

Problem Set 1 Answers

Due *in lecture* on Monday, September 23. Be sure to put your name on your problem set. Put “boxes” around your answers to the algebraic questions.

1. Suppose the economy is described by the following equations (so we are looking at a closed economy):

• Real Sector

- | | | |
|-----|---------------------------|---|
| (1) | $Y = Z$ | Output equals aggregate demand, an equilibrium condition |
| (2) | $Z = C + I + G$ | Definition of aggregate demand |
| (3) | $C = c_0 + c_1 Y_D$ | Consumption fn, c_1 is the marginal propensity to consume |
| (4) | $Y_D \equiv Y - T + Tr$ | Definition of disposable income |
| (5) | $T = t_1 Y$ | Tax function; t_1 is marginal tax rate. |
| (6) | $Tr = TR_0$ | Transfer payments; TR_0 is lump sum transfers. |
| (7) | $I = b_0 + b_1 Y - b_2 i$ | Investment function |
| (8) | $G = GO_0$ | Government spending on goods and services, exogenous |

• Asset Sector

- | | | |
|------|----------------------------------|-----------------------|
| (9) | $\frac{M^d}{P} = \frac{M^s}{P}$ | Equilibrium condition |
| (10) | $\frac{M^s}{P} = \frac{M_0}{P}$ | Real money supply |
| (11) | $\frac{M^d}{P} = \mu_0 + Y - hi$ | Real money demand |

1.1 Solve for the IS curve (Y as a function of i).

$$\begin{aligned}
 Y &= Z = C + I + G && \text{substitute in for } C, I, G \\
 Y &= c_0 + c_1 Y_D + b_0 + b_1 Y - b_2 i + GO_0 && \text{substitute in for } Y_D \\
 Y &= a_0 + c_1 (Y - T + Tr) + b_0 + b_1 Y - b_2 i + GO_0 && \text{substitute in for tax, transfers functions} \\
 Y &= a_0 + c_1 (Y - t_1 Y + TR_0) + b_0 + b_1 Y - b_2 i + GO_0 && \text{bring the "Y" terms to left hand side.}
 \end{aligned}$$

$$Y - b(Y - t_1 Y - \lambda Y) = Y(1 - b(1 - t - \lambda)) = c_0 + c_1 TR_0 + b_0 + GO_0 - b_2 i$$

divide both sides by $(1 - c_1(1 - t_1) - b_1)$ and let $A_0 \equiv c_0 + c_1 TR_0 + b_0 + GO_0$

$$\boxed{Y_0 = \bar{Y} \Lambda_0} \text{ let } \bar{Y} = \frac{1}{[1 - c_1(1 - t_1) - b_1]}$$

1.2 Solve for the LM curve (i as a function of Y). What is the channel by which monetary influences affect the real goods sector in this model?

$$\frac{M_0}{P_0} = \frac{M^s}{P} = \frac{M^d}{P} = \mu_0 + Y - hi$$

Solving for the interest rate, i , yields the LM curve:

$$i = \frac{\mu_0}{h} - \left(\frac{1}{h}\right)\left(\frac{M_0}{P_0}\right) + \left(\frac{1}{h}\right)Y$$

Monetary policy influences (in part) interest rates. Interest rates in turn affect investment, and via the simple Keynesian multiplier ($\bar{\gamma}$) affects the entire real sector.

1.3 Solve for the equilibrium values of Y .

To solve for the equilibrium value of income, substitute the LM into the IS equation from 1.1:

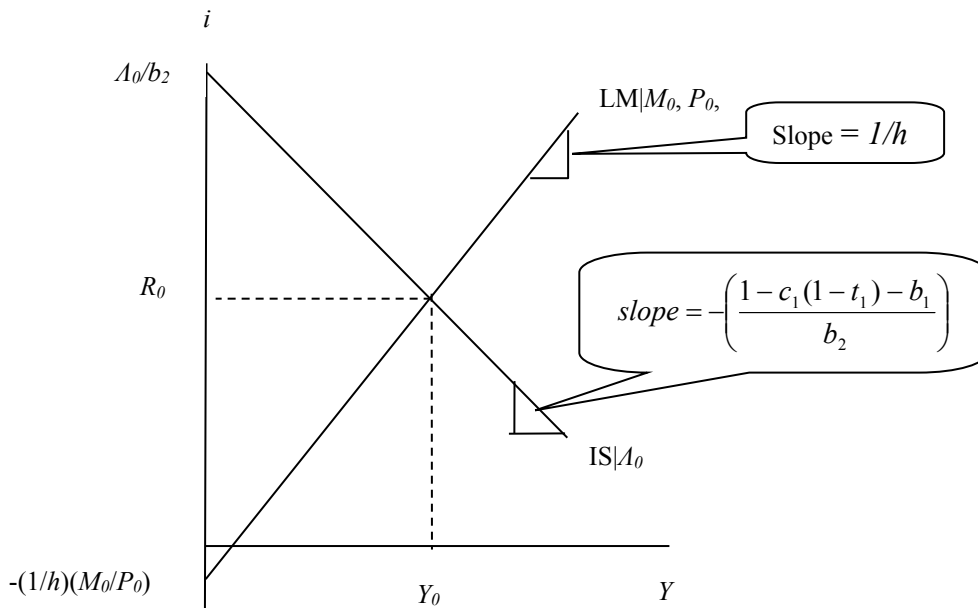
$$Y = \left(\frac{1}{1 - c_1(1 - t_1) - b_1}\right) \times \left[\Lambda_0 - b_2 \left\langle \frac{\mu_0}{h} - \frac{1}{h} \frac{M_0}{P_0} + \frac{1}{h} Y \right\rangle\right]$$

Move the term in parentheses (\cdot) and the $(b_2/h)Y$ term to the LHS; factoring out the Y 's on the LHS yields:

Dividing both sides by the term in the parentheses yields:

$$Y_0 = \hat{\gamma} \left[\Lambda_0 - \frac{b_2 \mu_0}{h} + \left(\frac{b_2}{h}\right)\left(\frac{M_0}{P_0}\right) \right] \text{ where } \hat{\gamma} \equiv \frac{1}{1 - c_1(1 - t_1) - b_1 + b_2/h}$$

1.4 Graph the IS and LM curves on one diagram. Clearly indicate the intercepts and the slopes.



2.1 Assume G increases by ΔGO , and is completely bond financed (no portfolio effects here). Calculate the government spending multiplier.

Take the total differential of your answer to 1.3.

$$\Delta Y = \hat{\gamma} \left[\Delta A - \frac{b_2 \mu_0}{h} + \left(\frac{b_2}{h} \right) \Delta \left(\frac{M}{P} \right) \right]$$

To find the government spending multiplier, set the changes in real money to zero and the money constant, and divide both sides by ΔGO :

$$\Delta Y = \hat{\gamma} \Delta GO \Rightarrow \frac{\Delta Y}{\Delta GO} = \hat{\gamma} \equiv \frac{1}{1 - c_1(1 - t_1) - b_1 + b_2 / h}$$

2.2 Suppose instead Tr increases by ΔTR . Calculate the government transfers multiplier.

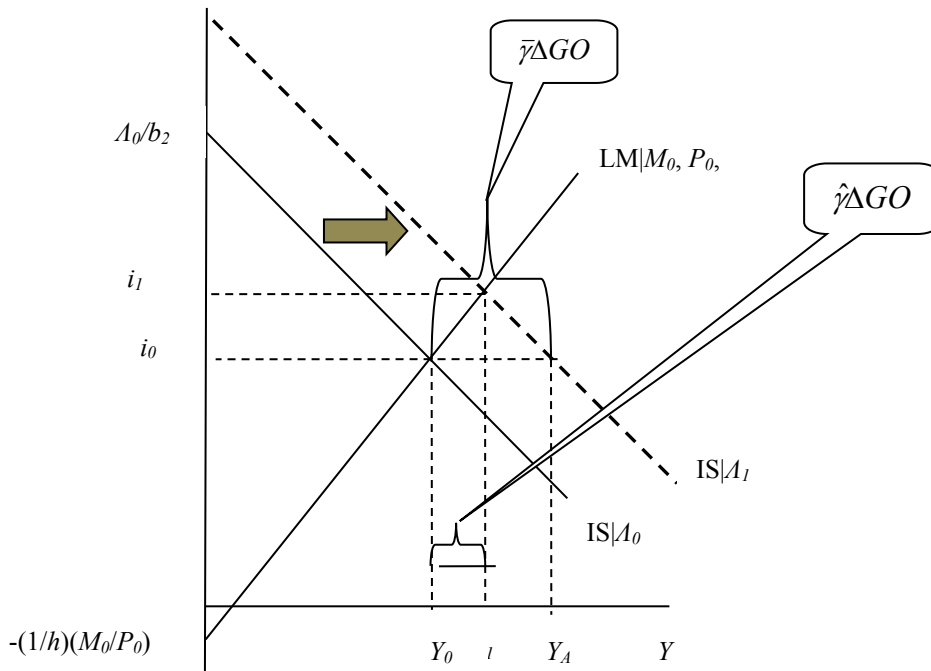
Take the total differential again:

$$\Delta Y = \hat{\gamma} \left[\Delta \Lambda - \frac{b_2 \mu_0}{h} + \left(\frac{b_2}{h} \right) \Delta \left(\frac{M}{P} \right) \right]$$

To find the government *transfers* multiplier, set the changes in real money to zero and the money constant, set the change in $\Delta \Lambda$ to equal $c_1 \Delta TR$, and divide both sides by ΔTR :

$$\Delta Y = \hat{\gamma} c_1 \Delta TR \Rightarrow \frac{\Delta Y}{\Delta TR} = \hat{\gamma} c_1 \equiv \frac{c_1}{1 - c_1(1 - t_1) + b_2 / h}$$

2.3 Redraw your answer to 1.4. Then in the same graph, show what happens to the equilibrium income and interest rate if government spending on goods and services is increased by ΔGO . Include in your graph the level of income that would be achieved if somehow the interest rate stayed constant (label this point Y_A).



2.4 At the new equilibrium, do we know if investment is higher or lower than the level it started out with? Do we know if it is higher or lower than at Y_A ?

Recall the investment function is given by:

$$I = b_0 + b_1Y - b_2i$$

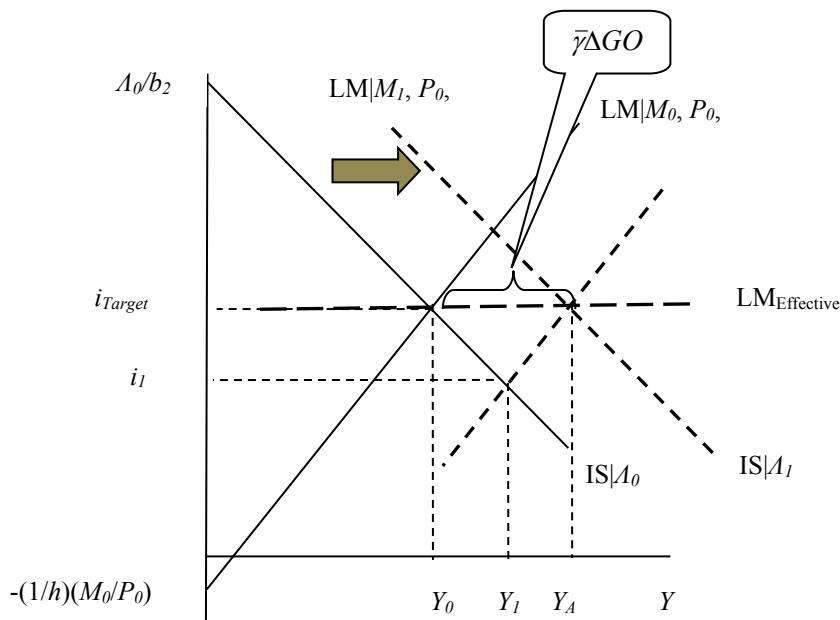
So the change in investment is given by:

$$\Delta I = \Delta b_0 + b_1\Delta Y - b_2\Delta i$$

Notice that at the new equilibrium, income is lower (Y_1), but the interest rate is lower as well (i_1). Hence, there are offsetting effects on investment, and the end results could be higher or lower, depending on the magnitudes of the changes in income and interest rates and the parameter values ($b_1, -b_2$).

Regarding the second question, if one were at Y_A , and interest rate i_0 , then investment would unambiguously be higher.

2.5 Suppose the Fed targets the interest rate at i_0 (call this i_{target}). Returning to 2.3, show graphically what happens if government is decreased. What happens to the level of investment?

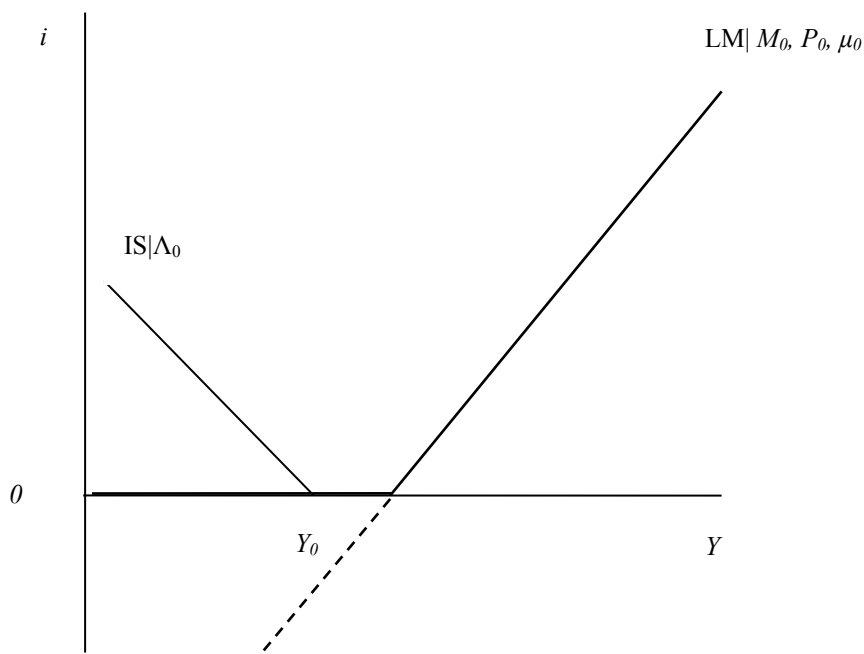


When the Fed targets the interest rate, and the target interest rate remains constant, then the LM is now the Effective LM. An increase in government spending, increasing autonomous spending, induces an unambiguous increase in investment, since income is higher, but interest rates are unchanged.

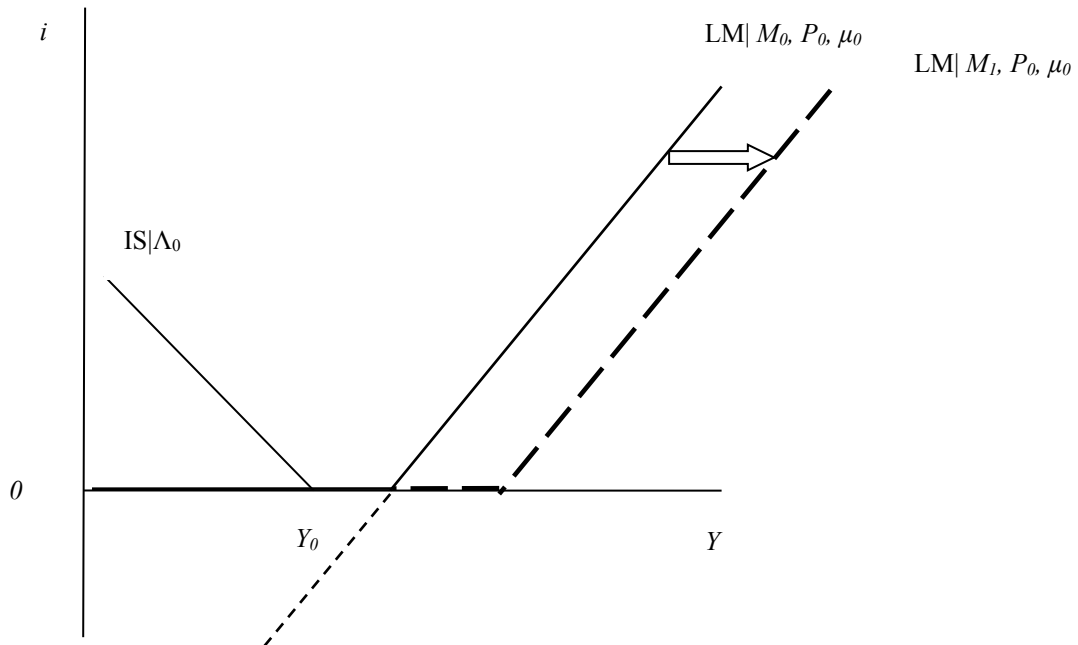
Note that the answer is the same if we are in a liquidity trap.

3. Consider a situation where the economy is in a liquidity trap.

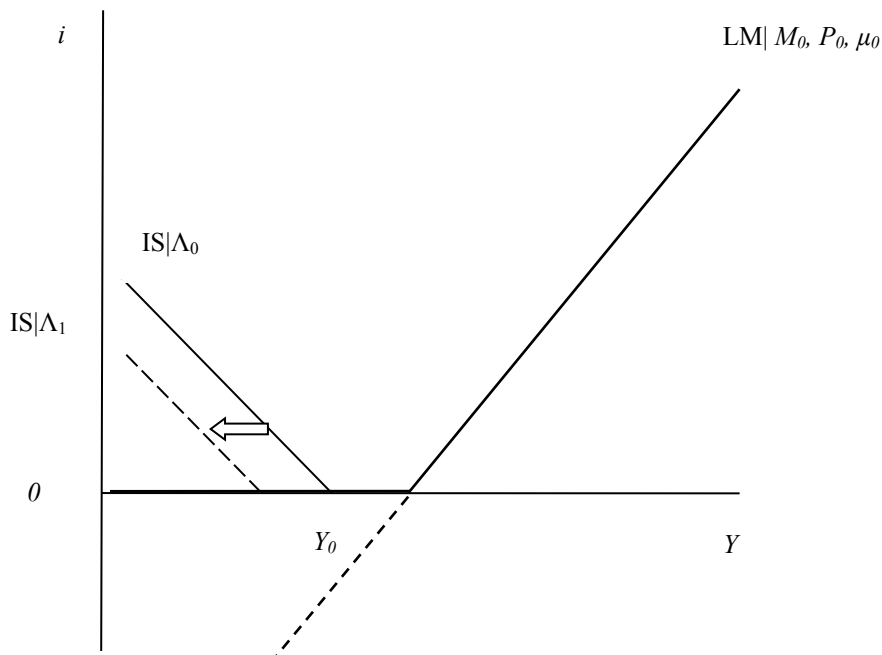
3.1 Draw a diagram illustrating the situation where interest rates are at the zero lower bound.



3.2 Show what happens to the equilibrium income and interest rate if the money supply is increased.



3.3 Show what happens if lump sum taxes are increased. Does investment rise or fall?



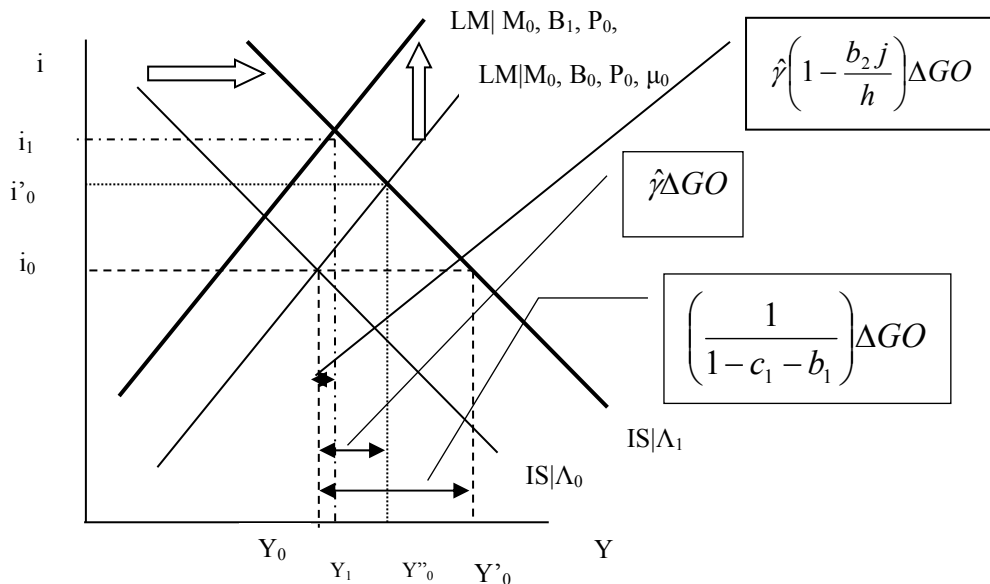
4. Consider an economy where the money demand function takes the following form:

$$\frac{M^d}{P} = \mu_0 + Y + j\left(\frac{M}{P} + \frac{B}{P}\right) - hi$$

4.1 Assume the budget is balanced to begin with. When government spending increases by ΔGO , the government must borrow $\Delta(M/P)$. What happens to the LM curve when government spending increases?

It shifts up, as shown in the figure below. That's because the increase in government spending induces a deficit, which must be financed. That means the government must sell bonds to the public; it can only induce the public to buy the additional bonds by dropping the price, i.e., raising the interest rate.

4.2 Can you tell what happens to equilibrium income and interest rates when the government spending increases? Show, using an IS-LM diagram.



4.3 What happens to investment?

The answer depends on what happens to income. Recall:

$$\Delta I = \Delta b_0 + b_1 \Delta Y - b_2 \Delta i$$

So, if income falls, then investment unambiguously declines.

4.4 Assume the Fed targets the interest rate at the level it was before the increase in government spending. What is the impact on equilibrium income and interest rates of the increase in government spending?

This makes the effective LM curve flat. Hence, the multiplier collapses to the simple Keynesian multiplier. Then:

$$\frac{\Delta Y}{\Delta GO} = \frac{1}{1 - c_1(1 - t_1) - b_1}$$