Economics 451 Exam 1 Spring 2010 Prof Montgomery

Answer all questions. 100 points possible. Explanations can be brief. You may be time-constrained, so please try to allocate your time optimally.

1. [22 points] Consider a marriage market with 100 males and 120 females.

Suppose that any married couple would receive joint income $Z_{mf} = 40$. Within marriages, this joint income will be split between the male and female so that $Z^m + Z^f = 40$. The equilibrium income levels Z^m* and Z^f* are determined by market supply and demand.

The value of being single differs across males and across females. (That is, Z_{sm} differs across males, and Z_{sf} differs across females.) For 80 males, the value of being single is 10. For the other 20 males, the value of being single is 25. For 70 females, the value of being single is 5. For the other 50 females, the value of being single is 25.

a) To analyze market equilibrium, plot the supply and demand for *husbands*. [HINT: Be sure to label your graph carefully (including labels on the axis and curves, and identifying numerical coordinates of important points on each curve).] What is the equilibrium income Z^m* received by married men? How many marriages occur in equilibrium? [HINT: You have enough information to give *numerical* answers.]

b) How does your result in part (a) differ from the outcome discussed in lecture (where we assumed that Z_{sm} is the same for all males, that Z_{sf} is the same for all females, and that $Z_{sm} + Z_{sf} < Z_{mf}$)? Briefly discuss.

2) [24 points] Consider a marriage market in which the surplus generated by each match depends only on the "ability" of the male and female involved. More formally, the surplus for a match is given by $s(A_m, A_f)$ where s is surplus, A_m is the ability of the male, and A_f is the ability of the female. Becker (*Treatise*, Chapter 4) shows that the equilibrium matching structure can display either positive assortative mating or negative assortative mating, depending on the functional form of the surplus function.

a) Briefly explain the distinction between positive and negative assortative mating. Then state the formal condition that determines whether sorting is positive or negative.

b) For each of the following surplus functions, will sorting be positive or negative?

(i) $s = (A_m \times A_f)^{1/2}$ (i.e., s equals the square root of A_m times A_f)
(ii) $s = (A_m \times A_f)^2$ (i.e., s equals the square of A_m times A_f)

[NOTE: You need to provide proof (or at least a numerical illustration) to support your answers. You won't receive many points merely for guessing correctly.]

3. [28 points] Consider a marriage market with 7 males and 7 females. The following matrix gives the payoffs that would be received by each male and each female in each potential match. [Males are placed along the rows of the matrix, and females are placed along the columns, so that each pair (m_{ij}, f_{ji}) specifies the payoff that male i and female j would receive if they married.] Assume that each individual would receive payoff 0 if he/she remains single, and assume that utility is NOT transferable.

		females						
		1	2	3	4	5	6	7
	1	(1,7)	(3,1)	(2,6)	(5,6)	(4,7)	(7,6)	(6,1)
	2	(2,1)	(1,7)	(4,5)	(3,1)	(7,6)	(5,7)	(6,2)
	3	(1,6)	(2,2)	(7,1)	(3,2)	(6,1)	(5,5)	(4,3)
males	4	(4,2)	(3,6)	(2,4)	(1,3)	(5,5)	(6,3)	(7,5)
	5	(4,4)	(2,3)	(3,2)	(5,4)	(1,2)	(7,4)	(6,4)
	6	(7,3)	(6,5)	(5,7)	(4,7)	(3,3)	(2,1)	(1,6)
	7	(6,5)	(3,4)	(1,3)	(4,5)	(2,4)	(7,2)	(5,7)

a) Use the Gale-Shapley algorithm to find 2 stable match structures.

b) In markets where match structures are determined by computer, a variety of other matching algorithms (alternatives to the Gale-Shapley algorithm) have been used. These algorithms may or may not generate stable match structures. To illustrate, consider the following simple algorithm:

> Male 1 is matched with his most preferred female.
> Excluding the female already "taken" by male 1, male 2 is matched with his most preferred female among those females remaining.
> Excluding the females already "taken" by males 1 and 2, male 3 is matched with his most preferred female among those females remaining.
> Continue in this way until all males are matched.

Obviously, given an equal number of men and women, this algorithm will always produce a *feasible* match structure (because everyone will have exactly one spouse). But it may be less obvious whether this algorithm does (or does not) produce a *stable* match structure. Given the payoff matrix above, what is the match structure generated by this new algorithm? Is this match structure stable? If not, give at least one reason why.

c) Suppose that female 4 is unhappy with both outcomes of the Gale-Shapley algorithm in part (a). She argues that there may be some other stable match structure in which she would receive a higher payoff. Is this claim true or false? Explain.

4. [26 points] A husband and wife disagree about the share of household income to allocate to themselves (the parents) versus their child. Assume that the husband's utility function is given by

$$U_H = u(Z_P) + (1/3)\, u(Z_C)$$

and that the wife's utility function is given by

$$U_W = u(Z_P) + (1/2)\, u(Z_C)$$

where Z_P denotes the parent's consumption, Z_c denotes the child's consumption, and the u function is given by

$$u(Z) = \sqrt{Z}$$

so that

$$u'(Z) = \frac{1}{2\sqrt{Z}}.$$

Further assume that household income is 100, so that the household budget constraint is

$$Z_P + Z_C = 100.$$

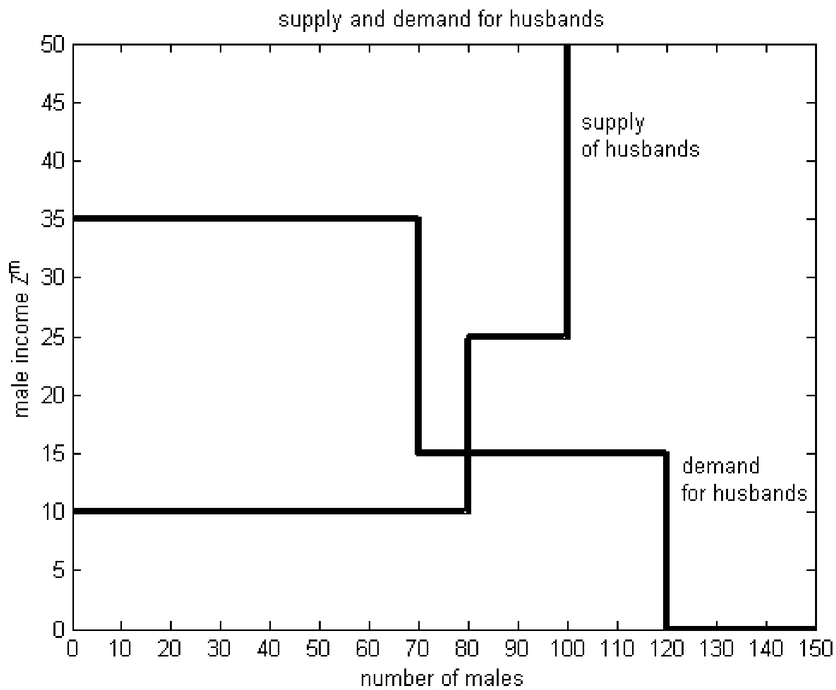
a) Suppose that the husband controls the household income, and chooses the consumption levels Z_P and Z_C in order to maximize his utility. (Assume that no further transfers can be made by the wife after the husband sets these levels.) What equation determines these optimal consumption levels? What fraction of household income would the husband spend on the parents? What fraction of household income would he spend on the child? [HINT: You have enough information to give a *numerical* solution.]

b) Now suppose that the wife controls the household income, and chooses the consumption levels Z_P and Z_C in order to maximize her utility. (Assume that no further transfers can be made by the husband after the wife sets these levels.) What equation determines these optimal consumption levels? What fraction of household income would the wife spend on the parents? What fraction of household income would she spend on the child? [HINT: Again, you should give a *numerical* answer.]

c) Conceptually (without computing a precise solution), how would we resolve the husband and wife's bargaining problem using the Nash bargaining solution? What would determine whether the negotiated outcome is closer to the husband's or wife's most preferred outcome? Briefly explain, using an efficiency frontier diagram.

Econ 451 Exam 1 Spring 2010 Solutions

1a) [16 pts]



[To derive the supply curve, consider the number of males willing to become husbands at each income level Z^m. For Z^m below 10, no male would be willing. If Z^m rises above 10, then 80 males become willing. If Z^m rises above 25, the remaining 20 males also become willing. To derive the demand curve, note that females will demand husbands only if $Z^f > Z_{sf}$ which can be rewritten as $Z_{mf} - Z^m > Z_{sf}$ which can be rewritten as $Z^m < 40 - Z_{sf}$. If Z^m is very high, then no females would demand husbands. If Z^m drops below 35, then 70 females would demand husbands. If Z^m drops below 15, then the remaining 50 females would also demand husbands.]

Market equilibrium is given by the intersection of the supply and demand curves. Thus, in equilibrium, married males receive $Z^{m*} = 15$, and 80 marriages occur.

b) [6 pts] Given the assumptions made in lecture (all males are the same; all females are the same), we obtain extreme results regarding the equilibrium number of marriages (everyone on the "short side" of the market is married) and the equilibrium income levels (the short side captures all the rents within marriage). In contrast, when we allow differences in the value of being single across males and across females (as in part a), we obtain less extreme results. Not everyone on the short side will necessarily marry, and the short side will not necessarily obtain all the rents. Indeed, for the particular example in part (a), most of the rents within most marriages go to females even though there are more females than men overall.

2a) [8 pts] Positive assortative mating occurs when "like marries like." That is, high-ability males marry high-ability females, and low-ability males marry low-ability females. Negative assortative mating occurs when "opposites attract." That is, high-ability males marry low-ability females, and vice versa.

As discussed by Becker (Treatise, Chap 4), the valence (positive or negative) of assortative mating depends on the cross-partial derivative of the surplus function. More precisely, positive assortative mating occurs when $\partial^2 s/\partial A_m \partial A_f > 0$, and negative assortative mating occurs when $\partial^2 s/\partial A_m \partial A_f < 0$.

b) [16 pts] Consider the general case where the surplus function is given by

$$s = (A_m \times A_f)^u = A_m^{\ u} A_f^{\ u}$$

where u is a constant. To compute the cross-partial derivative, we first differentiate s with respect to A_m, obtaining

$$\partial s/\partial A_m = u A_m^{u-1} A_f^{u}$$

We then differentiate again with respect to A_f, obtaining

$$\partial^2 s/\partial A_m \partial A_f = \partial [\partial s/\partial A_m]/\partial A_f = \partial [u A_m^{u-1} A_f^{u}]/\partial A_f = u^2 A_m^{u-1} A_f^{u-1}$$

Thus, we see that the cross-partial derivate will be positive for any u.

(i) positive assortative mating

given u = 1/2, the cross-partial derivative is $(1/4) A_m^{-1/2} A_f^{-1/2} > 0$

(ii) positive assortative mating

given u = 2, the cross-partial derivative is $4 A_m A_f > 0$

Both results could also have been illustrated with numerical examples. For instance, for part (bi), suppose we have 2 males (with ability 1 and ability 2) and 2 females (with ability 1 and ability 2). The surplus matrix is given by

	female 1 ($A_f = 1$)	female 2 ($A_f = 2$)
male 1 ($A_m = 1$)	1	1.41
male 2 ($A_m = 2$)	1.41	2

Recall that, given transferable utility, the equilibrium match structure maximizes the total surplus. Thus, for this example, we obtain positive assortative mating (with the match structure M1-F1, M2-F2). You get a similar result for part (bii).

3a) [14 pts] Outcome of G-S algorithm when males make proposals:

M1-F6, M2-F5, M3-F3, M4-F7, M5-F4, M6-F2, M7-F1

Outcome of G-S algorithm when females make proposals:

M1-F6, M2-F5, M3-F1, M4-F2, M5-F4, M6-F3, M7-F7

b) [8 pts] This alternative algorithm generates the match structure:

M1-F6, M2-F5, M3-F3, M4-F7, M5-F4, M6-F1, M7-F2

This match structure is *not* stable. You can find many violations of the stability condition. For instance, M7 and F1 would prefer to leave their current partners for each other. This would increase M7's payoff from 3 to 7, and would increase F1's payoff from 3 to 5, so both M7 and F1 are made better off.

c) [6 pts] This claim is false. The second outcome of the G-S algorithm (when females make proposals) generates the best stable match structure from the perspective of each female (including F4). Thus, there is no stable match structure that would generate a payoff higher than 4 for F4.

4a) [9 pts] Substituting the budget constraint into the husband's utility function,

$$U_H = u(Z_P) + (1/3)\, u(100 - Z_P).$$

Differentiating with respect to Z_P, we obtain $u'(Z_P^*) - (1/3)\, u'(100 - Z_P^*) = 0$. Given the functional form for the u function, this becomes

$$\frac{1}{2\sqrt{Z_P *}} = \frac{1}{3}\,\frac{1}{2\sqrt{100 - Z_P *}} \quad .$$

By squaring both sides of this equation and simplifying, we obtain

$$Z_P^* = 90 \quad \text{and hence} \quad Z_C^* = I - Z_P^* = 10 \quad .$$

Thus, the husband would spend 90% on the parents, and the remaining 10% on the child.

b) [9 pts] Substituting the budget constraint into the wife's utility function, and then differentiating with respect to Z_P, we obtain $u'(Z_P^*) - (1/2)\, u'(100 - Z_P^*) = 0$. Given the functional form for the u function, this becomes

$$\frac{1}{2\sqrt{Z_P *}} = \frac{1}{2}\,\frac{1}{2\sqrt{100 - Z_P *}} \quad .$$

By squaring both sides of this equation and simplifying, we obtain

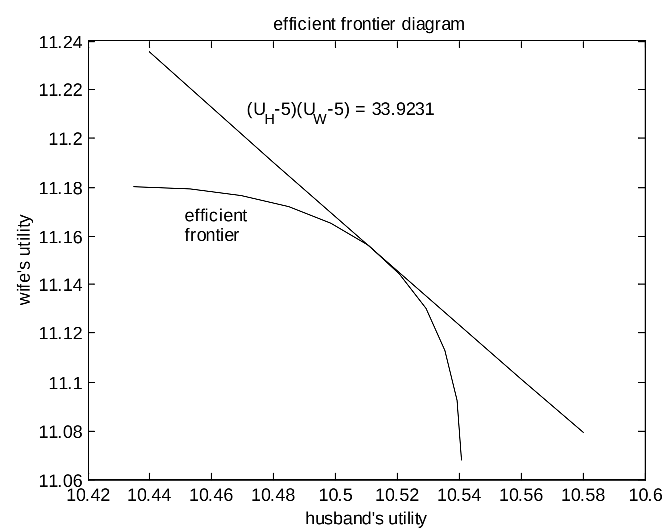
$Z_P^* = 80$ and thus $Z_C^* = I - Z_P^* = 20$.

Thus, the wife spends 80% on the parents and 20% on the child.

c) [8 pts] Substituting the budget constraint into both the husband's utility function and the wife's utility function, and holding constant household income I, each of these utility functions can be written as a function of Z_P. Plotting the pair $\{U_H(Z_P), U_W(Z_P)\}$ for every value of Z_P (between the wife's preferred Z_P^* and the husband's preferred Z_P^*), we obtain an efficient frontier curve. Intuitively, for each utility level received by one spouse, this curve shows the highest utility level that can be received by the other spouse. Applying the Nash bargaining solution, we find the point on the efficient frontier curve that maximizes $(U_H-T_H)(U_W-T_W)$ where T_H and T_W are "threat points" for the husband and wife. Graphically, plotting "indifference curves" corresponding to different levels of $(U_H-T_H)(U_W-T_W)$, the Nash bargaining solution is determined by a tangency between the efficient frontier and one of these "indifference curves." Obviously, the Nash bargaining solution will depend crucially on the threat points, which represent the payoff that each spouse would receive if there was no agreement (which might be interpreted as divorce).

[While you did not need to provide a precise solution, there is enough information given in the problem to find one. To permit a fully numerical solution, suppose $T_H = 5$ and $T_W = 5$. To plot the efficient frontier, you first find the value of U_H and U_W for every Z_P between the wife's $Z_P^* = 80$ and the husband's $Z_P^* = 90$. You can then find the value of the function $(U_H-T_H)(U_W-T_W) = (U_H-5)(U_W-5)$ for each Z_P.

Z_P	80.0000	81.0000	82.0000	83.0000	84.0000	85.0000	86.0000	87.0000	88.0000	89.0000	90.0000
U_W	11.1803	11.1794	11.1767	11.1720	11.1652	11.1560	11.1444	11.1302	11.1129	11.0923	11.0680
U_H	10.4350	10.4530	10.4696	10.4848	10.4985	10.5105	10.5208	10.5292	10.5355	10.5395	10.5409
$(U_H-T_H)\times(U_W-T_W)$	33.5900	33.6963	33.7841	33.8521	33.8990	33.9231	33.9225	33.8950	33.8381	33.7484	33.6222



Economics 451 Exam 2 Spring 2010 Prof Montgomery

Answer all 4 questions. 100 points possible. Explanations can be brief. You may be time-constrained, so please try to allocate your time optimally.

1) [12 points] Empirically, married couples tend to have higher levels of religious participation when both partners belong to the same religion. Iannaccone (*Journal for the Scientific Study of Religion*, 1990) discusses both "efficiency" and "sorting" explanations for this empirical finding. Briefly discuss both of these explanations. What empirical test does Iannaccone suggest to try to determine which explanation is correct? Which explanation does the empirical evidence seem to support?

2) [24 points] Consider a group with three individuals (A, B, C) attempting to choose between three outcomes (x, y, z). Each individual has a strict preference ordering (which is complete and transitive) over the outcomes. The social preference order is determined by a social welfare function. Consider the first row of the table below. Given the inputs to the social welfare function (in the first three columns), suppose the social welfare function gives the output in the final column.

A's prefs (P_A)	B's prefs (P_B)	C's prefs (P_C)	social prefs ($P = f(P_A, P_B, P_C)$)
$x \, P_A \, y \, P_A \, z$	$y \, P_B \, x \, P_B \, z$	$z \, P_C \, y \, P_C \, x$	$y \, P \, z \, P \, x$
$y \, P_A \, x \, P_A \, z$	$x \, P_B \, y \, P_B \, z$	$y \, P_C \, z \, P_C \, x$	

a) Comparing the inputs in the first and second rows of this table, which outcome (x or y or z) can be considered the "irrelevant alternative"? Briefly indicate why.

b) Assuming that this social welfare function satisfies the conditions T (Transitivity), P (Pareto Optimality), and I (Independence from Irrelevant Alternatives), what output must the social welfare function give for the *second* row of the table? Explain how you used the conditions T, P, and I to determine this answer.

b) Redo part (a), assuming that the output in the first row is now yPxPz (instead of yPzPx). Given the inputs in the second row, can you use Conditions T, P, and I to derive a *complete* social preference order for the second row? If so, state this social preference order. If not, what *can* you say about the social preferences using these conditions?

3) [40 points] Three friends (Al, Bill, Carl) are planning to take a road trip together. They will visit only one place, and are now trying to agree on this destination. Their four options are Florida, Georgia, Houston, and Iowa. Each individual's (strict) preference order is given by (the columns of) the following table:

	Al	Bill	Carl
rank 1	Florida	Georgia	Iowa
2	Georgia	Houston	Houston
3	Iowa	Florida	Florida
4	Houston	Iowa	Georgia

a) Use the Condorcet procedure to determine the group (social) preferences. Is this social preference order complete? Briefly explain why or why not. Is it transitive? Briefly explain why or why not.

b) In general, how can you tell if an outcome belongs to the "Condorcet set"? For this problem, which destination(s) are contained in the Condorcet set? Be sure to show/explain how you know this answer.

c) In order to resolve the group's choice problem, Al suggests that they use an agenda voting procedure. Suppose that everyone votes **sincerely** and that the group uses the **amendment procedure.** [HINT: Using amendment procedure, 2 outcomes are compared; the winner is compared to the 3rd outcome; that winner of that round is then compared to the 4th outcome.] If Al gets to choose the order in which the options are considered, can he construct an agenda which results in his top choice (Florida) as the final winner? If so, give the voting sequence. If not, explain why this is not possible.

d) Now suppose that Bill is allowed to choose the order of voting. Following amendment procedure, Bill suggests that they vote first between Iowa and Florida; the winner is compared to Houston; the winner of that round is then compared to Georgia. But in contrast to part (c), suppose that everyone votes **strategically**. Which destination is the final winner? Explain, using the appropriate diagram.

e) Now suppose that Carl is allowed to design the agenda voting procedure. Assume that everyone votes **strategically**. Is there any way that Carl can use the **amendment procedure** to obtain his top choice (Iowa) as the final winner? If so, give the voting sequence. If not, explain why this is not possible. Is there any way the Carl can obtain his top choice using the **successive procedure?** [HINT: Using successive procedure, each outcome is voted up or down, and voting ceases as soon as one outcome is voted up.] If so, give the voting sequence. If not, explain why this is not possible.

4) [24 points] Consider the (political-science version of the) Hotelling model developed in lecture. There is a large population of voters with ideal points distributed between 0 and 1; two candidates (1, 2) simultaneously choose locations (x_1, x_2); each voter votes for the candidate closest to the voter's own ideal point; each candidate wants to maximize the proportion of the vote that he/she receives; the candidates split the vote equally if they both choose the same location. As we learned in lecture, this game has a unique Nash equilibrium in which both candidates choose the location of the median voter.

a) Now consider the Hotelling model when there are *three* candidates (1, 2, 3) who simultaneously choose locations (x_1, x_2, x_3). [NOTE: The other assumptions of the model are unchanged. In particular, if more than one candidate chooses the same location, all candidates in that location split equally the votes generated by that location.] Logically, there are 3 possible types of outcomes for this game:

i) all candidates choose different locations ($x_1 \neq x_2$, $x_2 \neq x_3$, $x_1 \neq x_3$)
ii) all candidates choose the same location ($x_1 = x_2 = x_3$)
iii) exactly two candidates choose the same location (say $x_1 = x_2 \neq x_3$)

For each of these three types of outcomes (i, ii, iii), could there be a Nash equilibrium which takes this form? If so, describe the Nash equilibrium more precisely. If not, explain why some candidate would always deviate from this type of outcome.

[HINT: If it helps, you can assume that voters are distributed uniformly over the [0,1] interval. That is, for any locations a and b (where $0 \leq a \leq b \leq 1$), the proportion of voters with ideal points between a and b is equal to $b - a$. However, it is possible to answer this question without knowing the precise shape of the distribution of ideal points.]

b) Now suppose there are *four* candidates (1, 2, 3, 4) who simultaneously choose locations (x_1, x_2, x_3, x_4). Further assume that voters are distributed uniformly over the [0,1] interval. Could there be a Nash equilibrium in which $x_1 = x_2 \neq x_3 = x_4$? If so, describe the Nash equilibrium more precisely. If not, explain why some candidate would always deviate from this type of outcome.

Econ 451 Exam 2 Spring 2010 Solutions

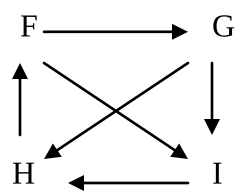
1) [12 pts] See Iannaccone (*JSSR* 1990), pp 303-309. The "efficiency" model assumes that households are able to produce more of the religious good when both members of the couple belong to the same religion. The "sorting" model assumes that individuals who are more "serious" about their religion will tend to marry spouses from their own religious tradition. To test between these explanations, Iannaccone considers three types of individuals: single, married to different-religion spouse, and married to same-religion spouse. He argues that, if the "pure sorting" model is correct, then we should observe the highest religious participation among those individuals married to a same-religion spouse (these individuals are the most "serious" about religion), the lowest participation among those individuals married to a different-religion spouse (these individuals are "less serious" about religion), and an intermediate level of participation among singles (who are a mixture of "serious" and "less serious"). In contrast, if the "pure efficiency" model is correct, Iannaccone argues that religious participation levels will be the same for individuals who are single or married to a different-religion spouse, which individuals who are married to a same-religion spouse would have higher participation. Iannaccone's empirical findings support the "efficiency" explanation.

2a) [6 pts] The "irrelevant alternative" is y. Removing this outcome from the individual preference orders, we find $x\ P_A z$ and $x\ P_B\ z$ and $z\ P_C\ x$ in both rows. Thus, moving from the first row to the second row, no individual changes his/her own preference for x versus z. Rather, each individual has merely shifted y in his/her preference order.

b) [9 pts] Condition I implies zPx. (In moving from the first row to the second row, there is no change by A or B or C in their rankings of x and z. Thus, given condition I, the social ranking of x and z shouldn't change either.) Inspection of the second row reveals that all three individuals prefer y to z. Thus, condition P implies yPz. Finally, having derived yPz and zPx, we can now apply condition T to obtain yPx. Thus, the social preferences in the second row are yPzPx.

c) [9 pts] Condition I now implies xPz. (Again, in moving from the first row to the second row, there is no change by A or B or C in their rankings of x and z. Thus, given condition I, the social ranking of x and z shouldn't change either.) Condition P again implies yPz. But having derived xPz and yPz, we cannot use condition T to determine the social preference between x and y. Assuming that social preferences are strict, the social preference order could be either xPyPz or yPxPz.

3a) [10 pts] Using arrows to indicate the outcome of each pairwise competition (so that F→G denotes F beats G), the Condorcet procedure generates the social preferences



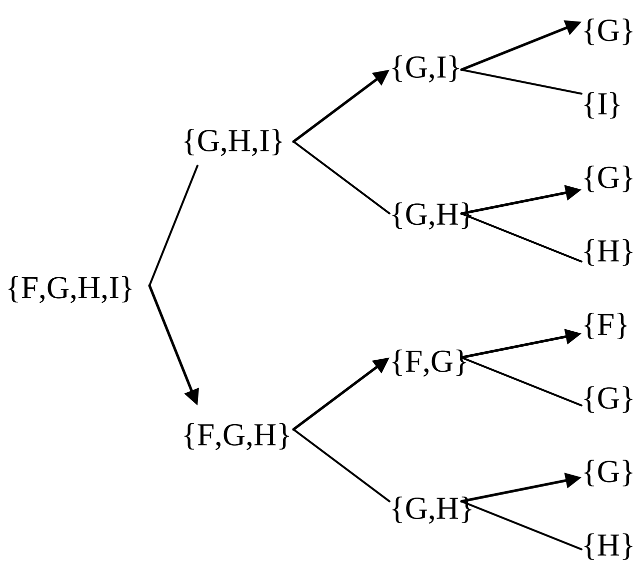
The social preferences are complete (because every pair of outcomes has been compared). The social preferences are not transitive (because there are voting cycles – e.g., F→G→I→H→F)

b) [6 pts] An outcome belongs to the Condorcet set if there is a (Hamiltonian) path that starts at that outcome and then goes through every other outcome. Here, all 4 destinations are in the Condorcet set. To verify, note the paths

F→G→I→H, G→I→H→F, I→H→F→G, H→F→G→I

c) [6 pts] Yes. One possible agenda is I vs H; winner vs G; winner vs F. [Note that I beats H in the first round; G beats I in the second round; F beats G in the final round.]

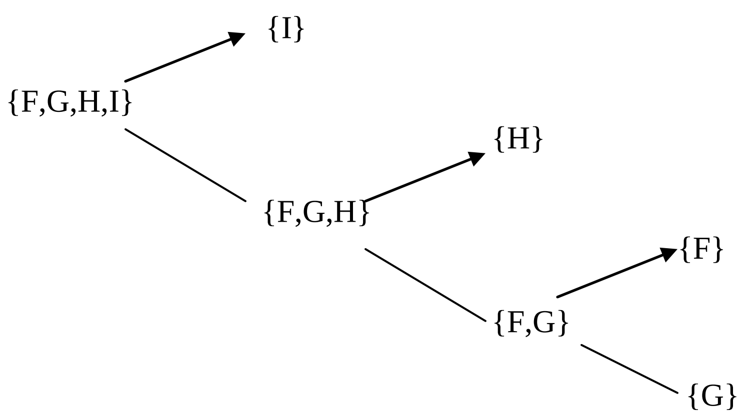
d) [8 pts] To answer this question, it is helpful to represent the agenda using the following tree diagram (giving the set of "live options" following each possible vote).



Applying backward induction, we first determine the outcome of every final (pairwise) contest. For instance, if the final contest is between G and I, the winner would be G (indicated by the arrow from {G,I} to {G}). Moving backwards, we can then determine the winners in earlier rounds. For instance, if F wins the first round and is then matched against H (in the lower part of the tree), the majority of voters will choose F over H because they know that the match is "really" between F and G (since voting for H would cause G to win in the final round). Overall, for this agenda with sophisticated voters, we find that F beats I in the first round; F beats H in the second round; F beats G in the final round. Thus, F is the final winner.

e) [10 pts] Given sophisticated voters, there is no way to use amendment procedure to make Iowa the final winner. The diagram in part (a) implies that I would need to face H in the final round (since I would be beaten by either F or G). However, in the next-to-last round, either F or G would beat H (and then go on to beat I in the final round).

In contrast, Carl can use successive procedure to make Iowa the winner. For instance, he could adopt the following sequence (which would cause I to win immediately).



4ai) [6 pts] There is no equilibrium in which all candidates choose different locations. Either of the "extreme" candidates could move closer to the "central" candidate, gaining some additional voters in the middle without losing any voters at the extremes.

aii) [6 pts] There is no equilibrium in which all candidates choose the same location. If all choose the same location, each receives 1/3 of the vote. By moving slightly away from the other two candidates, one candidate can obtain at least 1/2 of the vote.

aiii) [6 pts] There is no equilibrium with $x_1 = x_2 \neq x_3$. Candidate 3 could always move closer to the other two, gaining some additional voters in the middle without losing any voters at the extremes. [Note that, combining the answers to parts (ai)-(aiii), we have thus found that there is no (pure strategy) Nash equilibrium in the Hotelling model with 3 candidates.]

b) [6 pts] Yes, there is a Nash equilibrium in which $x_1 = x_2 = 1/4$ and $x_3 = x_4 = 3/4$. [In equilibrium, each candidate receives 1/4 of the vote. To verify the existence of this equilibrium, you need to show that no candidate could (strictly) improve his/her vote share by moving to a different location (holding fixed the other candidates' locations). Obviously, any candidate would do worse by moving to a more extreme location. If any candidate moved to location x between 1/4 and 3/4, he/she would attract voters in the interval [(x + 1/4)/2, (x + 3/4)/2], obtaining 1/4 of the vote.]

Economics 451 Exam 3 Spring 2010 Prof Montgomery

Answer all 4 questions. 100 points possible. Explanations can be brief. You may be time-constrained, so please try to allocate your time optimally.

1) [10 points] Based on the model of denominational secularization developed by Montgomery (*Rationality and Society* 1996), evaluate the following claim: "To maintain high membership levels over time, religious organizations should be democratic, paying close attention to the desires of members."

2) [27 points] Consider a small religious group with 6 members (indexed by i = 1, 2, 3, 4, 5, 6). Each member i spends time in religious participation (r_i) and spends time in non-religious activities (h_i) subject to the time constraint $r_i + h_i = 40$. Each member's utility depends on the average religious participation of group members and also the time spent individually in non-religious activities. More specifically, suppose that i's utility function can be written

$$U_i = (1/6)(r_1 + r_2 + r_3 + r_4 + r_5 + r_6) + \gamma\sqrt{h_i}$$

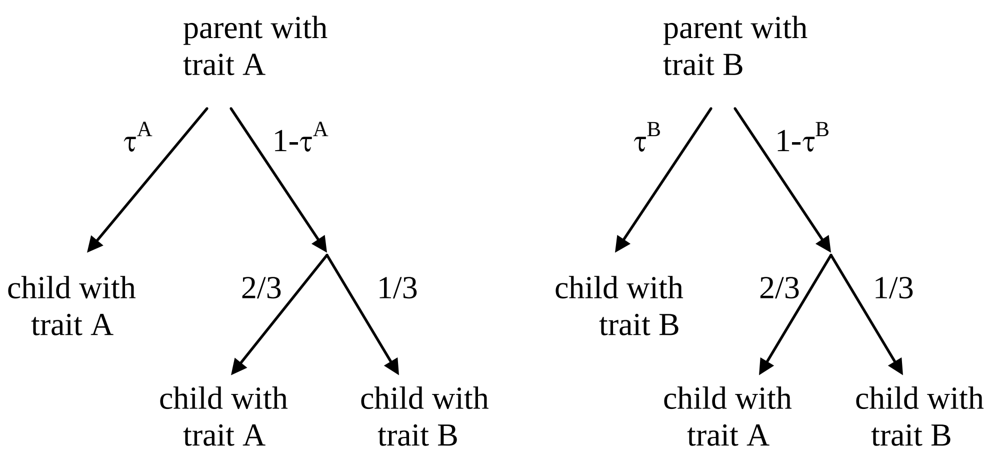
where γ is a utility parameter that can be manipulated by the religious group.

a) Assuming $\gamma = 2$, find the optimal time spent in religious participation by member 1. How much utility does member 1 receive, given that all members are making this same optimal choice? [NOTE: You have enough information to derive a numerical solution.]

b) Now suppose that the religious group reduces the utility that members receive from non-religious activities, decreasing γ from 2 to 1. Given $\gamma = 1$, find the optimal time spent in religious participation by member 1. How much utility does member 1 receive, given that all members are making this same individually optimal choice?

c) Comparing your results from parts (a) and (b), explain how this model illustrates the argument made by Iannaccone in his 1994 *American Journal of Sociology* paper.

3) [36 points] Following Bisin and Verdier (*Quarterly Journal of Economics* 2000), consider a society in which individuals acquire either trait A or trait B. Let q^i_t denote the proportion of the population with trait i (= A or B) in period t, and note that $q^A_t + q^B_t = 1$ for all t. Traits may be acquired from parents (through vertical socialization) or from other members of the society (through oblique socialization). Each parent has one child, and attempts to instill her own trait in the children. Suppose that vertical socialization succeeds with probability τ^A for parents with trait A, and that vertical socialization succeeds with probability τ^B for parents with trait B. If vertical socialization fails (for either type of parent), assume that the child acquires trait A with probability 2/3 and trait B with probability 1/3. [NOTE: Unlike Bisin and Verdier, we are assuming that oblique socialization probabilities do *not* depend on the proportion of the population having each trait.] Thus, the socialization process is described by the following probability trees:



a) Write q^A_{t+1} as a function of q^A_t, τ^A, and τ^B.

b) Suppose $\tau^A = \tau^B = 1/4$. Use the equation from part (a) to solve for the long-run proportion of A's in the population. [HINT: In the long run, $q^A_{t+1} = q^A_t$.]

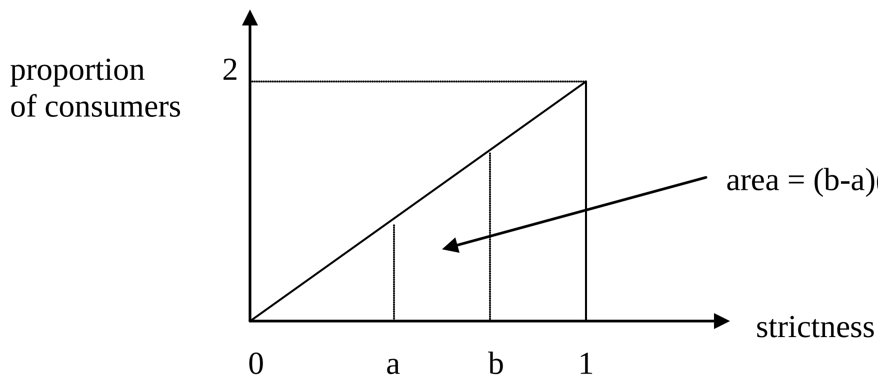
c) Now suppose that each parent chooses τ to maximize her expected utility. Assume that each parent receives utility equal to 1 if her child acquires the parent's own trait (through either vertical or oblique socialization), that the parent receives utility equal to 0 if her child acquires the other trait, and that the cost of vertical socialization equals $(\tau^i)^2$ for both types of parents (i = A and B). Write the expected utility function for a parent with trait A, and the expected utility function for a parent with trait B. Then differentiate each function to determine the optimal levels of socialization (τ^{A*} and τ^{B*}). [NOTE: You have enough information to give numerical solutions.]

d) Given the optimal levels τ^{A*} and τ^{B*} from part (c), use the equation from part (a) to solve for the long-run proportion of A's in the population.

e) Contrast your answer to part (b) with your answer to part (d). How does this problem illustrate the argument made by Bisin and Verdier?

4) [27 points] Consider McBride's (*Economica* 2010) application of the Hotelling model to religion. In this two-period model, there are an indefinite number of religious producers that could potentially enter the religious market. In period 1, potential entrants simultaneously choose to locate (choosing a strictness level between 0 and 1) or postpone the decision to period 2. (Once entry occurs, producers cannot change location nor exit.) In period 2, the remaining potential entrants choose a location or else choose never to enter. (Assume that potential entrants do not enter if they are indifferent between entry and non-entry.) After all entry decisions are made, each consumer chooses the religious producer that is closest to his/her ideal point. Each consumer also has the option of non-affiliation (corresponding to strictness level 0).

Suppose that each religious producer is willing to enter if and only if its membership exceeds 1/4 of the population. Further suppose that the ideal points of religious consumers are given by the "triangular" distribution below. [HINT: Given this distribution, the proportion of the population between a and b is equal to (b-a)(a+b).]



a) In equilibrium, what is the minimum location (i.e., the lowest possible strictness) of the strictest producer? [NOTE: You have enough information to give a numerical answer.] Briefly explain, giving the relevant computation.

b) In equilibrium, what is the maximum location (i.e., high possible strictness) of the least strict producer? [HINT: If a potential entrant was to locate between 0 and the least strict producer, what location would yield the highest membership?] Briefly explain, giving the relevant computation.

c) Consider an outcome where two producers enter the market, with the stricter producer choosing the location in part (a), and the less strict producer choosing the location in part (b). What proportion of the population would belong to the high-strictness producer? to the low-strictness producer? would choose no religion?

d) Is the outcome in part (c) an equilibrium? If so, briefly explain why, giving the relevant computations. In not, indicate how some producer or potential entrant would deviate from this outcome.

Econ 451 Exam 3 Spring 2010 Solutions

1) [10 pts] In their book on American religious history, Finke and Stark (*The Churching of America* 1992) describe how new denominations tend to enter the "religious economy" at a high level of strictness, but then become less strict over time. Consequently, older denominations tend to "pile up" at the low-strictness end of the continuum, and their "market share" tends to fall over time.

Finke and Stark argue that these dynamics could result from intergenerational social (i.e., income) mobility. In the formal model developed by Montgomery (*Rationality and Society* 1996), individuals with lower incomes tend to prefer denominations with higher strictness levels. Thus, new strict denominations tend to recruit low-income members. However, due to intergenerational income mobility, the children and grandchildren of the initial members may have higher income levels than their parents. Assuming that these children have enough loyalty (religious capital) to remain within their parent's denomination, the average income within the denomination will gradually rise over time. If a denomination is "democratic" – if its strictness level is determined by the preference of the median member – then the denomination will eventually become less strict. Over the course of a denomination's history, it will experience membership growth in the early (high strictness) years, when there are few competitors in that "market niche." But denominations will then experience membership decline in later (low strictness) years, because there are many competitors in the low-strictness market niche.

Ironically, denominations which are democratic (i.e., responsive to the desires of current members) are thus likely to experience long-term membership decline as they become less strict. While the reduction in strictness may benefit some current members (those with higher income), it pushes the denomination into the overcrowded low-strictness market niche. Further, assuming an inverse relationship between loyalty (religious capital) and strictness, the denomination will become less likely to retain future generations of children. In contrast, denominations which are less democratic – which ignore the "voice" of higher income second- and third-generation members – could remain stricter and have more members in the long run.

2a) [14 pts] Member 1's utility is

$$U_1 = (1/6)(r_1 + r_2 + r_3 + r_4 + r_5 + r_6) + \gamma (40-r_1)^{1/2}$$

Differentiating with respect to r_1, we obtain

$$\partial U_1/\partial r_1 = 1/6 + \gamma(1/2)(40-r_1)^{-1/2}(-1)$$

Setting this derivative equal to zero, we obtain

$$1/6 = \gamma(1/2)(40-r_1)^{-1/2}$$

Simplifying, we obtain the optimal solution

$$r_1^* = 40 - 9\gamma^2$$

Given $\gamma = 2$, individual 1 sets $r_1^* = 4$. Given the symmetry of the problem, the other individuals also make this same optimal choice: $r_i = 4$ for all i.

Thus, $U_1 = (1/6)(4 + 4 + 4 + 4 + 4 + 4) + 2(40-4)^{1/2} = 4 + 2(6) = 4 + 12 = 16$

b) [4 pts] Using the general formula above, $r_1^* = 40 - 9(1)^2 = 31$

and $U_1 = (1/6)(31 + 31 + 31 + 31 + 31 + 31) + 1(40-31)^{1/2} = 31 + 1(3) = 31 + 3 = 34.$

c) [9 pts] Strict denominations impose costs on their members, introducing behavioral norms which reduce the utility that members obtain from non-religious participation. (These costs are captured in a simple way in the present problem by a reduction in the utility parameter γ.) Thus, it might initially seem puzzling why anyone would (rationally) join a strict denomination. However, Iannaccone argues that higher costs (i.e., lower γ) help the denomination solve a free-rider problem that stems from positive externalities in religious participation. (This positive externality is captured in a simple way in the present problem by assuming that religious utility depends on *average* rather than *own* religious participation.) Given this positive externality (and high γ), members would spend too little time in religious participation. By decreasing γ, the denomination decreases the incentive for non-religious participation, and hence causes members to spend more time in religious participation. As illustrated in parts (a) and (b), the decrease in γ does cause utility from non-religious participation to fall from 12 to 3. However, the utility from religious participation rises from 4 to 31, and hence overall utility rises from 16 to 34.

3a) [8 pts] In period t, proportion q^A_t of parents have trait A. Their children may acquire trait A through vertical socialization (prob τ^A). If vertical socialization fails (prob $1-\tau^A$), their children may still acquire trait A through oblique socialization (prob 2/3). Proportion $1-q^A_t$ of parents have trait B. If vertical socialization fails (prob$1-\tau^B$), their children may acquire trait A through oblique socialization (prob 2/3). Thus,

$$q^A_{t+1} = [\tau^A + (1-\tau^A)(2/3)]\, q^A_t + [(1-\tau^B)(2/3)](1-q^A_t).$$

b) [4 pts] Given $\tau^A = \tau^B = 1/4$, the preceding equation becomes

$$q^A_{t+1} = [1/4 + (3/4)(2/3)]\, q^A_t + [(3/4)(2/3)](1-q^A_t) = (1/4)\, q^A_t + 1/2.$$

In the long run, $q^A = (1/4)\, q^A + 1/2$ which implies $(3/4)q^A = 1/2$ and hence $q^A = 2/3$.

c) [12 pts] A parent with trait A receives utility 1 if her child acquires trait A through vertical socialization (prob τ^A) or oblique socialization (prob $(1-\tau^A)(2/3)$). A parent with trait A receives utility 0 if her child acquires trait B (prob $(1-\tau^A)(1/3)$). The cost of socialization is $(\tau^A)^2$. Thus,

$$EU_A = [\tau^A + (1-\tau^A)(2/3)]\, 1 + [(1-\tau^A)(1/3)]\, 0 - (\tau^A)^2 = (1/3)\tau^A + 2/3 - (\tau^A)^2$$

Differentiating with respect to τ^A, we obtain $1/3 - 2\,\tau^A = 0$ which implies $\tau^{A*} = 1/6$.

A parent with trait B receives utility 1 if her child acquires trait B through vertical socialization (prob τ^B) or oblique socialization (prob $(1-\tau^B)(1/3)$). A parent with trait A receives utility 0 if her child acquires trait A (prob $(1-\tau^B)(2/3)$). The cost of socialization is $(\tau^B)^2$. Thus,

$$EU_B = [\tau^B + (1-\tau^B)(1/3)]\, 1 + [(1-\tau^B)(2/3)]\, 0 - (\tau^B)^2 = (2/3)\tau^B + 1/3 - (\tau^B)^2$$

Differentiating with respect to τ^B, we obtain $2/3 - 2\,\tau^B = 0$ which implies $\tau^{B*} = 1/3$.

d) [4 pts] $q^A_{t+1} = [1/6 + (5/6)(2/3)]\, q^A_t + [(2/3)(2/3)](1-q^A_t) = (5/18)q^A_t + 4/9$

In the long run, $q^A = (5/18)q^A + 4/9$ which implies $(13/18)q^A = 4/9$ and hence $q^A = 8/13$.

e) [8 pts] Because their children are less likely to acquire the desired trait through oblique socialization, minority parents have more incentive to undertake vertical socialization ($\tau^{B*} > \tau^{A*}$). On the macro level, this result implies a higher equilibrium share of the population in the minority group than would have occurred if both types of parents put the same level of effort in vertical socialization ($\tau^A = \tau^B$). Comparing parts (b) and (d), note that the equilibrium share of A's fell from 2/3 to 8/13.

4a) [6 pts] The strictest producer will move as far left as possible without allowing entry on the right. If entry occurs just to the right of this producer (at location x), the entrant would obtain membership level $(1-x)(x+1) = 1-x^2$. Thus, to prevent entry on the right, the strictest producer would set x so that

$$1-x^2 \leq 1/4 \quad \rightarrow \quad x^2 \geq 3/4 \quad \rightarrow \quad x \geq \text{sqrt}(3/4) \approx .8660$$

b) [6 pts] To determine the maximum location of the least strict producer, we consider how far right this producer can locate without allowing entry on the left. Note that, if entry did occur, it would occur just to the left of this producer. (Due to the triangular shape of the distribution, the entrant's membership rises as location increases from 0 to the location of the least strict producer.) If entry does occur just to the left (at location x), the entrant would attract all consumers with strictness levels between x/2 and x, and thus obtain membership $(x - x/2)(x/2 + x) = (x/2)(3x/2) = 3x^2/4$. To prevent entry on the left, the least strict producer would set x so that

$$3x^2/4 \leq 1/4 \quad \rightarrow \quad x^2 \leq 1/3 \quad \rightarrow \quad x \leq \text{sqrt}(1/3) \approx .5773$$

c) [6 pts] Given $s_1 = .8660$ and $s_2 = .5773$, producer 1 would attract consumers with strictness levels between $(s_1+s_2)/2 = .7216$ and 1, and thus have membership

$$m_1 = (1-.7216)(.7216+1) = .4793$$

Producer 2 would attract consumers with strictness levels between $(0+s_2)/2 = .2886$ and $(s_1+s_2)/2 = .7216$, and thus have membership

$$m_2 = (.7216-.2886)(.2886+.7216) = .4374$$

Consumers with strictness levels between 0 and $(0+s_2)/2 = .2886$ would choose no religion. Thus, the proportion of the population with no affiliation is

$$m_{NA} = (.2886-0)(0+.2886) = .0833$$

d) [9 pts] Yes, this outcome is an equilibrium. To verify, we first need to show that no other producer would wish to enter. We have already shown that no entrant would locate to the right of producer 1 (see part a) or to the left of producer 2 (see part b). Given the membership levels above, it is also apparent that no entrant would want to locate precisely at s_1 or s_2 (because this would yield 1/2 of m_1 or m_2, which is less than 1/4). Given entry between s_1 and s_2, the best location would be just to the left of s_1 (due to the triangular distribution). This entrant would obtain all consumers between $(s_1+s_2)/2 = .7216$ and $s_1 = .8660$, and thus membership of $(.8660-.7216)(.7216+.8660) = .2292$. Thus, there is not a large enough "niche" for new entry. To verify the existence of the equilibrium, we also need to show that neither producer 1 nor 2 would wish to deviate. But we have already shown that both producers have membership above 1/4 (see part c), that producer 1 is as far left as possible (see part a), and that producer 2 is as far right as possible (see part b). Thus, neither producer can do better.