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13 Neighborhood Effects

In his seminal 1987 book, *The Truly Disadvantaged*, William Julius Wilson attempted to explain the growth of an inner-city "underclass" and its attendant "social dislocations" such as joblessness, crime, and out-of-wedlock births. One important aspect of Wilson's argument was his emphasis on interpersonal influence. For instance, he argued that middle-class "role models" help to encourage labor-force participation within neighborhoods. Conversely, he suggested that joblessness becomes more socially acceptable in neighborhoods where unemployment is widespread. Following Wilson's lead, many social scientists have subsequently attempted to assess the importance of these "neighborhood effects."

Beyond emphasizing interpersonal influence, Wilson also identified two macro-structural changes contributing to the rise in joblessness. First, high-paying manufacturing jobs became increasingly scarce, as many employers moved production from the city to the suburbs (or outside the US altogether). Second, given the decline in barriers to residential mobility, many middle-class blacks also left the city for the suburbs. Crucially, the impact of these macro-structural changes may be compounded by neighborhood effects. Any rise in unemployment increases the social acceptablility of joblessness, leading to further unemployment. This vicious cycle might potentially account for the sudden, dramatic rise in joblessness observed in some inner-city neighborhoods.

Wilson's argument suggests a threshold model in which the actual neighborhood employment rate depends upon the expected neighbhorhood employment rate. In the present chapter, we formally develop a simple threshold model, and then analyze the effect of the two macro-structural changes identified by Wilson. Beyond providing an interesting application of threshold models, this chapter also introduces the notion of a *catastrophe* in a dynamical system.

13.1 The model

Following the structure of threshold models introduced in the previous chapter, we assume that each individual i faces a binary choice. Here, the "participation" decision involves the choice between employment and non-employment. Adopting terminology from economics, each of these options is associated with some level of "utility" (i.e., satisfaction) for the individual. More precisely, we'll assume that the utility of employment for individual i is given by

$$U_E(i) \;=\; w(i)$$

where $w(i)$ denotes the wage on the highest-paying job available to individual i. The utility of non-employment for individual i is given by

$$U_N = (1-x)b$$

where x is the expected neighborhood employment rate (hence $1-x$ is the expected unemployment rate) and b might be interpreted as the value of an unemployment benefit.

This specification of utility warrants several comments. First, while $U_E(i)$ depends solely on the monetary value of individual i's wage, U_N captures "social" as well as monetary considerations. Following Wilson's argument, joblessness becomes less socially acceptable as the neighborhood employment rate rises. Alternatively, we might say that greater "stigma" is attached to joblessness in neighbhorhoods where the employment rate is higher. Quantitatively, given our specification of U_N, we see that $U_N = b$ when $x = 0$, while $U_N = 0$ when $x = 1$. Second, having already interpreted the parameter b as an unemployment benefit, other interpretations are also possible. For instance, economists might interpret this parameter as the individual's non-monetary "value of leisure." Alternatively, recasting non-employment as "criminal activity," we might view b as the monetary benefits of crime. Presumably, just as the stigma associated with joblessness falls as the unemployment rate rises, the stigma associated with criminal activity falls as the proportion of criminals in the neighbhorhood rises. Finally, while we have allowed $U_E(i)$ to vary across individuals, note we have assumed (merely for simplicity) that U_N is the same for all individuals.

Given this specification of utility, individual i chooses employment if and only if

$$U_E(i) \geq U_N$$

which implies

$$w(i) \geq (1-x)b$$

or, equivalently,

$$x \geq 1-(w(i)/b).$$

The expression on the right-hand side of this last inequality can be interpreted as individual i's threshold level, since i chooses employment if and only if the expected employment rate x exceeds this level. Thus, in contrast to the preceding chapter where thresholds were simply taken as given, the present model derives each individual's threshold level from his wage level. Note that individuals with high wages will have low thresholds (choosing to work even if the expected employment rate is low), while individuals with low wages will have high thresholds (choosing not to work unless the expected employment rate is very high).

Moving to the neighbhorhood level, we now assume a wage distribution (where the "wage" is again the highest-paying job available to each individual) with probability density function $f(w)$ and cumulative distribution function $F(w)$.[1] As we have

[1] To emphasize: in contrast to the preceding chapter, we are specifying the *wage* distribution rather than *threshold* distribution.

already seen, those individuals in the lower tail of the wage distribution (with wages $w < (1-x)b$) will choose non-employment, while those in the upper tail of this distribution (with wages $w \geq (1-x)b$) will choose employment. Consequently, given the expected employment rate x, the actual unemployment rate is $F((1-x)b)$, and the actual employment rate is $1 - F((1-x)b)$. Further assuming adaptive expectations, the dynamics of the model are thus determined by the equation

$$x_{t+1} \;=\; 1 - F((1-x_t)b)$$

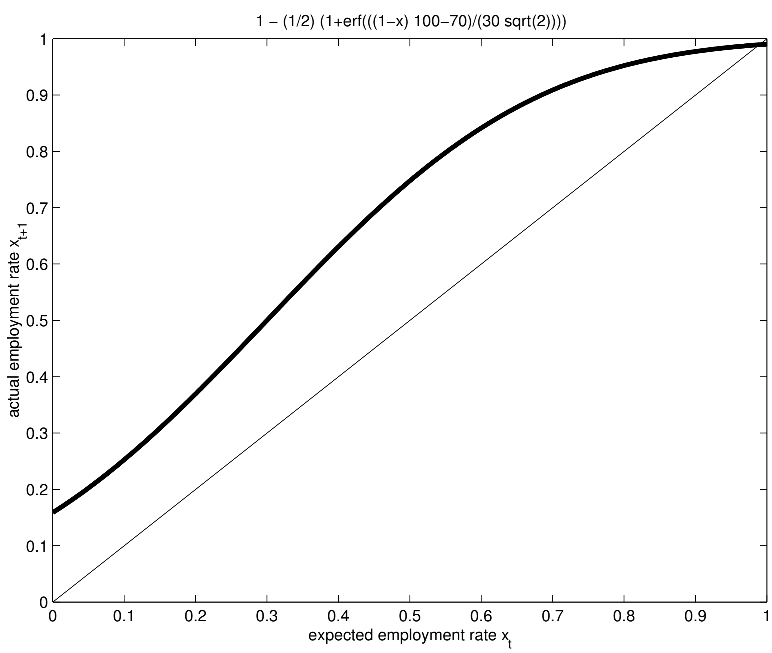
where x_t denotes the neighborhood employment rate in period t.

To illustrate, suppose that wages are normally distributed.[2] As we saw in Chapter 12, the cdf for the normal distribution is given by

$$F(w) \;=\; (1/2)\left(1 + \mathrm{erf}\left(\frac{w-\mu}{\sigma\sqrt{2}}\right)\right)$$

where μ is the mean of the distribution and σ is the standard deviation. Choosing the parameters b, μ, and σ, we can plot the threshold curve to determine the fixed points and their stability. For instance, given $b = 100$, $\mu = 70$, and $\sigma = 30$, we obtain the threshold diagram below.

```
>> s = '1 - (1/2)*(1+erf(((1-x)*100-70)/(30*sqrt(2))))';   % generator function as a string
>> hold on; ezplot(s,[0,1,0,1]); plot(0:1,0:1); hold off
>> % threshold diagram
```



[2]Because the normal density function $f(w)$ is positive for all $w \in (-\infty, \infty)$, some individuals will have negative wages. Following our interpretation of negative thresholds in Chapter 12, we can interpret these individuals as those who will never choose to work, regardless of the expected employment rate.

Given these parameter values, we thus find a unique stable equilibrium in which the neighborhood employment rate is close to 1. This result might be surprising given that most individuals have wages below the value of the unemployment benefit b. More precisely, because F(1) = .8413, we know that only 15.87% of the individuals in the neighborhood have wages exceeding $b = 100$. However, the employment of these individuals reduces the utility of non-employment for others, setting in motion a virtuous cycle. More precisely, if we assume the initial condition x_0, we obtain the time path below.

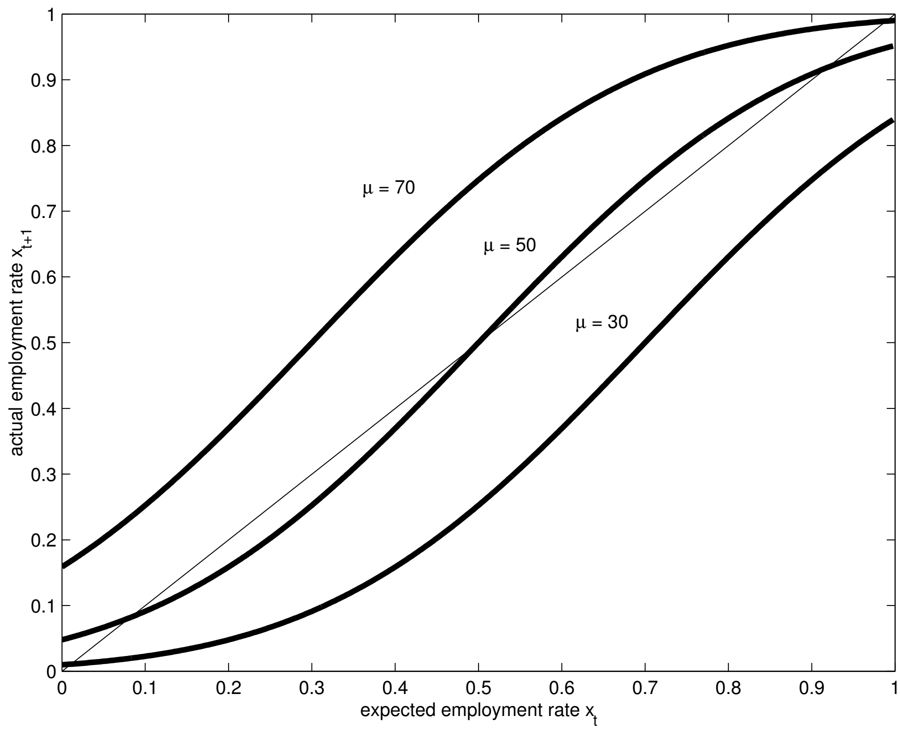
```
>> x = 0; y = x;
>> for t = 1:10; x = 1 - (1/2)*(1+erf(((1-x)*100-70)/(30*sqrt(2)))); y = [y; x]; end
>> y

y =
         0
    0.1587
    0.3188
    0.5249
    0.7733
    0.9427
    0.9839
    0.9887
    0.9892
    0.9892
    0.9892
```

Thus, given 15.87% who would always work (for any $x \in [0, 1]$), another 16.01% are induced to work, bringing the total to 31.88%. In turn, this induces another 20.61% to work, bringing the total to 52.49%. And so on, until almost everyone in the neighborhood is working. Note that this time path could also be viewed graphically as a cobweb on the threshold diagram.

13.2 Worsening of the wage distribution

Given this model, we can now address the impact of the macro-structural forces identified by Wilson. We'll begin by analyzing how the loss of "good jobs" might affect the equilibrium employment rate. To model this change in a very simple manner, we'll continue to assume that wages are normally distributed, and that the mean μ falls over time (while the standard deviation σ remains constant). For instance, if μ falls from 70 to 50 to 30, we obtain the three threshold curves shown on the diagram below.

```
>> s70 = '1 - (1/2)*(1+erf(((1-x)*100-70)/(30*sqrt(2))))';
>> s50 = '1 - (1/2)*(1+erf(((1-x)*100-50)/(30*sqrt(2))))';
>> s30 = '1 - (1/2)*(1+erf(((1-x)*100-30)/(30*sqrt(2))))';
>> hold on; ezplot(s70, [0,1,0,1]); ezplot(s50, [0,1,0,1]); ezplot(s30, [0,1,0,1]);
plot(0:1,0:1); hold off
>> % threshold curves for mu = 70, 50, 30
```



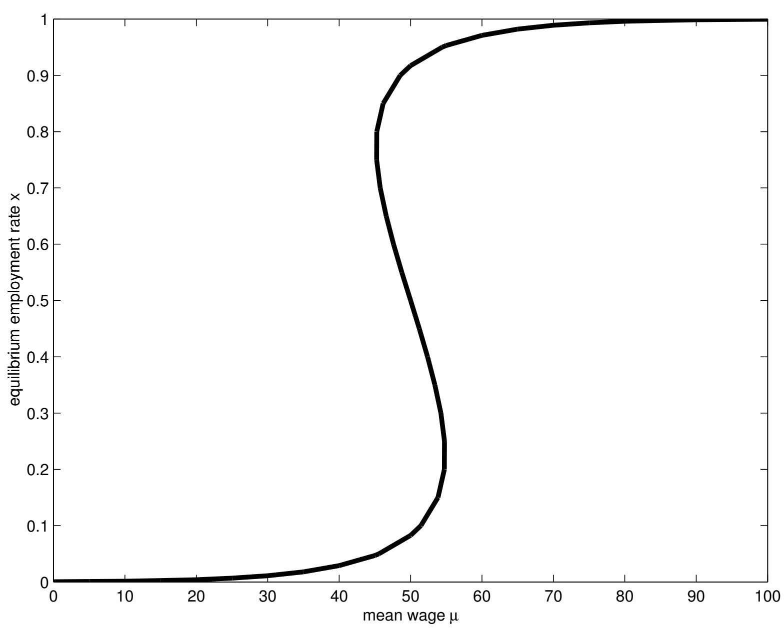
Graphically, a decrease in the mean of the wage distribution results in a downward shift of the threshold curve. Crucially, this downward shift is accompanied by a qualitative change in the number and location of fixed points. We've already seen that, when the mean wage $\mu = 70$, there is a unique, stable equilibrium at a high employment rate ($x^* = 0.9892$). If the mean wage drops to $\mu = 50$, there is now a stable upper equilibrium (which you can show is $x^* = 0.9185$), a stable lower equilibrium ($x^* = 0.0815$), and an intermediate unstable equilibrium ($x^* = 0.5$). If the mean wage drops further to $\mu = 30$, there is a unique, stable equilibrium at a low employment rate ($x^* = 0.0108$).

Given the preceding diagram, we can now envision the following scenerio. Suppose that the mean wage is initially $\mu = 70$, and that the neighborhood employment rate is in equilibrium. Further suppose that the mean of the wage distribution begins slowly to fall. Here, "slowly" indicates that the change in the wage distribution occurs much more gradually then the short-run dynamics captured by the generator function. Consequently, the employment rate quickly adjusts to the change in the wage distribution, always remaining close to the equilibrium determined by the intersection of the threshold curve and 45-degree line. More precisely, as μ falls slowly from 70 to 50, the employment rate also falls slowly from 98.92% to 91.85%. Thus, the gradual worsening of the wage distribution initially results in a small, gradual worsening of the equilibrium employment rate.

However, we can see from the diagram that further worsening of the wage distribution will eventually have more dramatic consequences. As the threshold curve continues to shift downwards, there is some value of μ for which the upper equilibrium ceases to exists. Once μ reaches this value, there is a unique equilibrium at a low employment rate, and the short-run dynamics will cause the employment rate to fall rapidly. This scenario might well be regarded as a "catastrophe" from a public-policy perspective. However, this term is also employed by mathematicans in the analysis of dynamical systems. In that context, a *catastrophe* occurs when a small (continuous) change in a parameter value causes a large (discontinuous) jump in the equilibrium.

To better visualize the catastophe in the current model, consider the diagram below, which shows the fixed point(s) x^* for each value of the parameter μ.

```
>> s = [ ]; srow = [ ];
for x = 0:.05:1;
    for mu = 0:5:100;
        fx = 1 - (1/2)*(1+erf(((1-x)*100-mu)/(30*sqrt(2)))) - x;
        srow = [srow, fx];
    end;
    s = [s; srow];
    srow = [ ];
end
>> contour(0:5:100,0:.05:1,s,[0 0])    % catastrophe diagram
```



Given $\mu = 70$, the unique equilibrium $x^* = 0.9892$ lies on the upper "arm" of this curve. As μ begins to fall, the equilibrium moves along this arm, reaching $x^* = 0.9185$ when $\mu = 50$. But from the diagram, we can see that further decreases in the mean wage will cause more rapid declines in employment rate, and that a catastrophe will occur when $\mu \approx 45$. Once the mean wage has fallen below this critical level, the neighborhood employment rate plunges from approximately 80% to 5%. Further declines in μ would have only small effects on x^*, as the equilibrium moves along the lower arm of the curve.

Interestingly, if we suppose that neighborhood is initially in equilibrium at a *low* employment rate, and assume a gradual *improvement* in the mean wage, the time series of employment rates is not quite the reverse of the preceding scenario. In this alternative scenario, the equilibrium slows moves upwards along the lower arm of the curve, until the mean wage reaches $\mu \approx 55$. Once the mean wage rises about this critical level, the employment rate jumps quickly from approximately 20% to 95%.[3] From a public-policy perspective, this suggest the importance of preventative action in the face of a worsening wage distribution. Once the catastrophe has already occurred, even larger policy initiatives may be necessary to induce a "positive catastrophe."

Having offered a conceptual discussion of the catastrophe diagram, let's now consider its computation. Fixing the parameters μ and σ, the fixed point(s) x^* are the solutions of the equation

$$1 - F((1-x)100) - x = 0.$$

That is, we need to "find the zeros" of the function on the left-hand side of this equation. We have assumed that $F(w)$ is the cdf for the normal distribution with mean μ and standard deviation σ. To make our notation more explicit, we can write $F(w)$ as $F(w, \mu, \sigma)$. Allowing μ to be set arbitrarily while fixing $\sigma = 30$, we are thus attempting to find the pairs $\{\mu, x\}$ such that

$$s(\mu, x) \;=\; 1 - F((1-x)100, \mu, 30) - x \;=\; 0.$$

Approaching this problem graphically, $s(\mu, x)$ can be viewed as a three-dimensional surface. To give a two-dimensional representation of this surface, we might plot the *contour lines* corresponding to different values of the function $s(\mu, x)$. However, for present purposes, we are concerned solely with the single contour line corresponding to $s(\mu, x) = 0$.[4] To use the `contour` function in Matlab, we first need to compute

[3] Here, there is a divergence between public-policy and dynamical systems terminology. The upward jump in the employment rate would seem desirable – hardly a "catastophe" – from a public-policy perspective, but remains a catastrophe from a dynamical systems perspective.

[4] In economics, contour plots are often used to represent a utility or production function with two inputs. In that context, the contour lines for utility functions are called "indifference curves," while the contour lines for production functions are called "isoquants." Readers unfamiliar with contour plots might consider the analogy to topographic maps, which indicate the shape of a mountain range (a three-dimensional surface) using elevation lines (contour lines). For present purposes, we are concerned solely with the particular elevation line denoting "sea level" (the contour line for which $s = 0$).

$s(\mu, x)$ numerically, finding the value of s for each point $\{\mu, x\}$ on a grid.[5] The resulting s matrix (along with the grid coordinates) can then be entered into the `contour` function to obtain a contour plot. The final input to this function `[0 0]` indicates that we want to plot only the single contour line corresponding to $s = 0$.

13.3 Outmigration of high-wage individuals

In addition to the worsening of the wage distribution, Wilson identified the exodus of the black middle-class as a second factor contributing to the rise in inner-city joblessness. The loss of employed individuals from a neighborhood will obviously have a direct effect on the neighborhood employment rate. But as we saw in the Chapter 12.4, the removal of individuals from a group can also have a large indirect effect, causing some of those who remain behind to change their behavior.

To proceed formally, suppose that the outmigration of the middle-class can be viewed as a truncation of the wage distribution. That is, all individuals with wages greater than some level z are removed from the population. Renormalizing the old wage distribution $F(w)$ to account for this truncation, we obtain the new wage distribution

$$G(w, z) = \begin{cases} F(w)/F(z) & \text{for all } w \leq z \\ 1 & \text{for all } w > z \end{cases}$$

which can be restated as

$$G(w, z) = \min\{F(w)/F(z), 1\}.$$

Fixed points are now determined by the equation

$$1 - G((1 - x)b, z) = x$$

or equivalently

$$1 - \min\{F((1 - x)b)/F(z), 1\} = x.$$

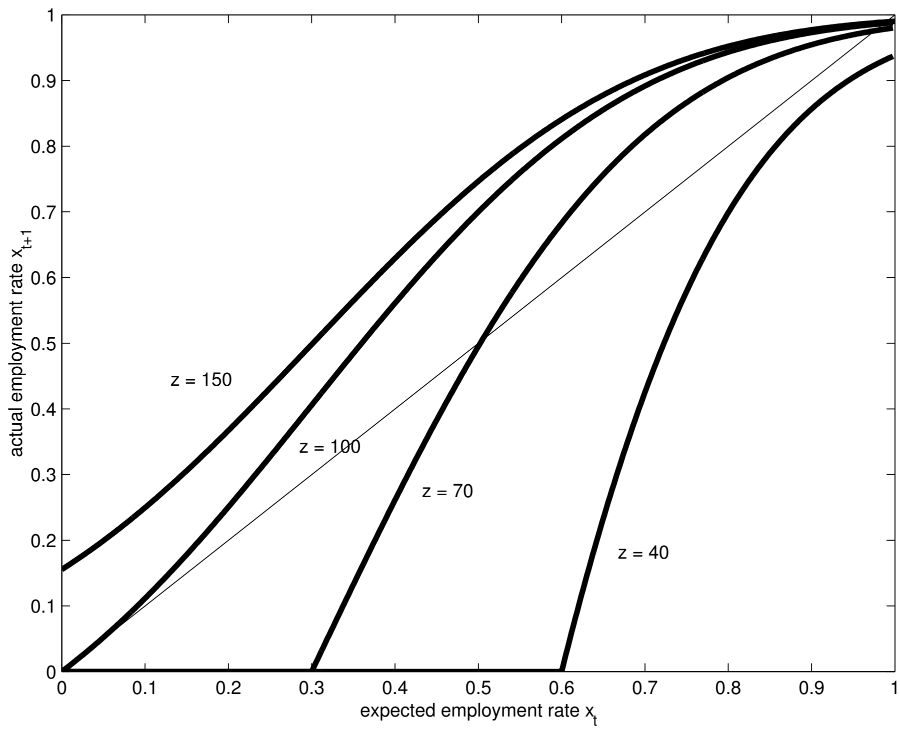
Our goal in this section is to understand how the truncation level z will affect these fixed points and their stability.

To develop numerical examples, we'll again assume that $F(w)$ is the cdf for the normal distribution, and further fix $\mu = 70$ and $\sigma = 30$ and $b = 100$. Given the truncation level z, we can now plot the threshold curve to determine the fixed points and their stability. The diagram below shows four threshold curves corresponding to four different values of z.

```
>> s150 = '1 -
min(.5*(1 + erf(((1-x)*100-70)/(30*sqrt(2)))) / (.5*(1 + erf((150-70)/(30*sqrt(2))))),1)';
```

[5] The precise grid is somewhat arbitrary. I computed $s(\mu, x)$ for all $\mu \in \{0, 5, 10, \ldots, 100\}$ and all $x \in \{0, 0.05, 0.1, \ldots, 1.0\}$, so that s is a 21×21 matrix. To generate the contour diagram, Matlab interpolates between these points. Thus, finer grids result in more accurate contour plots.

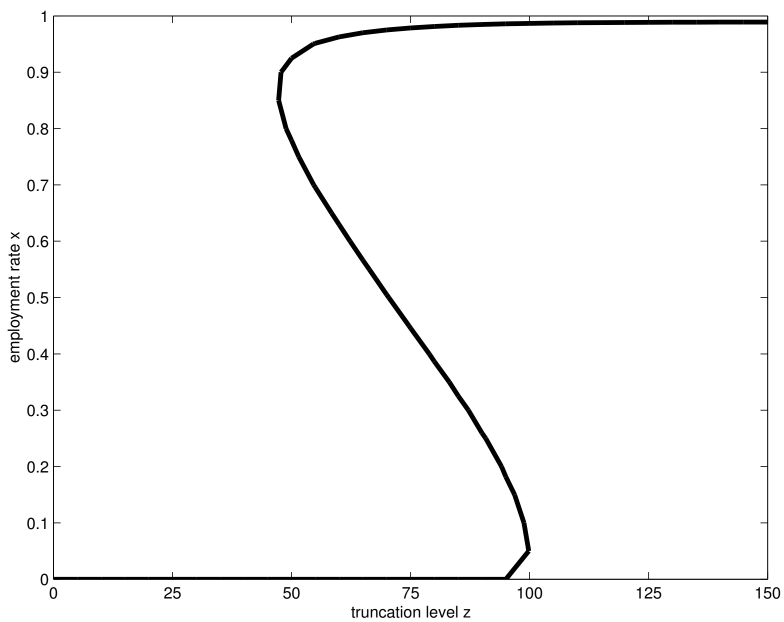
```
>> s100 = '1 -
min(.5*(1 + erf(((1-x)*100-70)/(30*sqrt(2)))) / (.5*(1 + erf((100-70)/(30*sqrt(2))))),1)';
>> s70 = '1 -
min(.5*(1 + erf(((1-x)*100-70)/(30*sqrt(2)))) / (.5*(1 + erf((70-70)/(30*sqrt(2))))),1)';
>> s40 = '1 -
min(.5*(1 + erf(((1-x)*100-70)/(30*sqrt(2)))) / (.5*(1 + erf((40-70)/(30*sqrt(2))))),1)';
>> hold on; ezplot(s150,[0,1,0,1]); ezplot(s100,[0,1,0,1]); ezplot(s70,[0,1,0,1]);
ezplot(s40,[0,1,0,1]); plot(0:1,0:1); hold off
>> % threshold diagrams for z = 150, 100, 70, 40
```



Having assumed $\mu = 70$ and $\sigma = 30$, few individuals have wages above 150. Thus, the threshold curve for $z = 150$ is very similar to the initial threshold curve shown in section 13.1 (which implicitly assumes $z = \infty$). However, as z falls, the threshold curve shifts downward, altering the number and location of the fixed points. Qualitatively, the consequences are quite similar to those induced by the decline in the mean wage μ. For z greater than 100, there is a unique stable equilibrium at a high employment rate. But for $z = 70$, there is a stable upper equilibrium, a stable lower equilibrium at $x^* = 0$, and an unstable intermediate equilibrium. For even lower truncation values such as $z = 40$, the only equililbrium is $x^* = 0$.

From this diagram, it is apparent that outmigration of high-wage individuals could also have catastropic consequences. Suppose that, due to gradual reduction in both legal and informal barriers to residential mobility, the truncation level z falls slowly over time. Initial decreases in z (from 150 to 100 to 70) would induce only small decreases in the equilibrium employment rate. But once z falls past a critical point, the upper stable equilibrium no longer exists, and the employment rate would fall dramatically. From the diagram below, we see that the catastrophe occurs when $z \approx 50$, causing the employment rate to fall from $x^* \approx 0.9$ to $x^* = 0$.

```
>> s = []; srow = [];
for x = 0:.05:1;
    for z = 0:5:150;
        fx = 1 - min((1/2)*(1+erf(((1-x)*100-70)/(30*sqrt(2))))
                     / (1/2)*(1+erf((z-70)/(30*sqrt(2)))),1) - x;
        srow = [srow, fx];
    end;
    s = [s; srow];
    srow = [];
end
>> contour(0:5:150,0:.05:1,s,[0 0])   % catastrophe diagram
```



Conceptually, the two macro-structural forces identified by Wilson – the loss of high-paying jobs and the outmigration of high-wage workers – might seem rather different. But our formal analysis suggests that these forces are actually quite similar. To clarify the nature of this similarity, we may once again employ the concept of stochastic dominance defined in Chapter 12. Given a decrease in the mean wage μ, the old wage distribution stochastically dominates the new distribution. Formally, $\mu \geq \mu'$ implies

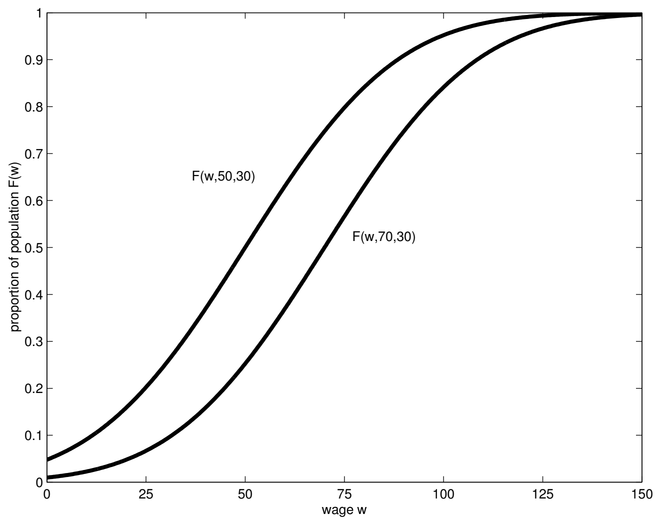
$$F(w, \mu, \sigma) \;\leq\; F(w, \mu', \sigma)$$

for any w and σ. Similarly, given a decrease in the truncation level z, the old wage distribution stochastically dominates the new distribution. Formally, $z \geq z'$ implies

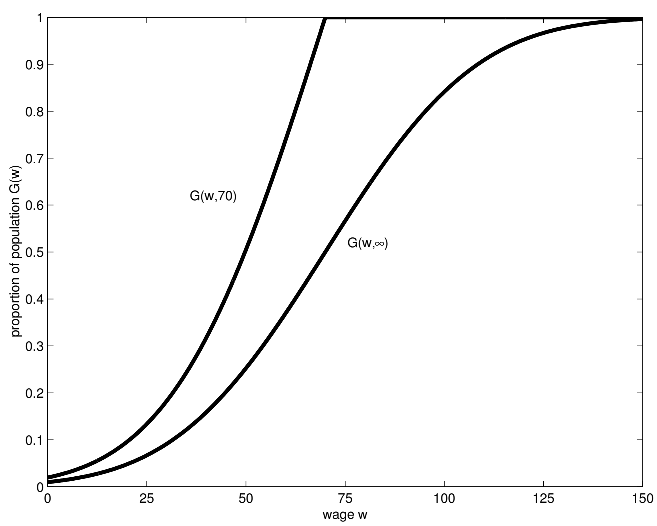
$$G(w, z) \;\leq\; G(w, z')$$

for any w. To illustrate, consider the cumulative wage distributions plotted below.

```
>> F70 = '.5*(1 + erf((w-70)/(30*sqrt(2))))';
>> F50 = '.5*(1 + erf((w-50)/(30*sqrt(2))))';
>> hold on; ezplot(F70,[0,150,0,1]); ezplot(F50, [0,150,0,1]); hold off
>> % cumulative wage distribution for mu = 70, 50
```




```
>> Fz70 = 'min(.5*(1 + erf((w-70)/(30*sqrt(2)))) / (.5*(1 + erf((70-70)/(30*sqrt(2))))),1)';
>> hold on; ezplot(F70,[0,150,0,1]); ezplot(Fz70, [0,150,0,1]); hold off
>> % cumulative wage distribution for z = inf, 70
```



Thus, we see that the cumulative wage function shifts upwards as the mean wage falls from $\mu = 70$ to $\mu = 50$, and also as the truncation level falls from $z = \infty$ to $z = 70$. Consequently, a decrease in either parameter has similar quantitive effects.

13.4 Further reading

This chapter draws on two unpublished working papers (Montgomery 1989, 1990) that I wrote after first reading *The Truly Disadvantaged*. Wilson developed his arguments further in other books including... Something on empirical developments....