Economics 390
Topics in Macroeconomics
(11/18/2013)

Instructor: Prof. Menzie Chinn
UW Madison
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The Government Budget Constraint

The Arithmetic of Deficits and Debt

- The budget deficit in year $t$ equals:

$$\text{deficit}_t = rB_{t-1} + G_t - T_t$$

- $B_{t-1}$ is government debt at the end of year, $t - 1$ or, equivalently, at the beginning of year $t$; $r$ is the real interest rate, which we shall assume to be constant here. Thus $rB_{t-1}$ equals the real interest payments on the government debt in year $t$.

- $G_t$ is government spending during year $t$.

- $T_t$ is taxes minus transfers during year $t$.

In words: The budget deficit equals spending, including interest payments on the debt, minus taxes net of transfers.
The Government Budget Constraint

• Note two characteristics of $\text{deficit}_t = rB_{t-1} + G_t - T_t$:

  – We measure interest payments as real interest payments rather than as actual interest payments. The correct measure of the deficit is sometimes called the inflation-adjusted deficit.

  – $G$ does not include transfer payments.
The Government Budget Constraint

The government budget constraint states that the change in government debt during year \( t \) is equal to the deficit during year \( t \):

\[
B_t - B_{t-1} = \text{Deficit}_t
\]

Using the definition of the deficit

\[
\text{deficit}_t = rB_{t-1} + G_t - T_t
\]

we can rewrite the government budget constraint as

\[
B_t - B_{t-1} = rB_{t-1} + G_t - T_t
\]
The Government Budget Constraint

Using this decomposition, we can rewrite

\[ B_t - B_{t-1} = rB_{t-1} + G_t - T_t \]

Change in the debt = Interest payments + Primary deficit

Primary Deficit

\[ B_t = (1+r)B_{t-1} + G_t - T_t \]
The Government Budget Constraint

*The Arithmetic of the Debt Ratio*

\[
\frac{B_t}{Y_t} = (1 + r) \frac{B_{t-1}}{Y_t} + \frac{G_t - T_t}{Y_t}
\]

\[
\frac{B_t}{Y_t} = (1 + r) \left( \frac{Y_{t-1}}{Y_t} \right) \frac{B_{t-1}}{Y_{t-1}} + \frac{G_t - T_t}{Y_t}
\]

\[
\frac{B_t}{Y_t} = (1 + r - g) \frac{B_{t-1}}{Y_{t-1}} + \frac{G_t - T_t}{Y_t}
\]

\[
\frac{B_t}{Y_t} - \frac{B_{t-1}}{Y_{t-1}} = (r - g) \frac{B_{t-1}}{Y_{t-1}} + \frac{G_t - T_t}{Y_t}
\]

\[
\frac{B_t}{Y_t} - \frac{B_{t-1}}{Y_{t-1}} = (r - g) \frac{B_{t-1}}{Y_{t-1}} + \frac{G_t - T_t}{Y_t}
\]
CBO, Long Term Budget Outlook, September 2013
Figure 1-1.
Federal Debt Held by the Public Under CBO’s Extended Baseline
(Percentage of gross domestic product)

Components of Total Spending

World War II
Great Depression
Civil War
World War I

Actual
Projected

Federal Spending on Major Health Care Programs
Other Noninterest Spending
Social Security
Net Interest

Interest Rates and Debt

\[ i_{t,k} - i_{t,j} = -\gamma (\pi_t - \bar{\pi}) - \phi (y_t - \bar{y}_t) + \sigma_t + \epsilon_t \]  

(2)

\[ \text{SPREAD}_t = \beta_0 + \beta_1 \text{UNGAP}_t + \beta_2 \text{INFL}_t + \beta_3 \text{STRSURP}_t + \beta_4 \text{FOROFFICIAL}_t + \beta_5 \text{FEDLT}_t + e_t, \]  

(3)

- SPREAD is long-short interest differential
- UNGAP is unemployment-NAIRU gap
- INFL is the PCE inflation gap (minus 1.8%)
- STRSURP is the structural budget balance, ratio to potential GDP
- FOROFFICIAL is purchases foreign official sector purchases, ratio to potential GDP
- FEDLT is Fed purchases of long term Treasurys, ratio to potential GDP
Empirical Estimates

Table 1: Regression Results for the Treasury Interest Rate Term Spread, Ten-Years and Three Months

<table>
<thead>
<tr>
<th></th>
<th>C</th>
<th>UNGAP</th>
<th>INFL</th>
<th>STRSURP</th>
<th>FOROFFICIAL</th>
<th>FEDLT</th>
<th>DISCMPOL</th>
<th>Adj. $R^2$</th>
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<tr>
<td>1</td>
<td>1.373**</td>
<td>0.416**</td>
<td>−0.276**</td>
<td>−0.190*</td>
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<td></td>
<td></td>
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<td></td>
<td>(0.278)</td>
<td>(0.143)</td>
<td>(0.072)</td>
<td>(0.106)</td>
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<tr>
<td>2</td>
<td>1.435**</td>
<td>0.481**</td>
<td>−0.383**</td>
<td>−0.291**</td>
<td>−0.445*</td>
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<td>0.588</td>
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<tr>
<td></td>
<td>(0.269)</td>
<td>(0.142)</td>
<td>(0.090)</td>
<td>(0.116)</td>
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<tr>
<td>3</td>
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<td>0.648**</td>
<td>−0.407**</td>
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<td>−0.561**</td>
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<td>0.680</td>
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<td></td>
<td>(0.238)</td>
<td>(0.137)</td>
<td>(0.080)</td>
<td>(0.102)</td>
<td>(0.237)</td>
<td>(0.189)</td>
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<td>4</td>
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<td>0.565**</td>
<td>−0.420**</td>
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<td>(0.091)</td>
<td>(0.070)</td>
<td></td>
<td>(0.086)</td>
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<td>5</td>
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<td></td>
<td>(0.238)</td>
<td>(0.135)</td>
<td>(0.079)</td>
<td>(0.099)</td>
<td>(0.239)</td>
<td>(0.198)</td>
<td>(0.461)</td>
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<td>6</td>
<td>1.223**</td>
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<td>(0.086)</td>
<td>(0.069)</td>
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<td>(0.082)</td>
<td>(0.413)</td>
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</table>

Note: Ordinary least squares, sample 1979–2010.

Source: Kitchen and Chinn (2012)
The Dangers of Very High Debt

• The higher the ratio of debt to GDP, the larger the potential for catastrophic debt dynamics.

• Expectations of higher and higher debt give a hint that a problem may arise, which will lead to the emergence of the problem, thereby validating the initial expectations.

• Debt repudiation consists of canceling the debt, in part or in full.
Debt Crises: GHHM

\[ B_{t+1} = (1 + R_t)(B_t - S_t). \] (1)

Dividing both sides of (1) by GDP in year \( t+1 \) gives

\[ \frac{B_{t+1}}{Y_{t+1}} = \frac{Y_t}{Y_{t+1}} \frac{1}{Y_t} (1 + R_t)(B_t - S_t). \] (2)

Let lower-case symbols denote magnitudes as a fraction of GDP,

\[ b_t = \frac{B_t}{Y_t} \]

\[ s_t = \frac{S_t}{Y_t}, \]

and let \( r_t \) be defined by

\[ 1 + r_t = \frac{(1 + R_t)Y_t}{Y_{t+1}} \] (3)

so that (2) can be written

\[ b_{t+1} = (1 + r_t)(b_t - s_t). \] (4)

If \( g_t \) denotes the nominal GDP growth rate between \( t \) and \( t+1 \),

\[ Y_{t+1} = (1 + g_t)Y_t, \] (5)
\[ r_i = R_i - g_i. \] (6)

Suppose that a country faces a constant net borrowing cost \( r^* \) and wants to maintain a constant debt-to-GDP ratio \( b^* \). Then (4) implies that this would require a primary surplus \( s^* \) satisfying \(^2\)

\[ b^* = (1 + r^*)(b^* - s^*) \] (7)

\[ s^* = \frac{r^*b^*}{1 + r^*}. \] (8)

\[ R_{it} = \hat{\alpha}_i + \hat{\gamma}_t + 0.0313 b_{i,t-1} + 0.0142 b_{i,t-1}^n - 0.184 c_{i,t-1} + e_{it} \]  
\[ (3.95) \quad (2.30) \quad (5.16) \]  
\[ R^2 = 0.69 \quad \text{log likelihood} = -288.32. \]  

\[ R_{it} = \hat{\alpha}_i + \hat{\gamma}_t + 0.0029 b_{i,t-1} + 0.245 c_{i,t-1} + 0.000203 b_{i,t-1}^2 \]  
\[ (0.30) \quad (4.29) \quad (4.81) \]  
\[ + 0.00793 c_{i,t-1}^2 - 0.00636 c_{i,t-1} b_{i,t-1} + e_{it} \]  
\[ (2.98) \quad (10.18) \]  
\[ R^2 = 0.82 \quad \text{log likelihood} = -224.28. \]  

\[ R_{it} = \hat{\alpha}_i + \hat{\gamma}_t + 0.0370 b_{i,t-1}^n - 0.157 c_{i,t-1} + 0.0000365 (b_{i,t-1}^n)^2 \]  
\[ (7.14) \quad (3.65) \quad (0.89) \]  
\[ + 0.0101 c_{i,t-1}^2 - 0.00124 c_{i,t-1} b_{i,t-1}^n + e_{it} \]  
\[ (2.35) \quad (2.10) \]  
\[ R^2 = 0.76 \quad \text{log likelihood} = -259.74. \]
Implications

Figure 3.1 Response of Sovereign yields to Debt Ratios under alternative current account balances
Do Other Factors Matter?

Source: IMF, WEO, October 2013
Gross Government Debt

Source: IMF, WEO, October 2013
Sovereign Yields

Source: OECD via FRED
Is It the Current Account?

Source: IMF, WEO, October 2013
Reinterpreting GHHM