itself to exist, $\text{Var}(\frac{1}{I} \sum_{i \in I} \theta_i) = \frac{1}{I^2} (I \sigma_\theta^2 + (I - 1) I \bar{\rho} \sigma_\theta^2) \geq 0$, which holds if and only if $\bar{\rho} \geq \bar{\rho}^-(\gamma) = -\frac{1}{I-1} = -(1 - \gamma)$. Thus, the lower bound is binding only for commonality equal to $\bar{\rho}^-(\cdot)$.

¹⁵In the absence of price inference, the nonexistence with $I = 2$ results from strategic interdependence in bid shading: The slope of the best response to an arbitrary (inverse) linear bid of the opponent is strictly greater than the slope of the opponent's (inverse) bid and, consequently, a Nash equilibrium does not exist in which inverse bids have finite slopes. When values are, on average, positively correlated, price informativeness amplifies the strategic interdependence of best responses, whereas when values are, on average, negatively correlated, price informativeness mitigates the interdependence, which gives existence in our model.

price is characterized by $p^* = \frac{1}{I} \sum_{i \in I} E(\theta_i | s_i, p^*)$. Given an affine information structure, the conditional expectation is linear, $E(\theta_i | s_i, p) = c_\theta E(\theta_i) + c_s s_i + c_p p$. The two conditions and the projection theorem applied to random vector (θ_i, s_i, p) determine the inference coefficients c_s , c_p , and c_θ in terms of commonality and market size.

PROPOSITION 2—Equilibrium Bids: *The equilibrium bid of trader i is*

$$(4) \quad q_i(p) = \frac{\gamma - c_p}{1 - c_p} \frac{c_\theta}{\mu} E(\theta_i) + \frac{\gamma - c_p}{1 - c_p} \frac{c_s}{\mu} s_i - \frac{\gamma - c_p}{\mu} p,$$

where inference coefficients in the conditional expectation $E(\theta_i | s_i, p)$ are given by

$$(5) \quad c_s = \frac{1 - \bar{\rho}}{1 - \bar{\rho} + \sigma^2},$$

$$(6) \quad c_p = \frac{(2 - \gamma)\bar{\rho}}{1 - \gamma + \bar{\rho}} \frac{\sigma^2}{1 - \bar{\rho} + \sigma^2},$$

$$(7) \quad c_\theta = 1 - c_s - c_p.$$

By ensuring that the equilibrium price is equally informative across bidders, equicommonality permits a symmetric equilibrium.

Propositions 1 and 2 allow us to introduce a convenient geometric representation of equicommonal auctions. Holding primitives other than \mathcal{C} fixed, the class of auctions characterized by a given pair $(\gamma, \bar{\rho})$ can be represented as a point in the Cartesian product $[0, 1] \times [-1, 1]$ (Figure 1(A)).¹⁶ By Proposition 2, all auctions in this class share the same equilibrium bids. Figure 1(B) depicts commonality functions for the sequences of auctions from the examples in Section 2.2.

4. INFORMATION AGGREGATION

How does market expansion affect the market’s ability to aggregate traders’ private information when trader values covary heterogeneously? A logically prior question is whether markets are informationally efficient in that prices convey to traders all available private payoff-relevant information. As such, the pool of information available in a market is nondecreasing with every new trader.¹⁷ In informationally efficient markets, since the additional piece of information contained in the new traders’ signals becomes fully incorporated in price, price informativeness improves as the market grows.

¹⁶Precisely, γ takes values from a countable set $\Gamma \equiv \{\gamma \in [0, 1] | \gamma = 1 - 1/(I - 1), I = 2, 3, \dots\}$. The commonality function is a map $\bar{\rho}(\cdot): \Gamma \rightarrow [-1, 1]$. The space of equicommonal auctions is given by a Cartesian product $\Gamma \times [-1, 1]$.

¹⁷Strictly speaking, this holds (as a sufficient condition) in all sequences of auctions in which, for all i and j , $\rho_{i,j}$ is not affected by introducing additional bidders $h \neq i, j$.

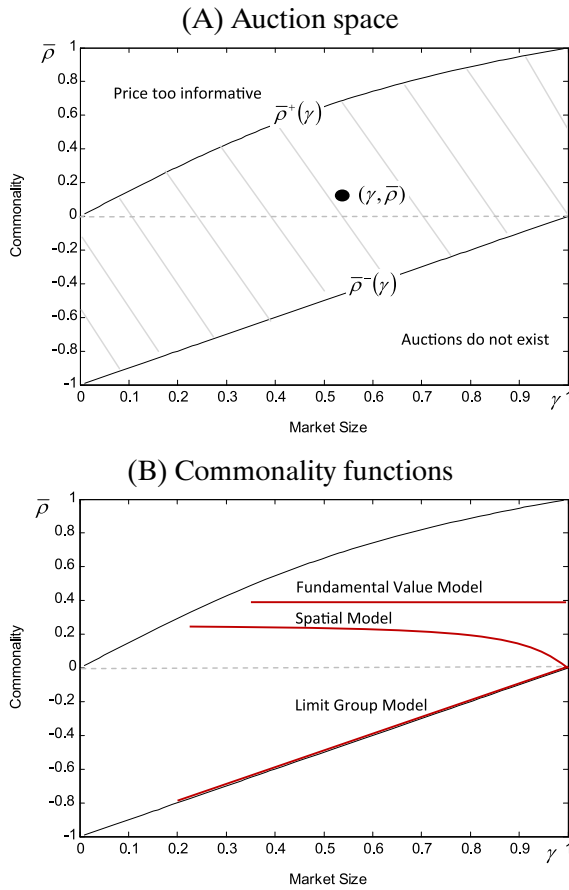


FIGURE 1.—Existence and commonality function.

4.1. Informational Efficiency

In the literature, informational efficiency is conceptualized by means of a privately revealing price: The market is set against an efficiency benchmark of the total available information, which corresponds to the profile of all bidders' signals, $s \equiv \{s_i\}_{i \in I}$.

DEFINITION 1: The equilibrium price is *privately revealing* if, for any bidder i , the conditional cumulative distribution functions of the posterior of θ_i satisfy $F(\theta_i | s_i, p^*) = F(\theta_i | s)$ for every state s , given the corresponding equilibrium price $p^* = p^*(s)$.

A privately revealing price allows every Bayesian player i , who also observes his own signal s_i , to learn about value θ_i as much as he would if he had access to

all the information available in the market, s . Proposition 3 determines which double auctions accomplish efficiency in this sense.

PROPOSITION 3—Aggregation of Private Information: *In a finite double auction, the equilibrium price is privately revealing if and only if $\rho_{i,j} = \bar{\rho}$ for all $j \neq i$.*

Proposition 3 extends to infinite auctions as long as $\lim_{\gamma \rightarrow 1} \bar{\rho}(\gamma) < 1$. Our result resonates with that of Vives (2009), who examined markets with $\rho_{i,j} = \bar{\rho}$ for all $j \neq i$ and proves the “if” part of our result. Even if learning through market does not suffice for traders to learn their values exactly, in markets such as those in which uncertainty is driven by fundamental shocks, traders learn all information that is available. One lesson from Vives (2009), and the small-market literature more generally, is that strategic behavior and the nonnegligibility of individual signals in price in finite markets can be consistent with informational efficiency. Our result complements the aggregation prediction of the literature by underscoring the role of heterogeneity in interdependence among trader values for (in)efficiency.

The lack of private revelation of information in Proposition 3 does not result from the presence of noise traders (e.g., Kyle (1989)) or uncertainty about aggregate endowment. As in Jordan (1983), the dimension of signals exceeds the dimension of the learning instrument (price). However, given the normally distributed signals, the dimension of (payoff-relevant) information is reducible to that of price and information is not too rich to be summarized by price: for any bidder, a statistic exists that is sufficient for the payoff-relevant information contained in the signals of other bidders. This sufficient statistic is a properly weighted average signal, where the weights depend on correlations \mathcal{C} and may differ across bidders. In an equicommonal auction, price deterministically reveals the equally weighted average signal \bar{s} (see (10)), which is, for each bidder, the sufficient statistic only in models with identical correlations.¹⁸

¹⁸The generic lack of private revelation does not stem from equilibrium symmetry. Even if asymmetric linear equilibria exist, they are not privately revealing when correlations are heterogeneous. Suppose that an asymmetric equilibrium exists. Heuristically, in any equicommonal auction, by the projection theorem, for any trader i , there exists a vector of weights $\omega^i = (\omega_1^i, \omega_2^i, \dots, \omega_j^i)$ satisfying $\sum_j \omega_j^i = 1$, such that a one-dimensional statistic $ss_i = \sum_j \omega_j^i s_j$ is sufficient for the signals of other bidders. Weights ω^i depend on the correlations of signals s_j with value θ_i and with heterogeneous correlations in values, they *differ across* bidders; $\omega^i \neq \omega^j$ for $i \neq j$. (For instance, in the spatial model from Example 2, the weights associated with immediate neighbors are higher than those of distant bidders.) Thus, ss_i is not perfectly correlated with ss_j for some $j \neq i$. Then, even if in some asymmetric equilibrium, price perfectly reveals sufficient statistic ss_i , it cannot simultaneously deterministically reveal ss_j . That is, price does not aggregate information for bidder j and, hence, is not privately revealing. When correlations in values are the same for all bidder pairs, the sufficient statistic coincides for all bidders and is revealed by price in a symmetric (but not asymmetric, if it exists) equilibrium. Thus, the private revelation property requires both identical correlations in values for all bidder pairs and equilibrium symmetry.

Proposition 3 demonstrates that, generically in equicommonal auctions, prices do not aggregate all available information. Informational inefficiency, in turn, severs the link between market size and price informativeness. The next section provides a condition under which market growth that increases heterogeneity still translates into more informative market prices.

4.2. Price Informativeness

To measure price informativeness, we examine how much inference through the market—that is, conditioning on the equilibrium price p^* as well as one’s own signal s_i —reduces the variance of the posterior of θ_i , conditional only on the signal. Define an index of *price informativeness* $\psi^+ \in [0, 1]$ as

$$(8) \quad \psi^+ \equiv \frac{\text{Var}(\theta_i|s_i) - \text{Var}(\theta_i|s_i, p^*)}{\text{Var}(\theta_i|s_i)}.$$

Index ψ^+ quantifies the market’s contribution to inference about a trader value θ_i . No reduction in variance ($\psi^+ = 0$) occurs when the price contains no payoff-relevant information beyond a private signal, whereas full reduction ($\psi^+ = 1$) is accomplished when the price, jointly with the private signal, precisely reveals the value θ_i to trader i . Index ψ^+ is not trader-dependent, an artifact of the equicommonality assumption in a symmetric equilibrium.

Proposition 4 pins down the necessary and sufficient condition, in any equicommonal auction, for a new bidder to increase the informational content of price. In a sequence of equicommonal auctions, let $\Delta\bar{\rho}(\gamma)$ be the change in commonality that results from including a new bidder in an auction of size γ .

PROPOSITION 4—Informational Impact: *Fix γ and $\bar{\rho} > 0$ ($\bar{\rho} < 0$). A threshold $\tau < 0$ (respectively, $\tau > 0$) exists such that, in any auction that satisfies $\bar{\rho}(\gamma) = \bar{\rho}$, the contribution of an additional bidder to price informativeness is strictly positive if and only if $\Delta\bar{\rho}(\gamma) > \tau$ (respectively, $\Delta\bar{\rho}(\gamma) < \tau$).*

Price informativeness increases provided a new trader participation does not induce too strong a reduction in the (absolute value of) commonality. The threshold τ is characterized in the [Appendix](#).

Proposition 4 can be interpreted geometrically by constructing a map of price informativeness curves (Figure 2). For each value $\psi^+ \in [0, 1]$, let a ψ^+ curve comprise all profiles $(\gamma, \bar{\rho})$ that give rise to price informativeness equal to ψ^+ . For $\bar{\rho} = 0$ (e.g., independent private value setting), price is uninformative ($\psi^+ = 0$) for any market size and the 0 curve coincides with the horizontal axis. For any $\psi^+ \in (0, 1)$, a ψ^+ curve consists of two segments, located in the positive and negative quadrants of $\bar{\rho}$, as bidders can learn from prices in environments with positive and negative dependence among values. For the slope of ψ^+ curves, ceteris paribus, price informativeness ψ^+ increases in market

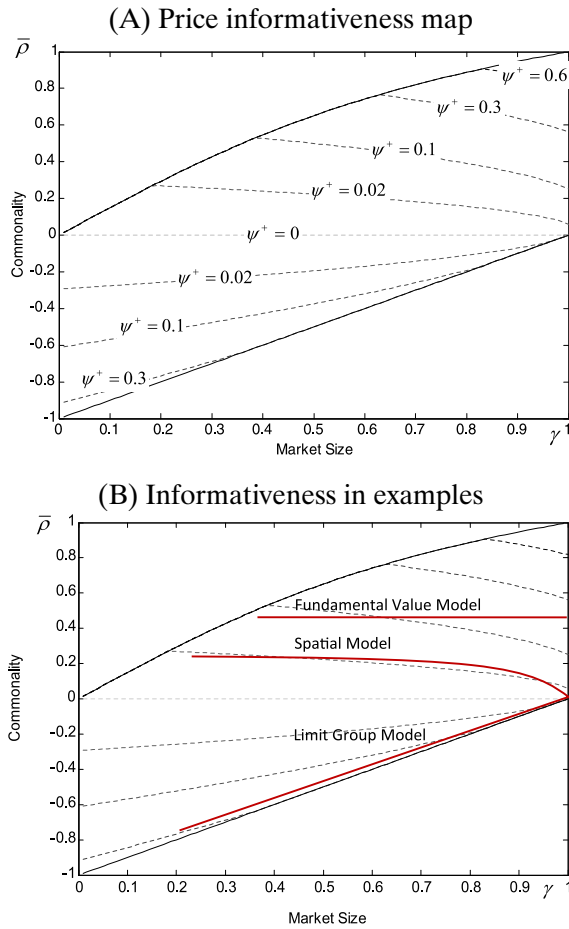


FIGURE 2.—Price informativeness map.

size γ and the absolute value of commonality $\bar{\rho}$. Consequently, the positive (negative) components of a ψ^+ curve slope down (up) and ψ^+ curves located farther away from the horizontal axis correspond to greater price informativeness, with the curve for the maximum price informativeness $\psi^+ = 1$ comprising one point $(1, 1)$. Take an arbitrary sequence of auctions represented by a commonality function $\bar{\rho}(\gamma)$. For any auction $(\gamma, \bar{\rho})$ in the sequence, the condition from Proposition 4 is approximately (cf. footnote 16) the commonality function crossing the ψ^+ curve at point $(\gamma, \bar{\rho})$ from below if $\bar{\rho} > 0$ or from above if $\bar{\rho} < 0$. Threshold τ measures the change in commonality that is just sufficient to maintain the price informativeness constant with an additional bidder.

Implications

By Proposition 4, since market growth can have an arbitrary impact on price informativeness,¹⁹ to assess this impact, it is essential first to determine the growth's effect on the structure of covariance in trader preferences. Specifically, if the fundamental value model provides a good approximation of preference interdependence in the considered market, then market growth unambiguously advances learning. Insofar as geographical, social, cultural, and other "distances" between traders are the chief determinant of the interdependencies among values, our spatial model suggests that additional traders enhance learning when the market is small, but when the market size exceeds a certain threshold, price becomes less informative as the market grows further. (See Figure 3(A).) In the group model, price informativeness monotonically increases with market size unless values are sufficiently negatively correlated across trader groups ($\alpha \in (-1, -1/3)$), in which case, price informativeness exhibits a U-shaped behavior (Figure 3(B)). Worth noting is an instance of the limit group model as $\alpha \rightarrow -1$,²⁰ where price informativeness decreases with every additional trader and learning from prices is most effective in the smallest market. Notably, in this model, $\bar{s} = \frac{1}{I} \sum_i \varepsilon_i$ and, hence, the equilibrium price is independent from each trader's value θ_i . Still, traders learn from prices in all but the infinite auction. Section 4.3 investigates the mechanisms that give rise to the behaviors of price informativeness described in this section.

Informational Content in Infinite Markets

Inference in infinite equicommonal auctions may feature three qualitatively different outcomes. Specifically, price is perfectly uninformative about θ_i if $\lim_{\gamma \rightarrow 1} \bar{\rho}(\gamma) = 0$; it contains information about θ_i if $\lim_{\gamma \rightarrow 1} \bar{\rho}(\gamma) \in (0, 1]$; only if $\lim_{\gamma \rightarrow 1} \bar{\rho}(\gamma) = 1$ does price deterministically reveal θ_i . For instance, in the spatial model, the price conveys no information about bidder values in the infinite market. Given that new traders add to the pool of payoff-relevant information, why does the price become uninformative in the infinite spatial model? For any decay rate $\beta \in (0, 1)$, any trader's value is strongly correlated only with a group of close neighbors and is essentially independent from the values of distant traders. A strong correlation of values in a neighborhood becomes negligible in an infinite market. With respect to price informativeness (but not with

¹⁹In his classic paper, Kremer (2002) offered an example of a (unit-demand) auction in which a large auction fails to aggregate all information. A key feature of the example is that the total amount of information is fixed (does not depend on market size) so that the accuracy of an individual signal decreases in the number of bidders. Our paper preserves signal accuracy (noise variance is constant) and, hence, total information available in the market increases with new traders. Still, price may reveal less information in larger markets due to heterogeneous interdependence among values.

²⁰In the group model with $\alpha = -1$, $\bar{\rho} = -(1 - \gamma)$ and, by Proposition 1, an equilibrium does not exist.

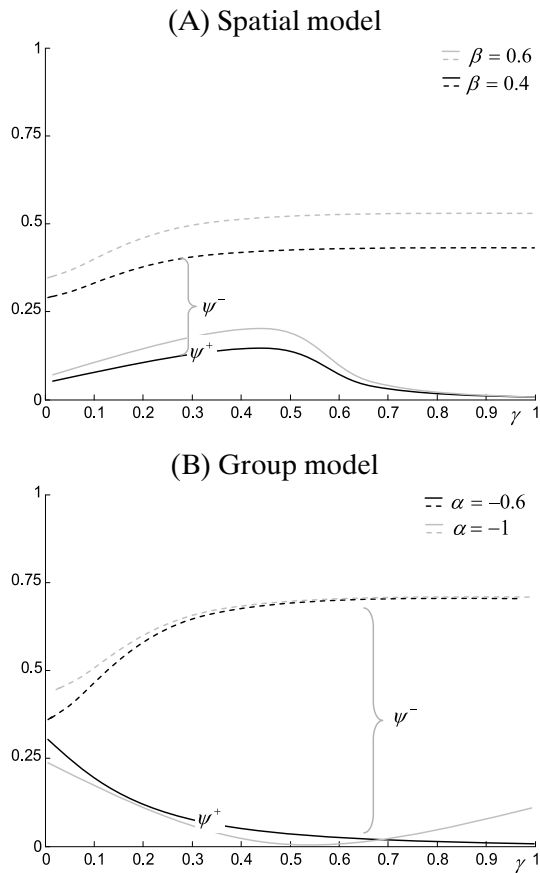


FIGURE 3.—Price informativeness and informational gap.

regard to informational efficiency), an infinite market operates effectively like one with independent private values. In Figure 3, informational inefficiency is measured by $\psi_i^- \equiv (\text{Var}(\theta_i|s_i, p^*) - \text{Var}(\theta_i|s)) / \text{Var}(\theta_i|s_i)$; $\psi_i^- \in [0, 1]$. Indices ψ^+ , defined in (8), and ψ_i^- quantify the contribution of the market to learning and the potential for learning outside the market, respectively; $\psi_i^- + \psi^+ \leq 1$.

4.3. Finite versus Infinite Auctions

The absence of the monotonicity of price informativeness suggests that some information that is lost in price in infinite auctions becomes revealed in finite auctions. This section identifies components of a trader signal s_i transmitted by price in finite and infinite auctions. For the purpose of comparing signal decompositions in finite as well as infinite auctions, this section assumes that an auction A^{l+1} results from adding a trader to an auction A^l and the pairwise

correlations in values from the auction A^I are preserved in A^{I+1} . Precisely, a random vector $\{\theta_i\}_{i=1}^I$ in auction A^I is a truncation of vector $\{\theta_i\}_{i=1}^\infty$ in the infinite auction.

For an infinite auction with trader values $\{\theta_i\}_{i=1}^\infty$, define a *common value component* as a random variable X such that for each trader i , θ_i can be decomposed into X and a *residual* $R_i \equiv \theta_i - X$, such that, for each trader i , (i) $X \perp R_i$ and (ii) $R_i \perp \lim_{I \rightarrow \infty} \frac{1}{I} \sum_i R_i$; that is, the common value component is independent from each bidder residual and each bidder residual is independent from the average residual in the infinite auction.²¹ A thus-defined common value component X represents randomness in values common to all traders,²² while residuals $\{R_i\}_{i=1}^\infty$, which can be mutually correlated, capture shocks in values that affect trader subgroups, but not the market as a whole. Lemma 1 in the Appendix demonstrates that in any infinite equicommonal auction—that is one such that $\lim_{I \rightarrow \infty} \frac{1}{I-1} \sum_{j \in I, j \neq i} \rho_{i,j} \equiv \bar{\rho}^\infty$ is the same for all i —a common value component X exists and is identified uniquely up to a constant, $X = \lim_{I \rightarrow \infty} \frac{1}{I} \sum_{i \in I} \theta_i \sim \mathcal{N}(E(\theta), \sigma_\theta^2 \bar{\rho}^\infty)$. Among the examples from Section 2.2, X is deterministic in the independent private value setting, the spatial model, and the limit group model ($\alpha \rightarrow -1$), and is nondegenerate in the fundamental value mode and the group model with $\alpha > -1$.

To explain Bayesian learning about value θ_i from price p^* and signal $s_i = X + R_i + \varepsilon_i$, and, more generally, to shed light on the results about price informativeness from Section 4.2, we examine Bayesian updating of each signal component $y = X, R_i, \varepsilon_i$. Proposition 5 determines price inference coefficient $c_{y,p}$ in the conditional expectation $E(y|s_i, p^*) = \text{const} + c_{y,s_i} s_i + c_{y,p} p^*$ in terms of the coefficients from the projection of the equilibrium price on the signal components, $\beta_y = \text{Cov}(y, p^*) / \text{Var}(y)$, which allows us to attribute price inference to each component y in a given auction.²³

PROPOSITION 5—Inference Coefficients: *For a signal component $y = X, R_i, \varepsilon_i$, inference coefficient $c_{y,p}$ is given by*

$$(9) \quad c_{y,p} = \bar{c} \sum_{y' \neq y} (\beta_y - \beta_{y'}) \sigma_y^2 \sigma_{y'}^2,$$

where constant $\bar{c} \equiv \text{Var}(s_i) \text{Var}(p^*) - (\text{Cov}(s_i, p^*))^2 > 0$ is the same for all y .

²¹Given the assumption that a finite auction is a truncation of the infinite auction, the common value component defined for an infinite auction can then be interpreted as a common value component for a subset of bidders as well.

²²Clearly, the common value component may be nondegenerate even if the underlying process does not involve a shock that is common to all bidders (e.g., the group model).

²³It can be shown that $\beta_y \geq 0$ for $y = X, R_i, \varepsilon_i$, even if $\bar{\rho} < 0$. In small auctions, price is strictly positively correlated with ε_i and R_i , as it is an increasing function of the average signal, $\bar{s} = \frac{1}{I} (X + R_i + \varepsilon_i) + \frac{1}{I} \sum_{j \neq i} s_j$. If the common value component is nondegenerate, $\beta_X > \beta_\varepsilon > 0$ holds by the positive correlation of X with s_j . Moreover, $\beta_R \in [0, \beta_X]$, where $\beta_R = 0$ for $\bar{\rho}(\gamma) - \bar{\rho}^\infty = -(1 - \gamma)$ and $\beta_R = \beta_X$ for $\bar{\rho}(\gamma) - \bar{\rho}^\infty = 1$; $\beta_R = \beta_\varepsilon$ if $\bar{\rho} = 0$.

As is important for our analysis, coefficient $c_{y,p}$ depends on the coefficients β_X , β_{R_i} , and β_ε only through differences $|\beta_y - \beta_y|$ and is zero when β_y 's coincide for all y . A Bayesian bidder i who makes inference based on s_i and p^* can be interpreted as decomposing the observed signal realization into the conditional expectations of the signal components $s_i = E(X|s_i, p^*) + E(R_i|s_i, p^*) + E(\varepsilon_i|s_i, p^*)$. Given the fixed sum of the three components, a higher price realization results in an upward revision of the expectation of y (i.e., $c_{y,p} > 0$) and, thus, a downward revision of the sum of the other two components only if price moves more strongly with y than with the other two components. When $\beta_X = \beta_R = \beta_\varepsilon$, the equilibrium price realizations do not affect signal decomposition and the price does not contain any information about y beyond s_i .

Infinite Auctions

As is well understood from the competitive literature, in large markets, the equilibrium price reflects only those elements of information that are common to all traders (e.g., Hellwig (1980)). Accordingly, in an infinite equicommonal auction, the equilibrium price reveals no information contained in signals $\{s_i\}_{i=1}^\infty$ other than that which results from the common value component X . Individual signal s_i has a negligible impact on the average signal \bar{s} and, hence, price p^* . Correlation of p^* with s_i —and Bayesian updating in general—originates from the common value component alone, present in the price and values of all traders. It follows that $\beta_R = \beta_\varepsilon = 0$ and, hence, from (9), the price is informative about value $\theta_i = X + R_i$ (i.e., $c_{p,X} + c_{p,R} = c\beta_X\sigma_X^2\sigma_\varepsilon^2 \neq 0$) only if the common value component is nondegenerate ($\sigma_X^2 = \sigma_\theta^2\bar{\rho}^\infty \neq 0$), as it is in the fundamental value and group models. The informational content of the common value component X about θ_i might vary from almost full revelation (which is obtained only with pure common values for almost all bidders) to nothing (e.g., the spatial model).

Finite Auctions

In finite auctions, the effect of an individual realization s_i on the equilibrium price is nonnegligible. Consequently, correlation between the signal and the price can arise not only from the presence of a common value component X , but also from the residual R_i and noise ε_i . This changes the nature of learning in finite auctions from that of infinite auctions, as follows.

First, a finite-auction price can be informative, even if X is deterministic ($\bar{\rho}^\infty = \sigma_X^2 = c_{p,X} = 0$); hence, $R_i = \theta_i$ (modulo a constant) and the price contains no information about value θ_i in the infinite auction.

Second, by Proposition 5, with a deterministic X , price inference in finite auctions can be interpreted as (“net”) *learning through residual* ($\beta_R > \beta_\varepsilon$) or *learning through noise* ($\beta_R < \beta_\varepsilon$). Learning through the residual occurs if and only if $\bar{\rho} > 0$; then, $c_{p,R} = -c_{p,\varepsilon} = c(\beta_R - \beta_\varepsilon)\sigma_R^2\sigma_\varepsilon^2 > 0$. This happens in the spatial model. Similarly, learning through noise occurs if and only if $\bar{\rho} < 0$;

then $c_{p,R} = -c_{p,\varepsilon} = c(\beta_R - \beta_\varepsilon)\sigma_R^2\sigma_\varepsilon^2 < 0$. In the latter case, bidder i attributes a higher p^* to a high realization of ε_i rather than R_i . This explains why, in the limit group model ($\alpha \rightarrow -1$), the price is informative even though $\bar{s} = \frac{1}{I} \sum_i \varepsilon_i$ and, hence, the price is independent from each trader's value θ_i and $\beta_R = 0$. In a finite independent private value auction (more generally, $\bar{\rho} = 0$), residual R_i and noise ε_i are positively correlated with the price ($\beta_R = \beta_\varepsilon > 0$); however, the comovements offset each other and the price is uninformative, $c_{p,R} = c_{p,\varepsilon} = c_{p,X} = 0$.

Third, learning about θ_i through the common value component reinforces learning through the residual or counterbalances learning through noise. In the group model, $\beta_R = 0$, and depending on β_X , β_ε , σ_X^2 , and σ_ε^2 , traders learn either through noise or through the common value component. In the fundamental value model, residuals are mutually independent; therefore, $\beta_R = \beta_\varepsilon > 0$ and bidders learn only through the common value component, as $\beta_X > \beta_\varepsilon$.

Fourth, whether and why price informativeness ψ^+ is monotone in γ depends on how differences $|\beta_y - \beta_{y'}|$ in inference coefficients (9) change as the market grows. For example, the nonmonotone (U-shaped) price informativeness ψ^+ in the group model with $\alpha > -1$ can be understood in terms of two countervailing effects that derive from $\beta_X > \beta_\varepsilon > \beta_R = 0$.²⁴

APPENDIX

The proof of Proposition 1 is provided after the proof of Proposition 2.

PROOF OF PROPOSITION 2: From (3) and market clearing, the equilibrium price is equal to $p^* = \frac{1}{I} \sum_{i \in I} E(\theta_i | s_i, p^*)$. Given that $E(\theta_i | s_i, p) = c_\theta E(\theta_i) + c_s s_i + c_p p$, the equilibrium price can be written as

$$(10) \quad p^* = \frac{c_\theta E(\theta_i)}{1 - c_p} + \frac{c_s}{1 - c_p} \bar{s},$$

where $\bar{s} = \frac{1}{I} \sum_{i \in I} s_i$. Using (10), random vector (θ_i, s_i, p^*) is jointly normally distributed,

$$(11) \quad \begin{pmatrix} \theta_i \\ s_i \\ p^* \end{pmatrix} = \mathcal{N} \left[\begin{pmatrix} E(\theta_i) \\ E(\theta_i) \\ E(\theta_i) \end{pmatrix}, \begin{pmatrix} \sigma_\theta^2 & & \\ & \sigma_\theta^2 & \\ & & \sigma_\theta^2 + \sigma_\varepsilon^2 \end{pmatrix}, \begin{pmatrix} \text{Cov}(\theta_i, p^*) \\ \text{Cov}(s_i, p^*) \\ \text{Var}(p^*) \end{pmatrix} \right].$$

²⁴If the variance of X is sufficiently small ($\alpha \in (-1, -1/3)$), then the comovement of p^* with X is outweighed by the effect of learning through noise in small markets. As the market grows and β_ε decreases, $|\beta_R - \beta_\varepsilon|$ monotonically decreases as well and ψ^+ diminishes. For any $\alpha \in (-1, -1/3)$, γ exists for which the two effects exactly balance and price informativeness attains its minimum of zero. For all market sizes beyond this threshold, bidders infer through the common value component and, because $|\beta_X - \beta_\varepsilon|$ is monotonically increasing, so is ψ^+ . In the group model with $\alpha \in (-1/3, 1)$, learning through the common value component dominates for all γ and ψ^+ monotonically increases in γ .

Covariances in (11) are given by

$$\begin{aligned} \text{Cov}(\theta_i, p^*) &= \frac{1}{I} \frac{c_s}{1 - c_p} (1 + (I - 1)\bar{\rho}) \sigma_\theta^2, \\ \text{Cov}(s_i, p^*) &= \frac{1}{I} \frac{c_s}{1 - c_p} ((1 + (I - 1)\bar{\rho}) + \sigma^2) \sigma_\theta^2, \end{aligned}$$

and

$$\text{Var}(p^*) = \frac{1}{I} \left(\frac{c_s}{1 - c_p} \right)^2 ((1 + (I - 1)\bar{\rho}) + \sigma^2) \sigma_\theta^2.$$

Applying the projection theorem²⁵ to random vector (11) and the method of undetermined coefficients yields the inference coefficients c_s and c_p in $E(\theta_i|s_i, p)$, (5) and (6); by $E(\theta_i) = E(s_i) = E(p)$, c_θ is as in (7). Using (3), the equilibrium bid is

$$(12) \quad q_i(p) = \frac{1}{(\mu - (\partial q_i(p)/\partial p)^{-1}/(I - 1))} [c_\theta E(\theta_i) + c_s s_i - (1 - c_p)p].$$

By the linearity of equilibrium, $\partial q_i(p)/\partial p$ is constant. Taking a derivative of (12) with respect to price and solving for the bid slope, we obtain $\partial q_i(p)/\partial p = -(\gamma - c_p)/\mu$, which gives the equilibrium bid (4). *Q.E.D.*

PROOF OF PROPOSITION 1: *Only if:* The profile of bids (4), $i \in I$, from Proposition 2 constitutes an equilibrium with downward-sloping bids only if the slope of the (aggregate) supply satisfies $\infty > -(\partial q_i(p)/\partial p)^{-1}/(I - 1) > 0$. This implies $\gamma > c_p > -\infty$, which, by (6), requires

$$(13) \quad \bar{\rho} \neq -(1 - \gamma).$$

Combined with the fact that $\bar{\rho} \geq -(1 - \gamma)$ holds for any random vector $\{\theta_i\}_{i \in I}$, condition (13) implies the desired lower bound on the commonality, $\bar{\rho} > -(1 -$

²⁵Let θ and s be random vectors such that $(\theta, s) \sim N(\mu, \Sigma)$, where

$$\mu \equiv \begin{pmatrix} \mu_\theta \\ \mu_s \end{pmatrix} \text{ and } \Sigma \equiv \begin{pmatrix} \Sigma_{\theta,\theta} & \Sigma_{\theta,s} \\ \Sigma_{s,\theta} & \Sigma_{s,s} \end{pmatrix}$$

are partial expectations and variance–covariance matrix, and $\Sigma_{s,s}$ is positive definite. The distribution of θ conditional on s is normal and given by $(\theta|s) \sim N(\mu_\theta + \Sigma_{\theta,s} \Sigma_{s,s}^{-1}(s - \mu_s), \Sigma_{\theta,\theta} - \Sigma_{\theta,s} \Sigma_{s,s}^{-1} \Sigma_{s,\theta})$.

$\gamma) \equiv \bar{\rho}^-(\gamma)$. The upper bound is derived from condition $\gamma > c_p$, which by (6) is equivalent to

$$\bar{\rho} < \frac{\gamma^2 - 2(1 - \gamma)\sigma^2 + (1 - \gamma)\sqrt{4\sigma^4 + \left(\gamma\frac{2 - \gamma}{1 - \gamma}\right)^2}}{2\gamma} \equiv \bar{\rho}^+(\gamma, \sigma^2).$$

For $\gamma = 0$, the upper bound $\bar{\rho}^+(0, \sigma^2)$ is defined as the limit of $\bar{\rho}^+(\gamma, \sigma^2)$ as $\gamma \rightarrow 0$, $\bar{\rho}^+(0, \sigma^2) \equiv 0$.

If: For any commonality $\bar{\rho}$ such that $\bar{\rho}^-(\gamma) < \bar{\rho} < \bar{\rho}^+(\gamma, \sigma^2)$, the first-order condition (3) is necessary and *sufficient* for optimality of the bid (4) (for any price) for each i , given that bidders $j \neq i$ submit bids (4). It follows that the bids from Proposition 2 constitute a unique symmetric linear Bayesian Nash equilibrium. Q.E.D.

PROOF OF PROPOSITION 3: *Only if:* Assume that the equilibrium price is privately revealing, that is, the posterior cumulative distribution functions coincide, $F(\theta_i|s_i, p^*) = F(\theta_i|s)$ for every i and s , given the equilibrium price $p^* = p^*(s)$. Fix i . Using that the price is a deterministic function of the average signal (by (10)), we have that $F(\theta_i|s_i, p^*) = F(\theta_i|s_i, \bar{s})$. By the projection theorem applied to (θ_i, s) , $E(\theta_i|s_i, \bar{s}) = c_0 + c \cdot s$, where $c^T = (c_{s_1}, c_{s_2}, \dots, c_{s_I})$ is a vector of constants in which all entries $j \neq i$ are identical. That the equality $E(\theta_i|s_i, \bar{s}) = E(\theta_i|s)$ holds for all s implies that the coefficients multiplying each s_k , $k \in I$, are the same in both conditional expectations. It follows that coefficients in $E(\theta_i|s)$ satisfy $c_{s_j} = c_{s_k}$ for all $j, k \neq i$. We now show that this implies $\rho_{i,j} = \bar{\rho}$ for all $j \neq i$. Let $\Sigma_{s,s} \equiv \sigma_\theta^2 C + \sigma_\varepsilon^2 I$ be the variance–covariance matrix of signals $\{s_i\}_{i \in I}$ and let $\Sigma_{\theta_i,s} = \{\text{Cov}(\theta_i, s_k)\}_{k \in I}$ be the row vector of covariances. By the positive semidefiniteness of C , $\Sigma_{s,s}$ is positive definite and, hence, invertible. Applying the projection theorem, coefficients $c \in R^I$ in expectation $E(\theta_i|s)$ are characterized by $c^T = \Sigma_{\theta_i,s} \Sigma_{s,s}^{-1}$, which gives

$$(14) \quad (\Sigma_{\theta_i,s})^T = \Sigma_{s,s} c.$$

For any $j \neq i$, the j th row of (14), $\text{Cov}(\theta_i, s_j) = \sum_{k \in I} \text{Cov}(\theta_j, \theta_k) c_{s_k} + c_{s_j} \sigma_\varepsilon^2$, using $\text{Cov}(\theta_i, s_j) = \text{Cov}(\theta_i, \theta_j)$ can be written as

$$(15) \quad \text{Cov}(\theta_i, \theta_j) = c_{s_j} \sum_{k \neq j} \text{Cov}(\theta_j, \theta_k) + (c_{s_i} - c_{s_j}) \text{Cov}(\theta_i, \theta_j) + c_{s_j} (\sigma_\theta^2 + \sigma_\varepsilon^2),$$

where we used that coefficients c_{s_j} are the same for all $j \neq i$. Equation (15) gives

$$\text{Cov}(\theta_i, \theta_j) = \frac{c_{s_j} (I - 1) \sigma_\theta^2 \bar{\rho}}{1 - (c_{s_i} - c_{s_j})} + \frac{c_{s_j} (\sigma_\theta^2 + \sigma_\varepsilon^2)}{1 - (c_{s_i} - c_{s_j})}.$$

Since c_{s_j} is the same for all $j \neq i$, $\text{Cov}(\theta_i, \theta_j)$ is identical for all $j \neq i$ and, hence, by equicommonality, $\rho_{i,j} = \bar{\rho}$ for all $j \neq i$. Since the argument holds for any i , it follows that $\rho_{i,j} = \bar{\rho}$ for all pairs $i, j, i \neq j$.

If: Assume that $\rho_{i,j} = \bar{\rho}$ for all $j \neq i$ in the correlation matrix \mathcal{C} . We derive the first two moments of $F(\theta_i|s)$. The variance–covariance matrix of signals $\Sigma_{s,s}$ can be written as

$$\Sigma_{s,s} = \begin{pmatrix} a & b & \dots & b \\ b & a & \dots & b \\ \vdots & \vdots & \ddots & \vdots \\ b & b & \dots & a \end{pmatrix},$$

where $a = \sigma_\theta^2 + \sigma_\varepsilon^2$ and $b = \bar{\rho}\sigma_\theta^2$. Its inverse is given by

$$\Sigma_{s,s}^{-1} = \begin{pmatrix} \tilde{a} & -\tilde{b} & \dots & -\tilde{b} \\ -\tilde{b} & \tilde{a} & \dots & -\tilde{b} \\ \vdots & \vdots & \ddots & \vdots \\ -\tilde{b} & -\tilde{b} & \dots & \tilde{a} \end{pmatrix},$$

where

$$\tilde{a} = \frac{\sigma_\theta^2 + \sigma_\varepsilon^2 + (I - 2)\bar{\rho}\sigma_\theta^2}{(\sigma_\theta^2 + \sigma_\varepsilon^2)^2 + (I - 2)(\sigma_\theta^2 + \sigma_\varepsilon^2)\bar{\rho}\sigma_\theta^2 - (I - 1)\bar{\rho}^2\sigma_\theta^4}$$

and

$$\tilde{b} = \frac{\bar{\rho}\sigma_\theta^2}{(\sigma_\theta^2 + \sigma_\varepsilon^2)^2 + (I - 2)(\sigma_\theta^2 + \sigma_\varepsilon^2)\bar{\rho}\sigma_\theta^2 - (I - 1)\bar{\rho}^2\sigma_\theta^4}.$$

Assuming without loss of generality that $i = 1$, one can write $\Sigma_{\theta_i,s} = \sigma_\theta^2(1, \bar{\rho}, \bar{\rho}, \dots, \bar{\rho})$. From the projection theorem, the coefficients in expectation $E(\theta_i|s)$ are characterized by $c^T = \Sigma_{\theta_i,s}\Sigma_{s,s}^{-1}$, which gives

$$(16) \quad c_{s_i} = \frac{\sigma_\theta^4 + \sigma_\varepsilon^2\sigma_\theta^2 + (I - 2)\bar{\rho}\sigma_\theta^4 - (I - 1)\bar{\rho}^2\sigma_\theta^4}{(\sigma_\theta^2 + \sigma_\varepsilon^2)^2 + (I - 2)(\sigma_\theta^2 + \sigma_\varepsilon^2)\bar{\rho}\sigma_\theta^2 - (I - 1)\bar{\rho}^2\sigma_\theta^4},$$

$$(17) \quad c_{s_j} = \frac{\bar{\rho}\sigma_\varepsilon^2\sigma_\theta^2}{(\sigma_\theta^2 + \sigma_\varepsilon^2)^2 + (I - 2)(\sigma_\theta^2 + \sigma_\varepsilon^2)\bar{\rho}\sigma_\theta^2 - (I - 1)\bar{\rho}^2\sigma_\theta^4}.$$

We now show that expectation $E(\theta_i|s_i, p^*(s))$, with the coefficients derived in (5) and (6), assigns the same weight to all individual signals as coefficients (16) and (17). To see this, using (10), write the equilibrium price as a function of

signals. The expectation becomes

$$\begin{aligned}
 & E(\theta_i | s_i, p^*(s)) \\
 &= E(\theta_i) + \left[c_s + \frac{c_p c_s}{1 - c_p} \frac{1}{I} \right] [s_i - E(s_i)] + \frac{c_p c_s}{1 - c_p} \frac{1}{I} \sum_{j \neq i} (s_j - E(s_j))
 \end{aligned}$$

and, hence, $E(\theta_i | s_i, p^*(s)) = E(\theta_i | s)$ if and only if for all $j \neq i$,

$$(18) \quad c_{s_i} = c_s + \frac{c_p c_s}{1 - c_p} \frac{1}{I},$$

$$(19) \quad c_{s_j} = \frac{c_p c_s}{1 - c_p} \frac{1}{I}.$$

That conditions (18) and (19) hold can be verified from (5), (6), (16), and (17). This proves the equality of expectations $E(\theta_i | s_i, p^*(s))$ and $E(\theta_i | s)$ for all s .

Next, we demonstrate the equality of variances in the posterior cumulative distribution functions $F(\theta_i | s_i, p^*)$ and $F(\theta_i | s)$. Let ϕ^s be defined by $\text{Var}(\theta_i | s) = (1 - \phi^s) \sigma_\theta^2$. From the projection theorem, $\phi^s = (\Sigma_{\theta_i, s} \Sigma_{s, s}^{-1} (\Sigma_{\theta_i, s})^T) / \sigma_\theta^2$ and, therefore,

$$\begin{aligned}
 \phi^s &= c_{s_i} + (I - 1) \bar{\rho} c_{s_j} \\
 &= \frac{\sigma_\theta^4 + \sigma_\varepsilon^2 \sigma_\theta^2 + (I - 2) \bar{\rho} \sigma_\theta^4 - (I - 1) \bar{\rho}^2 \sigma_\theta^4 + (I - 1) \bar{\rho}^2 \sigma_\varepsilon^2 \sigma_\theta^2}{[\sigma_\varepsilon^2 + (1 + (I - 1) \bar{\rho}) \sigma_\theta^2] (\sigma_\theta^2 (1 - \bar{\rho}) + \sigma_\varepsilon^2)} \\
 &= \frac{(1 - \bar{\rho})}{(1 - \bar{\rho} + \sigma^2)} \\
 &\quad \times \left[1 + \frac{\bar{\rho} \sigma^2 + (I - 1) \bar{\rho} (1 - \bar{\rho}) + (I - 1) \bar{\rho}^2 \sigma^2 - (1 - \bar{\rho}) (I - 1) \bar{\rho}}{(1 - \bar{\rho}) (\sigma^2 + (1 + (I - 1) \bar{\rho}))} \right] \\
 &= \phi^p,
 \end{aligned}$$

where ϕ^p is defined by $\text{Var}(\theta_i | s_i, p^*) = (1 - \phi^p) \sigma_\theta^2$ and, hence, the posterior variances coincide, and by the normality of distributions, $F(\theta_i | s_i, p^*) = F(\theta_i | s)$. *Q.E.D.*

PROOF OF PROPOSITION 4: Applied twice, the projection theorem gives conditional variances $\text{Var}(\theta_i | s_i)$ and $\text{Var}(\theta_i | s_i, p^*)$, from which price informativeness ψ^+ is derived,

$$(20) \quad \psi^+ = \frac{\sigma^2 \bar{\rho}^2}{[(1 + \sigma^2)(1 - \gamma) + \bar{\rho}][1 + \sigma^2 - \bar{\rho}]}.$$

For any ψ^+ and γ , equation (20) is quadratic in $\bar{\rho}$ with roots

$$(21) \quad \bar{\rho} = \frac{(\sigma^2 + 1)}{2(\sigma^2 + \psi^+)} [\psi^+ \gamma \pm \sqrt{\psi^{+2} \gamma^2 + 4\psi^+(1 - \gamma)(\sigma^2 + \psi^+)}].$$

For any $\psi^+ \in [0, 1]$ and $\gamma \in [0, 1]$, equation (21) gives the values of $\bar{\rho}$ that, jointly with γ , give rise to price informativeness equal to ψ^+ . For $\psi^+ > 0$, equation (20) has a positive and a negative root. For a given pair $(\gamma, \bar{\rho})$ and the corresponding ψ^+ , the threshold τ is determined as the change of $\bar{\rho}$ that maintains constant the value of price informativeness ψ^+ with an additional trader, whose inclusion increases γ by

$$\Delta\gamma \equiv \frac{1}{I(I - 1)} = \frac{(1 - \gamma)^2}{2 - \gamma}.$$

Using (21) for $\bar{\rho} > 0$, threshold τ can be found,

$$\begin{aligned} \tau &= \frac{(\sigma^2 + 1)}{2(\sigma^2 + \psi^+)} \\ &\times \left[\psi^+ \Delta\gamma + \sqrt{\psi^{+2}(\gamma + \Delta\gamma)^2 + 4\psi^+(1 - \gamma - \Delta\gamma)(\sigma^2 + \psi^+)} \right. \\ &\quad \left. - \sqrt{\psi^{+2}\gamma^2 + 4\psi^+(1 - \gamma)(\sigma^2 + \psi^+)} \right]. \end{aligned}$$

Since the positive root in (21) is decreasing in γ and increasing in ψ^+ , then $\tau < 0$. The threshold for $\bar{\rho} < 0$ can be derived analogously. *Q.E.D.*

PROOF OF PROPOSITION 5: From the projection theorem applied to (y, s_i, p^*) for any $y = X, R_i, \varepsilon_i$, the vector of coefficients in the conditional expectation $E(y|s_i, p^*)$ is the product

$$(22) \quad (c_{y,s_i}, c_{y,p^*}) = (\text{Cov}(y, s_i), \text{Cov}(y, p^*))\Sigma^{-1},$$

where Σ is the variance–covariance matrix of vector (s_i, p^*) . The inverse of Σ is

$$\Sigma^{-1} = \frac{1}{\det(\Sigma)} \begin{pmatrix} \text{Var}(p^*) & -\text{Cov}(s_i, p^*) \\ -\text{Cov}(s_i, p^*) & \text{Var}(s_i) \end{pmatrix}.$$

Using that $s_i = X + R_i + \varepsilon_i$, we have that $\text{Var}(s_i) = \sum_y \sigma_y^2$ and $\text{Cov}(y, s_i) = \sigma_y^2$. From the projection of p^* on the signal components X, R_i , and ε_i , we obtain $\text{Cov}(y, p^*) = \beta_y \sigma_y^2$, $y = X, R_i, \varepsilon_i$, and $\text{Cov}(s_i, p^*) = \sum_y \beta_y \sigma_y^2$. Using (22),

$$c_{y,p^*} = \frac{1}{\det(\Sigma)} \left[\beta_y \sigma_y^2 \sum_{y'} \sigma_{y'}^2 - \sigma_y^2 \sum_{y'} \beta_{y'} \sigma_{y'}^2 \right]$$

$$= \frac{1}{\det(\Sigma)} \sum_{y' \neq y} (\beta_y - \beta_{y'}) \sigma_y^2 \sigma_{y'}^2.$$

Letting $\bar{c} \equiv 1/\det(\Sigma) = \text{Var}(s_i) \text{Var}(p^*) - (\text{Cov}(s_i, p^*))^2$ and observing that $\bar{c} > 0$, by the positive definiteness of Σ , we obtain (9). Q.E.D.

LEMMA 1—Identification: *A common value component exists if and only if the infinite auction is equicommonal, that is, $\lim_{I \rightarrow \infty} \frac{1}{I-1} \sum_{j \in I, j \neq i} \rho_{i,j} \equiv \bar{\rho}^\infty$ is the same for all i . Moreover, $X = \bar{\theta}^\infty \equiv \lim_{I \rightarrow \infty} \frac{1}{I} \sum_{i \in I} \theta_i$, where the common value component is unique up to a constant.*

PROOF: Let $\{\theta_i\}_{i=1}^\infty$ be a jointly normally distributed random vector. Define $R \equiv \lim_{I \rightarrow \infty} \frac{1}{I} \sum_i R_i$. Only if: Let X be such that, for all i , $\theta_i = X + R_i$, $X \perp R_i$, and $R_i \perp R$. Then

$$\begin{aligned} & \lim_{I \rightarrow \infty} \frac{1}{I-1} \sum_{j \in I; j \neq i} \text{Cov}(\theta_i, \theta_j) \\ &= \text{Cov}\left(X + R_i, X + \lim_{I \rightarrow \infty} \frac{1}{I-1} \sum_{j \in I; j \neq i} R_j\right) \\ &= \text{Cov}\left(X + R_i, X + \lim_{I \rightarrow \infty} \frac{I}{I-1} \frac{1}{I} \sum_{j \in I} R_j - \lim_{I \rightarrow \infty} \frac{1}{I-1} R_i\right) \\ &= \text{Cov}\left(X + R_i, X + R - \lim_{I \rightarrow \infty} \frac{1}{I-1} R_i\right) \\ &= \text{Cov}(X + R_i, X + R) = \text{Var}(X). \end{aligned}$$

Since $\lim_{I \rightarrow \infty} \frac{1}{I-1} \sum_{j \in I, j \neq i} \rho_{i,j} = \lim_{I \rightarrow \infty} \frac{1}{I-1} \sum_{j \in I; j \neq i} \text{Cov}(\theta_i, \theta_j) / \sigma_\theta^2 = \text{Var}(X) / \sigma_\theta^2$ is independent across bidders, the auction is equicommonal. If: Consider an infinite equicommonal auction. We show that $X = \bar{\theta}^\infty$ and $R_i = \theta_i - \bar{\theta}^\infty$ satisfy conditions (i) and (ii) in the definition of a common value component. For a vector $\{\theta_i\}_{i=1}^I$ of the first $I < \infty$ elements of $\{\theta_i\}_{i=1}^\infty$, define $\bar{\theta}^I \equiv \frac{1}{I} \sum_{i \in I} \theta_i$:

$$\begin{aligned} \text{Cov}(\bar{\theta}^I, \theta_i - \bar{\theta}^I) &= \frac{1}{I} \sum_{j \in I} \text{Cov}(\theta_j, \theta_i) - \frac{1}{I^2} \sum_{j \in I} \sum_{k \in I} \text{Cov}(\theta_j, \theta_k) \\ &= \frac{1}{I} \sum_{j \in I, j \neq i} \text{Cov}(\theta_j, \theta_i) - \frac{1}{I^2} \sum_{j \in I} \sum_{k \in I, k \neq j} \text{Cov}(\theta_j, \theta_k). \end{aligned}$$

Taking the limit as $I \rightarrow \infty$ and using that $\lim_{I \rightarrow \infty} \frac{1}{I} \sum_{k \in I, k \neq j} \text{Cov}(\theta_j, \theta_k) = \bar{\rho}^\infty \sigma_\theta^2$ for all j , we have $\text{Cov}(\bar{\theta}^\infty, R_i) = \lim_{I \rightarrow \infty} \text{Cov}(\bar{\theta}^I, \theta_i - \bar{\theta}^I) = 0$. Since $\bar{\theta}^\infty$

and R_i are normally distributed, they are independent. In addition,

$$\begin{aligned}\bar{\rho}^\infty \sigma_\theta^2 &= \lim_{I \rightarrow \infty} \frac{1}{I-1} \sum_{j \in I, j \neq i} \text{Cov}(\theta_i, \theta_j) \\ &= \text{Cov}(\bar{\theta}^\infty + R_i, \bar{\theta}^\infty + R) = \text{Var}(\bar{\theta}^\infty) + \text{Cov}(R_i, R)\end{aligned}$$

and $\text{Var}(\bar{\theta}^\infty) = \bar{\rho}^\infty \sigma_\theta^2$ imply $\text{Cov}(R_i, R) = 0$ and, hence, the normally distributed R_i and R are independent. For the uniqueness of the decomposition (up to a constant), observe that for any random variable X that satisfies the two conditions in the definition of a common value component, $\bar{\theta}^\infty \equiv \lim_{I \rightarrow \infty} \frac{1}{I} \sum_{i \in I} \theta_i = X + R$ holds. For any $I < \infty$,

$$\begin{aligned}\text{Var}\left(\frac{1}{I} \sum_{i \in I} R_i\right) &= \text{Cov}\left(\frac{1}{I} \sum_{i \in I} R_i, \frac{1}{I} \sum_{j \in I} R_j\right) \\ &= \frac{1}{I} \sum_{i \in I} \text{Cov}\left(R_i, \frac{1}{I} \sum_{j \in I} R_j\right).\end{aligned}$$

Since $\text{Cov}(R_i, \lim_{I \rightarrow \infty} \frac{1}{I} \sum_{j \in I} R_j) = 0$, taking the limit as $I \rightarrow \infty$ gives $\text{Var}(R) = 0$. It follows that R is a deterministic constant and X is equal to $\bar{\theta}^\infty$ modulo a (deterministic) constant. On the other hand, for any common value component X , the random variable $\bar{\theta}^\infty + \text{const}$, where const is an arbitrary constant, satisfies the definition of a common value component. *Q.E.D.*

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