Public Affairs 974 Menzie D. Chinn
Fall 2012 Social Sciences 7418
University of Wisconsin-Madison

Problem Set 1 Answers

Due ***in lecture*** on Thursday, October 25. Be sure to put your name on your problem set. Put "boxes" around your answers to the algebraic questions.

1. Suppose the economy is described by the following equations (so we are looking at a closed economy):

• Real Sector

(1)	$Y = Z$	Output equals aggregate demand, an equilibrium condition
(2)	$Z = C + I + G$	Definition of aggregate demand
(3)	$C = c_o + c_1 Y_D$	Consumption fn, c_1 is the marginal propensity to consume
(4)	$Y_D \equiv Y - T + Tr$	Definition of disposable income
(5)	$T = t_1 Y$	Tax function; t_1 is marginal tax rate.
(6)	$Tr = TR_0$	Transfer payments; TR_0 is lump sum transfers.
(7)	$I = b_0 + b_1 Y - b_2 i$	Investment function
(8)	$G = GO_0$	Government spending on goods and services, exogenous

• Asset Sector

(9)	$\frac{M^d}{P} = \frac{M^s}{P}$	Equilibrium condition
(10)	$\frac{M^s}{P} = \frac{M_0}{P}$	Real money supply
(11)	$\frac{M^d}{P} = \mu_0 + Y - hi$	Real money demand

1.1 Solve for the IS curve (Y as a function of i).

$Y = Z = C + I + G$ substitute in for *C, I, G*

$Y = c_0 + c_1 Y_D + b_0 + b_1 Y - b_2 i + GO_0$ substitute in for Y_d

$Y = a_0 + c_1 (Y - T + Tr) + b_0 + b_1 Y - b_2 i + GO_0$ substitute in for tax, transfers functions

$Y = a_0 + c_1 (Y - t_1 Y + TR_0) + b_0 + b_1 Y - b_2 i + GO_0$ bring the "Y" terms to left hand side.

$Y - b(Y - t_1 Y - \lambda Y) = Y(1 - b(1 - t - \lambda)) = c_0 + c_1 TR_0 + b_0 + GO_0 - b_2 i$

divide both sides by $(1 - c_1 (1 - t_1) - b_1)$ and let $\Lambda_0 \equiv c_0 + c_1 TR_0 + b_0 + GO_0$

$$\boxed{Y_0 = \bar{\gamma}[\Lambda_0 - b_2 i]} \text{ let } \bar{\gamma} = \frac{1}{[1 - c_1(1 - t_1) - b_1]}$$

1.2 Solve for the LM curve (i as a function of Y). What is the channel by which monetary influences affect the real goods sector in this model?

$$\frac{M_0}{P_0} = \frac{M^s}{P} = \frac{M^d}{P} = \mu_0 + Y - hi$$

Solving for the interest rate, i, yields the LM curve:

$$\boxed{i = \frac{\mu_0}{h} - \left(\frac{1}{h}\right)\left(\frac{M_0}{P_0}\right) + \left(\frac{1}{h}\right)Y}$$

Monetary policy influences (in part) interest rates. Interest rates in turn affect investment, and via the simple Keynesian multiplier ($\bar{\gamma}$) affects the entire real sector.

1.3 Solve for the equilibrium values of *Y*.

To solve for the equilibrium value of income, substitute the LM into the IS equation from 1.1:

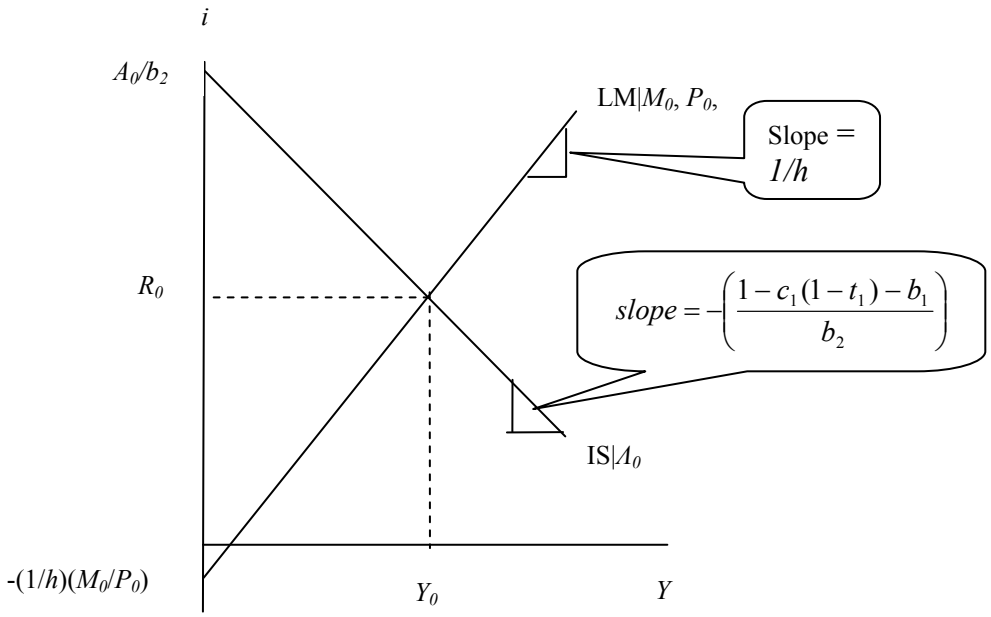
$$Y = \left(\frac{1}{1 - c_1(1 - t_1) - b_1}\right) \times \left[\Lambda_0 - b_2\left\langle \frac{\mu_0}{h} - \frac{1}{h}\frac{M_0}{P_0} + \frac{1}{h}Y \right\rangle\right]$$

Move the term in parentheses (.) and the ***(b₂/h)Y*** term to the LHS; factoring out the *Y*'s on the LHS yields:

Dividing both sides by the term in the parentheses yields:

$$\boxed{Y_0 = \hat{\gamma}[\Lambda_0 - \frac{b_2\mu_0}{h} + \left(\frac{b_2}{h}\right)\left(\frac{M_0}{P_0}\right)]} \quad \text{where} \quad \hat{\gamma} \equiv \frac{1}{1 - c_1(1 - t_1) - b_1 + b_2 / h}$$

1.4 Graph the IS and LM curves on one diagram. Clearly indicate the intercepts and the slopes.



2.1 Assume G increases by ΔGO, and is completely bond financed (no portfolio effects here). Calculate the government spending multiplier.

Take the total differential of your answer to 1.3.

$$\Delta Y = \hat{\gamma}\left[\Delta\Lambda - \frac{b_2\mu_0}{h} + \left(\frac{b_2}{h}\right)\Delta\left(\frac{M}{P}\right)\right]$$

To find the government spending multiplier, set the changes in real money to zero and the money constant, and divide both sides by ΔGO:

$$\boxed{\Delta Y = \hat{\gamma}\Delta GO \Rightarrow \frac{\Delta Y}{\Delta GO} = \hat{\gamma} \equiv \frac{1}{1 - c_1(1-t_1) - b_1 + b_2/h}}$$

2.2 Suppose instead Tr increases by ΔTR. Calculate the government transfers multiplier.

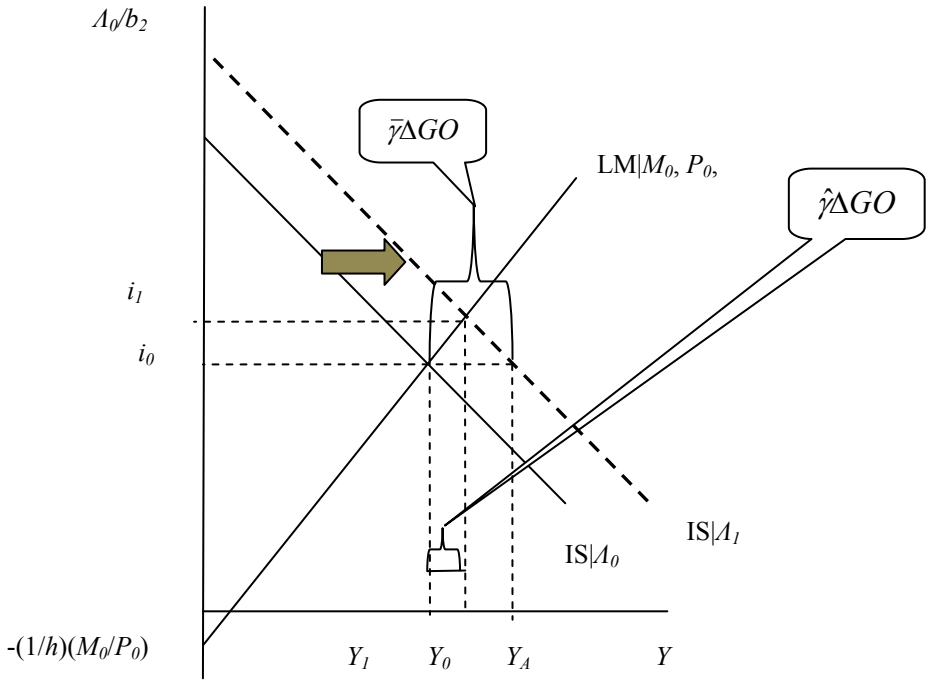
Take the total differential again:

$$\Delta Y = \hat{\gamma}\left[\Delta\Lambda - \frac{b_2\mu_0}{h} + \left(\frac{b_2}{h}\right)\Delta\left(\frac{M}{P}\right)\right]$$

To find the government *transfers* multiplier, set the changes in real money to zero and the money constant, set the change in $\Delta\Lambda$ to equal $c_1\,\Delta TR$, and divide both sides by ΔTR:

$$\boxed{\Delta Y = \hat{\gamma}c_1\Delta TR \Rightarrow \frac{\Delta Y}{\Delta TR} = \hat{\gamma}c_1 \equiv \frac{c_1}{1 - c_1(1-t_1) + b_2/h}}$$

2.3 Redraw your answer to 1.4. Then in the same graph, show what happens to the equilibrium income and interest rate if government spending on goods and services is decreased by ΔGO. Include in your graph the level of income that would be achieved if somehow the interest rate stayed constant (label this point Y_A).



2.4 At the new equilibrium, do we know if investment is higher or lower than the level it started out with? Do we know if it is higher or lower than at Y_A?

Recall the investment function is given by:

$$I = b_0 + b_1 Y - b_2 i$$

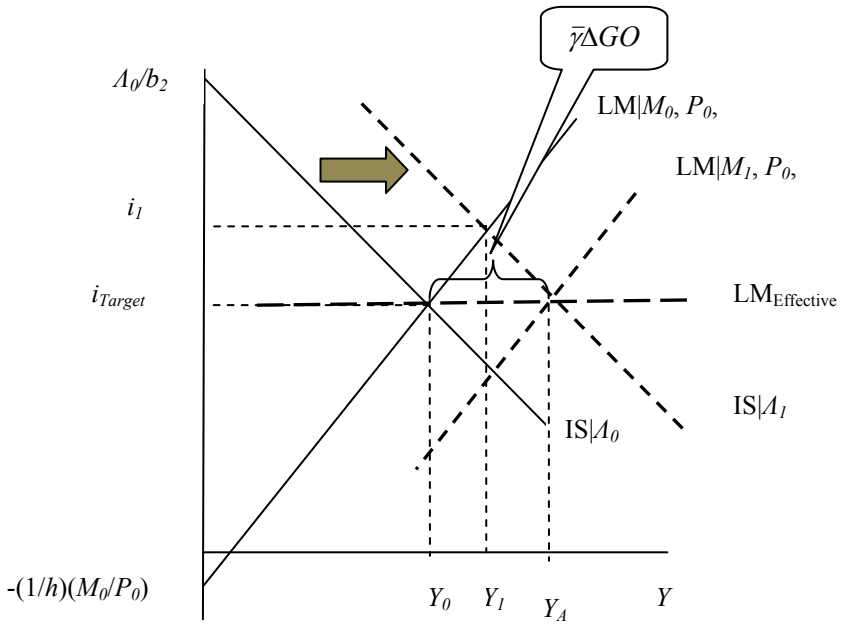
So the change in investment is given by:

$$\Delta I = \Delta b_0 + b_1 \Delta Y - b_2 \Delta i$$

Notice that at the new equilibrium, income is higher (Y_1), but the interest rate is higher as well (i_1). Hence, there are offsetting effects on investment, and the end results could be higher or lower, depending on the magnitudes of the changes in income and interest rates and the parameter values (b_1, $-b_2$).

Regarding the second question, if one were at Y_A, *and* interest rate i_0, then investment would unambiguously be lower.

2.5 Suppose the Fed targets the interest rate at i_0 (call this i_{target}). Returning to 2.3, show graphically what happens if government is increased. What happens to the level of investment?



When the Fed targets the interest rate, and the target interest rate remains constant, then the LM is now the Effective LM. An increase in government spending, increasing autonomous spending, induces an unambiguous increase in investment, since income is higher, but interest rates are unchanged.

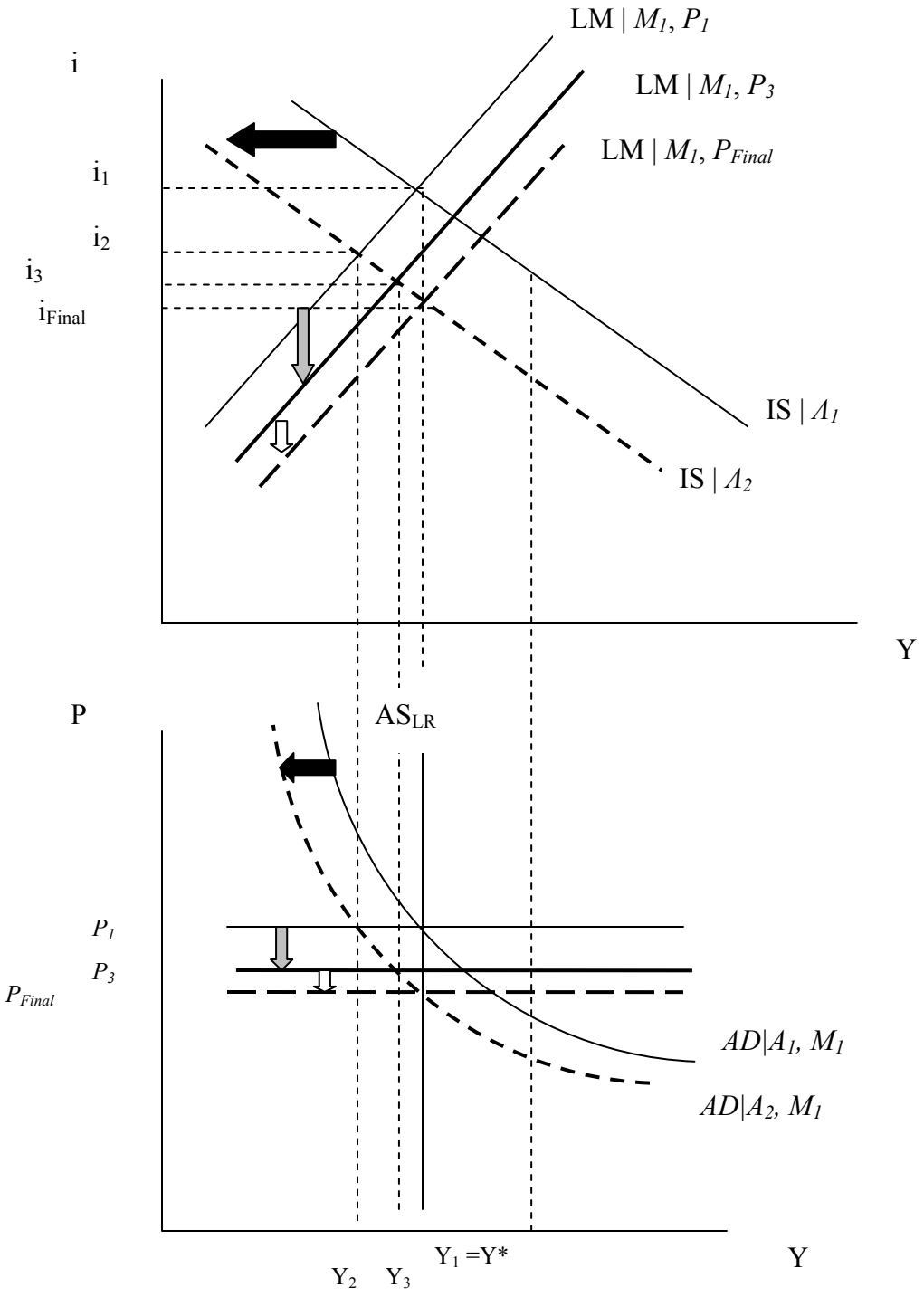
Note that the answer is the same if we are in a liquidity trap.

3. Consider the Aggregate Demand-Aggregate Supply framework. Suppose investment spending decreases when we are not in a liquidity trap (and do not end up in a liquidity trap), and the Fed does ***NOT*** target the interest rate. You can assume for simplicity expected inflation is always zero.

3.1 Show what happens in an IS-LM and AD-AS graph in the period the investment spending reduction occurs.

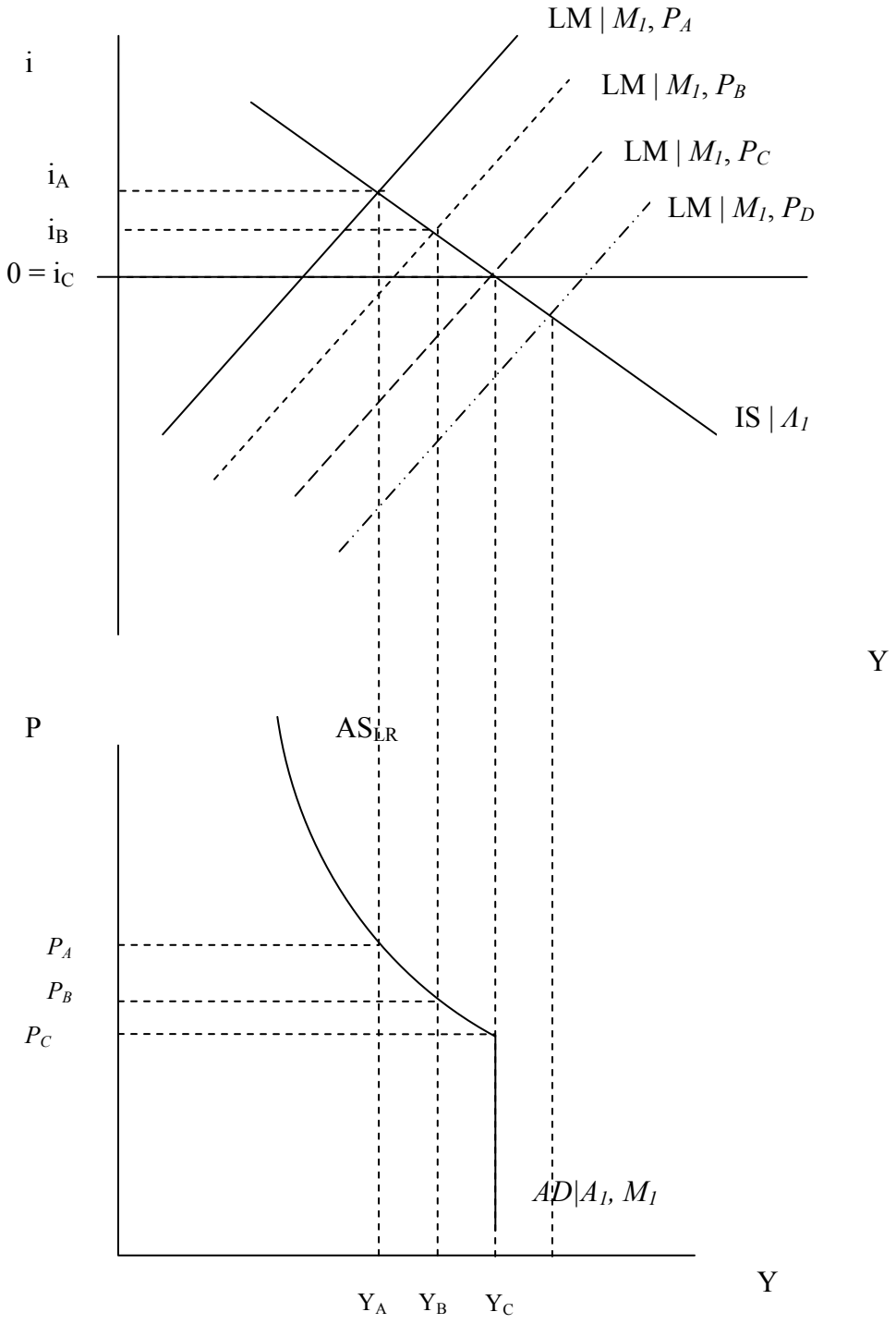
3.2 Show what happens over time to output, the price level, and the interest rate.

I will answer questions 3.1 and 3.2 starting at full employment, to simplify the exposition. The reduction in investment spending shifts in the IS, AD curves (black arrows). Interest rates fall from i_1 to i_2; income falls from Y_1 to Y_2. The price level remains fixed at P_1. In period 3, the price level falls, driving up the real money stock, dropping interest rates to i_3, spurring investment, so that income rises to Y_3. (gray arrows). Over time (white arrows), the price level continues to fall, so that M/P continues to rise, driving the interest rate down to i_{Final}, and output back to Y^*.



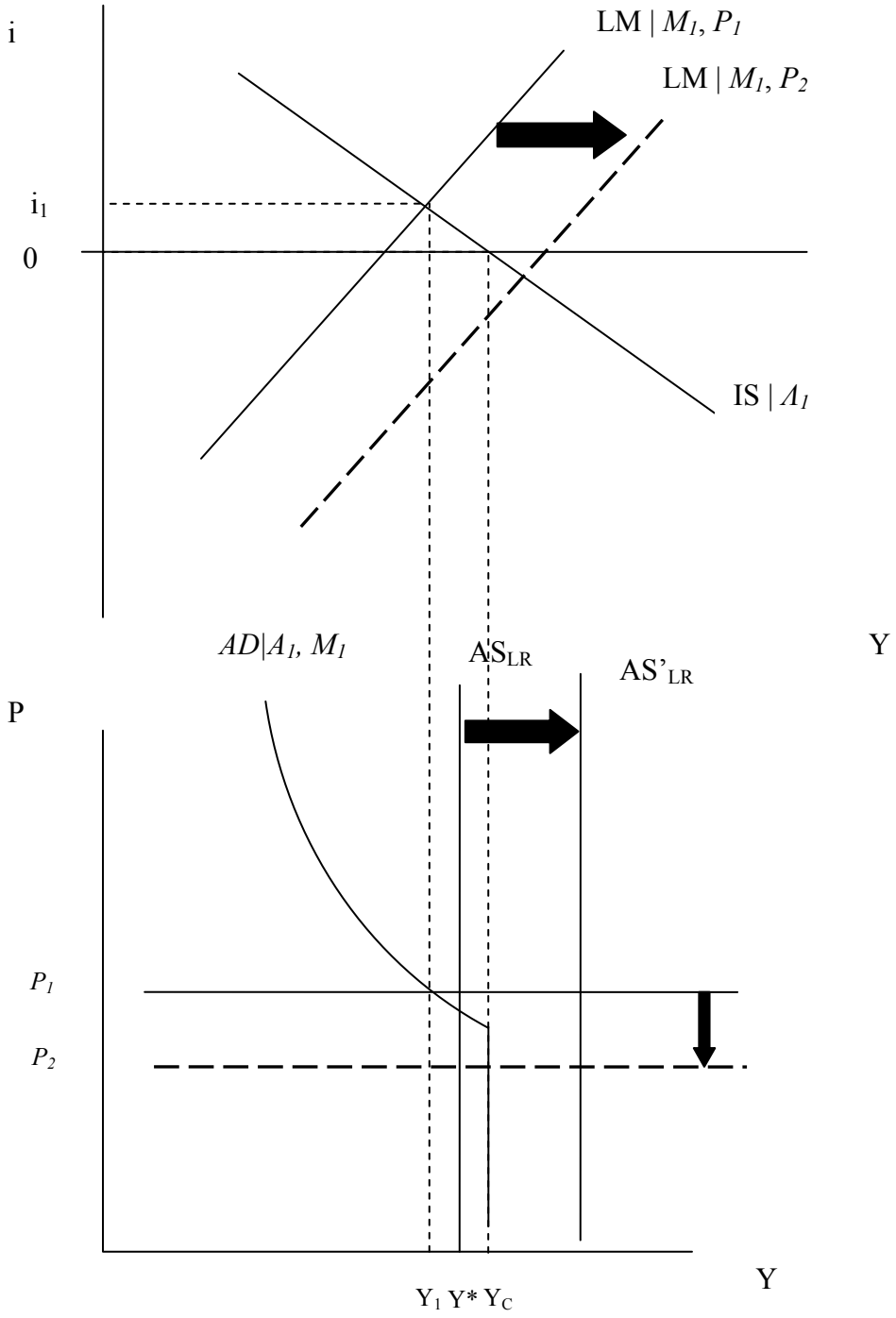
3.3 Show what the IS-LM and AD-AS graphs look like if initially, the economy is not in a liquidity trap, but interest rates are very close to zero, and output is below potential GDP/full employment. Then show what happens if potential GDP shifts out *a lot* because entrepreneurs develop new products due to tax rate reductions and deregulation.

First, let's determine the shape of the AD curve. Consider the LM curve for a given money stock, but different price levels (P_A, P_B, P_C, and P_D), and a given IS curve, thus deriving the AD curve.



Notice that when the price level falls sufficiently, the interest rate no longer falls (we are in a liquidity trap). The resulting Aggregate Demand curve is kinked at Y_C (where the economy hits the liquidity trap).

Second, consider the question. When potential GDP rises, then the downward pressure on the price level increases. The predetermined price line falls (black arrow in AD-AS graph), increasing the real money supply, shifting out the LM curve (black arrow in IS-LM graph). But at some point, interest rates can fall no more (than below zero). Hence, increasing potential GDP has no impact on output when the liquidity trap is encountered.



4.1. Look up on Bloomberg or elsewhere the yield on a one year T-bill. Calculate the price as if the bond were to mature one year from now (specify the date you looked up the data). Show your calculations.

Government Bonds

U.S. UK Germany Japan Hong Kong Australia Brazil

U.S. Treasuries

	COUPON	MATURITY	PRICE/YIELD	PRICE/YIELD CHANGE	TIME
3-Month	0.000	01/24/2013	0.11 / 0.11	0.000 / 0.000	10/26
6-Month	0.000	04/25/2013	0.14 / 0.15	0.000 / 0.000	10/26
12-Month	0.000	10/17/2013	0.17 / 0.17	-0.005 / -0.005	10/26
2-Year	0.250	10/31/2014	99-28+ / 0.30	0-00¾ / -0.014	10/26
3-Year	0.250	10/15/2015	99-17½ / 0.40	0-03 / -0.031	10/26
5-Year	0.750	10/31/2017	99-30+ / 0.76	0-10 / -0.064	10/26
7-Year	1.250	10/31/2019	100-12+ / 1.19	0-18+ / -0.086	10/26
10-Year	1.625	08/15/2022	98-29½ / 1.75	0-22+ / -0.078	10/26
30-Year	2.750	08/15/2042	96-30 / 2.90	1-13 / -0.073	10/26

Using the formula, $P = \frac{100}{1+i} = \frac{100}{1.0011}$, one can solve for $\boxed{P = 99.89}$

4.2 Once again, look up the one year and two year yields. Assuming the expectations hypothesis of the term structure holds, what is the expected interest one year interest rate, one year from today. Show your calculations.

According to the expectations hypothesis of the term structure, for a two year bond,

$i^e_{1t+1} = 2i_{2t} - i_{1t}$

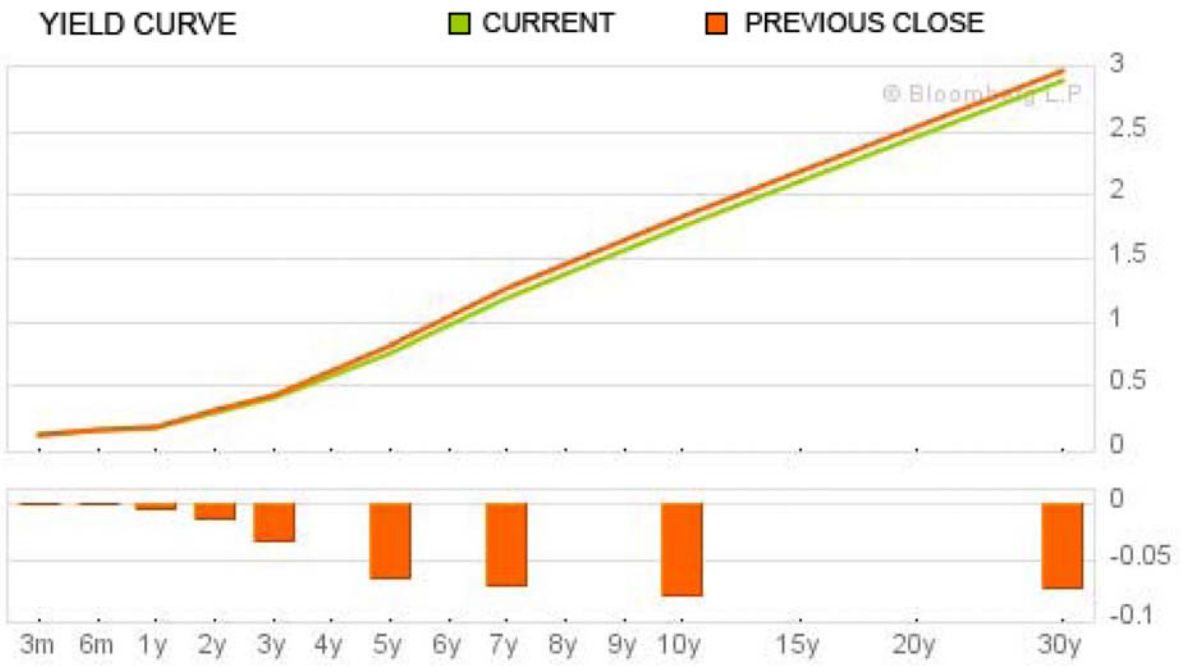
Substituting in 0.26 and 0.17 for the two and one year bond yields results in:

$i^e_{1t+1} = 2 \times 0.0030 - 0.0017 = 0.0043$

Or

$\boxed{0.43\%}$

4.3 Draw the yield curve, for 3 months to 30 years.



4.4 Assuming the expectations hypothesis of the term structure holds (i.e., there is no liquidity premium), do you expect the US economy to go into recession in the next year? Why or why not?

No, since the one year interest rate one year from now is expected to be higher than the current one year interest rate.

5. Compute the price of a share of stock that pays a $1 per year dividend and that you expect to be able to sell in one year for $20, assuming you require a 15% return.

Answer. Recall one expression for the present value of a stock is given by:

$$P_t = \frac{D_{t+1}}{1+(+rf+rp)} + \frac{E_t P_{t+1}}{1+(rf+rp)}$$

Substituting in the relevant information leads to:

$$P_t = \frac{1}{1+0.15} + \frac{20}{1+0.15} = \frac{21}{1.15} \approx 18.26$$

6.1 Calculate the price of a share of stock, assuming dividends are expected to be constant at $D_0 = 1$ and $(rf + rp)$ is also expected to be constant at 0.05. Show your algebraic work.

Answer. Recall the Gordon model is given by:

$$P_t = D_t \times \left[\frac{(1+g)^1}{(1+(rf+rp))^1} + \frac{(1+g)^2}{(1+(rf+rp))^2} + \ldots + \frac{(1+g)^\infty}{(1+(rf+rp))^\infty} \right] = \frac{D_t}{(rf+rp-g)}$$

Substituting in the numbers yields:

$$P_t = \frac{1}{(0.05 - 0)} = 20$$

6.2 Suppose that you revise your expectations regarding (*rf* + *rp*) downward by 2 percentage points. What immediately happens to the price of the share of stock? Once again, show your work.

If the discount rate falls by 0.02, then one obtains:

$$P_t = \frac{1}{(0.03 - 0)} = 33.3\overline{3}$$

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