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Information and strategic behavior ☆

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Abstract

Does encouraging trader participation enhance market competitiveness? This paper shows that, when trader preferences are interdependent, trader market power does not necessarily decrease with greater participation, and traders need not become price takers in large markets. Thus, larger markets can be less liquid and associated with lower ex ante welfare. In the linear-normal model, the necessary and sufficient condition on the information structure is provided under which price impact is monotone in market size. A condition is given when the rational expectations equilibrium, which is typically not fully revealing within the considered class of preference interdependencies, is obtained in large markets.

JEL classification: D43; D44; D47; D82; G14; L13

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1. Introduction

Two central lessons from industrial organization and auction theory relate to the notion that markets with a larger number of traders are more competitive: (1) greater participation reduces

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the impact of each individual trader on the market as a whole and (2) traders act as price takers in large markets. An implication of these results for market design is the recommendation to encourage market participation, which is viewed as an enhancement to competition, liquidity, and welfare. The two predictions hold robustly for markets with complete information or independent private values (e.g., Rustichini, Satterthwaite, and Williams [15]). This paper shows that, when trader values are interdependent, these two predictions need not hold in general. Relative to the existing literature on strategic trading with common values, we examine a richer class of interdependencies in preferences that are common in economic settings.

For markets with interdependent values, (non)competitiveness has been studied in a number of influential papers. In strategic settings, Wilson [21], Milgrom [10], Pesendorfer and Swinkels [11], Reny and Perry [12], and Vives [18] established the convergence of Nash equilibrium to the competitive rational expectations equilibrium. However, the question of monotonicity of price impact in finite markets with interdependent preferences has received less attention. Research on finite and large (infinite) markets with interdependent values has primarily focused on markets in which there is an underlying fundamental value that defines, for all agents, the values derived from the exchanged good. In particular, apart from idiosyncratic shocks, the preferences of all market participants are affected only by aggregate shocks to the fundamental value. In these markets, the effect of an additional trader on market competitiveness is unambiguous; that is, in this canonical information structure, where the correlations among values, induced by common and idiosyncratic shocks, are the same for all trader pairs for any market size, increasing the market size decreases market power, and the interdependence in trader values through aggregate shocks does not alter conclusions (1) and (2) about price impact.

Model. We examine the relationship between non-competitiveness and interdependencies in values in a market for a perfectly divisible good[1] with an arbitrary number of traders, based on the standard *uniform-price* double auction. The analysis is cast in a linear-normal setting; we analyze the unique symmetric linear Bayesian Nash equilibrium. Thus, relative to the large-market rational-expectations literature on information aggregation, we deal with a more modest class of quadratic utilities but are able to analyze a relatively rich class of interdependencies in trader values for markets of any size. Specifically, we adopt the class of information structures introduced in Rostek and Weretka ([13]; equicommonal information structures) to study information aggregation; here, we analyze market power and welfare. Beyond markets with aggregate and idiosyncratic shocks alone, the equicommonal class allows accommodation of a variety of environments with heterogeneously interdependent preferences, including group or spatial dependence in values and networks with size externalities on interdependence among values. Moreover, unlike models with aggregate and idiosyncratic shocks, negative dependence of values is allowed. All traders—buyers and sellers—are Bayesian and strategic in that they (endogenously) have price impact and take it into account in their trading decisions; in particular, there are no noise traders, uninformed or (by assumption) price-taking traders.[2]

[1] Examples of perfectly divisible goods include assets, electricity, gold, emission permits, etc.

[2] That all market participants are fully strategic is a feature of the model that is shared by the double-auction models of Dubey, Geanakoplos and Shubik [4], Reny and Perry [12], and Vives [18–20]. Dubey, Geanakoplos and Shubik [4] model a market as a dynamic strategic market game, in which during every period traders choose nominal spending that is not contingent on price. In a static strategic market game, information revealed in price cannot be incorporated into decisions; with multiple trading rounds, traders are able to use information contained in prices from prior trading rounds. A Walrasian (double) auction accounts for feedback between inference and market depth, even though the game is static. By allowing choices that are contingent on prices, downward-sloping demands enable traders to take advantage of the

RESULTS. In many economic settings, preferences of agents are *heterogeneously* correlated; income, endowment, technology, and liquidity shocks affect various market participants differently, such as consumers and producers, institutional investors, countries, industries or demographic groups.[3] The main result of this paper shows that, in markets with heterogeneous correlations, the price impact (measured by the slope of a trader's residual supply; Kyle's lambda) is not monotonic in market size in general. In contrast to markets in which values are affected solely by aggregate and idiosyncratic shocks, by affecting adversely the informativeness of market price, a new trader may increase the market power of all traders and lower gains to trade. Agents' ex ante welfare from trading can be lower in larger markets. Moreover, under extreme conditions, the effect of private information on inference may give rise to price making, even in large (infinite) markets. Consequently, policy implications and empirical predictions in markets with heterogeneously correlated values may differ markedly from those for markets with a fundamental value of the good.

The first of the two main paper's results establishes the necessary and sufficient condition under which non-competitiveness decreases with the introduction of a new trader. Specifically, the price impact of each trader decreases provided that a new market participant does not increase too much the commonality in values of all market participants, measured by the average correlation. We provide examples of markets in which, as the number of traders increases, price impact is non-monotone.

The second result shows that, under mild assumptions, as the number of bidders increases, the linear Bayesian Nash equilibrium converges to the unique competitive rational expectations equilibrium, in which bidders become price takers. Since price typically does not fully aggregate information in the considered class of auctions, even though inefficiency due to market power disappears in large (limit) markets, private information inefficiency does not. Our result contributes to the literature: (1) by providing a (fully strategic) foundation for not fully privately revealing equilibria in the linear-Gaussian model with perfectly divisible goods and heterogeneous interdependencies in preferences[4] and (2) by separating full revelation from price-taking behavior, demonstrating that price taking is predicted robustly in large markets for all but perfectly correlated values (pure common values for almost all traders), whereas price is fully revealing only if the heterogeneity in correlations among values is absent for *all* trader pairs. We give an example of a market that violates our convergence condition and exhibits price making in the limit.[5]

information contained in prices even though they choose strategies before (without) knowing the equilibrium price. This feature of the Walrasian auction contrasts also with the Cournot competition in quantities (e.g., Vives [17]), in which traders can learn from prices, but cannot incorporate the information conveyed by prices into their bids.

Reny and Perry [12] provide a fully strategic foundation for a fully revealing rational expectations equilibrium in a model of a large ($I \to \infty$) double auction with unit demands, more general utility functions than quadratic and dependence on the values that stems from a fundamental value of the good. Instead, the model presented in this paper adopts a linear-normal setting and multi-unit demands (divisible goods), while permitting an arbitrary number of bidders and more general preference interdependencies.

[3] A growing body of empirical research shows that trading strategies vary with geographical proximity or social affiliation—cultural or linguistic (e.g., Coval and Moskowitz [3]; Harrison, Kubik and Stein [5]; Cohen, Frazzini and Malloy [2]; Veldkamp [16] provides an overview of the empirical evidence).

[4] "Privately" indicates that the private information of a trader (his signal) and price provide a sufficient statistic for the joint information in the market (profile of all signals). In our model, in markets with (only) aggregate and idiosyncratic preference shocks, equilibrium is fully privately revealing.

[5] A contemporaneous paper by Vives [20] studies strategic foundations for the competitive rational expectations equilibrium in a linear-Gaussian model of a double auction for a divisible good with a continuum of traders. Vives [20] shows that, in a demand schedule game, a unique symmetric linear Bayesian Nash equilibrium exists that is privately revealing

STRUCTURE OF THE PAPER. Section 2 lays out the model of a double auction. Section 3 studies the monotonicity of price impact in market size and convergence to the competitive rational expectations equilibrium. Section 4 discusses additional implications of our results. Proofs of all results are contained in Appendix A.

2. A model of a double equicommonal auction

We model the market as a double auction in the familiar linear-normal setting. $I \geq 2$ agents trade a divisible good. Trader i has a quasilinear and quadratic utility function

$$U_i(q_i, m_i) = \theta_i q_i - \frac{\mu}{2} q_i^2 + m_i, \tag{1}$$

where q_i is the obtained quantity of the good auctioned, m_i is money, and $\mu > 0$. Each bidder is uncertain about the utility he derives from the good and observes only a noisy signal about his own value θ_i, $s_i = \theta_i + \varepsilon_i$. Asymmetric information is captured by random intercepts of marginal utility functions $\{\theta_i\}_{i \in I}$, referred to as *values* and interpreted to arise from shocks to preferences, endowment or any other shock that shifts the marginal utility of a trader. The affine information structure maintains of the linearity of the model: Random vector $\{\theta_i, \varepsilon_i\}_{i \in I}$ is jointly normally distributed, noise ε_i is mean-zero i.i.d. with variance σ_ε^2, and the expectation $E(\theta_i)$ and the variance σ_θ^2 of θ_i are the same for all i. The variance ratio $\sigma^2 \equiv \sigma_\varepsilon^2 / \sigma_\theta^2$ measures the relative importance of noise in the signal. We define an index of *market size* as a monotone function of the number of traders,

$$\gamma = \frac{I-2}{I-1}, \tag{2}$$

which ranges between zero for $I = 2$ and one as $I \to \infty$; $\gamma \in \Gamma$, where

$$\Gamma \equiv \big\{\gamma \in [0,1] \big| \gamma = (I-2)/(I-1) \text{ for } I = 2, 3, \ldots\big\}. \tag{3}$$

PREFERENCE INTERDEPENDENCIES. Much of the literature on trading with private information, in strategic and rational-expectations models alike, has examined the following specification of preference interdependence

$$\theta_i = \theta + \tilde{\theta}_i, \tag{4}$$

where θ is a fundamental (aggregate) shock and $\tilde{\theta}_i$ is an idiosyncratic shock that is independent across traders, and θ and $\tilde{\theta}_i$ are independent as well. Information structures characterized by (4) include the *pure common value* model, $\tilde{\theta}_i = 0$ for all i, and the *independent (private) value* model, $\theta = 0$. More generally, in terms of the correlation matrix of the joint distribution of values $\{\theta_i\}_{i \in I}$ (i.e., the variance–covariance matrix of $\{\theta_i\}_{i \in I}$ normalized by variance σ_θ^2),

if the correlation of traders' valuations is less than 1. (When the correlation is 1, there is a fully revealing REE, but it is not implementable as a game.) The analysis by Vives [20] is developed for information structures in which pairwise correlations are the same across traders—a common assumption in the information aggregation literature, which we analyze as the Fundamental Value Model benchmark. Whereas, as described above, we provide new results on the possibility of non-monotonicity of price impact and welfare in markets with heterogeneous preference interdependence, unlike the contribution of Vives [20], we do not study the rate of convergence to REE, but we give the condition for the convergence in more general information structures than the Fundamental Value Model. Namely, for heterogeneous interdependent values, we show that a condition on the *average* correlation suffices.

$$\mathcal{C} \equiv \begin{pmatrix} 1 & \rho_{1,2} & \cdots & \rho_{1,I} \\ \rho_{2,1} & 1 & \cdots & \rho_{2,I} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{I,1} & \rho_{I,2} & \cdots & 1 \end{pmatrix}, \tag{5}$$

specification (4) implies that $\rho_{i,j} = \bar{\rho} \in [0, 1]$ for all $j \neq i$, where $\bar{\rho} = Var(\theta)/(Var(\theta) + Var(\tilde{\theta}_i))$ (e.g., Kyle [8]; Vives [18,20]; the *Fundamental Value Model*[6,7]). Thus, while trader preferences may only be imperfectly aligned, correlations in values are identical for all pairs of traders in the market.

We consider markets in which correlations $\rho_{i,j}$ may differ across pairs of traders. We assume that each trader i's value θ_i is *on average* correlated with other traders' values θ_j, $i \neq j$, in the same way,[8]

$$\frac{1}{I-1} \sum_{j \neq i} \rho_{i,j} = \bar{\rho}, \tag{6}$$

for some $\bar{\rho} \in [-1, 1]$; statistic $\bar{\rho}$ can be interpreted as a measure of *commonality* in values of the traded good to market participants (the *Equicommonal Model*; Rostek and Weretka [13]). Essentially, relative to specification (4), the Equicommonal Model permits heterogeneous correlations among $\{\tilde{\theta}_j\}_{j \neq i}$ as long as the average correlation of $\tilde{\theta}_i$ with $\{\tilde{\theta}_j\}_{j \neq i}$ is the same for all agents i, which introduces local—apart from aggregate and idiosyncratic—interdependencies among trader values. Throughout the analysis, the results are illustrated in a family of equicommonal auctions with two groups.

Example 1 (*Group model*). There are two groups of traders of equal size, A and B; the total number of traders adds up to an even number I. The values that members of a given group derive from the good are perfectly correlated ($\rho_{i,j} = 1$); cross-group correlation may depend on market size γ and can be positive or negative, or values can be independent; $\rho_{i,j} = \alpha_1 + \alpha_2(\gamma^{\alpha_3} - 1)$, where α_1, α_2 and α_3 are such that $\rho_{i,j} \in [-1, 1]$ for all γ. α_1 measures the size-independent cross-group correlation, and α_2 and α_3 measure the strength and convexity of the correlation's dependence in γ. The behavior of cross-group correlation $\rho_{i,j}$ for different parameters is depicted in Fig. 1a.

The dependence of cross-group correlation on market size ($\alpha_2, \alpha_3 \neq 0$) allows capturing of the size externality that results from the increased or decreased number and strength of linkages between the groups as populations grow. The Group Model permits negative (average) interdependence of values.[9] Section 4 illustrates our results in other examples of equicommonal markets.

A SEQUENCE OF AUCTIONS. Since market-size effects are of primary interest, instead of taking an auction with a fixed number of traders as the object of analysis, we analyze sequences of

[6] Kyle [8] considered $\bar{\rho} = 1$, assuming the presence of noise traders apart from strategic traders. Vives [18] allowed $\bar{\rho} < 1$.

[7] The specification of preferences based on a fundamental value is commonly adopted in asset pricing or macroeconomic literature to study the impact of aggregate shocks to fundamentals.

[8] This ensures that the equilibrium price is equally informative across agents (i.e., the price inference coefficient is the same in all agents' conditional expectations), which is necessary for the symmetry of equilibrium and, hence, the tractability of the model.

[9] With negative correlations (e.g., without externalities, $\alpha_1 < -\frac{1}{3}, \alpha_2 = 0$), the model can accommodate interactions in which groups of traders compete for a pool of resources (for instance, government transfers) outside the market, and

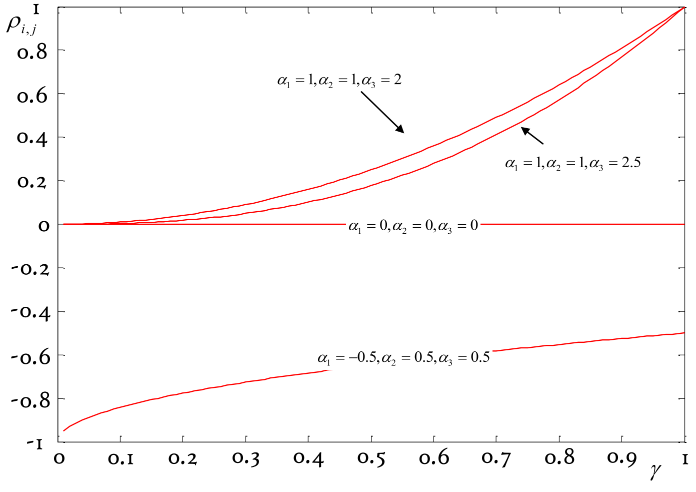

Fig. 1a. Cross-group correlation.

auctions indexed by the number of market participants $\{A^I\}_{I=1}^{\infty}$. In all auctions in the sequence, the utility function is the same; with the number of traders I, the correlation matrix $\mathcal{C}$ may change in the sequence in an arbitrary way provided that, for a given market size, the average correlation, which may vary with market size, is the same across traders. A sequence of auctions can be conveniently summarized by the *commonality function* $\bar{\rho} : \Gamma \to [-1, 1]$, which specifies commonality for any γ. In the model with values described by (4) or, more generally, in the Fundamental Value Model, a new trader is neutral for the commonality, as he does not change the variance ratio $Var(\theta)/(Var(\theta) + Var(\tilde{\theta}_i))$; the commonality function is constant $\bar{\rho}$. In the Group Model, additional traders increase populations in both groups, and the commonality function is given by

$$\bar{\rho}^{GM}(\gamma) = \frac{1}{2}\left[\gamma + (2-\gamma)\left(\alpha_1 - \alpha_2 + \alpha_2\gamma^{\alpha_3}\right)\right] \tag{8}$$

and is depicted in Fig. 1b.

Double auction. We study double auctions based on the classical uniform-price mechanism. Traders learn their signals and submit strictly downward-sloping (net) demand schedules, $\{q_i(p, s_i)\}_{i \in I}$; the part of a bid with negative quantities is interpreted as a supply schedule. The market-clearing price is the price p^* at which the aggregate demand $Q(p) \equiv \sum_{i=1}^{I} q_i(p, s_i)$ equals zero, $Q(p^*) = 0$. Trader i obtains the quantity determined by his submitted bid evaluated

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the division of the pool is uncertain during the trade stage. In the extreme case of $\alpha_1 = -1$ and $\alpha_2 = 0$, the pool is fixed. Consider the following example. Let trades be characterized by a quasilinear utility function

$$U_i(q_i, m_i) = (q_i + t_i) - \frac{1}{2}\mu(q_i + t_i)^2 + m_i, \tag{7}$$

where q_i is the quantity of a good obtained from trade in the market and t_i is the uncertain-at-the-time-of-trade transfer of a commodity determined by the government. This model gives rise to preferences as in (1), up to a constant, where $\theta_i \equiv 1 - \mu t_i$. A model with a balanced government budget (a fixed pool of resources), $t_A = -t_B$, corresponds to $\alpha_1 = -1$ and $\alpha_2 = 0$. An imperfect negative correlation of transfers gives rise to $\alpha \in (-1, 0)$. In a model where t_A and t_B are determined independently, $\alpha = 0$.

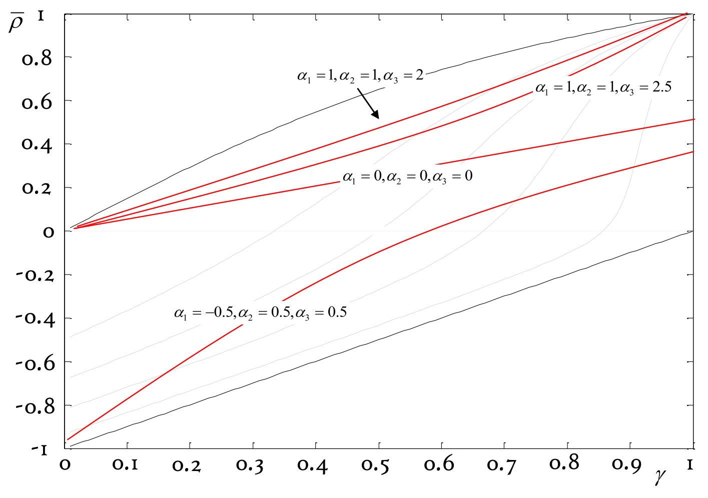

Fig. 1b. Commonality in group model.

at the equilibrium price, $q_i^* = q_i(p^*, s_i)$, for which he pays $q_i^* \cdot p^*$, and his payoff is given by the utility function (1) evaluated at (q_i^*, p^*). As a solution concept, we use the unique symmetric linear[10] Bayesian Nash equilibrium, where "symmetric linear" means that bids have the functional form of $q_i(p, s_i) = \alpha_0 + \alpha_s s_i + \alpha_p p$ and that coefficients α_0, α_s, and α_p are the same across traders.

In the Equicommonal Model of a double auction, statistic $\bar{\rho}$ is sufficient for $\mathcal{C}$ in the linear Bayesian Nash equilibrium; i.e., *ceteris paribus*, any two equicommonal auctions with the same commonality and otherwise arbitrary correlation matrices have the same Bayesian Nash equilibrium (see the derivation of the equilibrium bid in Section 3).[11] It follows that equilibrium bids $\{q_i(p, s_i)\}_{i \in I}$ are the same in the class of all auctions characterized by a given profile $(\gamma, \bar{\rho}) \in \Gamma \times [-1, 1]$. As is well understood, when trader preferences are too strongly correlated, price may be too informative for equilibrium to exist. In this paper's model, for any level of market size γ, one can find an upper bound $\bar{\rho}^+(\gamma, \sigma^2)$ and a lower bound $\bar{\rho}^-(\gamma)$ on the commonality with the property that equilibrium exists if, and only if, the commonality of an auction is strictly between the two bounds. These two bounds are given in Lemma 1 in Appendix A and are depicted in Fig. 1c.

3. Market power with interdependent values

We measure the market power of trader i by his *price impact*—a price change resulting from a unilateral deviation of trader i from his equilibrium bid at the margin (Kyle's λ). The price impact of trader i is defined as the slope of his residual supply, resulting from the aggregation

[10] The assumption that bids are strictly downward sloping rules out trivial (no-trade) equilibria.

[11] Equilibrium was derived in Rostek and Weretka [13] and hence is not stated as a proposition. We present the key steps to facilitate the analysis of equilibrium price impact. For the questions studied in this paper, let us note that auctions in the class characterized by a given profile $(\gamma, \bar{\rho})$ exhibit the same equilibrium price impact. Nevertheless, $\bar{\rho}$ is not a sufficient statistic for other properties of equilibria; for instance, informational efficiency.

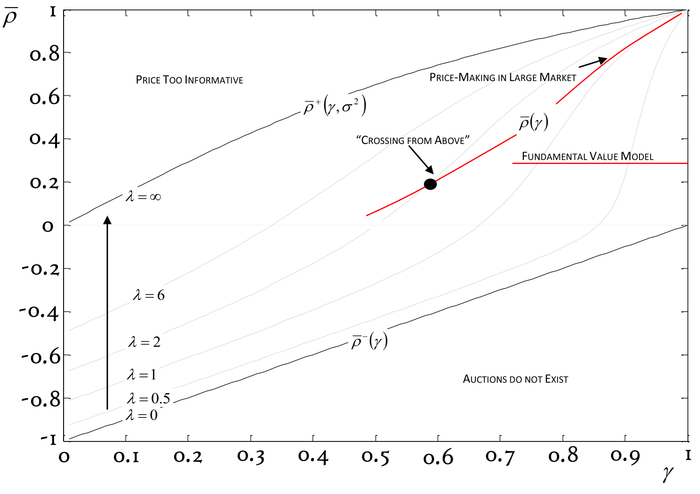

Fig. 1c. Price impact curves.

of schedules submitted by other agents $\{q_j(p,s_i)\}_{j\neq i}$.[12] In a symmetric linear Bayesian Nash equilibrium, individual bids have random intercepts and deterministic slopes that coincide for all i, and these properties are inherited by residual supplies faced by all agents. The slope of the residual supply faced by trader i is the sum of $I-1=\frac{1}{1-\gamma}$ bids and are given by

$$\lambda_i=\left(\sum_{j\neq i}\left(\frac{\partial q_j(p,s_j)}{\partial p}\right)^{-1}\right)^{-1}=\frac{1}{I-1}\left(\frac{\partial q_i(p,s_i)}{\partial p}\right)^{-1}=(1-\gamma)\left(\frac{\partial q_i(p,s_i)}{\partial p}\right)^{-1}. \tag{9}$$

In equilibrium, trader i accounts for his price impact λ_i by equalizing his expected marginal utility with his marginal expenditure for every price. Thus, his demand schedule $q_i(p,s_i)$ is derived from the first-order condition

$$E(\theta_i|p,s_i)-\mu q_i=p+\lambda_i q_i. \tag{10}$$

From (10), the bid schedule is

$$q_i(p,s_i)=\frac{1}{\lambda_i+\mu}\big[E(\theta|s_i,p)-p\big]. \tag{11}$$

It is through inference from prices that the interdependence of values affects market non-competitiveness. Knowing the map from equilibrium prices to states of the world, traders condition their bids on prices—and, hence, states of the world—by updating their expectations $E(\theta_i|p,s_i)$. Aggregating condition (10) to apply market clearing, the equilibrium price is equal to $p^*=\frac{1}{I}\sum_{i\in I}E(\theta_i|p^*,s_i)$. Given the affine information structure, the conditional expectation of θ_i is linear in the bidder's own signal s_i and price p,

[12] The residual supply of trader i is defined as a horizontal sum of the other traders' bid schedules $\{q_j,s_j(p)\}_{j\neq i}$, the slopes of which coincide with $\partial q_i(\cdot)/\partial p$ for all $j\neq i$ in a symmetric equilibrium. The slope of the sum of $I-1=\frac{1}{1-\gamma}$ bids is given by (9).

$$E(\theta_i|p, s_i) = c_\theta E(\theta_i) + c_p p + c_s s_i, \tag{12}$$

where coefficients c_θ, c_s, c_p are identical across traders and, since $E(\theta_i) = E(s_i) = E(p)$, also satisfy $c_\theta = 1 - c_s - c_p$. It follows that the equilibrium price is given by

$$p^* = \frac{c_\theta E(\theta_i)}{1 - c_p} + \frac{c_s}{1 - c_p}\bar{s}, \tag{13}$$

where $\bar{s} = \frac{1}{I}\sum_{i \in I} s_i$. Applying the Projection Theorem to the random vector (θ_i, s_i, p^*) gives equilibrium inference coefficients in closed form in terms of exogenous parameters (see Appendix A); in particular,

$$c_p = \frac{2 - \gamma}{1 - \gamma + \bar{\rho}} \frac{\bar{\rho}\sigma^2}{1 - \bar{\rho} + \sigma^2}. \tag{14}$$

The stochastic process underlying joint distribution of valuations $\{\theta_i\}_{i \in I}$ affects price impact whenever commonality satisfies $\bar{\rho} \neq 0$, in which case bidders learn from prices. The equilibrium price p^* is positively correlated with each trader's value θ_i, even in markets with negative commonality $\bar{\rho} < 0$. The coefficient of price inference c_p in (12) can be negative; this occurs if (and only if) $\bar{\rho}$ is negative.

Using (10) and (12), the equilibrium bid is

$$q_i(p, s_i) = \frac{1}{(\mu + \lambda_i)}\left[c_\theta E(\theta_i) + c_s s_i + (1 - c_p)p\right], \tag{15}$$

and, by (9),

$$\lambda_i = (1 - \gamma)\frac{\lambda_j + \mu}{1 - c_p}, \tag{16}$$

the solution to which, given the symmetry $\lambda_i = \lambda_j = \lambda$, is the equilibrium price impact λ.

It is useful to decompose an equilibrium price impact in the symmetric linear Bayesian Nash equilibrium as follows:

$$\lambda = \frac{(1 - \gamma)\mu}{\gamma - c_p} = \underbrace{\frac{\gamma}{\gamma - c_p}}_{\text{Inference Effect}} \times \underbrace{\frac{1}{\gamma}}_{\text{Strategic Effect}} \times \underbrace{(1 - \gamma)\mu}_{\text{Risk Aversion Effect}}. \tag{17}$$

In any finite market ($\gamma < 1$) with decreasing marginal utility ($\mu > 0$), each trader has a positive equilibrium price impact that is determined by three components. The risk aversion (or inventory) component characterizes the price impact of trader i in auctions in which his trading partners submit competitive bids and his price impact arises due to the other traders' decreasing marginal utilities (risk aversion). (This component coincides with the price impact of a monopsony who trades with $I - 1$ competitive buyers whose marginal utilities have slope μ.) In double auctions with independent private values ($\bar{\rho} = 0$), the risk aversion effect is reinforced by the strategic effect resulting from the complementarity in the equilibrium price impacts $\{\lambda_j\}_j$ across traders (cf. the fixed point in price impacts in Eq. (9) and Eq. (16)). Finally, when trader values are interdependent ($\bar{\rho} \neq 0$), the two effects can be amplified or weakened by a change in the posterior beliefs that they elicit, depending on the sign of commonality.[13] The mechanism behind it is as follows: When bidder i unilaterally deviates from equilibrium, other traders interpret

[13] Vives [18] interprets the impact of the inference effect as "adverse" or "favorable" selection, depending on whether ρ is positive or negative.

(incorrectly) the changed price as corresponding to a different realization of the average signal $\bar{s}$. Revision of conditional expectations of $\{\theta_j\}_{j\neq i}$ and market clearing imply an additional change to price adjustment, which, by the same mechanism, revises expectations, etc. The *inference effect* in (17) measures the overall impact of the revision.[14]

Let us make a couple of observations about information and strategic behavior: (1) It is not the correlation between price and trader values[15] that affects market non-competitiveness in (17), nor is it interdependence among values $\{\theta\}_{i\in I}$ *per se*, but the non-zero *average* correlation $\bar{\rho}$. (2) With interdependent values, in small markets, market power can be lower than the independent private value level. Whether learning about values from prices makes the market more or less competitive depends on whether bidders make inferences about their values, on average, from the correlation of price p^* with noise ε_i or value θ_i. Precisely, private information enhances market competitiveness through price inference if, and only if, the correlation of price p^* with noise ε_i dominates its correlation with value θ_i; that is, if and only if, $\bar{\rho} < 0$ ($c_p < 0$) (for the identification result, see Rostek and Weretka [13], Proposition 5).

3.1. Price impact in small markets

Absent private information, encouraging participation by a market designer increases competitiveness with every new trader, as does it in markets in which traders have private information and their values are independent ($\rho_{i,j} = 0$ for all $i \neq j$). In decomposition (17), underlying the monotonicity is the aggregation of a greater number of bids (diminished risk aversion effect in (17) and the reduction in the complementarity of bilateral market power brought by a new bidder). Robust in the absence of interdependencies in values, the monotonicity result needs not carry over to markets in which trader values are interdependent: By affecting informativeness of the equilibrium price,[16] new market participants may increase the market power of all traders.

Proposition 1 provides the necessary and sufficient condition for an increase in the number of traders to improve competitiveness in an arbitrary equicommonal auction. For any sequence of equicommonal auctions, define $\Delta\rho(\gamma)$ as the change in commonality that results from increasing the number of traders by one.

Proposition 1 *(Monotonicity of price impact). There exists a threshold function $\pi_{\gamma,\bar{\rho}} > 0$ such that, in any equicommonal auction, for any market size γ and commonality $\bar{\rho}$, price impact decreases with an additional trader if, and only if, $\Delta\rho(\gamma) < \pi_{\gamma,\bar{\rho}}$.*

The threshold $\pi_{\gamma,\bar{\rho}}$ is characterized in terms of primitives of an auction in Appendix A, Eq. (28). Proposition 1 states that, whether the market becomes more competitive with a new

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[14] The price impact of a trader, defined as a price change following an off-equilibrium quantity deviation, changes the expectations of other traders, who act according to the equilibrium map between prices and signals. If agents could observe the signal vector, a deviation would not impact beliefs, and the price impact would be as with independent private values, with the private value adjusted accordingly to condition on all signals.

[15] E.g., in the Group Model with $\alpha_1 \simeq -1$ and $\alpha_2 = 0$, price is uncorrelated with each bidder's value θ_i and yet bidders learn from prices, which increases competitiveness compared to the IPV setting.

[16] As in Rostek and Weretka [13], we measure price informativeness by looking at how much conditioning on the equilibrium price p^* as well as one's own signal s_i reduces the variance of the posterior of θ_i, relative to variance conditional only on the signal. Formally, this gives the following index of *price informativeness*: $[Var(\theta_i|s_i) - Var(\theta_i|p^*, s_i)]/Var(\theta_i|s_i)$.

bidder depends on the induced change in the (average) correlation in preferences that his participation brings about in the market.

GEOMETRIC INTERPRETATION. Proposition 1 has a transparent geometric interpretation. For any level of price impact $\lambda \in (0, \infty)$, let a λ-curve comprise all profiles $(\gamma, \bar{\rho}) \in \Gamma \times [-1, 1]$ such that auctions of size γ and commonality $\bar{\rho}$ are characterized by equilibrium price impact λ (Fig. 1c). The positive slope of λ-curves reflects that an inclusion of a trader while maintaining the same level of commonality in the market lowers price impact, whereas increasing commonality for a given number of traders increases price impact. The map of λ-curves in Fig. 1c spans the price impact ranging from 0 (λ-curves close to the lower bound $\bar{\rho}^-(\gamma)$) to ∞ (λ-curves close to upper bound $\bar{\rho}^+(\gamma, \sigma^2)$).

The condition from Proposition 1 can now be seen in terms of "crossing from above." Consider any sequence of equicommonal auctions represented by a commonality function $\bar{\rho}(\gamma)$. For any auction of size γ in the sequence, the market becomes more competitive with a new bidder if the schedule $\bar{\rho}(\gamma)$ crosses the corresponding λ-curve at point $(\gamma, \bar{\rho}(\gamma))$ from above. The bound $\pi_{\gamma,\bar{\rho}}$ in Proposition 1 corresponds to the change in commonality induced by a new bidder that just suffices to leave price impact intact (see Fig. 1c).[17]

Being invariant to changes in μ, the map of λ-curves allows comparison of price impact in auctions with different convexity parameters μ. An auction with any $\mu > 0$ gives rise to the same λ-curve map as with $\mu = 1$, with the value of price impact for each curve normalized by μ. It follows that, in Proposition 1, the threshold function is independent from convexity μ; if price impact is monotone for some μ, it is also monotone in a model with any (fixed) $\mu' > 0$.

IMPLICATIONS. By Proposition 1, equilibrium price impact in equicommonal markets may exhibit essentially arbitrary, possibly non-monotone, behavior as markets grow. While arbitrary, the behavior of market power in a sequence of equicommonal auctions $\{A^I\}_{I=1}^{\infty}$ is fully characterized by the behavior of the sufficient statistic given by commonality. For more specific implications, suppose one wished to determine the market size that achieves the maximal market competitiveness. In any Equicommonal Model with non-increasing $\bar{\rho}(\gamma)$, such as the Fundamental Value Model, price impact strictly decreases in market size as in models with independent private values. Therefore, in markets where interdependence in trader preferences arises from aggregate (fundamental) and/or idiosyncratic shocks alone, price impact is monotone. By Proposition 1, the monotonicity of market competitiveness in the number of traders is consistent with preferences becoming more aligned in a larger market. This is, for example, the case in the Group Model with zero cross-group correlation ($\alpha_1 = 0, \alpha_2 = 0$). Here, the commonality function increases in market size even without the size externality, yet the commonality impact is not sufficiently large for price impact to increase for any market size (Fig. 2, Scenario A). The monotonicity of price impact holds also when the cross-group correlation decreases in market size (e.g., negative size externalities such as congestion effects). However, when the size externality on the cross-group correlation is positive and sufficiently strong (e.g., positive taste synergies in a market), then the Group Model gives rise to a non-monotone price impact (Fig. 2, Scenario B depicts the price impact for $\alpha_2, \alpha_3 > 0$; the market is least competitive at an intermediate market size).

[17] Note that, since the domain of the commonality function $\bar{\rho}(\cdot)$, $\Gamma \subset [0, 1]$, is countable, the threshold $\pi_{\gamma,\bar{\rho}}$ in the proposition corresponds to the change in (rather than the slope of) commonality along the appropriate λ-curve that follows from the change in γ due to an inclusion of one trader, $\Delta\gamma = 1/(I - 1)$.

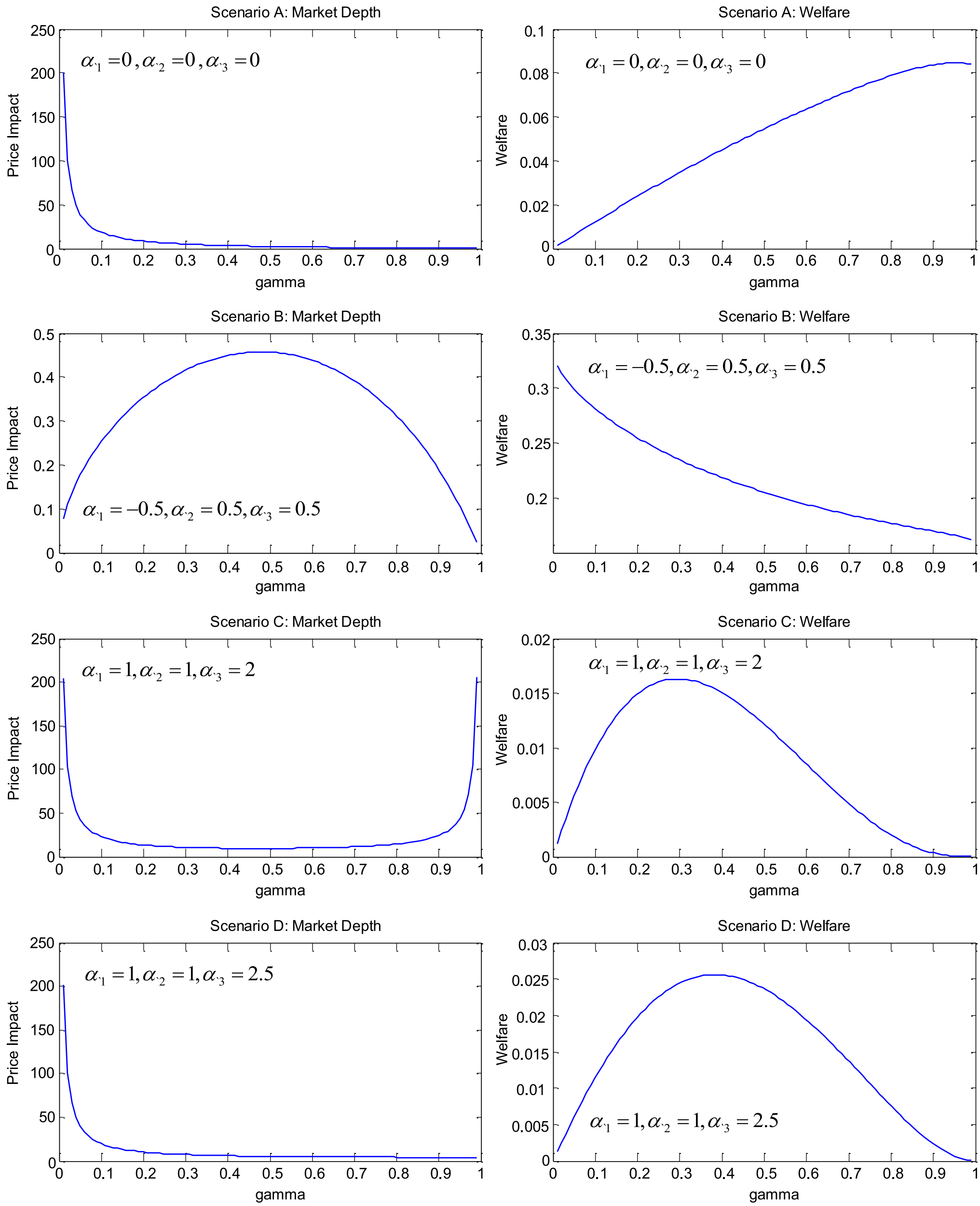

Fig. 2. Market depth and welfare in group model.

3.2. Information and welfare in small markets

We now discuss the way in which market size affects welfare, as measured by the *ex ante* utility of a trader in a symmetric equicommonal auction. It is often argued that encouraging trader participation improves efficiency. Here, we show that, in markets with private informa-

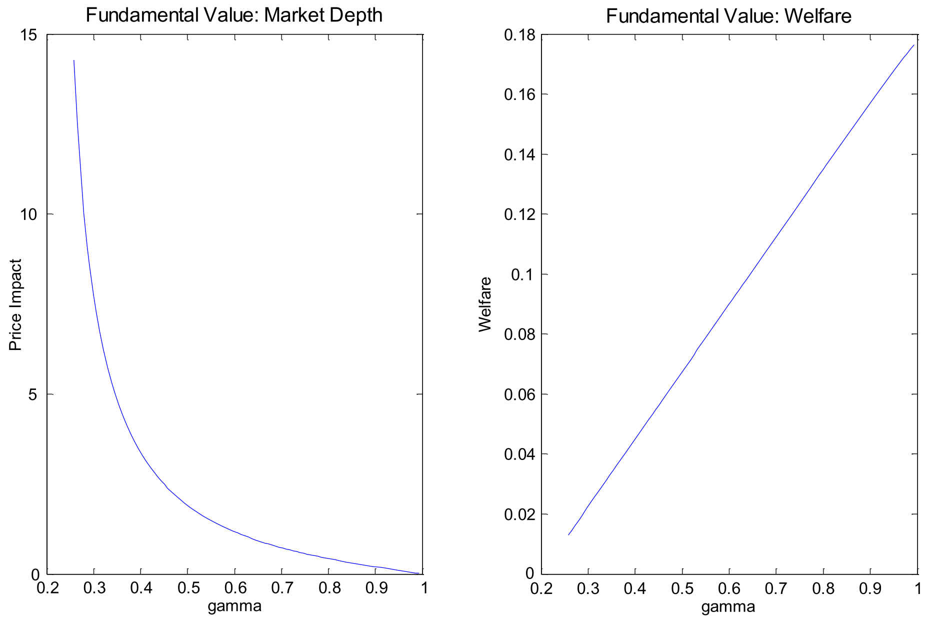

Fig. 3. Market depth and welfare in fundamental value model.

tion, this need not be the case in general. Specifically, when preferences are heterogeneously interdependent, the positive effect of the increased number of traders on price impact can be counterbalanced with adverse effects resulting from the change in preference commonality on price impact, price informativeness and gains to trade.

Consider an increase in market size γ that increases commonality, $\bar{\rho}(\gamma) \in [-1, 1]$. Information can affect welfare through the following channels.

- *Surplus*: When agents' preferences are more correlated on average, the gains to trade are reduced.
- *Information*: A larger *absolute value of* commonality increases the total information about values contained in the signal vector. Depending on the heterogeneity in value correlations, information is aggregated (or not) in equilibrium price.
- *Competitiveness*: By Proposition 1, a higher commonality may increase price impact and reduce demand and trade.

The direct competitiveness effect resulting from the larger market size and three indirect effects resulting from preference interdependence jointly determine whether increasing the number of traders has a positive or negative impact on the *ex ante* expected utility of the incumbents.

In the Fundamental Value Model, commonality is independent of market size and, thus, does not affect welfare indirectly through price impact and gains to trade or price informativeness, and larger markets have unambiguous (positive) effects on welfare (see Fig. 3), as shown by Vives [18]. Let us use Example 1 to illustrate how heterogeneity in preferred interdependence changes the impact of market size on welfare. In the Group Model with zero cross-group correlation ($\alpha_1 = \alpha_2 = \alpha_3 = 0$), welfare exhibits a hump-shaped pattern for values of γ close to one (Fig. 2, Scenario A). This happens even though price impact is monotonically decreasing in market size, and so for all γ. The non-monotonicity in welfare can be attributed to a reduction in the gains to trade. Likewise, in the Group Model with $\alpha_1 = 1$, $\alpha_2 = 1$ and $\alpha_3 = 2$, welfare is maximal for the intermediate market size (Fig. 2, Scenario C). The adverse effects of market size on welfare are particularly stark in the Group Model with parameters $\alpha_1 = -0.5$, $\alpha_2 = \alpha_3 = 0.5$. Here, an increase in the value of commonality, as the number of traders grows, reduces gains to trade for all market sizes γ, and only in large enough markets increases price informativeness. In this model, welfare decreases in trader participation for all market sizes (Fig. 2, Scenario B).

These simple, illustrative examples suggest that, in assessing whether an increase in market size improves or reduces welfare, indirect effects on preference interdependence (i.e., the commonality statistics in equicommonal auctions) are essential. Indeed, outside of the Fundamental Value Model, entry of new market participants changes the joint distribution of preferences and gives rise to the indirect effects of market size on welfare through private information.

3.3. Large markets

A voluminous body of research has examined the question of whether strategic traders become price takers in large markets and, hence, whether models based on competitive equilibrium accurately describe large-market interactions. In this section, we analyze competitiveness in large equicommonal markets. In the presence of asymmetric information, the appropriate solution concept in a large market is a *(competitive) rational expectations* equilibrium in which traders have correct conditional expectations about values θ_i given the observed prices. A competitive rational expectations equilibrium[18] is a profile of trades $\{q_i^{REE}(p, s_i)\}_i$ and a price function measurable with respect to values and signal noise such that the market clears and all agents optimize: for each i,

$$q_i^{REE}(p, s_i) \equiv \arg\max_q E\big(U_i(q, p \times q)\big| p, s_i\big). \tag{18}$$

Analogous to the Bayesian Nash equilibrium, the competitive rational expectations equilibrium entails the knowledge of the map between the equilibrium price and states of the world $p^{REE}(s)$ by each trader, and the conditional expectations reflect this knowledge. Unlike in the Nash equilibrium, in the competitive equilibrium, traders do not recognize their price impact and, for all prices, their demands coincide with expected marginal utility.

In models where only one (buying or selling) side of the market is strategic, order reduction by the strategic side of the market changes the equilibrium price—and the map from states to prices—relative to the competitive model; in turn, this distorts bidders' (models of) expectations. Conveniently, in the double auction studied in this paper, symmetric traders reduce their

[18] Concerning the implementability of the competitive rational expectations equilibrium as a demand function equilibrium, it can be shown that, for any finite number of traders, the necessary and sufficient conditions for the existence of the Linear Bayesian Equilibrium in demand schedules from Rostek and Weretka [13, Proposition 1; see the Appendix], which involve bounds on the commonality statistics, are also sufficient for the implementability of the competitive rational expectations as equilibrium in demand functions. (The results in Rostek and Weretka [13] on inference and Rostek and Weretka [14] on the duality between a (Bayesian Nash) equilibrium in demand *functions* and a general-equilibrium (REE) representation of equilibrium in quantity *levels* (a duality based on a formulation of a double auction as the model of "trading against price impact") allow for the implementability of the *non-competitive and competitive* rational expectations in markets with information structures with heterogeneously *interdependent* preferences.) Given the feature of the model that each agent has mass equal to one, since the market clearing condition is not well defined in the limit with a continuum of traders, we characterize the limit of equilibria in the sequence of finite auctions.

demands and supplies for every price by the same factor (cf. (15)). As a result, the market clears at the competitive price $p^*(s) = p^{REE}(s)$, regardless of the number of traders, even though the equilibrium trade is non-competitive.[19]

Proposition 2 gives the conditions under which market power vanishes in large auctions and the demand schedules of strategic Bayesian players converge (pointwise) to bids (18) in equicommonal markets.

Proposition 2 *(Convergence to competitive REE). In the symmetric Bayesian Nash equilibrium, each bidder i's bid converges pointwise to bid $q_i^{REE}(p, s_i)$, if*

$$\lim_{\gamma \to 1} \sup \bar{\rho}(\gamma) < 1. \tag{19}$$

The competitive rational expectations equilibrium obtains under weak conditions: As long as preferences do not become perfectly aligned in large markets (i.e., pure common values for almost all traders), traders treat prices parametrically and markets become competitive. Conversely, only when commonality converges to one might price impact in large markets prevail, depending on the convergence rate of $\bar{\rho}(\gamma)$, relative to that of γ. Note that condition (19) does not require that the commonality function converges in a given model.

Our result (with fully strategic, Bayesian players) contributes to the literature that seeks the foundations for the competitive REE as follows. The existing results have been obtained for markets with particular types of interdependencies in trader values; namely, those in which there is a fundamental value of the good that determines the values derived from the good for all bidders or, more generally, in which bidder values are derived from aggregate and idiosyncratic shocks. Such a shock structure generates a Fundamental Value matrix $\mathcal{C}$ and, in large markets, rules out interdependence in values in groups of traders except for those present in the market as a whole. Proposition 2 provides a foundation for the competitive REE in models with equicommonal correlation structures $\mathcal{C}$. Additionally, in equicommonal markets, price fully privately reveals all the available information only if $\rho_{i,j} = \bar{\rho}$, for all $j \neq i$ (i.e., the Fundamental Value Model, Rostek and Weretka [13, Proposition 3]), which are non-generic within the class of equicommonal auctions. Proposition 2 holds in any Equicommonal Model with identical and heterogeneous interdependencies, and therefore provides a foundation also for a not fully privately revealing rational expectations equilibrium in equicommonal markets.[20]

Proposition 2 can be interpreted geometrically in the map of λ-curves from Fig. 1c. For any level of price impact $\lambda > 0$, the λ-curves approach $\bar{\rho} = 1$ as markets grow large. Therefore, in any market where condition (19) holds, the commonality function eventually crosses from above each λ-curve, and the price impact becomes negligible in the limit.

The significance of condition (19) can be seen in the Group Model with $\alpha_1 = \alpha_2 = 1, \alpha_3 = 2$; the corresponding commonality function and price impact are depicted in Fig. 1a and Fig. 2,

[19] Essentially, in the symmetric model, market power is balanced between the buyer and seller sides of the market. In a Nash equilibrium, price impacts are strictly positive, demand schedules are below the expected marginal utility and the equilibrium allocation is inefficient for all $\gamma < 1$.

[20] Partial revelation of information in the competitive limit of our model does not result from the presence of noise traders (e.g., Hellwig [6]), or uninformed traders (e.g., Ausubel [1]), or uncertainty of dimension greater than that of price (e.g., Jordan [7]; Messner and Vives [9]). In particular, in any equicommonal auction, for any agent, a one-dimensional statistics exists that is sufficient for the payoff-relevant information contained in the signals of other agents. Nevertheless, except when correlations for all pairs of traders are identical, the statistics differs from the average signal that the equilibrium price reveals.

Scenario C. As $\gamma \to 1$, the price impact λ of trader i is bounded away from zero. In fact, properly choosing α_1 and the positive externality coefficients α_2 and α_3 may yield an arbitrary equilibrium price impact in the large, limit market. Fig. 2 depicts the examples of markets in the Group Model with the following distinct monotonicity and convergence properties of price impact: markets with price impact monotonically decreasing in the number of traders or non-monotone price impact that is positive in the limit large market (Fig. 2, Scenarios C and D), a market with non-monotone price impact and price-taking behavior with a large number of traders (Fig. 2, Scenario B), and a market with $\alpha_1 = \alpha_2 = 1$ and $\alpha_3 = 2$, where price impact is maximal in the large limit market (Fig. 2, Scenario C).

In a market with infinitely many traders, how can an individual bidder affect prices? As the market becomes effectively one of pure common values ($\bar{\rho}(\gamma) \to 1$), price becomes the principal source of inference in expectations $E(\theta_i|p, s_i)$; since traders interpret price changes as resulting from changes in the fundamental value, with pure common values, this translates (almost) one-to-one to an adjustment of conditional expectation $E(\theta_i|p, s_i)$. Thus, while the risk aversion and strategic effects in (17) are negligible, so is the difference $\gamma - c_p$; the inference effect grows without bound, and the joint impact of the three effects is bounded away from zero.

3.4. Other equicommonal markets

The model of equicommonal auctions extends to markets with asymmetric group sizes in which the correlation of trader values in the smaller group exceeds the small-group correlation to preserve equicommonality; for instance, a small number of producers with strongly correlated cost parameters trades with a large number of consumers whose preferences correlate only weakly. Then, the residual market is not *ex ante* identical for each agent. As in the case of the symmetric Group Model, although commonality is increasing, the commonality impact is not sufficient to upset the monotonicity property of price impact. In addition, the competitive rational expectations equilibrium is predicted in large markets. We describe two other economic environments that are common in economic applications and can give rise to equicommonal models.

- *(*FUNDAMENTAL VALUE MODEL WITH SIZE DEPENDENCE*)* Consider a model given by (4) (e.g., Vives [18]), modified so that the variances of the aggregate and idiosyncratic components of $\{\theta_i\}_{i \in I}$ and value correlations between any trader pair are not constant, but can vary with the pool of participants through the sufficient statistic of market size. This specification subsumes a variant, common in applications, in which the true value θ_i is a (weighted) sum of the signals of all traders. Depending on the commonality function associated with a particular model, price impact can be non-monotone, and if within the limit commonality converges to one, price-making behavior might be observed for the large limit market.
- *(*SPATIAL MODEL*)* In many markets, correlation of endowments, preferences or productivity varies with geographical or cultural proximity in a systematic manner. This motivates a spatial model in which the correlation of values decays according to the distance among the agents. Consider I *traders* located on a circle with the distance between any two immediate neighbors normalized to one. Let $d_{i,j}$ be the shorter of the two distances between traders i and j (measured along the circle). To capture stronger interdependence among the values of closer neighbors, correlation between any two *traders* $\rho_{i,j}$ is assumed to decay with distance, $\rho_{i,j} = \beta^{d_{i,j}}$, where $\beta \in (0, 1)$ is the decay rate. The model takes the decay rate β as a primitive and assumes that a new trader enlarges the market by increasing the circumference

of the circle by one. This facilitates analysis of market interactions where trader preferences become less and less alike as the market expands. The commonality function is given by $\bar{\rho}^{CC}(\gamma) = 2(1-\gamma)\beta(1-\beta^{\frac{1}{2}\frac{1}{1-\gamma}})/(1-\beta)$, assuming that I is an odd number and is monotonically decreasing to zero in market size. In this model, price impact is monotone, and the competitive equilibrium is obtained in the large market.[21] Since equilibrium price in the large market is deterministic, in the competitive limit market, price reveals no information about the values of $\{\theta_i\}_{i\in I}$ to traders (even though some of the information is available in the market).

4. Discussion

Market power is a key concern in practical market design. We examine how the monotonicity of market power and welfare is affected by private information through price inference in markets with heterogeneous independencies in values. Analysis of information aggregation remains implicit in the present paper, as it is studied in Rostek and Weretka [13]. In light of the results about information aggregation, the results established in this paper reveal additional insights:

(1) A number of studies from the information aggregation literature examined the relative contribution of market power versus private information to the inefficiency of equilibrium allocation. This paper finds that, for information structures that admit symmetric models of inference by traders (i.e., the Equicommonal Model), market power depends on the information structure through *commonality* in trader preferences. In turn, Rostek and Weretka [13] show that the amount of the total information that is available in the market fails to be transmitted through prices to bidders (and, therefore, the potential to learn via non-market modes of learning) depends instead on the *heterogeneity* in preference interdependence: information is aggregated into prices if, and only if, $\bar{\rho}_{i,j} = \bar{\rho}$ for all $i \neq j$. This sheds light on the sources of inefficiency: commonality in values leads to—and, indeed determines—inefficiency due to market power, whereas heterogeneity in preference interdependencies induces inefficiency due to private information.

(2) Our results qualify the robustness of predictions regarding price-taking behavior in large markets and the ability of prices to aggregate information (in small as well as large markets). In markets described by the Fundamental Value Model, bidders become price takers and markets are informationally efficient (i.e., price is fully privately revealing) in equilibrium of the limit large market. In markets described by the Equicommonal Model, price taking is predicted robustly in large markets and are independent of details of information structure other than a uniform bound on the large-market commonality. On the other hand, even in large markets, aggregation of information is not generic and is obtained only in markets described by the Fundamental Value Model.

(3) In the Fundamental Value Model, additional market participants lower the price impact and increase the price informativeness. In this sense, there is a comonotone relationship between market competitiveness and learning from prices. Our analysis suggests that this relationship is not inherent in double auctions and, in particular, does not extend to equicommonal markets.

[21] In this experiment, we assume that a new trader increases the circumference of the circle by one so that a city with I traders has a circumference equal to I. W.l.o.g. a trader can be added at an arbitrary position on the circle. Alternatively, one could assume that the circumference is fixed and that additional traders increase the density of the population. The latter formulation would imply that preferences comove more closely in pairs of traders in larger markets and, therefore, would not capture the decay in distance commonality, which we intend to analyze. In this case, price impact would still be monotonic and, because $\lim_{\gamma\to 1}\bar{\rho}(\gamma) < 1$, the competitive equilibrium would be obtained in the large market.

Rather, the relation is shaped by how the commonality in trader values evolves with new market participants in the following way. For moderate changes in commonality of values, one should expect price informativeness to increase and price impact to decrease with additional bidders. Large enough increases in commonality increase both price impact as well as price informativeness, whereas sufficiently large decreases diminish both price impact and the informational content of prices.

(4) Learning from prices (and, hence, information acquisition) may or may not improve welfare for a given market size: Improved estimation of agent values improves individual decisions, but may also increase market power. The policy that maximizes learning from prices need not correspond to the maximization of liquidity or welfare. The policy that aims at minimizing market power need not maximize welfare.

Appendix A

Lemma 1 (EXISTENCE OF EQUILIBRIUM). *(See Rostek and Weretka [13].) In an equicommonal auction of size γ and commonality $\bar{\rho}$, a symmetric linear Bayesian Nash equilibrium exists, if and only if, $\bar{\rho}^-(\gamma) < \bar{\rho} < \bar{\rho}^+(\gamma, \sigma^2)$, where*

$$\bar{\rho}^+(\gamma,\sigma^2) = \frac{\gamma^2 - 2(1-\gamma)\sigma^2 + (1-\gamma)\sqrt{4\sigma^4 + (\gamma\frac{2-\gamma}{1-\gamma})^2}}{2\gamma}, \tag{20}$$

$$\bar{\rho}^-(\gamma) = -(1-\gamma). \tag{21}$$

INFERENCE COEFFICIENTS. In the symmetric linear equilibrium, bids take the functional form of $q_i(p, s_i) = \alpha_0 + \alpha_s s_i + \alpha_p p$, where constants α_0, α_s, and α_p are the same across all traders. Given linear strategies, trader i faces a residual supply with a deterministic slope λ and a stochastic intercept that is a function of other traders' signals. Following up on the derivation in Section 3, the random vector (θ_i, s_i, p^*) is jointly normally distributed,

$$\begin{pmatrix} \theta_i \\ s_i \\ p^* \end{pmatrix} = \mathcal{N}\left[\begin{pmatrix} E(\theta_i) \\ E(\theta_i) \\ E(\theta_i) \end{pmatrix}, \begin{pmatrix} \sigma_\theta^2 & \sigma_\theta^2 & cov(\theta_i, p^*) \\ \sigma_\theta^2 & \sigma_\theta^2 + \sigma_\varepsilon^2 & cov(s_i, p^*) \\ cov(p^*, \theta_i) & cov(p^*, s_i) & Var(p^*) \end{pmatrix}\right]. \tag{22}$$

(13) allows determination of variances and covariances in (22). Applying the Projection Theorem, the method of undetermined coefficients yields the inference coefficients

$$c_s = \frac{1-\bar{\rho}}{1-\bar{\rho}+\sigma^2}, \tag{23}$$

$$c_p = \frac{(2-\gamma)\bar{\rho}}{1-\gamma+\bar{\rho}}\frac{\sigma^2}{1-\bar{\rho}+\sigma^2}. \tag{24}$$

Proof of Proposition 1. (MONOTONICITY OF PRICE IMPACT). Eqs. (17), (23) and (24) implicitly define $\bar{\rho}$, for any value of μ/λ and γ, through a quadratic equation, the roots of which are

$$\bar{\rho} = \frac{1}{2}\left[\gamma + \sigma^2 - \frac{(2-\gamma)\sigma^2}{\gamma-(1-\gamma)\mu/\lambda}\right] \pm \frac{1}{2}\left[\left(\gamma + \sigma^2 - \frac{(2-\gamma)\sigma^2}{\gamma-(1-\gamma)\mu/\lambda}\right)^2 + 4\left[1+\sigma^2\right][1-\gamma]\right]^{\frac{1}{2}}. \tag{25}$$

Each root is discontinuous in γ at the market size $\gamma^* = (\mu/\lambda)/(1+\mu/\lambda)$, for which $c_p = 0$ and inference from prices does not occur. Next, it is demonstrated that for any given price impact λ, a unique λ-curve exists within the bounds (21) and (20) that is increasing in γ and defined by the negative root for $\gamma < \gamma^*$ and by the positive root for $\gamma > \gamma^*$. Fix λ. This also determines the threshold $\gamma^* = \mu/\lambda/(1+\mu/\lambda)$. We first argue that for $\gamma < \gamma^*$, the positive root exceeds one, and therefore, cannot be part of a λ-curve: when $\gamma < \gamma^*$, the denominator in (25) is negative and, hence, the root is strictly increasing in σ^2. The value of the positive root is thus bounded from below by the value of (25) at $\bar{\sigma}^2 = 0$, which is equal to $\frac{1}{2}\gamma + \frac{1}{2}[\gamma^2 + 4[1-\gamma]]^{\frac{1}{2}}$. This bound itself is, in turn, monotonically increasing in γ and, hence, bounded from below by one (at $\gamma = 0$). We now argue that, for any $\lambda \in (0, \infty)$ and $\gamma < \gamma^*$, the negative root gives a value of $\bar{\rho}$ within the bounds (21) and (20). The negative root is non-positive for all $\gamma < \gamma^*$ and one can write

$$\frac{1}{2}\bar{\rho} = x - \sqrt{x^2 + c}, \tag{26}$$

where $c = 4[1+\sigma^2][1-\gamma]$ and

$$x = \gamma + \sigma^2 - \frac{(2-\gamma)\sigma^2}{\gamma - (1-\gamma)\mu/\lambda}. \tag{27}$$

From (26), the negative root is increasing in x. Since x is decreasing in μ/λ, for any γ the value of $\bar{\rho}$ is bounded from below by the negative root evaluated at the limit $\mu/\lambda \to \infty$. For any fixed γ, the limit equals $-(1-\gamma)$ and, therefore, coincides with $\bar{\rho}^-$. It follows that for any $\gamma < \gamma^*$, exactly one value of $\bar{\rho} \in (\bar{\rho}^-(\gamma), \bar{\rho}^+(\gamma, \sigma^2))$ exists such that the price impact is equal to λ and given by the negative root (25). By (25), a λ-curve is continuous and strictly increasing in γ. Mimicking the argument for $\gamma > \gamma^*$, the λ-curve and its properties are uniquely determined by the positive root (25). Finally, that $c_p = 0$ for $\gamma = \gamma^*$ implies $\bar{\rho} = 0$. Note that since for $\gamma < \gamma^*$ and $\gamma > \gamma^*$, the λ-curves converge to 0 as $\gamma \to \gamma^*$ and, hence, the uniquely defined λ-curve is continuous on the whole interval [0, 1]. Notice that the λ-curve converges to the upper bound (20) as $\lambda \to \infty$ and to the lower bound (21) as $\lambda \to 0$. Tedious algebra (see the end of Appendix A) reveals that for all profiles of $(\gamma, \bar{\rho})$ for which equilibrium exists except $(\gamma^*, \bar{\rho})$, the following derivatives of price impact can be established: $\partial\lambda/\partial\bar{\rho} > 0$ and $\partial\lambda/\partial\gamma < 0$. By the implicit function theorem, it follows that the λ-curve is monotonically increasing in γ. Threshold $\pi_{\gamma,\bar{\rho}}$ equals the increase of commonality that preserves the same price impact after an increase of γ that results from increasing the number of traders by one, $\Delta\gamma = 1/(I-1)$. For $\gamma < \gamma^* - \Delta\gamma$, the threshold is given by

$$\begin{aligned}
\pi_{\gamma,\bar{\rho}} = {}& \frac{1}{2}\left[\gamma + \Delta\gamma - \frac{(2-\gamma)\sigma^2}{\gamma - (1-\gamma)\mu/\lambda} + \frac{(2-\gamma-\Delta\gamma)\sigma^2}{\gamma + \Delta\gamma - (1-\gamma-\Delta\gamma)\mu/\lambda}\right] \\
& - \frac{1}{2}\left[\left(\gamma + \Delta\gamma + \sigma^2 - \frac{(2-\gamma-\Delta\gamma)\sigma^2}{\gamma + \Delta\gamma - (1-\gamma-\Delta\gamma)\mu/\lambda}\right)^2 \right. \\
& \left. + 4\big[1+\sigma^2\big][1-\gamma-\Delta\gamma]\right]^{\frac{1}{2}} \\
& + \frac{1}{2}\left[\left(\gamma + \sigma^2 - \frac{(2-\gamma)\sigma^2}{\gamma - (1-\gamma)\mu/\lambda}\right)^2 + 4\big[1+\sigma^2\big][1-\gamma]\right]^{\frac{1}{2}}.
\end{aligned} \tag{28}$$

For $\gamma > \gamma^*$, the sign of the last two expressions is reverted, while for $\gamma \in (\gamma^* - \Delta\gamma, \gamma^*)$ the two last expressions have positive signs. By the monotonicity of λ-curves, $\pi_{\gamma,\bar{\rho}} > 0$. Moreover, since equilibrium price impact is proportional to convexity coefficient, λ-curve map is the same for all μ, and threshold does not depend on risk aversion. □

Proof of Proposition 2. (Convergence to competitive REE). If $\lim_{\gamma\to 1}\sup\bar{\rho}(\gamma) < 1$, then

$$\lim_{\gamma\to 1}\sup c_p = \frac{\sigma^2}{1-\lim_{\gamma\to 1}\sup\bar{\rho}(\gamma)+\sigma^2} < 1, \tag{29}$$

which when combined with

$$\lim_{\gamma\to 1}\sup\lambda = \frac{1}{1-\lim_{\gamma\to 1}\sup c_p} \times \lim_{\gamma\to 1}(1-\gamma)\mu \tag{30}$$

gives $\lim_{\gamma\to 1}\sup\lambda = 0$, and the first of the two elements in (30) is bounded. It follows that the optimal bids (15) pointwise converge to the rational expectation bids (18), given by (15) with $\lambda = 0$ and the price function is as in the strategic model. □

We show that $\partial\lambda/\partial\bar{\rho} > 0$ and $\partial\lambda/\partial\gamma < 0$, which are used in the proof of Proposition 1. We will use that

$$\frac{\lambda}{\mu} = \frac{1-\gamma}{\gamma - c_p}, \tag{31}$$

$$c_p = \frac{2-\gamma}{1-\gamma+\bar{\rho}}\frac{\bar{\rho}\sigma^2}{1-\bar{\rho}+\sigma^2}. \tag{32}$$

Claim 1. *Price impact is increasing in $\bar{\rho}$, ceteris paribus. Using*

$$\frac{\partial c_p}{\partial\bar{\rho}} = -\frac{2-\gamma}{(1-\gamma+\bar{\rho})^2}\frac{\bar{\rho}\sigma^2}{1-\bar{\rho}+\sigma^2} + \frac{2-\gamma}{1-\gamma+\bar{\rho}}\left(\frac{\sigma^2(1+\sigma^2)}{(1-\bar{\rho}+\sigma^2)^2}\right), \tag{33}$$

we have that

$$sign\left\{\frac{\partial c_p}{\partial\bar{\rho}}\right\} = sign\left\{\frac{\sigma^2(1+\sigma^2)}{(1-\bar{\rho}+\sigma^2)} - \frac{\bar{\rho}\sigma^2}{(1-\gamma+\bar{\rho})}\right\}. \tag{34}$$

The sign (34) is positive for $\bar{\rho} < 0$; it is also positive for $\bar{\rho} > 0$, since

$$\frac{\sigma^2(1+\sigma^2)}{(1-\bar{\rho}+\sigma^2)} - \frac{\bar{\rho}\sigma^2}{(1-\gamma+\bar{\rho})} > \frac{\sigma^2(1+\sigma^2)}{(1-\bar{\rho}+\sigma^2)} - \sigma^2 = \frac{\bar{\rho}\sigma^2}{(1-\bar{\rho}+\sigma^2)} > 0. \tag{35}$$

This implies $\partial\lambda/\partial\bar{\rho} > 0$.

Claim 2. *Price impact is decreasing in γ, ceteris paribus. The sign of the derivative of price impact with respect to γ*

$$\partial\frac{\lambda}{\mu}/\partial\gamma = -\frac{1}{(\gamma-c_p)^2}\left[1-c_p-(1-\gamma)\frac{\partial c_p}{\partial\gamma}\right] \tag{36}$$

is the negative of the sign of expression $1-c_p-(1-\gamma)\frac{\partial c_p}{\partial\gamma}$. Since

$$\frac{\partial c_p}{\partial\gamma} = \frac{1-\bar{\rho}}{(1-\gamma+\bar{\rho})^2}\frac{\bar{\rho}\sigma^2}{1-\bar{\rho}+\sigma^2}, \tag{37}$$

the expression $1 - c_p - (1-\gamma)\frac{\partial c_p}{\partial \gamma}$ *is given by*

$$\frac{\bar{\rho}\sigma^2}{1-\bar{\rho}+\sigma^2}\left[\frac{1-\bar{\rho}+\sigma^2}{\bar{\rho}\sigma^2} - \frac{2-\gamma}{1-\gamma+\bar{\rho}} - \frac{(1-\bar{\rho})(1-\gamma)}{(1-\gamma+\bar{\rho})^2}\right]. \tag{38}$$

The derivative of the first term in brackets with respect to σ^2 *is given by*

$$\frac{\bar{\rho}(\bar{\rho}-1)}{\bar{\rho}^2\sigma^4}, \tag{39}$$

which implies that for all $\bar{\rho} > 0$*, the whole term in brackets in (38) is bounded from below by the limit as* $\sigma^2 \to \infty$ *by*

$$\frac{1}{\bar{\rho}} - \frac{2-\gamma}{1-\gamma+\bar{\rho}} - \frac{(1-\bar{\rho})(1-\gamma)}{(1-\gamma+\bar{\rho})^2}, \tag{40}$$

which gives

$$\frac{(1-\gamma+\bar{\rho})(1-\gamma+\gamma\bar{\rho}) + (2-\gamma)\bar{\rho}^2}{(1-\gamma+\bar{\rho})^2\bar{\rho}} > 0 \tag{41}$$

and, therefore, for all $\bar{\rho} > 0$*,* $\partial\lambda/\partial\gamma < 0$*. For* $\bar{\rho} < 0$*, we need to show that the term in brackets in (38) is negative for all values of parameters. Since the term* $\frac{1-\bar{\rho}+\sigma^2}{\bar{\rho}\sigma^2}$ *is increasing in* σ^2 *for all* $\bar{\rho} < 0$*, it is bounded from above by*

$$\frac{1}{\bar{\rho}} - \frac{2-\gamma}{1-\gamma+\bar{\rho}} - \frac{(1-\bar{\rho})(1-\gamma)}{(1-\gamma+\bar{\rho})^2}, \tag{42}$$

which implies

$$\frac{(1-\gamma+\bar{\rho})(1-\gamma+\gamma\bar{\rho}) + (2-\gamma)\bar{\rho}^2}{(1-\gamma+\bar{\rho})^2\bar{\rho}}, \tag{43}$$

which is negative for any $\bar{\rho} \in (-(1-\gamma), 0)$*. It follows that* $\partial\lambda/\partial\gamma > 0$*.*

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