

**Stock Prices, “News” and the Efficient Markets Hypothesis (rev’d 11/2/2012)**

***The Present Value Model Approach to Asset Pricing***

The textbook expresses the stock price as the present discounted value of the dividend paid and the price of the stock next period.

$$P_t = \frac{D_{t+1}}{1+i} + \frac{P_{t+1}}{1+i} \quad [8.1]$$

In reality, we don’t know the price next period. Rather we have an expectation of the price next period, and for now we assume for now Rational Expectations (such that the market expectation is the mathematical conditional expectation). Hence the expectations operator refers to the conditional mathematical expectations operator. In one-period:

$$P_t = \frac{D_{t+1}}{1+i} + \frac{E_t P_{t+1}}{1+i} \quad [8.1']$$

Where for simplicity, the interest rate used to discount the future is a constant  $i$ , and  $E_t(Z_{t+1}) = E(Z | \text{information available at time } t+1)$ . Assume at time  $t$ , that  $D_t$  is known.

Note that the price next period is given by:

$$P_{t+1} = \frac{D_{t+2}}{1+i} + \frac{E_{t+1} P_{t+2}}{1+i} \quad [8.2']$$

Substituting into [8.1'] yields:

$$P_t = \frac{D_{t+1}}{1+i} + \frac{E_t(E_{t+1} D_{t+2})}{(1+i)(1+i)} + \frac{E_t(E_{t+1} P_{t+2})}{(1+i)(1+i)} \quad (1)$$

Note that by the “Law of Iterated Expectations”, viz.,

$$E_t(E_{t+1}(Z_{t+2})) = E_t Z_{t+2}$$

Equation (1) becomes:

$$P_t = \frac{D_{t+1}}{1+i} + \frac{E_t D_{t+2}}{(1+i)^2} + \frac{E_t P_{t+2}}{(1+i)^2} \quad [8.3']$$

One can continue to substitute out for  $P_{t+1}$  to obtain the **Generalized Dividend Valuation Model:**

$$P_t = \frac{D_{t+1}}{1+i} + \frac{E_t D_{t+2}}{(1+i)^2} + \dots + \frac{E_t D_{t+n}}{(1+i)^n} + \frac{E_t P_{t+n}}{(1+i)^n} \quad [8.4']$$

Note that this expression implies, under certain conditions:

$$P_t = \frac{D_{t+1}}{1+i} + \frac{E_t D_{t+2}}{(1+i)^2} + \frac{E_t D_{t+3}}{(1+i)^3} + \dots + \frac{E_t D_{t+\infty}}{(1+i)^\infty} = \sum_{n=1}^{\infty} \frac{E_t D_{t+n}}{(1+i)^n} \quad (2)$$

Equation (2) rules out “bubbles”. That is, it is assumed that  $\lim_{n \rightarrow \infty} \frac{E_t P_{t+n}}{(1+i)^n} = 0$

The **Gordon Growth Model** assumes that dividends are expected to grow deterministically at rate  $g$ , such that  $D_{t+n} = (1+g)^n \times D_t$  (which is equation [8.6]). Substituting [8.6] into (2) yields, for  $n < \infty$ :

$$P_t = \frac{D_t \times (1+g)^1}{(1+i)^1} + \frac{D_t \times (1+g)^2}{(1+i)^2} + \dots + \frac{D_t \times (1+g)^n}{(1+i)^n} \quad [8.7]$$

If one allows  $n$  to go to infinity:

$$P_t = \frac{D_t \times (1+g)^1}{(1+i)^1} + \frac{D_t \times (1+g)^2}{(1+i)^2} + \dots + \frac{D_t \times (1+g)^\infty}{(1+i)^\infty} \quad (3)$$

$$P_t = D_t \times \left[ \frac{(1+g)^1}{(1+i)^1} + \frac{(1+g)^2}{(1+i)^2} + \dots + \frac{(1+g)^\infty}{(1+i)^\infty} \right] = \frac{D_t}{(i-g)} \quad (4)$$

Note the textbook re-formulates the timing to get a slightly different expression:

$$P_t = \frac{D_t(1+g)}{(i-g)} \quad [8.8]$$

In general,  $D$  will not grow in a smooth deterministic fashion, nor will  $i$  be constant. As a consequence, the fluctuations in prices will not move one for one with contemporaneous dividends.

The previous calculations assume that the interest rate used to discount the future values is constant, and equal to a risk free rate. In general, the interest rate used is the sum of the risk free rate and a risk premium ( $rf$  and  $rp$ , respectively, in the textbook). Substituting into (4) and [8.8] yields:

$$P_t = \frac{D_t}{(rf + rp - g)} \quad (5)$$

Note the textbook re-formulates the timing to get a slightly different expression:

$$P_t = \frac{D_t(1+g)}{(rf + rp - g)} \quad [8.11]$$

In the below figures, monthly data from Robert Shiller's website ([http://www.econ.yale.edu/~shiller/data/ie\\_data.xls](http://www.econ.yale.edu/~shiller/data/ie_data.xls)) are used to highlight the relationships between stock prices, dividends and interest rates. In Figure 1, real (CPI deflated) stock prices and dividends are shown; and in Figure 2 real prices and ten year interest rates.

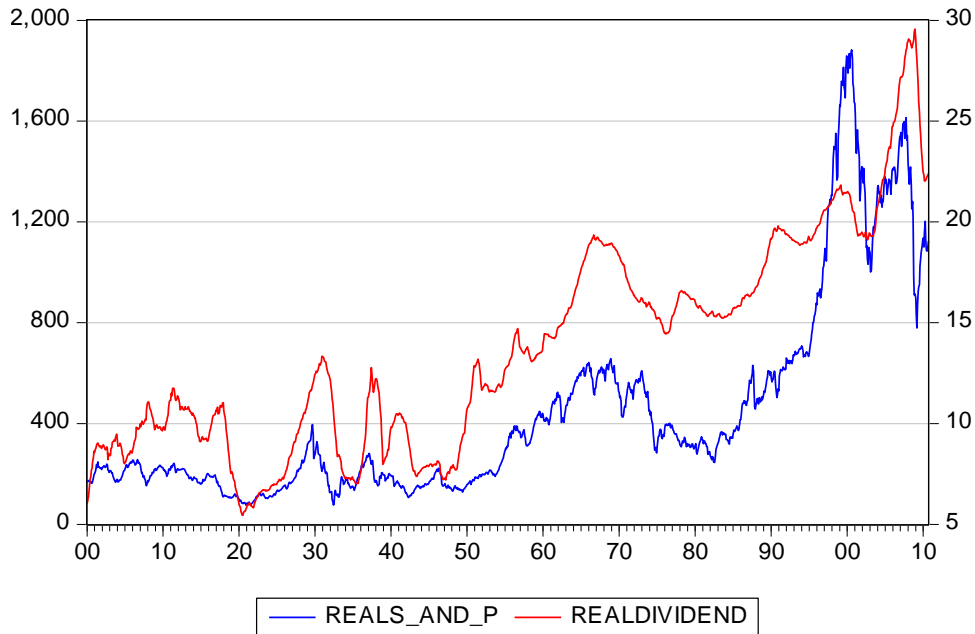


Figure 1: Real (CPI deflated) Standard and Poor index (left scale), and real dividends (right scale). Source: Robert Shiller, [http://www.econ.yale.edu/~shiller/data/ie\\_data.xls](http://www.econ.yale.edu/~shiller/data/ie_data.xls).

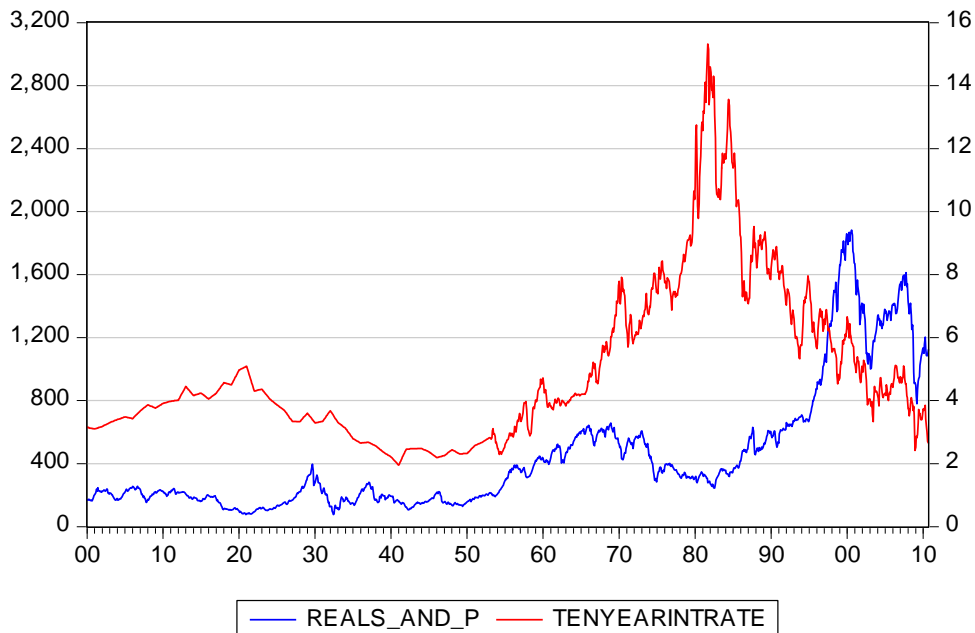


Figure 2: Real (CPI deflated) Standard and Poor index (left scale), and ten year interest rate (right scale). Source: Robert Shiller, [http://www.econ.yale.edu/~shiller/data/ie\\_data.xls](http://www.econ.yale.edu/~shiller/data/ie_data.xls).

As predicted, the stock price index covaries positively with dividends (dividends are highly serially correlated, so a movement up in dividends persists), and is negatively related to the interest rate. Note that dividing both sides of equation 5 by  $D$  leads to the relationship that the price/dividend ratio is inversely related to the interest rate (minus the growth rate of dividends).

$$\frac{P_t}{D_t} = \left[ \frac{(1+g)^1}{(1+rf+rp)^1} + \frac{(1+g)^2}{(1+rf+rp)^2} + \dots + \frac{(1+g)^\infty}{(1+rf+rp)^\infty} \right] = \frac{1}{(rf+rp-g)} \quad (6)$$

The posited inverse relationship is shown in Figure 3.

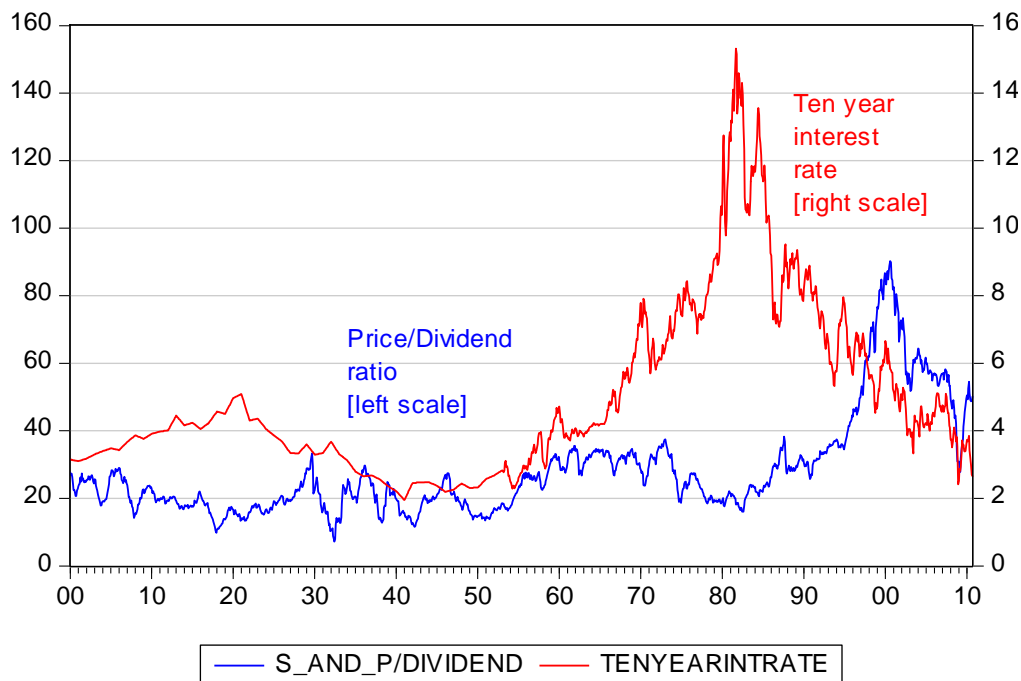


Figure 3: Standard and Poor price to dividend ratio (left scale), and ten year interest rate (right scale). Source: Robert Shiller, [http://www.econ.yale.edu/~shiller/data/ie\\_data.xls](http://www.econ.yale.edu/~shiller/data/ie_data.xls).

“News”

Let’s return to a risk neutral model. Recall the present value of a stock is given by:

$$P_t = \frac{D_{t+1}}{1+i} + \frac{E_t P_{t+1}}{1+i} \quad [8.1']$$

$$E_t P_{t+1} = \frac{E_t D_{t+2}}{1+i} + \frac{E_t (E_{t+1} P_{t+2})}{1+i} = \frac{E_t D_{t+2}}{1+i} + \frac{E_t P_{t+2}}{1+i} \quad (7)$$

The last term after the second equal sign in (7) obtains by the “Law of Iterated Expectations”, viz.,

$$E_t(E_{t+1}(Z_{t+2})) = E_t Z_{t+2}$$

Now decompose the change in the price of the asset:

$$P_{t+1} - P_t \equiv (E_t P_{t+1} - P_t) + [(P_{t+1} - E_t P_{t+1})] \quad (8)$$

The first term is the expected portion of the price change. The second term in the brackets is the unexpected portion. This second portion in square brackets can be further broken up.

$$P_{t+1} - P_t = (E_t P_{t+1} - P_t) + \left[ \frac{D_{t+2} - E_t D_{t+2}}{(1 + rf + rp)} + \frac{E_{t+1} P_{t+2} - E_t P_{t+2}}{(1 + rf + rp)} \right] \quad (9)$$

“News” includes the dividends announced for period t+2. It is unforecastable. This news may also affect people’s expectations regarding  $D$  in the future, and hence  $P$  in the future (which in turn affects expectations of  $P$  in period t+2). Hence, new information directly results in a new price, and revisions in expectations. Notice the second term in the square bracket is

$$\frac{E_{t+1} P_{t+2} - E_t P_{t+2}}{(1 + rf + rp)}$$

which is the change in the expectations regarding the asset price in period t+2, based upon what the market knew in period t+1 versus what it knew in period t.

Note that other “news” that doesn’t affect  $D$  in period t+2 could still affect expected asset prices in the future, and hence the asset price today.

### ***An example: The stock market***

(Goldman Sachs sued by SEC, announcement approx. 10:30am)



## Goldman Sachs Group, Inc. (The)



What equation (2) says is that the price will evolve as expectations of dividends into the future change over time.

$$P_t = \frac{D_{t+1}}{1+(rf+rp)} + \frac{E_t D_{t+2}}{(1+(rf+rp))^2} + \frac{E_t D_{t+3}}{(1+(rf+rp))^3} + \dots + \frac{E_t D_{t+\infty}}{(1+(rf+rp))^\infty} = \sum_{n=1}^{\infty} \frac{E_t D_{t+n}}{(1+(rf+rp))^n} \quad (2)$$

Those dividend streams depend in part upon the earnings that firms are expected to earn in the future. As the economy looks more likely to slow down, expectations of earnings (and hence dividends) are likely to be revised downward.

In addition, there is no reason the required return on equity has to remain constant. If it varies over time, then (2) becomes:

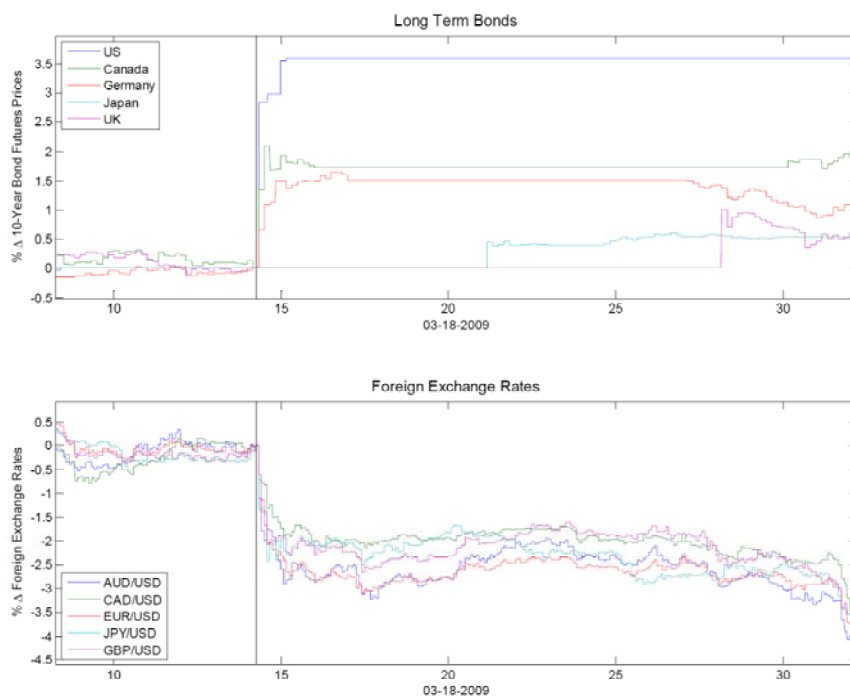
$$P_t = \frac{D_{t+1}}{1+(rf_t+rp_t)} + \frac{E_t D_{t+2}}{(1+(rf_t+rp_t))(1+(rf_{t+1}+rp_{t+1}))} + \frac{E_t D_{t+3}}{(1+(rf_t+rp_t))(1+(rf_{t+1}+rp_{t+1}))(1+(rf_{t+2}+rp_{t+2}))} + \dots + \frac{E_t D_{t+\infty}}{(1+(rf_t+rp_t))\dots(1+(rf_{t+\infty-1}+rp_{t+\infty-1}))} \quad (2')$$

To the extent that the required return varies with the interest rate (say on the 3 month Treasury) and risk aversion, an additional source of variation is introduced into the stock price.

### *Announcement Effects in Other Markets*

This framework for analyzing asset prices has been applied in innumerable cases. Most recently, it has been applied to the Fed's implementation of large scale asset purchases of long term Treasuries and mortgage backed securities. Gagnon et al. (2010) is the most well known analysis of the LSAP's effect on long term bond rates. Neely (2010) analyses the impact of LSAP announcements on bond rates and exchange rates.

Figure 6: High frequency international bond yield and exchange rate movements on March 18, 2009



Notes: The figures show the high frequency movements of international long yields (top panel) and exchange rates (bottom panel) in the hours around the FOMC release (vertical line) on March 18, 2009. The zero value on the x-axis denotes midnight, U.S. Eastern time, of the day of the announcement and the vertical line denotes the time of the announcement.

Table 1: Announcements associated with the LSAP programs

Announcements or suggestions of future purchases.					
Date	Event	Time	Bloomberg time	Event	Other significant news from t-1 through t+1
11/25/2008	Initial LSAP announcement	08:15	08:08	Fed announces purchases of \$100 billion in GSE debt and up to \$500 billion in MBS.	
12/1/2008	Bernanke Speech	13:40	13:45	Chairman Bernanke mentions that the Fed could purchase long-term Treasuries.	NBER dating committee officially declares a recession. Alistair Darling, Chancellor of the Exchequer, promises backing to retail deposits at London Scottish Bank, effectively backing all retail bank deposits in the UK.
12/16/2008	FOMC Statement	14:15	14:21	FOMC statement first mentions possible purchase of long-term Treasuries.	Federal funds rate target reduced from 1 percent to a 0-25 bp target range.
1/28/2009	FOMC Statement	14:15	14:16	FOMC statement says that it is ready to expand agency debt and MBS purchases, as well as to purchase long-term Treasuries. TALF will be implemented.	
3/18/2009	FOMC Statement	14:15	13:17	FOMC will purchase an additional \$750 billion in agency MBS, to increase its purchases of agency debt by \$100 billion, and \$300 billion in long-term Treasuries.	

### ***Efficient Markets Hypothesis***

In the above discussion, I have assumed that the subjective market expectation of future dividends and prices equals the mathematical expectations. This assumption is called the rational expectations hypothesis. One implication of rational expectations is:

$$X_t = E_{t-1}X_t + u_t$$
$$u_t \sim iid(0, \sigma_u)$$

That is, the actual realization of  $X$  equals the expectation of  $X$  based on time  $t-1$  information, plus an unforecastable random error,  $u$ .

Consider for instance if dividends were paid only every year, and each period was one day. Then equation (1) becomes:

$$P_t = \frac{E_t P_{t+1}}{1 + (rf + rp)}$$

Use the definition of rational expectations above, and assume log-normality of the error term:

$$p_t = E_t p_{t+1} - \ln(1 + rf + rp)$$

$$p_{t+1} = p_t + (rf + rp) + \tilde{u}_{t+1}$$

So stock prices follow a random walk (with drift). It is not quite right to say the best predictor of tomorrow's stock price is today's. Rather there is a small predictable component ( $k_e$ ), which from one day to another is quantitatively very small.

### ***References***

Joseph Gagnon, Matthew Raskin, Julie Remache, and Brian Sack, 2010, "Large-Scale Asset Purchases by the Federal Reserve: Did They Work?" *Staff Report* no. 441 (Federal Reserve Bank of New York).

Christopher J. Neely, 2010, "The Large Scale Asset Purchases Had Large International Effects," *Working Paper* 2010-018A (Federal Reserve Bank of St. Louis).