Economics 435
The Financial System
(2/12/14)

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UW Madison
Spring 2014
Future Value and Present Value

• If the present value is $100 and the interest rate is 5%, then the future value one year from now is:

\[ \text{FV} = \text{PV} + \text{PV}(i) = \text{PV}(1 + i) \]

\[ $100 + $100(0.05) = $105 \]

• This also shows that the higher the interest rate, the higher the future value.

• In general:

\[ PV = \frac{FV}{(1 + i)} \]
Future Value and Compound Interest

• What if you leave your $100 in the bank for two years at 5\% yearly interest rate?

• The future value is:

\[
100 + 100(0.05) + 100(0.05) + 5(0.05) = 110.25
\]

\[
100(1.05)(1.05) = 100(1.05)^2
\]

• In general

\[
FV_n = PV(1 + i)^n
\]

\[
PV = \frac{FV}{(1 + i)^n}
\]
Complications

• What if payments, $X_t$, occur all the way along until the end?

• What if the interest rate, $i_t$, is not constant?

$$PV_t = \left[ \frac{X_{t+1}}{(1+i_t)} + \frac{X_{t+2}}{(1+i_t)(1+i_{t+1})} + \ldots + \frac{X_{t+n}}{(1+i_t)(1+i_{t+1})\ldots(1+i_{t+n-1})} \right]$$

• But at time $t$, one doesn’t know $t+n$ information … so:

$$PV_t = \left[ \mathcal{E}_t \frac{X_{t+1}}{(1+i_t)} + \mathcal{E}_t \frac{X_{t+2}}{(1+i_t)(1+i_{t+1})} + \ldots + \mathcal{E}_t \frac{X_{t+n}}{(1+i_t)(1+i_{t+1})\ldots(1+i_{t+n-1})} \right]$$
Bond Basics

• The most common type of bond is a coupon bond.
  – Issuer is required to make annual payments, called coupon payments.
  – The annual interest the borrower pays \((i_c)\), is the coupon rate.
  – The date on which the payments stop and the loan is repaid \((n)\), is the maturity date or term to maturity.
  – The final payment is the principal, face value, or par value of the bond.
Bond Prices

1. Zero-coupon or discount bond
   – Promise a single payment on a future date
   – Example: Treasury bill

2. Fixed-payment loan
   – Sequence of fixed payments
   – Example: Mortgage or car loan

3. Coupon bond
   – Periodic interest payments + principal repayment at maturity
   – Example: U.S. Treasury Bonds and most corporate bonds

4. Consol
   – Periodic interest payments forever, principal never repaid
   – Example: U.K. government has some outstanding
Zero-Coupon Bonds

• U.S. Treasury bills (T-bills) are the most straightforward type of bond.
  – Each T-bill represents a promise by the U.S. government to pay $100 on a fixed future date.
  – No coupon payments - zero-coupon bonds
  – Also called pure discount bonds (or discount bonds) since the price is less than face value - they sell at a discount.

• Price of $100 face value zero-coupon bond

\[
= \frac{100}{(1 + i)^n}
\]
Zero-Coupon Bonds

Assume $i = 5\%$

Price of a One-Year Treasury Bill

\[
\frac{100}{(1 + 0.05)} = $95.24
\]

Price of a Six-Month Treasury Bill

\[
\frac{100}{(1 + 0.05)^{1/2}} = $97.59
\]
Zero-Coupon Bonds

• For a zero-coupon bond, the relationship between the price and the interest rate is the same as we saw on present value calculations.

• When the price moves, the interest rate moves with it, in the opposite direction.

• We can compute the interest rate from the price using the present value formula.

The price of a one-year T-bill is $95.

\[ i = \left( \frac{100}{95} \right) - 1 = 0.0526 = 5.26\% \]
Fixed-Payment Loans

- Home mortgages and car loans are fixed-payment loans.
  - They promise a fixed number of equal payments at regular intervals.
  - Amortized loans - the borrower pays off part of the principal along with the interest for the life of the loan.

- Value of a Fixed Payment Loan =

\[
\frac{Fixed\text{Payment}}{(1+i)} + \frac{Fixed\text{Payment}}{(1+i)^2} + \cdots + \frac{Fixed\text{Payment}}{(1+i)^n}
\]

- The sum of the present value of the payments.
Coupon Bonds

• The issuer of a coupon bond promises to make a series of periodic interest payments (coupon payments), plus a principal payment at maturity.

Price of Coupon Bond =

\[
P_{CB} = \left[ \frac{\text{CouponPayment}}{(1+i)^1} + \frac{\text{CouponPayment}}{(1+i)^2} + \ldots + \frac{\text{CouponPayment}}{(1+i)^n} \right] + \frac{\text{FaceValue}}{(1+i)^n}
\]
Consols

- Consols or perpetuities, are like coupon bonds whose payments last forever.
- The borrower pays only interest, never repaying the principal.
- The U.S. government sold consols once in 1900, but the Treasury has bought them all back.
- The price of a consol is the present value of all future interest payments.

\[ P_{\text{Consol}} = \frac{\text{Yearly Coupon Payment}}{i} \]
Bond Yields

• We know how to calculate bond prices given an interest rate.

• We also need to be able to go in the other direction.
  – Calculate the return to an investment, implicit in the bond’s price.

• We will combine information about the promised payments with the price to obtain the yield:
  – A measure of the cost of borrowing and the reward for lending.
  – We will use the terms yield and interest rate interchangeably.
Yield to Maturity

• The most useful measure of the return on holding a bond is called the yield to maturity:
  – The yield bondholders receive if they hold the bond to its maturity when the final principal payment is made.

\[
\text{Price of 1yr 5\% Coupon Bond} = \frac{\$5}{(1 + i)} + \frac{\$100}{(1 + i)}
\]

• The value of \( i \) that solves the equation is the yield to maturity.
Current Yield

Example:
1 year, 5% coupon bond selling for $99

\[
\text{Current Yield} = \frac{5}{99} = 0.0505, \text{ or } 5.05\%
\]

Yield to maturity for this bond is 6.06 percent found as the solution to:

\[
\frac{5}{(1+i)} + \frac{100}{(1+i)} = 99
\]
Holding Period Returns

• The *one-year holding period return* is the sum of the yearly coupon payment divided by the price paid for the bond and the change in the price divided by the price paid.

\[
\text{One-year holding period return} = \frac{\text{Yearly Coupon Payment}}{\text{Price Paid}} + \frac{\text{Change in Price of the Bond}}{\text{Price of the Bond}}
\]

\[
= \text{Current Yield} + \text{Capital Gain (as a %)}
\]
### Treasury Notes & Bonds

**Go to:** Bills

Friday, February 07, 2014

Find Historical Data | WHAT'S THIS?

Treasury note and bond data are representative over-the-counter quotations as of 3pm Eastern time. For notes and bonds callable prior to maturity, yields are computed to the earliest call date for issues quoted above par and to the maturity date for issues below par.

<table>
<thead>
<tr>
<th>Maturity</th>
<th>Coupon</th>
<th>Bid</th>
<th>Asked</th>
<th>Chg</th>
<th>Asked yield</th>
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Data on Treasury Bills

TREASURY BILLS
GO TO: Notes and Bonds

Friday, February 07, 2014

Treasury bill bid and ask data are representative over-the-counter quotations as of 3pm Eastern time quoted as a discount to face value. Treasury bill yields are to maturity and based on the asked quote.

<table>
<thead>
<tr>
<th>Maturity</th>
<th>Bid</th>
<th>Asked</th>
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<th>Asked yield</th>
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“On the run”
### US Treasury Bonds Rates

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<th>Maturity</th>
<th>Yield</th>
<th>Yesterday</th>
<th>Last Week</th>
<th>Last Month</th>
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<td>0.05</td>
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<tr>
<td>6 Month</td>
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<td>0.05</td>
<td>0.05</td>
<td>0.06</td>
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<td>3 Year</td>
<td>0.63</td>
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<td>5 Year</td>
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<td>3.67</td>
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</table>

Secondary Market, Constant Maturity

Graph showing the trend of Ten year constant maturity Treasury yield, % and Three month Treasury, secondary market, % from 2000 to 2014.
Real and Nominal Interest Rates

• The nominal interest rate you agree on \((i)\) must be based on expected inflation \((\pi^e)\) over the term of the loan plus the real interest rate you agree on \((r)\).

\[
i = r + \pi^e
\]

• This is called the *Fisher Equation*.
• The higher expected inflation, the higher the nominal interest rate.
Data on Treasury Inflation Protected Securities (TIPS)

http://online.wsj.com/mdc/public/page/2_3020-tips.html

**Treasury Inflation-Protected Securities**

<table>
<thead>
<tr>
<th>Maturity</th>
<th>Coupon</th>
<th>Bid</th>
<th>Asked</th>
<th>Chg</th>
<th>Yield*</th>
<th>Accrued principal</th>
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*Yld to maturity on accrued principal.
Nominal vs. Real

Ten year constant maturity Treasurys

Ten year constant maturity TIPS

Ten year Treasurys adjusted by expected inflation

GS10-INFLCPI10YR_SPFAVG1
GS10
FII10
Constant Maturity vs. On the Run
Factors That Shift Bond Supply

Figure 6.2: A Shift in the Supply of Bonds

The graph illustrates the shift in the supply of bonds. Initially, the supply curve is $S_0$ at point $E_0$ with price $P_0$ and quantity $Q_0$. An increase in the desire to borrow shifts the supply curve to $S_1$, resulting in a new equilibrium at $E_1$ with price $P_1$ and quantity $Q_1$. This shift indicates an increase in the supply of bonds due to the increased desire to borrow.
Factors That Shift Bond Demand

Figure 6.3  A Shift in the Demand for Bonds

![Graph showing the shift in demand for bonds](image)

- Increased Demand for Bonds
- Price per Bond: $P_0$, $P_1$
- Quantity of Bonds: $D_0$, $D_1$